Національний Технічний Університет України "КПІ" Навчально-науковий комплекс «Інститут прикладного системного аналізу»

ЛАБОРАТОРНА РОБОТА № 2

3 дисципліни: Основи системного аналізу

Виконали:

Барзій Ілля Лєсніков Богдан Шрам Владислав (Бригада 1)

група КА-41

Задание

В каждом из вариантов заданы:

- исходные таблицы данных;
- размерности векторов x₁, x₂, x₃ n₁,n₂,n₃;
- количество выборок n (для расчета);
- количество целевых функций т;
- веса целевых функций;
- метод решения несовместных систем уравнений, в которых число неизвестных не равно числу уравнений;

Наборы P1, P2, P3 — степени полинома Чебышева (Лежандра, Лагера, Эрмита и др.) для x_1, x_2, x_3 — следует подобрать самостоятельно, исходя из критерия минимума максимального отклонения функций: $10^{-6} \le \Delta^0 \le 10^{-3}$.

Требуется:

- 1) сформировать целевые функции и вывести на печать
 - значения всех промежуточных коэффициентов (λ , a, c) и функций (Ψ , Φ);
 - вид полученных функций $\Phi_i(x_1,x_2,x_3)$ через: 1) $\Phi_{i1}(x_1)$, $\Phi_{i2}(x_2)$, $\Phi_{i3}(x_3)$; 2) полиномы Чебышева;
- 3) в форме обычных многочленов (целесообразно это предусмотреть в файле результатов) как в нормированном, так и в восстановленном виде;
- 2) построить графики исходных (по выборкам) и полученных функций;
- 3) оценить погрешность восстановленных функций $\Phi_i(x_1, x_2, x_3)$ по отношению к исходной заданной выборке.
- 4) сделать письменный отчет о выполненной работе, включив листинг программы.

Варіант 1

Размерности векторов X1, X2, X3 – n_1 =2, n_2 =2, n_3 =3; количество выборок для расчета = 45; количество целевых функций m=4; b_{iq_0} принимаются равными нормированным значениям $Y_i[q],\ i=\overline{1,m}$; метод решения несовместной системы уравнений: метод сопряженных направлений.

Метод спряженних напрямків розв'язку СЛНР

Ненульові вектори u и v спряжені (відносно A), якщо:

$$\mathbf{u}^{\mathrm{T}}\mathbf{A}\mathbf{v} = 0.$$

Оскільки A — симетрична та додатньо визначена, визначимо внутрішній добуток:

$$\langle \mathbf{u}, \mathbf{v} \rangle_{\mathbf{A}} := \langle \mathbf{A}\mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{u}, \mathbf{A}^{\mathrm{T}}\mathbf{v} \rangle = \langle \mathbf{u}, \mathbf{A}\mathbf{v} \rangle = \mathbf{u}^{\mathrm{T}}\mathbf{A}\mathbf{v}.$$

Два вектора спряжені, якщо вони ортогональні за вінутрішнім добутком. Спряженість — симетрична властивість.

Нехай, $P = \{\mathbf{p}_k : \forall i \neq k, i, k \in [1, n], \langle \mathbf{p}_i, \mathbf{p}_k \rangle_A = 0\}$ - набір n попарно спряжених напрямків. Тоді P - базис в \mathbb{R}^n , так можемо визначити розв'язок \mathbf{x}_* 3 $\mathbf{A}\mathbf{x} = \mathbf{b}$:

$$\mathbf{x}_* = \sum_{i=1}^n \alpha_i \mathbf{p}_i$$

Видно, що:

$$\mathbf{b} = \mathbf{A}\mathbf{x}_* = \sum_{i=1}^n \alpha_i \mathbf{A}\mathbf{p}_i.$$

Для будь-якого $\mathbf{p}_k \in P$,

$$\mathbf{p}_k^{\mathrm{T}} \mathbf{b} = \mathbf{p}_k^{\mathrm{T}} \mathbf{A} \mathbf{x}_* = \sum_{i=1}^n \alpha_i \mathbf{p}_k^{\mathrm{T}} \mathbf{A} \mathbf{p}_i = \alpha_k \mathbf{p}_k^{\mathrm{T}} \mathbf{A} \mathbf{p}_k.$$

(так як $\forall i \neq k, p_i, p_k$ попарно спряжені)

$$\alpha_k = \frac{\mathbf{p}_k^{\mathrm{T}} \mathbf{b}}{\mathbf{p}_k^{\mathrm{T}} \mathbf{A} \mathbf{p}_k} = \frac{\langle \mathbf{p}_k, \mathbf{b} \rangle}{\langle \mathbf{p}_k, \mathbf{p}_k \rangle_{\mathbf{A}}} = \frac{\langle \mathbf{p}_k, \mathbf{b} \rangle}{\|\mathbf{p}_k\|_{\mathbf{A}}^2}.$$

Це дає спосіб розв'язання рівняння $\mathbf{A}\mathbf{x} = \mathbf{b}$: знайти множину з n спряжених напрямків а потім вырахуівти α_k .

Ми бачимо, що матриця A для даного методу повинна бути симетрична та додатньовизначена. Щоб застосувати цей метод (у нашому випадку наша матриця A зовсім не ϵ симетричною), домножимо рівняння $\mathbf{A}\mathbf{x} = \mathbf{b}$ зліва і

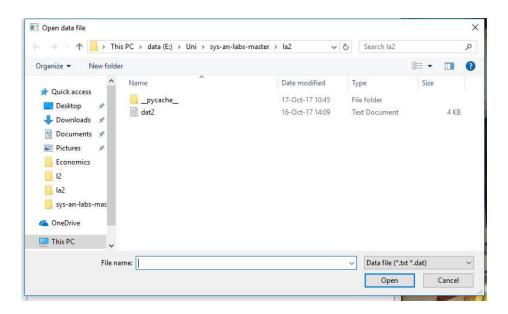
справа на A^T : A^T $Ax = A^T$ b. Тепер матриця A^T A ϵ симетричною і додатньовизначеною.

Інтерфейс користувача

Інтерфейс після запуску програми. Інтерфейс для задання бажаного методу при виконанні, завантаження файлу тощо.



Вікон вибору файлу з вхіднимиданими та данними на вихід.



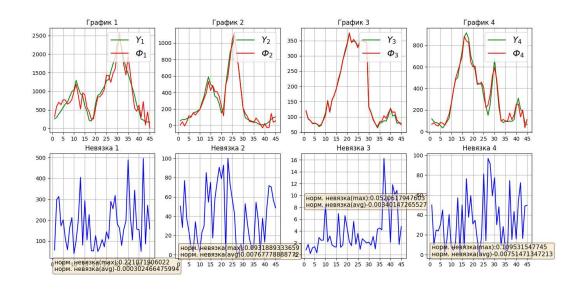


Інтервфейс після завантаження даних та виконання програми.

У поле виводу записаний результат роботи прорами. А також резултат виведений до електронної табилці Excel.

У вікні початкових параметрів можна обрати порядок для кожної із змінних X_i , розмірність вибірки, розмірність змінних.

При натисканні відповідної кнопки виводиться вікно з графіками: функцій, їх знайдених апрокимацій та нев'язки: (виведений приклад роботи за умов використанная зміщених поліномів Чебишева першого порядку)



У виведеному вікні ϵ можливість збереження графіків, їх збільшення тощо.

Розглянемо результати апроксимації нашої функції. Було взято апроксимацію многочленом Чебишева з параметрами для відповідних X_i 8 9 10

Стандартний вигляд таблиць:

Вводные данные: Х

6.05 12.015 1.05 9.015 10 8.15 10.1 1.15 9.109 15.8 2.1 4.2 10.2 8.125 1.192 9.125 22.5 2.5 3.5 12.25 6.175 2.25 9.175 25.7 3.51 2.72 14.325 5.2 4.325 9.198 32.5 4.2 2.53 16.35 4.25 6.35 9.251 35 5.02 2.1 8.411 9.495 40.7 8.2 1.15 18.49 3.4 20.698 2.5 10.505 10.498 51.8 10.1 0.72 22.9 2.7 12.61 11.598 65 12.8 0.54 18.45 3.7 14.695 13.699 82 14.4 0.15 16.75 4.75 15.75 15.748 95.4 14.7 0.55 14.8 5.775 17.804 17.775 102.8 15.5 1.76 12.95 6.8 19.85 19.798 117 16.3 2.23 10.84 7.85 18.05 21.85 125.78 16.7 3.61 8.91 8.855 16.91 23.855 97 16.9 5.16 15.925 10.865 14.925 25.865 95.5 17.5 8.25 14.929 12.885 12.011 27.875 93.9 17.7 11.37 13.933 14.915 10.933 25.899 91.5 18.2 13.26 12.935 16.95 8.935 23.951 79.58 19.1 15.51 11.95 18.975 6.95 21.975 55.4 19.5 17.74 10.81 20.995 4.95 19.015 31.5 21 8.75 22.975 2.108 17.975 12.5 23.56 11.35 6.15 19.95 1.251 15.95 10.8 25.3 8.58 5.2 18.9 3.204 13.915 8.5 28.7 6.74 4.45 17.875 5.248 11.875 4.4 31.56 4.85 7.325 15.865 8.325 9.865 2.5 27.1 6.21 8.35 13.855 11.351 7.855 5.3 24.7 9.52 9.4 11.85 15.408 5.85 8.7 26.2 10.75 9.775 17.495 3.775 11.2 23.7 8.1 12.6 7.75 15.607 1.75 14.7 20.36 6.1

14.7 5.71 13.697 3.697 17.8 17.7 4.15 16.75 3.603 11.75 5.605 20.1 13.34 2.36 18.8 2.495 9.798 7.495 40.52 11.72 1.35 19.85 4.394 7.85 9.415 65.2 9.9 2.13 17.907 6.245 5.913 11.255 80.76 7.74 4.57 15.91 8.192 3.91 13.205 91.1 6.36 6.75 13.925 10.175 2.925 11.175 109.5 5.7 9.26 11.929 12.125 1.929 9.125 122.9 4.75 11.79 9.01 14.105 3.933 7.091 108.3 3.65 13.12 7.935 12.01 5.935 5.985 84.5 3.52 15.36 5.95 10.11 7.95 3.115 58.6 2.72 12.85 5.02 8.115 9.995 1.115 35.8 2.34 10.34 4.05 6.128 11.95 2.12 15.26 2.16 8.68 5.935 4.131 13.935 4.13 9.52 1.76 5.32 6.925 2.135 15.925 6.135 4.8 1.48 2.16 ----- ----- ----- -----

Вводные данные: Ү

254.621 58.145 119.406 117.683 298.163 73.368 92.651 90.123 387.411 71.084 87.691 83.576 467.197 83.567 78.793 74.789 566.547 93.813 79.497 54.316 653.789 101.378 77.082 32.817 710.926 155.579 67.758 57.425 851.381 160.432 71.956 89.519 987.364 176.283 91.123 121.374 1036.12 193.657 112.859 249.173 1292.34 278.624 153.717 384.136 1088.32 354.324 117.965 479.152 926.939 478.926 155.912 501.239 877.128 588.675 169.359 625.482 605.327 499.367 192.924 740.976 458.386 468.567 218.549 875.846 218.859 353.932 247.354 916.124 195.737 335.124 284.167 863.928 306.168 261.946 316.375 703.153 685.761 151.387 341.326 631.195 890.639 210.519 375.651 571.588 923.784 485.142 344.856 436.847 1031.44 688.125 348.314 441.842 1121.32 883.435 344.716 439.425 1291.85 972.834 329.942 322.147 1308.61 1080.56 349.316 235.954 1529.96 887.987 348.231 150.492 1730.13 688.951 347.987 254.897 1917.15 455.494 342.967 458.289 2278.65 211.209 132.856 672.164 2412.14 96.197 115.632 453.356 2186.24 77.325 93.135 227.168 1862.35 64.615 77.824 106.123 52.534 63.453 82.659 1632.88 1467.16 45.178 79.167 93.834 36.176 80.836 91.345 1270.53 20.364 87.192 96.841 1084.24 881.956 10.428 85.834 93.952 8.475 101.985 109.463 616.829 473.329 10.924 128.591 233.415 249.421 24.183 102.861 308.613 225,356 46,324 105,817 207,319 176.578 76.457 78.473 182.263 170.948 95.814 81.417 84.132

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Y1-4

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Нормализованная невязка(тах) (Ү - Ф)
    -----
0.221072 0.0931889 0.0520618 0.109532
Нормализованная невязка(avg) (Y - Ф)
    ------
-0.000302466 \ 0.00767779 \ -0.00340147 \ -0.00751471
Невязка(max) (Y_ - \Phi_)
496.044 99.9066 16.2536 96.75
Невязка(avg) (Y_ - \Phi_)
-0.678678 8.23126 -1.06193 -6.6378
  (Psi11)[1] = 0.024984*T0(x11) + 0.071399*T1(x11) + -0.102491*T2(x11) + -0.063426*T3(x11) + 0.017830*T4(x11) + 0.017830*T4(x11
  + -0.047989*T5(x11) + 0.115281*T6(x11) + -0.020163*T7(x11) + 0.076135*T8(x11)
    (Psi12)[1]=0.024984*T0(x12) + -0.016306*T1(x12) + -0.052224*T2(x12) + 0.018404*T3(x12) + -0.05224*T2(x12) + 0.018404*T3(x12) + -0.052224*T2(x12) + -0.018404*T3(x12) + -0.052224*T2(x12) + -0.018404*T3(x12) + -0.052224*T2(x12) + -0.018404*T3(x12) + -0.018
  0.046023*T4(x12) + -0.071517*T5(x12) + 0.025037*T6(x12) + -0.031791*T7(x12) + -0.040087*T8(x12) + -0.04008*T8(x12) + -0.04008*T8(x12) + -0.04008*T8(x12) + -0.04008*
  (Psi21)[1] = 0.024984*T0(x21) + -0.029453*T1(x21) + -0.111641*T2(x21) + 0.047239*T3(x21) + 0.059290*T4(x21) + 0.047239*T3(x21) + 0.047239*T3(x21) + 0.059290*T4(x21) + 0.047239*T3(x21) + 0.047239*T3(x21) + 0.047239*T3(x21) + 0.059290*T4(x21) + 0.047239*T3(x21) + 0.047239*T3(x21
  + -0.072684*T5(x21) + -0.002097*T6(x21) + 0.013452*T7(x21) + -0.020132*T8(x21) + -0.056543*T9(x21)
  (Psi22)[1] = 0.024984*T0(x22) + -0.260993*T1(x22) + -0.065959*T2(x22) + 0.070835*T3(x22) + 0.082398*T4(x22) + 0.082398*T4(x22
  +0.052606*T5(x22) + -0.060877*T6(x22) + -0.126314*T7(x22) + 0.083884*T8(x22) + -0.027180*T9(x22)
  (Psi31)[1] = 0.024984*T0(x31) + 0.015578*T1(x31) + 0.139685*T2(x31) + 0.012091*T3(x31) + -0.007447*T4(x31) + 0.012091*T3(x31) + -0.007447*T4(x31) + 0.012091*T3(x31) + -0.007447*T4(x31) + 0.012091*T3(x31) + -0.007447*T4(x31) + -0.00747*T4(x31) + -0.00747*T4(x31) + -0.00747*T4(x31) + -
    + -0.013370*T5(x31) + -0.084279*T6(x31) + 0.022364*T7(x31) + -0.011321*T8(x31) + -0.003572*T9(x31) + -
  0.042344*T10(x31)
  (Psi32)[1] = 0.024984*T0(x32) + 0.179880*T1(x32) + -0.217257*T2(x32) + 0.075561*T3(x32) + 0.018224*T4(x32) + 0.01824*T4(x32) + 0.01824*T4
  + -0.000150*T5(x32) + -0.016114*T6(x32) + -0.065379*T7(x32) + -0.019076*T8(x32) + -0.097792*T9(x32) + -0.019076*T8(x32) + -
  0.104180*T10(x32)
  (Psi33)[1] = 0.024984*T0(x33) + -0.034146*T1(x33) + -0.116045*T2(x33) + 0.123045*T3(x33) + 0.000185*T4(x33) + 0.000185*T4(x33
    +\ 0.012658*T5(x33) + 0.041184*T6(x33) + -0.049333*T7(x33) + -0.012243*T8(x33) + 0.034756*T9(x33) + -0.012658*T5(x33) + -0.012688*T5(x33) + 0.034756*T9(x33) + -0.012688*T5(x33) + -0.012688*T
  0.011948*T10(x33)
  (Psi11)[2] = 0.033838*T0(x11) + -0.054204*T1(x11) + 0.018769*T2(x11) + 0.058173*T3(x11) + -0.061085*T4(x11) + -0.061085*T4(x
  +0.022813*T5(x11) + -0.038946*T6(x11) + 0.034572*T7(x11) + -0.054513*T8(x11)
  (Psi12)[2] = 0.033838*T0(x12) + 0.055062*T1(x12) + -0.062931*T2(x12) + -0.006520*T3(x12) + 0.011973*T4(x12) + -0.006520*T3(x12) + 0.011973*T4(x12) + -0.006520*T3(x12) + 0.011973*T4(x12) + -0.006520*T3(x12) + -0.00620*T3(x12) + -0
  +0.048745*T5(x12) +0.029748*T6(x12) +0.042422*T7(x12) +0.001750*T8(x12)
  (Psi21)[2] = 0.033838*T0(x21) + 0.073653*T1(x21) + -0.010265*T2(x21) + 0.025921*T3(x21) + -0.032050*T4(x21) + 0.025921*T3(x21) + -0.032050*T4(x21) + 0.025921*T3(x21) + -0.032050*T4(x21) + -0.032050*T4(x21
  +0.065490*T5(x21) + -0.001785*T6(x21) + 0.002362*T7(x21) + 0.030985*T8(x21) + 0.041385*T9(x21) + 0.041285*
  (Psi22)[2] = 0.033838*T0(x22) + 0.066063*T1(x22) + -0.002839*T2(x22) + 0.002127*T3(x22) + 0.020006*T4(x22) + 0.002127*T3(x22) + 0.002006*T4(x22) + 0.002127*T3(x22) + 0.002006*T4(x22) + 0.002006*T4(x22)
  +0.001011*T5(x22) + -0.005904*T6(x22) + 0.018860*T7(x22) + -0.083711*T8(x22) + 0.050485*T9(x22)
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(Psi31)[2] = 0.033838*T0(x31) + -0.093145*T1(x31) + 0.038071*T2(x31) + -0.015143*T3(x31) + 0.067287*T4(x31) + -0.015143*T3(x31) + 0.067287*T4(x31) + -0.015143*T3(x31) + 0.067287*T4(x31) + -0.015143*T3(x31) + -0.015143*T3(x31
  + -0.001958*T5(x31) + 0.038443*T6(x31) + 0.018167*T7(x31) + -0.000732*T8(x31) + -0.001335*T9(x31) +
0.000924*T10(x31)
  (Psi32)[2]=0.033838*T0(x32) + 0.179295*T1(x32) + -0.048604*T2(x32) + -0.021530*T3(x32) +
  0.035578*T4(x32) + -0.023380*T5(x32) + 0.003626*T6(x32) + 0.085141*T7(x32) + 0.114418*T8(x32) +
  0.075475*T9(x32) + -0.020217*T10(x32)
  (Psi33)[2]=0.033838*T0(x33) + -0.044868*T1(x33) + 0.009405*T2(x33) + -0.005623*T3(x33) +
  0.012978*T4(x33) + -0.035087*T5(x33) + -0.046109*T6(x33) + -0.002074*T7(x33) + 0.021424*T8(x33) + -0.046109*T6(x33) + -0.046
  0.004665*T9(x33) + -0.013785*T10(x33)
  (Psi11)[3] = 0.047916*T0(x11) + -0.004156*T1(x11) + -0.038284*T2(x11) + 0.054103*T3(x11) + 0.066806*T4(x11) + 0.054103*T3(x11) + 0.066806*T4(x11) + 0.066806*T4(x11
  + -0.043225*T5(x11) + 0.001020*T6(x11) + -0.028150*T7(x11) + -0.034936*T8(x11)
  (Psi12)[3] = 0.047916*T0(x12) + 0.110147*T1(x12) + -0.037295*T2(x12) + -0.070732*T3(x12) + -0.070732*T3(
0.012143*T4(x12) + 0.035525*T5(x12) + -0.016671*T6(x12) + 0.027845*T7(x12) + 0.038076*T8(x12) + 0.012143*T4(x12) + 0.012143*T
  (Psi21)[3] = 0.047916*T0(x21) + -0.001084*T1(x21) + -0.021877*T2(x21) + 0.066965*T3(x21) + 0.004833*T4(x21) + 0.004833*T4(x21
  + -0.046840*T5(x21) + 0.000160*T6(x21) + 0.060373*T7(x21) + 0.026384*T8(x21) + 0.000512*T9(x21) + 0.000512
  (Psi22)[3] = 0.047916*T0(x22) + 0.131596*T1(x22) + -0.044491*T2(x22) + -0.005684*T3(x22) + 0.008733*T4(x22) + -0.005684*T3(x22) + 0.008733*T4(x22) + -0.005684*T3(x22) + -0.005684*T3(x2
  +0.026920*T5(x22) + -0.014462*T6(x22) + 0.028110*T7(x22) + 0.007460*T8(x22) + -0.064278*T9(x22) + -0.064
   (Psi31)[3] = 0.047916*T0(x31) + -0.116178*T1(x31) + -0.004211*T2(x31) + -0.033831*T3(x31) 
  0.005626*T4(x31) + 0.019475*T5(x31) + 0.025375*T6(x31) + -0.009423*T7(x31) + -0.057764*T8(x31) + -0.05766*T8(x31) + -0.0
0.018799*T9(x31) + -0.007465*T10(x31)
(Psi32)[3] = 0.047916*T0(x32) + 0.159739*T1(x32) + -0.058008*T2(x32) + -0.155638*T3(x32) + 0.010345*T4(x32) + -0.155638*T3(x32) + 0.010345*T4(x32) + -0.058008*T2(x32) + -0.155638*T3(x32) + 0.010345*T4(x32) + -0.058008*T2(x32) + -0.058008*T3(x32) + -0.058008*T3(x32
  + \ -0.027894*T5(x32) \ + \ 0.020552*T6(x32) \ + \ 0.075547*T7(x32) \ + \ 0.001405*T8(x32) \ + \ 0.022104*T9(x32) \ + \ 0.022104*T9(x32) \ + \ 0.02104*T9(x32) \ + \ 0.02104*T
0.042266*T10(x32)
(Psi33)[3] = 0.047916*T0(x33) + 0.016814*T1(x33) + -0.074451*T2(x33) + 0.002153*T3(x33) + 0.038692*T4(x33) + 0.016814*T1(x33) + 0.016814*T1(x33)
+\ 0.017596*T5(x33)\ +\ -0.009040*T6(x33)\ +\ -0.007470*T7(x33)\ +\ -0.006000*T8(x33)\ +\ -0.012884*T9(x33)\ +\ -0.012884*T9(x33)\
0.042207*T10(x33)
    (Psi11)[4]=0.047442*T0(x11) + -0.025495*T1(x11) + -0.003148*T2(x11) + 0.033626*T3(x11) + -0.003148*T2(x11) + 0.0031626*T3(x11) + -0.003148*T2(x11) + 0.0031626*T3(x11) + -0.0031626*T3(x11) + -0.003166*T3(x11) + -0.003166*T3(x11) + -0.003666*T3(x11) + -0.00366*T3(x11) + -0.00366*T3(x11) + -0.00366*T3(x11) + -0.00366*T3(x11) + -0.00366*T3(x11) 
  0.026815*T4(x11) + 0.043067*T5(x11) + -0.055965*T6(x11) + 0.079858*T7(x11) + -0.011498*T8(x11)
  (Psi12)[4] = 0.047442*T0(x12) + 0.052262*T1(x12) + -0.017697*T2(x12) + 0.006497*T3(x12) + 0.035450*T4(x12) + 0.006497*T3(x12) + 0.006497*T3(x12)
+0.021463*T5(x12) + 0.025336*T6(x12) + 0.069224*T7(x12) + 0.039696*T8(x12)
  (Psi21)[4] = 0.047442*T0(x21) + 0.049482*T1(x21) + -0.012631*T2(x21) + -0.005431*T3(x21) + -0.00541*T3(x21) + -0.00541*T3(x21) + -0.00541*T3(x21) + -0.00541*T3(x21) + -0.00541*T3(x21) 
  0.039187*T4(x21) + 0.017137*T5(x21) + 0.033944*T6(x21) + 0.010440*T7(x21) + 0.031236*T8(x21) + 0.010440*T7(x21) + 0.01040*T7(x21) + 0.010
  0.060153*T9(x21)
  (Psi22)[4] = 0.047442*T0(x22) + 0.106098*T1(x22) + 0.108677*T2(x22) + -0.028434*T3(x22) + 0.002009*T4(x22) + 0.002009*T4(x22)
  +0.008805*T5(x22) + -0.007706*T6(x22) + 0.012705*T7(x22) + -0.014109*T8(x22) + 0.065260*T9(x22)
   (Psi31)[4] = 0.047442*T0(x31) + -0.015487*T1(x31) + -0.041199*T2(x31) + -0.021300*T3(x31) + -0.02100*T3(x31) + -0.02100*T3(x31) + -0.02100*T3(x31) + -0.02100*T3(x31) + -0.02100*T3(x31) + -0.02100*T3(x31)
  0.003298*T4(x31) + 0.030494*T5(x31) + 0.051906*T6(x31) + 0.005058*T7(x31) + 0.025666*T8(x31) + -
  0.038345*T9(x31) + 0.040987*T10(x31)
  (Psi32)[4] = 0.047442*T0(x32) + 0.100475*T1(x32) + -0.092765*T2(x32) + -0.031762*T3(x32) + 0.051298*T4(x32) + -0.051298*T4(x32) + -0.051298*T4(x
  + 0.016346*T5(x32) + -0.019403*T6(x32) + -0.021990*T7(x32) + 0.045598*T8(x32) + 0.077126*T9(x32) + 0.07712
0.024733*T10(x32)
  (Psi33)[4]=0.047442*T0(x33) + 0.000269*T1(x33) + -0.023081*T2(x33) + -0.022661*T3(x33) +
  0.014478*T4(x33) \ + \ 0.009518*T5(x33) \ + \ -0.014596*T6(x33) \ + \ 0.002609*T7(x33) \ + \ 0.016188*T8(x33) \ + \ -0.016188*T8(x33) \ + \ -0.0161888*T8(x33) \ + \ -0.016188*T8(x33) \ + \ -0.0161
  0.029088*T9(x33) + -0.034131*T10(x33)
  (Phi1)[1] = 0.033665*T0(x11) + 0.096206*T1(x11) + -0.138101*T2(x11) + -0.085463*T3(x11) + 0.024025*T4(x11) + -0.085463*T3(x11) + 0.024025*T4(x11) + -0.085463*T3(x11) + 0.024025*T4(x11) + -0.085463*T3(x11) + -0.085665*T3(x11) + -0.085665*T3(x11)
    + -0.064663*T5(x11) + 0.155335*T6(x11) + -0.027169*T7(x11) + 0.102588*T8(x11) + 0.034739*T0(x12) + -
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0.022673*T1(x12) + -0.072614*T2(x12) + 0.025589*T3(x12) + -0.063992*T4(x12) + -0.099440*T5(x12) +
0.034812*T6(x12) + -0.044204*T7(x12) + -0.055739*T8(x12)
(Phi2)[1] = 0.019644*T0(x21) + -0.023158*T1(x21) + -0.087779*T2(x21) + 0.037142*T3(x21) + 0.046618*T4(x21) + 0.046618*T4(x21)
  + -0.057149*T5(x21) + -0.001649*T6(x21) + 0.010577*T7(x21) + -0.015829*T8(x21) + -0.044458*T9(x21) +
  0.022260*T0(x22) + -0.232535*T1(x22) + -0.058767*T2(x22) + 0.063111*T3(x22) + 0.073413*T4(x22) + 0.063111*T3(x22) + 0.073413*T4(x22) + 0.063111*T3(x22) + 0.073413*T4(x22) + 0.063111*T3(x22) + 0.063111*
  0.046870*T5(x22) + -0.054239*T6(x22) + -0.112541*T7(x22) + 0.074737*T8(x22) + -0.024217*T9(x22)
(Phi3)[1]=0.005049*T0(x31) + 0.003148*T1(x31) + 0.028229*T2(x31) + 0.002443*T3(x31) + -0.001505*T4(x31)
  + -0.002702*T5(x31) + -0.017032*T6(x31) + 0.004520*T7(x31) + -0.002288*T8(x31) + -0.000722*T9(x31) + -0.002288*T8(x31) + -0.000722*T9(x31) + -0.002288*T8(x31) + -0.000722*T9(x31) + -0
0.008557*T10(x31) \ + \ 0.024729*T0(x32) \ + \ 0.178044*T1(x32) \ + \ -0.215039*T2(x32) \ + \ 0.074789*T3(x32) \ + \ -0.008557*T10(x31) \ + \ 0.008557*T10(x31) \ + \ 0.00857*T10(x31) \ + \ 0.00857*T10(x31
  0.018038*T4(x32) + -0.000149*T5(x32) + -0.015950*T6(x32) + -0.064712*T7(x32) + -0.018882*T8(x32) + -0.01
  0.096794*T9(x32) + 0.103117*T10(x32) + 0.042304*T0(x33) + -0.057818*T1(x33) + -0.196492*T2(x33) + -0.042304*T0(x33) + -0.042504*T0(x33) + -0.042504*T0(x33) + -0.042504*T0(x33) + -0.042
  0.208345*T3(x33) \ + \ 0.000313*T4(x33) \ + \ 0.021433*T5(x33) \ + \ 0.069734*T6(x33) \ + \ -0.083533*T7(x33) \ + \ -0.08353
0.020731*T8(x33) + 0.058850*T9(x33) + -0.020231*T10(x33)
  (Phi1)[2] = 0.022715*T0(x11) + -0.036387*T1(x11) + 0.012600*T2(x11) + 0.039051*T3(x11) + -0.041006*T4(x11) + -0.041006*T4(x1
    + 0.015314*T5(x11) + -0.026144*T6(x11) + 0.023208*T7(x11) + -0.036594*T8(x11) + 0.092415*T0(x12) + 0.015314*T5(x11) + 0.01531
  0.150381*T1(x12) + -0.171871*T2(x12) + -0.017807*T3(x12) + 0.032700*T4(x12) + 0.133129*T5(x12) +
  0.081245*T6(x12) + 0.115858*T7(x12) + 0.004780*T8(x12)
(Phi2)[2] = 0.049914*T0(x21) + 0.108646*T1(x21) + -0.015141*T2(x21) + 0.038236*T3(x21) + -0.047278*T4(x21) + -0.047278*T4(x2
  + \ 0.096606*T5(x21) \ + \ -0.002633*T6(x21) \ + \ 0.003484*T7(x21) \ + \ 0.045706*T8(x21) \ + \ 0.061048*T9(x21) \ + \ 0.061048*T9(x21
0.040405*T0(x22) + 0.078886*T1(x22) + -0.003390*T2(x22) + 0.002540*T3(x22) + 0.023889*T4(x22) + 0.002540*T3(x22) + 0.002540*T
0.001207*T5(x22) + -0.007050*T6(x22) + 0.022521*T7(x22) + -0.099959*T8(x22) + 0.060284*T9(x22) + -0.060284*T9(x22) + -0.060285*T9(x22) + -0.060285*T9(x22) + -0.060285*T9(x22) + -0.0602
(Phi3)[2]=0.060329*T0(x31) + -0.166069*T1(x31) + 0.067877*T2(x31) + -0.026999*T3(x31) + 0.119967*T4(x31)
  + -0.003491*T5(x31) + 0.068540*T6(x31) + 0.032390*T7(x31) + -0.001305*T8(x31) + -0.002381*T9(x31) +
0.001648*T10(x31) + 0.042080*T0(x32) + 0.222969*T1(x32) + -0.060443*T2(x32) + -0.026775*T3(x32) + -0.026
  0.044244*T4(x32) + -0.029075*T5(x32) + 0.004510*T6(x32) + 0.105880*T7(x32) + 0.142288*T8(x32) +
  0.093860*T9(x32) + -0.025141*T10(x32) + 0.039087*T0(x33) + -0.051829*T1(x33) + 0.010864*T2(x33) + -0.051829*T1(x33) + -0.051829*T1(x3
  0.006495*T3(x33) + -0.014991*T4(x33) + -0.040530*T5(x33) + -0.053262*T6(x33) + -0.002396*T7(x33) + -0.002596*T7(x33) + -0.002596*T7(x33) + -0.002596*T7(x33) + -0.002596*T7(x33) + -0.002596*T7(x33) + -0.00
0.024747*T8(x33) + -0.005389*T9(x33) + -0.015924*T10(x33)
(Phi1)[3] = 0.113414*T0(x11) + -0.009836*T1(x11) + -0.090615*T2(x11) + 0.128060*T3(x11) + 0.158126*T4(x11) + 0.158126*T4(x11)
+ \ -0.102312*T5(x11) \ + \ 0.002414*T6(x11) \ + \ -0.066629*T7(x11) \ + \ -0.082692*T8(x11) \ + \ 0.120595*T0(x12) \ + \ -0.082692*T8(x11) \ + \ -0
0.277223*T1(x12) + -0.093865*T2(x12) + -0.178020*T3(x12) + -0.030561*T4(x12) + 0.089410*T5(x12) + -0.078020*T3(x12) + -0.093865*T2(x12) + -0.093
0.041957*T6(x12) + 0.070081*T7(x12) + 0.095830*T8(x12)
(Phi2)[3] = 0.091901*T0(x21) + -0.002079*T1(x21) + -0.041959*T2(x21) + 0.128437*T3(x21) + 0.009270*T4(x21) + 0.009270*T4(x21)
+ \  \, -0.089839*T5(x21) \  \, + \  \, 0.000308*T6(x21) \  \, + \  \, 0.115794*T7(x21) \  \, + \  \, 0.050605*T8(x21) \  \, + \  \, 0.000982*T9(x21) \  \, + \  \, 0.0009
0.085621*T0(x22) + 0.235150*T1(x22) + -0.079502*T2(x22) + -0.010156*T3(x22) + 0.015605*T4(x22) + 0.015605*
  0.048103*T5(x22) + -0.025842*T6(x22) + 0.050229*T7(x22) + 0.013331*T8(x22) + -0.114859*T9(x22) + -0.013331*T8(x22) + -0.014859*T9(x22) + -0.0148
(Phi3) \lceil 3 \rceil = 0.063776*T0(x31) + -0.154634*T1(x31) + -0.005605*T2(x31) + -0.045030*T3(x31) + 0.007489*T4(x31) + -0.007489*T4(x31) + -0.007489*T
+ 0.025921*T5(x31) + 0.033775*T6(x31) + -0.012542*T7(x31) + -0.076884*T8(x31) + 0.025021*T9(x31) + -0.012542*T7(x31) + -0.01
0.009937*T10(x31) + 0.066218*T0(x32) + 0.220757*T1(x32) + -0.080167*T2(x32) + -0.215089*T3(x32) + -0.080167*T2(x32) + -0.080
  0.014296*T4(x32) + -0.038549*T5(x32) + 0.028403*T6(x32) + 0.104404*T7(x32) + 0.001942*T8(x32) +
  0.030548*T9(x32) + 0.058411*T10(x32) + 0.082667*T0(x33) + 0.029009*T1(x33) + -0.128448*T2(x33) +
  0.003714*T3(x33) + 0.066754*T4(x33) + 0.030357*T5(x33) + -0.015596*T6(x33) + -0.012887*T7(x33) + -0.0128
  0.010352*T8(x33) + -0.022228*T9(x33) + -0.072818*T10(x33)
(Phi1)[4] = 0.054363*T0(x11) + -0.029214*T1(x11) + -0.003607*T2(x11) + 0.038531*T3(x11) + -0.030727*T4(x11) + -0.03072*T4(x11) + -0.03072*T4
  + \ 0.049350*T5(x11) \ + \ -0.064130*T6(x11) \ + \ 0.091508*T7(x11) \ + \ -0.013175*T8(x11) \ + \ 0.104647*T0(x12) \ + \ -0.013175*T8(x11) \ + \ -0.
0.115278*T1(x12) + -0.039036*T2(x12) + 0.014330*T3(x12) + 0.078196*T4(x12) + 0.047344*T5(x12) + 0.04744*T5(x12) + 0.04744*
0.055886*T6(x12) + 0.152693*T7(x12) + 0.087560*T8(x12)
  (Phi2)[4]=0.141879*T0(x21) + 0.147981*T1(x21) + -0.037775*T2(x21) + -0.016242*T3(x21) + -0.117191*T4(x21)
    + 0.051249*T5(x21) + 0.101511*T6(x21) + 0.031223*T7(x21) + 0.093414*T8(x21) + 0.179892*T9(x21) + 0.179892*
0.078783*T0(x22) + 0.176188*T1(x22) + 0.180471*T2(x22) + -0.047218*T3(x22) + 0.003335*T4(x22) + 0.00335*T4(x22) + 0.0035*T4(x22) + 0.003*T4(x22) + 0.003*T4(x22
  0.014621*T5(x22) + -0.012796*T6(x22) + 0.021098*T7(x22) + -0.023430*T8(x22) + 0.108372*T9(x22) + -0.023430*T8(x22) + 0.0108372*T9(x22) + -0.0108372*T9(x22) + -0.01082*T9(x22) + -0.01082*T9
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(Phi3)[4]=0.044625*T0(x31) + -0.014568*T1(x31) + -0.038753*T2(x31) + -0.020036*T3(x31) + -0.02005*T3(x31) + -0.02005*T3(x31) + -0.02005*T3(x31) + -0.02005*T3(x31) + -0.02005*T3(x31) + -0.02005*T3(x31) + -
  0.003102*T4(x31) + 0.028684*T5(x31) + 0.048825*T6(x31) + 0.004758*T7(x31) + 0.024142*T8(x31) + -0.048825*T6(x31) + 0.004758*T7(x31) + 0.00475
  0.036068*T9(x31) + 0.038553*T10(x31) + 0.082619*T0(x32) + 0.174975*T1(x32) + -0.161548*T2(x32) + -0.1615
  0.055312*T3(x32) + 0.089334*T4(x32) + 0.028466*T5(x32) + -0.033791*T6(x32) + -0.038295*T7(x32) + -0.008295*T7(x32) + -0.0085
  0.050411*T2(x33) + -0.049494*T3(x33) + -0.031621*T4(x33) + 0.020788*T5(x33) + -0.031880*T6(x33) + -0.03180*T6(x33) + -0.03180*T6(x33) + -0.03180*T6(x33) + -0.03180*T6(x33) + -0.03180*T6(x33) + -0.03180*T6
  0.005699*T7(x33) + 0.035357*T8(x33) + -0.063531*T9(x33) + -0.074544*T10(x33)
  (F1) = 0.017790*T0(x11) + 0.050841*T1(x11) + -0.072981*T2(x11) + -0.045164*T3(x11) + 0.012696*T4(x11) + -0.012696*T4(x11) + -0.01266*T4(x11) + -0.01266*T4(x11) + -0.01266*T4(x11) + -0.
  0.034172*T5(x11) + 0.082088*T6(x11) + -0.014358*T7(x11) + 0.054214*T8(x11) + 0.018358*T0(x12) + -0.014358*T0(x12) + -0.01435
  0.011982*T1(x12) + -0.038373*T2(x12) + 0.013523*T3(x12) + -0.033817*T4(x12) + -0.052550*T5(x12) + -0.05250*T5(x12) 
  0.018397*T6(x12) + -0.023360*T7(x12) + -0.029456*T8(x12) + 0.021818*T0(x21) + -0.025721*T1(x21) + -0.025
  0.097492*T2(x21) + 0.041252*T3(x21) + 0.051776*T4(x21) + -0.063473*T5(x21) + -0.001832*T6(x21) + -0.063473*T5(x21) + -0.0634
  0.011748*T7(x21) \ + \ -0.017581*T8(x21) \ + \ -0.049377*T9(x21) \ + \ 0.024723*T0(x22) \ + \ -0.258266*T1(x22) \ + \ -0.017581*T8(x21) \ + \ -0.017
  0.065269*T2(x22) + 0.070095*T3(x22) + 0.081537*T4(x22) + 0.052057*T5(x22) + -0.060241*T6(x22) + -0.06024
0.124994*T7(x22) \ + \ 0.083007*T8(x22) \ + \ -0.026896*T9(x22) \ + \ 0.004522*T0(x31) \ + \ 0.002820*T1(x31) \ + \ 0.0028200*T1(x31) \ + \ 0.0028200*
0.025283*T2(x31) + 0.002188*T3(x31) + -0.001348*T4(x31) + -0.002420*T5(x31) + -0.015255*T6(x31) + -0.0015255*T6(x31) + -0.001525*T6(x31) + -0.00150
0.004048*T7(x31) + -0.002049*T8(x31) + -0.000647*T9(x31) + -0.007664*T10(x31) + 0.022149*T0(x32) + -0.007664*T10(x31) + -0.007664*T10
0.159467*T1(x32) + -0.192602*T2(x32) + 0.066986*T3(x32) + 0.016156*T4(x32) + -0.000133*T5(x32) + -0.00013*T5(x32) + -0.00013*T5(x32) + -0.00013*T5(x32) + -0.00013*T5(x32) + -0.00013*T5(x32) + -0.00015*T5(x32) + -0.00015*T5(x
0.014286*T6(x32) + -0.057960*T7(x32) + -0.016912*T8(x32) + -0.086694*T9(x32) + 0.092358*T10(x32) + -0.016912*T8(x32) + -0.01
  0.037890*T0(x33) + -0.051785*T1(x33) + -0.175990*T2(x33) + 0.186606*T3(x33) + 0.000281*T4(x33) + 0.000281*
  0.019197*T5(x33) + 0.062458*T6(x33) + -0.074817*T7(x33) + -0.018568*T8(x33) + 0.052710*T9(x33) + -0.018568*T8(x33) + 0.052710*T9(x33) + -0.018568*T8(x33) + 0.052710*T9(x33) + -0.018568*T8(x33) + -0.018568
0.018120*T10(x33)
  (F2) = 0.012356*T0(x11) + -0.019793*T1(x11) + 0.006854*T2(x11) + 0.021243*T3(x11) + -0.022306*T4(x11) + -0.019793*T1(x11) + 0.006854*T2(x11) + 0.021243*T3(x11) + -0.022306*T4(x11) + -0.019793*T1(x11) + 0.006854*T2(x11) +
  0.008330*T5(x11) + -0.014222*T6(x11) + 0.012624*T7(x11) + -0.019906*T8(x11) + 0.050271*T0(x12) + -0.019906*T8(x11) + -0.019906*T8(x11
  0.081804*T1(x12) + -0.093493*T2(x12) + -0.009686*T3(x12) + 0.017788*T4(x12) + 0.072419*T5(x12) +
  0.044195*T6(x12) + 0.063024*T7(x12) + 0.002600*T8(x12) + 0.031885*T0(x21) + 0.069402*T1(x21) + -
  0.009672*T2(x21) + 0.024425*T3(x21) + -0.030201*T4(x21) + 0.061711*T5(x21) + -0.001682*T6(x21) + -0.00162*T6(x21) + -0.00162*T6(x21) + -0.00162*T6(x21) + -0.00162*T
  0.002166*T2(x22) + 0.001623*T3(x22) + 0.015260*T4(x22) + 0.000771*T5(x22) + -0.004504*T6(x22) + -0.004004*T6(x22) + -0.004004*T6(x22) + -0.00400
0.014386*T7(x22) \ + \ -0.063853*T8(x22) \ + \ 0.038509*T9(x22) \ + \ 0.044864*T0(x31) \ + \ -0.123497*T1(x31) \ + \ -0.1234
0.050477*T2(x31) + -0.020078*T3(x31) + 0.089214*T4(x31) + -0.002596*T5(x31) + 0.050970*T6(x31) + -0.0100076*T2(x31) + 0.000076*T3(x31) + 0.00007
0.024087*T7(x31) \ + \ -0.000971*T8(x31) \ + \ -0.001770*T9(x31) \ + \ 0.001226*T10(x31) \ + \ 0.031293*T0(x32) \ + \ 0.001226*T10(x31) \ + \ 0.0012
0.165811*T1(x32) + -0.044948*T2(x32) + -0.019911*T3(x32) + -0.032902*T4(x32) + -0.021622*T5(x32) + -0.019911*T3(x32) + -0.01
  0.003354*T6(x32) + 0.078738*T7(x32) + 0.105813*T8(x32) + 0.069799*T9(x32) + -0.018696*T10(x32) + 0.069799*T9(x32) + 0.018696*T10(x32) + 0.018696
0.029067*T0(x33) + -0.038543*T1(x33) + 0.008079*T2(x33) + -0.004830*T3(x33) + -0.011148*T4(x33) + -0.01148*T4(x33) + -0.0148*T4(x33) + -0
  0.030140*T5(x33) + -0.039609*T6(x33) + -0.001781*T7(x33) + 0.018403*T8(x33) + -0.004008*T9(x33) + -0.004
0.011842*T10(x33)
(F3) = 0.045546*T0(x11) + -0.003950*T1(x11) + -0.036390*T2(x11) + 0.051427*T3(x11) + 0.063502*T4(x11) + -0.063502*T4(x11) + -0.065002*T4(x11) + -0.065002*T4(x11) + -0.065002*T4(x11) + -0.065002*T4(x11) + -0.065002*T4(x11) + 
0.041087*T5(x11) + 0.000969*T6(x11) + -0.026758*T7(x11) + -0.033208*T8(x11) + 0.048430*T0(x12) + -0.048430*T0(x12) + -0.04840*T0(x12) + -0.
0.111330*T1(x12) + -0.037695*T2(x12) + -0.071491*T3(x12) + -0.012273*T4(x12) + 0.035906*T5(x12) + -0.012273*T4(x12) + 0.0035906*T5(x12) + -0.012273*T4(x12) + 0.0035906*T5(x12) + -0.001273*T4(x12) + 0.001273*T4(x12) + 0.001273*T4(x12
0.016849*T6(x12) + 0.028144*T7(x12) + 0.038484*T8(x12) + 0.050054*T0(x21) + -0.001132*T1(x21) + -0.00112*T1(x21) + -0.00112*T1(x21) + -0.00112*T1(x21) + -0.00112*T1(x21) + -0.00112*T1(
  0.022853*T2(x21) + 0.069954*T3(x21) + 0.005049*T4(x21) + -0.048931*T5(x21) + 0.000168*T6(x21) + 0.000168*T
  0.063068*T7(x21) + 0.027562*T8(x21) + 0.000535*T9(x21) + 0.046633*T0(x22) + 0.128075*T1(x22) + -0.000535*T1(x22) + 0.000535*T1(x22) + 0.000555*T1(x22) + 0.00055*T1(x22) + 0.00055*T1(x22) + 0.00055*T1(x22) + 0.00055*T1(x22) + 0.00055*T1(x22
  0.043301*T2(x22) + -0.005532*T3(x22) + 0.008499*T4(x22) + 0.026200*T5(x22) + -0.014075*T6(x22) + 0.008499*T4(x22) + 0.008499*
  0.027357*T7(x22) + 0.007261*T8(x22) + -0.062558*T9(x22) + 0.044309*T0(x31) + -0.107433*T1(x31) + -0.1074
  0.003894*T2(x31) + -0.031285*T3(x31) + 0.005203*T4(x31) + 0.018009*T5(x31) + 0.023465*T6(x31) + -0.018009*T5(x31) + 0.018009*T5(x31) + 0.018009*
  0.008714*T7(x31) + -0.053416*T8(x31) + 0.017384*T9(x31) + -0.006904*T10(x31) + 0.046006*T0(x32) +
  0.153373*T1(x32) + -0.055696*T2(x32) + -0.149435*T3(x32) + 0.009932*T4(x32) + -0.026782*T5(x32) + -0.026
  0.019733*T6(x32) \ + \ 0.072536*T7(x32) \ + \ 0.001349*T8(x32) \ + \ 0.021223*T9(x32) \ + \ 0.040582*T10(x32) \ + \ 0.040582
  0.057433*T0(x33) + 0.020154*T1(x33) + -0.089240*T2(x33) + 0.002581*T3(x33) + 0.046378*T4(x33) + 0.04678*T4(x33) 
  0.021091*T5(x33) + -0.010835*T6(x33) + -0.008954*T7(x33) + -0.007192*T8(x33) + -0.015443*T9(x33) + -0.015445*T9(x33) + -0.015445*T9(x33) + -0.01545*T9(x33) + -0.01545*T9(x33) + -0.01545*T9(x33) + -0.01545*T9(x33) + -0.01545*
  0.050591*T10(x33)
  (F4)=0.028996*T0(x11) + -0.015582*T1(x11) + -0.001924*T2(x11) + 0.020551*T3(x11) + -0.016389*T4(x11) + -
0.026322*T5(x11) + -0.034205*T6(x11) + 0.048808*T7(x11) + -0.007027*T8(x11) + 0.055815*T0(x12) + -0.048808*T7(x11) + -0.0488
  0.061486*T1(x12) + -0.020821*T2(x12) + 0.007643*T3(x12) + 0.041707*T4(x12) + 0.025252*T5(x12) +
  0.014029*T2(x21) \ + \ -0.006032*T3(x21) \ + \ -0.043521*T4(x21) \ + \ 0.019032*T5(x21) \ + \ 0.037698*T6(x21) \ + \ 0.037698*T6(x21) \ + \ 0.019032*T5(x21) \ + \ 0.0087698*T6(x21) \ + \ 0.0087698
  0.011595*T7(x21) \ + \ 0.034691*T8(x21) \ + \ 0.066806*T9(x21) \ + \ 0.029257*T0(x22) \ + \ 0.065430*T1(x22) \ + \ 0.065430*T1(x22) \ + \ 0.066806*T9(x21) \ +
  0.067021*T2(x22) + -0.017535*T3(x22) + 0.001239*T4(x22) + 0.005430*T5(x22) + -0.004752*T6(x22) + -0.0047
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0.025806*T2(x31) + -0.013342*T3(x31) + -0.002066*T4(x31) + 0.019101*T5(x31) + 0.032513*T6(x31) + -0.019101*T5(x31) + 0.019101*T5(x31) + 0.019101
0.003168*T7(x31) + 0.016076*T8(x31) + -0.024018*T9(x31) + 0.025673*T10(x31) + 0.055016*T0(x32) +
0.116516*T1(x32) + -0.107575*T2(x32) + -0.036833*T3(x32) + 0.059488*T4(x32) + 0.018955*T5(x32) + -0.018955*T5(x32) + -0.01895*T5(x32) + -0.0189
0.022501*T6(x32) + -0.025500*T7(x32) + 0.052877*T8(x32) + 0.089440*T9(x32) + 0.028682*T10(x32) + 0.02868
0.068999*T0(x33) + 0.000392*T1(x33) + -0.033569*T2(x33) + -0.032958*T3(x33) + -0.021057*T4(x33) + -0.021057*T4(x35) + -0.021
 0.013842*T5(x33) + -0.021229*T6(x33) + 0.003795*T7(x33) + 0.023544*T8(x33) + -0.042305*T9(x33) + -0.042505*T9(x33) + -0.0425
0.049639*T10(x33)
(Ф1) трансформированный:
-16.499035343923882(x11)^7
                                                                                                                                                                                        254.47729204770297(x11)^6
                                                                                                                                                                                                                                                                                                                                                                            1638.0588035380672(x11)^5
                                                                                                                                                                                    10858.475991092877(x11)<sup>3</sup>
                                                                                                                                                                                                                                                                                                                                                                                12126.84250895369(x11)^2
 5578.659300657564(x11)^4
                                                                                                                                                                                                                                                                                                                                                 +
 7223.501579485039(x11) + 1776.4706029896959 +
 -0.8624015183095648(x12)^7
                                                                                                                                                                                           12.44247625327192(x12)^6
                                                                                                                                                                                                                                                                                                                                                  +
                                                                                                                                                                                                                                                                                                                                                                                297.9472108751881(x12)^5
 1670.4145464051235(x12)^4 + 4247.75476194035(x12)^3 - 5566.381139345729(x12)^2 + 3669.45411634083(x12) - 5566.381139345729(x12)^2 + 5666.381139345729(x12)^2 + 5666.381139345729(x12)^2 + 5666.381139406(x12)^2 + 5666.3811396(x12)^2 + 56666.3811396(x12)^2 + 566660(x12)^2 + 56660(x12)^2 + 56660(x12)^2 + 56660(x12)^2 + 56660(x12)^2 + 56660(x12)^2 + 56660(x12)^2
- 965.2042633510679 +
-7.825094341562885(x21)^8
                                                                                                                                                                                     177.00718926105043(x21)^7
                                                                                                                                                                                                                                                                                                                                                                          1777.8612673076861(x21)^6
9536.967310860196(x21)^5
                                                                                                                                                                                    28926.47941594365(x21)^4
                                                                                                                                                                                                                                                                                                                                                                          51128.914017826035(x21)^3
 52206.73370290521(x21)^2 + 28547.81994751869(x21) - 6471.980127025229 +
 -21.633253078488558(x22)^8
                                                                                                                                                                                         473.04090522884394(x22)^7
                                                                                                                                                                                                                                                                                                                                                                           3898.2640819479143(x22)^6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
 16034.667469910077(x22)^5
                                                                                                                                                                                           37214.09894163614(x22)^4
                                                                                                                                                                                                                                                                                                                                                                                     51216.0950566887(x22)<sup>3</sup>
41649.1488152903(x22)^2 + 18584.135019658086(x22) - 3525.369369179477 +
2.950331392289957(x31)^9
                                                                                                                                                                                     71.61058406748978(x31)^8
                                                                                                                                                                                                                                                                                                                                                                              753.3262126564871(x31)^7
4445.565758990215(x31)^6
                                                                                                                                                                                    15907.485514025086(x31)^5
                                                                                                                                                                                                                                                                                                                                                                          35570.246036076256(x31)^4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
49815.457427311885(x31)^3 - 42380.25526419885(x31)^2 + 20006.77572944458(x31) - 4018.305592366355 + 20006.77572944458(x31) - 40006.77572944458(x31) - 40006.775740(x31) - 40006.775740(x
-32.46095490592202(x32)^9
                                                                                                                                                                                     1015.3839130540568(x32)^8
                                                                                                                                                                                                                                                                                                                                                                          11779.992327567137(x32)^7
70522.76909143194(x32)^6 - 245613.542701798(x32)^5 + 525285.10280286(x32)^4 - 699409.5561481034(x32)^3
+565063.45178139(x32)^2 - 253472.9384111401(x32) + 48421.946284878424 +
8.324650665563219(x33)^9
                                                                                                                                                                                245.42734977687815(x33)^8
                                                                                                                                                                                                                                                                                                                                                                           3367.4929211131575(x33)^7
                                                                                                                                                                                       71902.09259720835(x33)^5
21123.16797690851(x33)^6
                                                                                                                                                                                                                                                                                                                                                                             145046.0165672307(x33)<sup>4</sup>
 178863.65023201166(x33)^3 - 132635.50461563678(x33)^2 + 54408.83487629122(x33) - 9500.0149461516 +
0.118898830317
(Ф2) трансформированный:
6.227447547975761(x11)^7
                                                                                                                                                                                        93.1156136222755(x11)^6
                                                                                                                                                                                                                                                                                                                                                                             613.5274197991897(x11)^5
2132.8501025795263(x11)^4
                                                                                                                                                          +
                                                                                                                                                                                         4176.946337701111(x11)^3
                                                                                                                                                                                                                                                                                                                                                                         4630.9807160833225(x11)^2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
2712.580186530753(x11) - 652.2901504659715 +
6.4503850985978675(x12)^7
                                                                                                                                                                                       81.11270601498921(x12)^6
                                                                                                                                                                                                                                                                                                                                              +
                                                                                                                                                                                                                                                                                                                                                                           454.86470155289896(x12)^5
 1259.3616541075046(x12)^4
                                                                                                                                                                                       1781.5808266799431(x12)^3
                                                                                                                                                                                                                                                                                                                                                                           1162.6866650671732(x12)^2
 175.4757309813853(x12) + 85.20450109745401 +
                                                                                                                                                                                                                                                                                                                                                                           1206.3551954277118(x21)^6
 7.627390896841483(x21)^8
                                                                                                                                                                                  125.65615596635669(x21)^7
 6582.88165326275(x21)^5
                                                                                                                                                                                    20564.72032802502(x21)^4
                                                                                                                                                                                                                                                                                                                                                                          37455.61129028283(x21)^3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
39319.17673300985(x21)^2 - 22044.78006988838(x21) + 5111.443044866554 +
                                                                                                                                                                                                                                                                                                                                                                           3229.5373849751295(x22)^6
 15.843288351613907(x22)^8
                                                                                                                                                                                       362.2980411242293(x22)^7
                                                                                                                                                                                                                                                                                                                                              +
                                                                                                                                                           +
                                                                                                                                                                                                                                                                                                                                                                               57082.82162310493(x22)^3
 14594.891162580612(x22)^5
                                                                                                                                                                                         37478.271690497364(x22)^4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
 51074.92462026819(x22)^2 - 24805.787768067934(x22) + 5047.432972579094 +
 -5.713844227509689(x31)^9
                                                                                                                                                                                       34.85453324048016(x31)^8
                                                                                                                                                                                                                                                                                                                                                                          143.63937794609132(x31)^7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
714.3882868632231(x31)^6
                                                                                                                                                                                  2868.9800861596923(x31)^5
                                                                                                                                                                                                                                                                                                                                             +
                                                                                                                                                                                                                                                                                                                                                                           6896.7617680751455(x31)^4
 9665.097279756654(x31)^3 + 7839.375388261031(x31)^2 - 3444.7332478754374(x31) + 642.5361508160423 + 3444.7332478754374(x31) + 3444.733247875474(x31) + 3444.7332478754(x31) + 3444.7332478754(x31) + 3444.7332478754(x31) + 3444.733247876(x31) + 3444.733247876(x31) + 3444.733247876(x31) + 3444.733247876(x31) + 3444.733247876(x31) + 3444.733247876(x31) + 3444.7332476(x31) + 3444.733476(x31) + 3444.736(x31) + 3444476(x31) + 344476(x31) + 344476(x31) + 344476(x31) + 344476(x31) +
 9.282006626607057(x32)^9
                                                                                                                                                                                  257.16373702982935(x32)^8
                                                                                                                                                                                                                                                                                                                                                                           3112.0860166524226(x32)^7
 19318.04839188441(x32)^6
                                                                                                                                                                                     68324.12468228056(x32)^5
                                                                                                                                                                                                                                                                                                                                                                         144872.45831642463(x32)^4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
 186495.4259809366(x32)^3 - 141850.20608272363(x32)^2 + 58159.71078750076(x32) - 9802.20731680929 + 9802.20731680929 + 9802.20731680929 + 9802.20731680929 + 9802.20731680929 + 9802.2073168090 + 9802.2073168090 + 9802.2073168090 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20731600 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.20700 + 9802.2000 + 9802.20000 + 9802.2000 + 9802.20000 + 9802.200
0.6617976315738954(x33)^9
                                                                                                                                                                                      31.283206296584517(x33)^8
                                                                                                                                                                                                                                                                                                                                                                                 521.0348790872956(x33)^7
4222.412184957497(x33)^6
                                                                                                                                                                                      18476.20944767647(x33)^5
                                                                                                                                                                                                                                                                                                                                                                        46801.724136937984(x33)^4
-0.355209657303
 (Ф3) трансформированный:
 -1.3189121923651257(x11)^7
                                                                                                                                                                                       22.657636109204404(x11)^6
                                                                                                                                                                                                                                                                                                                                                                             398.26896110184373(x11)^5
2018.7567073571313(x11)^4
                                                                                                                                                           +
                                                                                                                                                                                       4903.0141195305505(x11)<sup>3</sup>
                                                                                                                                                                                                                                                                                                                                                                               6303.881741450737(x11)<sup>2</sup>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      +
4133.454172721236(x11) - 1088.1629913279776 +
```

0.007835*T7(x22) + -0.008701*T8(x22) + 0.040246*T9(x22) + 0.029716*T0(x31) + -0.009701*T1(x31) + -0.0097

```
0.47068013859210733(x12)^7
                                                                   31.96692438821532(x12)^6
                                                                                                                                   461.3346378949201(x12)^5
2330.665869518791(x12)^4
                                                                 5704.374413783256(x12)<sup>3</sup>
                                                                                                                        +
                                                                                                                                   7355.437614500572(x12)^2
4813.684147581718(x12) + 1261.059888405153 +
1.5589248885332307(x21)^8
                                                                10.134447830401761(x21)^7
                                                                                                                                  30.080602773850124(x21)^6
                                                                                                                                                                                         +
575.5813924952051(x21)^5
                                                               2154.5015257178966(x21)^4
                                                                                                                                  3464.3221230155627(x21)^3
2504.3094879610753(x21)^2 + 587.6115897809382(x21) + 70.11902024076146 +
                                                                                                                                    2595.90957449268(x22)^6
-5.828423711255424(x22)^8
                                                                    225.848813999456(x22)^7
                                                                                                                                                                                         +
14017.597776333387(x22)^5
                                                                  41352.583131974265(x22)^4
                                                                                                                                     70686.5119565798(x22)^3
69912.09660828854(x22)^2 + 37136.333178210305(x22) - 8199.646902551875 +
                                                               235.77044853813825(x31)^8
8.47781038496583(x31)^9
                                                                                                                                   2354.635243021246(x31)^7
11894.603436612988(x31)^6
                                                                   34755.90502472879(x31)^5
                                                                                                                                   62682.94255726783(x31)^4
71398.04575492913(x31)^3 - 50460.25364955292(x31)^2 + 20375.72001719932(x31) - 3619.4378367570057 +
                                                                  101.64923683863844(x32)^8
                                                                                                                                  1941.6769356575103(x32)^7
                                                                                                                                                                                         +
-2.7575611389293426(x32)^9
15779.389177581084(x32)^6
                                                                 66727.76687748509(x32)^5
                                                                                                                                  163593.89230005592(x32)^4
242067.1176173419(x32)^3 + 213588.58166246983(x32)^2 - 103600.50585580175(x32) + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.454298757286 + 21276.45429875728 + 21276.45429875728 + 21276.45429875728 + 21276.45429875728 + 21276.45429875728 + 21276.45429875728 + 21276.45429875728 + 21276.45429875728 + 21276.4542987728 + 21276.4542987728 + 21276.454298 + 21276.454298 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 + 21276.45429 +
8.810849813577844(x33)^9
                                                               283.43571475006064(x33)^8
                                                                                                                       +
                                                                                                                                  3710.0951256991725(x33)^7
24720.96098924303(x33)^6
                                                                94558.67601661969(x33)^5
                                                                                                                                 219742.91439394862(x33)^4
                                                                                                                                                                                         +
-0.116726037685
(Ф4) трансформированный:
10.340062642384913(x11)^7
                                                                 138.30438619912715(x11)^6
                                                                                                                                    767.2199077346595(x11)^5
2177.8267303533385(x11)^4
                                                                 3414.3098379430335(x11)^3
                                                                                                                                  2966.2231336958403(x11)^2
                                                                                                                                                                                         +
1320.9156667325806(x11) - 230.27102705498004 +
0.21234290937930034(x12)^7
                                                                  18.911723812934625(x12)^6
                                                                                                                                  327.05473866715533(x12)^5
                                                                                                                                                                                         +
                                                                  5376.229076310934(x12)<sup>3</sup>
                                                                                                                         +
                                                                                                                                   7673.041078551894(x12)^2
1935.2626936451893(x12)^4
5454.111998594761(x12) + 1530.3196178527287 +
7.262006700268323(x21)^8
                                                               196.26557187167714(x21)^7
                                                                                                                                  2223.2184571515713(x21)^6
12388.483488590406(x21)^5
                                                                   38004.38651204436(x21)^4
                                                                                                                                   67569.11873557333(x21)<sup>3</sup>
69429.82634102342(x21)^2 - 38266.90007430129(x21) + 8756.36726613028 +
8.254499513629128(x22)^8
                                                                214.12426870705673(x22)^7
                                                                                                                                   2068.044347278305(x22)^6
10165.2521244752(x22)^5
                                                               28428.827712742255(x22)^4
                                                                                                                                  47090.95591969687(x22)<sup>3</sup>
                                                                                                                                                                                         +
45713.420030302965(x22)^2 - 24023.12373188455(x22) + 5275.1122669356655 +
-12.145604238263925(x31)^9
                                                                   331.21619971777284(x31)^8
                                                                                                                                   3644.999240799688(x31)^7
                                                                                                                                                                                         +
21048.74992792879(x31)^6
                                                                71419.09711456882(x31)^5
                                                                                                                                  149871.05710076328(x31)^4
196891.60338744923(x31)^3 + 157704.3864293453(x31)^2 - 70447.50116239837(x31) + 13459.887268830451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 1246936836451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 12469366451 + 124693664 + 124693664 + 124693664 + 124693664 + 124693664 + 124693664 + 124693664 + 124693664 + 124693664 + 124693664 + 124693664 + 124693666 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 12469366 + 1246966 + 1246966 + 1246966 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124696 + 124606 + 124606 + 124606 + 124606 + 124606 + 124
0.5803612447938074(x32)^9
                                                                 31.016394296141442(x32)^8
                                                                                                                                   252.2491820135051(x32)^7
329.4407318283065(x32)^6
                                                                 10251.12801068215(x32)^5
                                                                                                                                   45571.13074438827(x32)^4
96197.76184347773(x32)^3 + 108754.61674929845(x32)^2 - 63465.495751096714(x32) + 15037.709387259598 +
                                                                 112.94739433046499(x33)^8
                                                                                                                                  1942.5138393301077(x33)^7
3.0031071497559045(x33)^9
                                                                  69929.48811965442(x33)<sup>5</sup>
15888.261808659108(x33)^6
                                                                                                                                  179109.63870829443(x33)^4
-0.200838369483
(Ф1) трансформированный денормированный:
-4599.124277761342(x11)^7
                                                                  78.09779273258569(x11)^6
                                                                                                                                 0.7219858135533982(x11)^5
0.003999586409081503(x11)^4
                                                             1.3659936430073235e-05(x11)<sup>3</sup>
                                                                                                                               2.8193394250983826e-08(x11)^2
3.224719006924024e-11(x11) + 1.568782099813455e-14 +
108.49303389758154(x12)^7
                                                               3.1521105358839128(x12)^6
                                                                                                                               0.042804353913984086(x12)^5
0.0003070198129334486(x12)^4 +
                                                               1.2395770678133567e-06(x12)^3
                                                                                                                                2.835872060696867e-09(x12)^2
3.4340228349127955e-12(x12) - 1.7109687217354351e-15 +
-333.08012720986545(x21)^8
                                                                  12.786544150994871(x21)^7
                                                                                                                                  0.2544885173926728(x21)^6
                                                                                                                                                                                         +
0.0029254292212973765(x21)^5
                                                               2.0246231483310194e-05(x21)<sup>4</sup>
                                                                                                                                 8.532421705835877e-08(x21)^3
2.1392843676239542e-10(x21)^2 + 2.930230413720595e-13(x21) +
                                                                 5.0613046728000475(x22)^7
                                                                                                                                0.04457787130917294(x22)^6
-271.34327332287165(x22)^8
                                                       +
0.00021529211287335952(x22)^5
                                                                6.150154040629564e-07(x22)^4 +
                                                                                                                               1.0697610956676978e-09(x22)^3
1.1168694920396912e-12(x22)^2 +
1.0255911598531355(x31)^9
                                                            0.0012612242917637653(x31)^8
                                                                                                                               7.578164332054659e-07(x31)^7
2.6466800509500023e-10(x31)^6 + 5.72445821386612e-14(x31)^5 +
-355.9914393771211(x32)^9
                                                                6.998867479089318(x32)^8
                                                                                                                               0.06621634593155033(x32)^7
0.00035409454755581596(x32)^6 - 1.1547302715783045e-06(x32)^5
                                                                                                                                 2.378035201650968e-09(x32)<sup>4</sup>
3.1043835102261267e-12(x32)^3 + 2.4894201070132485e-15(x32)^2 +
```

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96.44677964358209(x33)^9
                                 7.998282664599994(x33)^8
                                                                  0.313772522464019(x33)^7
0.005961801840054881(x33)^6 +
                               6.298651968875157e-05(x33)^5
                                                                3.9894148902816983e-07(x33)<sup>4</sup>
125040.502148
(Ф2) трансформированный денормированный:
829.8130584698233(x11)^7
                                14.176528054200599(x11)^6
                                                                 0.13173452794522858(x11)^5
0.0007307907728948401(x11)^4
                                 2.486969523604905e-06(x11)^3
                                                                 5.089416466853185e-09(x11)<sup>2</sup>
5.7477220209684174e-12(x11) - 2.752258222856249e-15 +
108.29257681503226(x12)^7
                                1.8263932313443567(x12)^6
                                                                 0.015489215150316185(x12)^5
7.11432327334554e-05(x12)^4
                               1.755257353044725e-07(x12)^3
                                                                2.0184966827298826e-10(x12)^2
                                                                                              +
3.8860370732068746e-14(x12) +
117.3013242403875(x21)^8
                                4.228900383816952(x21)^7
                                                                 0.08408988606972168(x21)^6
                                6.980831300681834e-06(x21)<sup>4</sup>
0.0009847257334973055(x21)^5
                                                                 3.015351604812649e-08(x21)^3
7.740193826175305e-11(x21)^2 - 1.0836482380224529e-13(x21) +
                                1.9816431062154247(x22)^7
101.33748555902662(x22)^8
                                                                 0.01868744405857649(x22)^6
9.794604122018109e-05(x22)<sup>5</sup>
                                3.058104248234318e-07(x22)<sup>4</sup>
                                                                 5.821083070298784e-10(x22)^3
6.62369556691106e-13(x22)^2 +
-0.5171462644511157(x31)^9 +
                              0.00021171476735298736(x31)^8
                                                                 6.553817761885988e-08(x31)^7
2.111482606067017e-11(x31)^6 - 5.018276929908661e-15(x31)^5 +
                                                                0.008530747811915616(x32)^7
45.04837247843453(x32)^9
                               0.8768534858425531(x32)^8
4.659701745905531e-05(x32)^6 +
                               1.5278584730055276e-07(x32)^5 -
                                                                3.1026273906335033e-10(x32)<sup>4</sup>
                                                                                              +
3.9086127574394853e-13(x32)^3 +
                               0.521559951698434(x33)^8
                                                                0.024578969441882217(x33)^7
4.573680695776743(x33)^9
                                                           +
0.0005922359836085081(x33)^6 +
                                7.932308648794978e-06(x33)^5 - 6.250652555818357e-08(x33)^4
                                                                                              +
-26182.613937
(Ф3) трансформированный денормированный:
190.00404951898813(x11)^7
                                3.8798759735340322(x11)^6
                                                                 0.041638613167215094(x11)^5
0.00025947403486339184(x11)^4 + 9.723824211158121e-07(x11)^3 -
                                                                2.1604871098340514e-09(x11)<sup>2</sup>
2.6211938641988632e-12(x11) - 1.3370338275963192e-15 +
-24.66271959535307(x12)^7
                                0.6541056178798178(x12)^6
                                                                0.008356265229455014(x12)^5
5.7928391754610155e-05(x12)^4 -
                                2.2957673919400212e-07(x12)^3 +
                                                                5.200250666118661e-10(x12)^2
6.263420972810898e-13(x12) +
1.6709675606205754(x21)^8
                               0.02490817333399907(x21)^7
                                                               0.0016886746314248016(x21)^6
2.710855621998293e-05(x21)^5 -
                                1.9727335904124013e-07(x21)^4 +
                                                                 7.196805123697831e-10(x21)<sup>3</sup>
1.2501715290418767e-12(x21)^2 +
-18.705186703901436(x22)^8
                                0.44422432765639663(x22)^7
                                                                0.004897073545991418(x22)^6
2.9275722474557048e-05(x22)^5 -
                                1.0233911674150614e-07(x22)^4 +
                                                                2.1516167085412892e-10(x22)<sup>3</sup>
2.6771487878065965e-13(x22)^2 +
0.4390393478500424(x31)^9
                              0.0005561055731529721(x31)^8
                                                                3.0889885629711803e-07(x31)^7
9.26151728039998e-11(x31)^6 + 1.656565513221482e-14(x31)^5 +
-6.1109034866200425(x32)^9
                                0.1503465632902456(x32)^8
                                                               0.0018574834320967371(x32)^7
1.2163706389281536e-05(x32)^6
                                4.601976216558418e-08(x32)^5
                                                                 1.062250977603846e-10(x32)^4
1.5218829725126513e-13(x32)^3 +
14.649064819654553(x33)^9
                                1.2709898723180324(x33)^8
                                                           +
                                                                0.048745627701920875(x33)^7
0.000987852551009717(x33)^6 +
                               1.1702444440747316e-05(x33)^5
                                                              - 8.50889105329792e-08(x33)^4
                                                                                              +
-2775.17963982
(Ф4) трансформированный денормированный:
765.408644971413(x11)^7
                               11.773390344103987(x11)^6
                                                                 0.09663440335266645(x11)^5
0.0004650645106541401(x11)^4
                                 1.348468111781951e-06(x11)<sup>3</sup>
                                                                 2.304491257934846e-09(x11)^2
2.1205593168802336e-12(x11) +
-52.493137275438585(x12)^7
                                                                0.020178241295985593(x12)^5
                                 1.466352247818114(x12)^6
0.00015179908779571136(x12)^4
                                6.509427598699939e-07(x12)^3 +
                                                                1.5839869945559074e-09(x12)^2
2.0330962950879735e-12(x12) + 1.067899501533055e-15 +
151.7196777753417(x21)^8
                                6.205038265485021(x21)^7
                                                                  0.1287721524507929(x21)^6
0.0015064083633963476(x21)^5 +
                               1.0502689771995762e-05(x21)^4 -
                                                                4.4489415689222575e-08(x21)<sup>3</sup>
1.1219279757699921e-10(x21)^2 - 1.547719148922098e-13(x21) +
```

Тестування на власній вибірці.

Була обрана вибірка за сайту: https://archive.ics.uci.edu

За допомогою даної вибірки ми спробували встановити залежність між цінами на житло в різних частинах Америки в залежності від таких факторів, як: середній дохід мешканця, якість навколишнього середовища та 10 інших критерієв.

Дана вибірка була створена Harrison, D. and Rubinfeld, D.L.

При виконанні роботи 'Hedonic prices and the demand for clean air' в 1978 році.

Опис данних:

Виборка має 12 змінних:

- 1. Рівень злочинів у місті
- 2. Кількісь парків тощо
- 3. Пропорція не рентальної землі в місті.
- 4. Концентрація оксидів натрію.
- 5. Середня кількість кімнат у житлі
- 6. Пропорція будинків, що ϵ заселеними з1940
- 7. Відстань до робочих центрів
- 8. Індекс доступності транспортних розв'язків
- 9. Кількість зтягнених налогів
- 10. Співвідношення учнів до вчителів
- 11. Індекс благополучча міста
- 12. Процентне співвідношення бідних верств населення

Та 1 вектор вихідних данних

1. Власне, середня вартість житла

Розмірність вибірки – 100 зразків

Для того, щоб більш наочно продемонструвати можливості алгоритма, який запропонований у лабораторній роботі, ми розбили параметри наступним чином:

Множині змінних X1 (показники, які можливо контролювати при проектуванні або виборі житла) було віднесено:

5. Середня кількість кімнат у житлі

Множині змінних X2 (зовнішні фактори, які можливо контролювати, чи ті які не змінюються з часом) ми віднесли наступні:

- 2. Кількісь парків тощо
- 3. Пропорція не рентальної землі в місті.
- 6. Пропорція будинків, що ϵ заселеними з1940
- 7. Відстань до робочих центрів
- 8. Індекс доступності транспортних розв'язків

Множині змінних X3 (зовнішні фактори, які неможливо контролювати або передбачити) ми віднесли наступні:

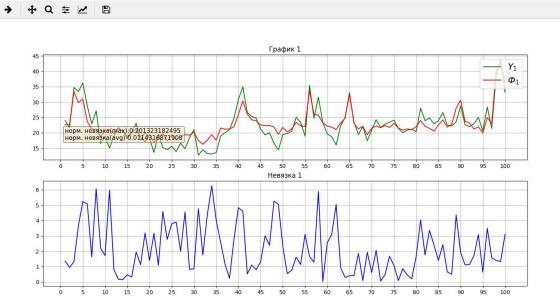
- 1. Рівень злочинів у місті
- 4. Концентрація оксидів натрію.
- 9. Кількість зтягнених налогів
- 10. Співвідношення учнів до вчителів
- 11. Індекс благополучча міста
- 12. Процентне співвідношення бідних верств населення

Вихідні параметри:

1. Власне, середня вартість житла

Для відновлення функціональної залежності найкраще себе показав поліном ступенів 4 8 9.

Отже, результати для роботи з поліномами Чебишева порядку 4 8 9:



Висновок

Отже, в нашій роботі була розв'язана задача пошуку функціональної залежності у вигляді узагальнених многочленів, і як критерій пошуку використовувався Чебишевський критерій наближення системи. Згідно з теоремою Вейерштрасса, зі збільшенням степені полінома, наближення моделі повинно покращуватися, але як ми виявили (на власній вибірці) — зі збільшенням від оптимального значення похибка зростатиме.

Для нашої моделі многочлен порядку 7 7 4 для наших змінних вже дає прийнятну похибку. Для власної вибірки оптимальним виявився многочлен 8 порядку для всіх змінних.

Знайдена функціональна залежність дає змогу знайти значення функції як і в середині інтервалу області значення змінних данної вибірки (мається на увазі ті, значення Хі, що не вказані у вибірці), так і дає змогу прогнозування за межі інтервалів області значень змінних у даній вибірці.

Література:

Праці з рішення неузгоджених систем:

L. V. Vojtíšek

Отыскание наилучшего в смысле Чебышева решения несовместной системы линейных алгебраических уравнений

Aplikace matematiky, Vol. 11 (1966), No. 3, 232—237

С. И. Зуховицкий

О наилучшем в смысле П. Л. Чебышева приближении конечной системы несовместных линейных уравнений, Матем. сб., 1953, том 33(75), номер 2, 327–342

Сучасні алгоритми основані на методі спряжених градієнтів:

Van der Vorst, H. A

Iterative Krylov Methods for Large Linear systems. Cambridge University Press, Cambridge. ISBN 0-521-81828-1. (2003).

Наближення за допомогою поліномів Чебишева різного роду. Відновлення рівнянь:

Відновлення разривних функцій:

О. В. Жучко, Ю. П. Пытьев,

Восстановление функциональной зависимости

теоретико-возможностными методами, Ж. вычисл. матем. и матем. физ., 2003, том 43, номер 5, 767–783

Під час віиконання роботи була використана теорія по методу спряжених напрямків.

Методичний посібник до лабораторних робіт з курсу «Методи оптимізації» А.П. Яковлева, І.Я. Спекторський. – К.: НТУУ «КПІ» ННК «ІПСА», 2000. – 65 с.

Інтернет – ресурси. Виеористані для уточнення алгоритмів. Вікіпедія : сторінки про поліноми Лежандра, Лагера, Ерміта, Чебишева.

Відновлення функціональних залежностей для різнотипних невизначеностей:

da Silva, R.B., Bulska, E., Godlewska-Zylkiewicz, B., Hedrich, M., Majcen, N., Magnusson, B., Marincic, S., Papadakis, I., Patriarca, M., Vassileva, E., Taylor, P., Analytical measurement: measurement uncertainty and statistics; ISBN 978-92-79-23070-7, 2012

Ronald M. KAPLAN and John T. MAXWEI,L I[I, An Algorithm for Functional Uncertainty, Xerox Pale Alto Research Center.

Sergey K. Korovin, Vasily V. Fomichev, State Observers for Linear Systems with Uncertainty, ISBN 978-3-11-021812-1,2000

Лістинг програми:

Main.py

```
author = 'KA 41 1'
# coding: utf8
import sys
from PyQt5.QtCore import pyqtSlot, pyqtSignal
from PyQt5.QtGui import QTextDocument, QFont
from PyQt5.QtWidgets import QApplication, QDialog, QFileDialog, QMessageBox
from PyQt5.uic import loadUiType
from presentation import PolynomialBuilder
from solve import Solve
app = QApplication(sys.argv)
app.setApplicationName('lab2 sa')
form class, base class = loadUiType('main window.ui')
class MainWindow(QDialog, form class):
  # signals:
  input changed = pyqtSignal('QString')
  output changed = pyqtSignal('QString')
  # x1 dim changed = pyqtSignal(int)
  # x2_dim_changed = pyqtSignal(int)
  # x3_dim_changed = pyqtSignal(int)
  # x1_deg_changed = pyqtSignal(int)
  # x2_deg_changed = pyqtSignal(int)
  # x3_deg_changed = pyqtSignal(int)
  # type cheb = pyqtSignal()
  # type lege = pyqtSignal()
  # type lagg = pyqtSignal()
  # type herm = pyqtSignal()
  def init (self, *args):
     super(MainWindow, self).__init__(*args)
    # setting up ui
     self.setupUi(self)
     # other initializations
     self.dimensions = [self.x1 dim.value(), self.x2 dim.value(),
                     self.x3 dim.value(), self.y dim.value()]
     self.degrees = [self.x1_deg.value(), self.x2_deg.value(), self.x3_deg.value()]
     self.type = 'null'
```

```
if self.radio cheb1.isChecked():
     self.type = 'chebyshev1'
  elif self.radio cheb2.isChecked():
     self.type = 'chebyshev2'
  elif self.radio legend.isChecked():
     self.type = 'legendre'
  elif self.radio lagg.isChecked():
     self.type = 'laguerre'
  elif self.radio herm.isChecked():
     self.type = 'hermit'
  self.input path = "
  self.output_path = "
  self.samples_num = self.sample_spin.value()
  self.lambda multiblock = self.lambda check.isChecked()
  self.weight_method = self.weights_box.currentText().lower()
  self.solution = None
  doc = self.results field.document()
  assert isinstance(doc, QTextDocument)
  font = doc.defaultFont()
  assert isinstance(font, QFont)
  font.setFamily('Courier New')
  font.setPixelSize(12)
  doc.setDefaultFont(font)
  return
@pyqtSlot()
def input clicked(self):
  filename = QFileDialog.getOpenFileName(self, 'Open data file', '.', 'Data file (*.txt *.dat)')[0]
  if filename == ":
     return
  if filename != self.input path:
     self.input path = filename
     self.input_changed.emit(filename)
  return
@pyqtSlot('QString')
def input modified(self, value):
  if value != self.input path:
     self.input path = value
  return
@pyqtSlot()
def output clicked(self):
  filename = QFileDialog.getSaveFileName(self, 'Save data file', '.', 'Spreadsheet (*.xlsx)')[0]
  if filename == ":
     return
  if filename != self.output path:
     self.output path = filename
     self.output changed.emit(filename)
  return
@pyqtSlot('QString')
def output_modified(self, value):
  if value != self.output_path:
     self.output path = value
  return
@pyqtSlot(int)
def samples modified(self, value):
  self.samples num = value
  return
@pyqtSlot(int)
```

```
def dimension modified(self, value):
  sender = self.sender().objectName()
  if sender == 'x1 dim':
     self.dimensions[0] = value
  elif sender == 'x2 dim':
     self.dimensions[1] = value
  elif sender == 'x3 dim':
     self.dimensions[2] = value
  elif sender == 'y dim':
     self.dimensions[3] = value
  return
@pyqtSlot(int)
def degree_modified(self, value):
  sender = self.sender().objectName()
  if sender == 'x1_deg':
     self.degrees[0] = value
  elif sender == 'x2 deg':
     self.degrees[1] = value
  elif sender == 'x3_deg':
     self.degrees[2] = value
  return
@pyqtSlot(bool)
def type modified(self, isdown):
  if (isdown):
     sender = self.sender().objectName()
     if sender == 'radio cheb1':
       self.type = 'chebyshev1'
     elif sender == 'radio cheb2':
       self.type = 'chebyshev2'
     elif sender == 'radio legend':
       self.type = 'legendre'
     elif sender == 'radio_lagg':
       self.type = 'laguerre'
     elif sender == 'radio_herm':
       self.type = 'hermit'
  return
@pyqtSlot()
def plot clicked(self):
  if self.solution:
     try:
       self.solution.plot graphs()
     except Exception as e:
       QMessageBox.warning(self, 'Error!', 'Error happened during plotting: ' + str(e))
  return
@pyqtSlot()
def exec clicked(self):
  self.exec\_button.setEnabled(False)
  try:
     solver = Solve(self.__get_params())
     solver.prepare()
     self.solution = PolynomialBuilder(solver)
     self.results field.setText(solver.show()+'\n\n'+self.solution.get results())
  except Exception as e:
     QMessageBox.warning(self,'Error!','Error happened during execution: ' + str(e))
  self.exec button.setEnabled(True)
  return
@pyqtSlot(bool)
def lambda_calc_method_changed(self, isdown):
```

```
self.lambda multiblock = isdown
     return
  @pyqtSlot('QString')
  def weights modified(self, value):
     self.weight method = value.lower()
     return
  def get params(self):
     return dict(poly type=self.type, degrees=self.degrees, dimensions=self.dimensions,
            samples=self.samples num, input file=self.input path, output file=self.output path,
            weights=self.weight_method, lambda_multiblock=self.lambda_multiblock)
form = MainWindow()
form.setWindowTitle('System Analysis - Lab 2')
form.show()
sys.exit(app.exec ())
input_data.py
__author__ = 'KA_41_1'
def read data(filename = 'data 2.txt'):
  f = open(filename, 'r')
  data = []
  for line in f:
     newline = str(line)
     data.append([float(i) for i in newline.split()])
  f.close()
  return data
package sample;
basis_geperator.py
from numpy.polynomial import Polynomial as pm
author = 'KA 41 1'
def basis sh chebyshev1(degree):
  basis = [pm([-1, 2]), pm([1])]
  for i in range(degree):
     basis.append(pm([-2, 4])*basis[-1] - basis[-2])
  del basis[0]
  return basis
def basis sh chebyshev2(degree):
  basis = [pm([1]), pm([-2, 4])]
  for i in range(degree):
     basis.append(pm([-2, 4])*basis[-1] - basis[-2])
  del basis[0]
  return basis
def basis sh legendre(degree):
  basis = [pm([1])]
  for i in range(degree):
    if i == 0:
       basis.append(pm([-1, 2]))
       continue
     basis.append((pm([-2*i - 1, 4*i + 2])*basis[-1] - i*basis[-2]) / (i + 1))
```

```
return basis
```

```
def basis hermite(degree):
  basis = [pm([0]), pm([1])]
  for i in range(degree):
     basis.append(pm([0,2])*basis[-1] - 2 * i * basis[-2])
  del basis[0]
  return basis
def basis_laguerre(degree):
  basis = [pm([1])]
  for i in range(degree):
     if i == 0:
       basis.append(pm([1, -1]))
        continue
     basis.append(pm([2*i+1,-1])*basis[-1] - i * i * basis[-2])
  return basis
choose_p.py
  author_ = 'KA_41_1'
from lab 2.solve import *
a= Solve({'samples':50, 'input file': 'data 2.txt', 'dimensions': [3, 1, 2, 2], 'output file': ", 'degrees': [3, 3, 3],
   'lambda multiblock': False, 'weights': 'average', 'poly type': 'hermit'})
#a= Solve({'samples': 100, 'input_file': 'data_2_our_sample.txt', 'dimensions': [1, 2, 1, 1], 'output_file': ", 'degrees': [3,
   'lambda_multiblock': False, 'weights': 'average', 'poly_type': 'hermit'})
a.define data()
a.norm_data()
a.define norm vectors()
a.built B()
a.poly_func()
def test p(a,p1,p2,p3):
  d = list()
  \#d = dict()
  for i in range(1,p1):
     for j in range(1,p2):
        for k in range(1,p3):
          a.p = [i+1,j+1,k+1]
          print(a.p)
          a.built_A()
          a.lamb()
          a.psi()
          a.built a()
          a.built Fi()
          a.built c()
          a.built F()
          a.built F ()
          \#d[str(i)+''+str(j)+''+str(k)] = [np.linalg.norm(a.F-a.Y), np.std(a.F_-a.Y_-, axis=0), \
                        np.linalg.norm(a.F_ - a.Y_)]
          d.append((str(i)+' '+str(j)+' '+str(k),np.linalg.norm(a.norm error)))
  return d
d = test p(a, 15, 15, 15)
f = open('test p.txt', 'w')
miner = d[0]
for i in d:
  f.write(str(i[0])+': '+str(i[1]))
  f.write('\n')
```

```
if i[1] < miner[1]:
     miner = i
print(miner)
debug.py
  author = 'KA 41 1'
from lab 2.solve import *
a= Solve({'samples': 50, 'input_file': 'data_2.txt', 'dimensions': [3, 1, 2, 2], 'output_file': 'data_611_average.xlsx',
'degrees': [3, 3, 3],
   'lambda_multiblock': False, 'weights': 'average', 'poly_type': 'laguerre'})
a.define_data()
a.norm data()
a.define_norm_vectors()
a.built_B()
a.poly func()
\#i,j,k = 2,15,1
\#i,j,k = 6,1,1 \# best for data 2.txt
i,j,k = 6,1,1
a.p = [i+1,j+1,k+1]
a.built_A()
a.lamb()
a.psi()
a.built a()
a.built Fi()
a.built c()
a.built F()
a.built_F_()
#a.save_to_file()
print(str(i)+' '+str(j)+' '+str(k),a.norm_error,np.linalg.norm(a.norm_error))
presentation.py
import numpy as np
import matplotlib.pyplot as plt
from os import name as os name
from solve import Solve
import basis_generator as b_gen
from show polynomial import Polynom
  _author__ = 'vlad'
class PolynomialBuilder(object):
  def init (self, solution):
     assert isinstance(solution, Solve)
     self._solution = solution
     max_degree = max(solution.p) - 1
     if solution.poly_type == 'chebyshev1':
       self.symbol = 'T'
       self.basis = b gen.basis sh chebyshev1(max degree)
     if solution.poly type == 'chebyshev2':
       self.symbol = 'U'
       self.basis = b gen.basis sh chebyshev2(max degree)
     elif solution.poly type == 'legendre':
       self.symbol = 'P'
       self.basis = b gen.basis sh legendre(max degree)
     elif solution.poly_type == 'laguerre':
       self.symbol = 'L'
```

```
self.basis = b gen.basis laguerre(max degree)
  elif solution.poly type == 'hermit':
     self.symbol = 'H'
     self.basis = b_gen.basis_hermite(max_degree)
  self.a = solution.a.T.tolist()
  self.c = solution.c.T.tolist()
  self.minX = [X.min(axis=0).getA1() for X in solution.X]
  self.maxX = [X.max(axis=0).getA1() for X in solution.X]
  self.minY = solution.Y .min(axis=0).getA1()
  self.maxY = solution.Y_.max(axis=0).getA1()
def _form_lamb_lists(self):
  Generates specific basis coefficients for Psi functions
  self.psi = list()
  for i in range(self. solution.Y.shape[1]): # 'i' is an index for Y
     psi i = list()
     shift = 0
     for j in range(3): # 'j' is an index to choose vector from X
       psi i j = list()
       for k in range(self. solution.deg[j]): # 'k' is an index for vector component
          psi i jk = self. solution.Lamb[shift:shift + self. solution.p[j], i].getA1()
          shift += self. solution.p[j]
          psi_i_j.append(psi_i_jk)
       psi i.append(psi i j)
     self.psi.append(psi i)
def transform to standard(self, coeffs):
  Transforms special polynomial to standard
  :param coeffs: coefficients of special polynomial
  :return: coefficients of standard polynomial
  std coeffs = np.zeros(coeffs.shape)
  for index in range(coeffs.shape[0]):
     cp = self.basis[index].coef.copy()
     cp.resize(coeffs.shape)
     std coeffs += coeffs[index] * cp
  return std coeffs
def _print_psi_i_jk(self, i, j, k):
  Returns string of Psi function in special polynomial form
  :param i: an index for Y
  :param j: an index to choose vector from X
  :param k: an index for vector component
  :return: result string
  strings = list()
  for n in range(len(self.psi[i][j][k])):
     strings.append(\{0:.6f\}*{symbol} {deg}(x{1}{2})'.format(self.psi[i][j][k][n], j + 1, k + 1,
                                        symbol=self.symbol, deg=n))
  return ' + '.join(strings)
def _print_phi_i_j(self, i, j):
  Returns string of Phi function in special polynomial form
  :param i: an index for Y
  :param j: an index to choose vector from X
  :return: result string
  strings = list()
```

```
for k in range(len(self.psi[i][j])):
     shift = sum(self. solution.deg[:j]) + k
     for n in range(len(self.psi[i][j][k])):
       strings.append('\{0:.6f\}*\{symbol\}\{deg\}(x\{1\}\{2\})'.format(self.a[i][shift]*self.psi[i][j][k][n],
                                           j + 1, k + 1, symbol=self.symbol, deg=n))
  return ' + '.join(strings)
def _print_F_i(self, i):
  Returns string of F function in special polynomial form
  :param i: an index for Y
  :return: result string
  strings = list()
  for j in range(3):
     for k in range(len(self.psi[i][j])):
       shift = sum(self. solution.deg[:j]) + k
        for n in range(len(self.psi[i][j][k])):
          strings.append(\{0:.6f\}*{symbol}{deg}(x{1}{2})'.format(self.c[i][j] * self.a[i][shift] *
                                              self.psi[i][j][k][n],
                                              j + 1, k + 1, symbol=self.symbol, deg=n))
  return ' + '.join(strings)
def _print_F_i_transformed_denormed(self, i):
  Returns string of F function in special polynomial form
  :param i: an index for Y
  :return: result string
  strings = list()
  constant = 0
  for j in range(3):
     for k in range(len(self.psi[i][j])):
        shift = sum(self. solution.deg[:j]) + k
       raw\_coeffs = self\_transform\_to\_standard(self.c[i][j] * self.a[i][shift] * self.psi[i][j][k])
       diff = self.maxX[j][k] - self.minX[j][k]
       mult poly = np.poly1d([1 / diff, - self.minX[j][k]] / diff)
       add poly = np.poly1d([1])
        current poly = np.poly1d([0])
        for n in range(len(raw coeffs)):
          current poly += add poly * raw coeffs[n]
          add poly *= mult_poly
       current poly = current poly * (self.maxY[i] - self.minY[i]) + self.minY[i]
       constant += current poly[0]
       current_poly[0] = 0
       current poly = np.poly1d(current poly.coeffs, variable='(x{0}{1})'.format(j + 1, k + 1))
        strings.append(str( Polynom(current poly, (x\{0\}\{1\})).format(j + 1, k + 1))))
  strings.append(str(constant))
  return '+\n'.join(strings)
def _print_F_i_transformed(self, i):
  Returns string of F function in special polynomial form
  :param i: an index for Y
  :return: result string
  strings = list()
  constant = 0
  for i in range(3):
     for k in range(len(self.psi[i][j])):
       shift = sum(self. solution.deg[:j]) + k
       current_poly = np.poly1d(self._transform_to_standard(self.c[i][j] * self.a[i][shift] *
                                          self.psi[i][j][k])[::-1],
```

```
variable=(x\{0\}\{1\})'.format(j + 1, k + 1)
          constant += current poly[0]
          current poly[0] = 0
          strings.append(str( Polynom(current poly, (x\{0\}\{1\})).format(j+1, k+1))))
     strings.append(str(constant))
     return '+\n'.join(strings)
  def get_results(self):
     Generates results based on given solution
     :return: Results string
     self._form_lamb_lists()
     psi\_strings = ['(Psi\{1\}\{2\})[\{0\}] = \{result\} \land i.format(i+1, j+1, k+1, result=self\_print\_psi\_i\_jk(i, j, k))
               for i in range(self._solution.Y.shape[1])
               for j in range(3)
               for k in range(self. solution.deg[j])]
     phi strings = \lceil (Phi\{1\}) \lceil \{0\} \rceil = \{result\} \setminus (i+1, j+1, result = self. print phi i j(i, j))
               for i in range(self. solution.Y.shape[1])
               for j in range(3)]
     f strings = ['(F\{0\})=\{result\} \setminus n'.format(i+1, result=self. print F i(i))
             for i in range(self. solution.Y.shape[1])]
     f strings transformed
                                           ['(\Phi\{0\})]
                                                          трансформированный:\n{result}\n'.format(i
                                                                                                                         1.
result=self._print_F_i_transformed(i))
                     for i in range(self._solution.Y.shape[1])]
     f strings transformed denormed = ['(\Phi\{0\})] трансформированный '\
                           'денормированный:\n{\text{result}}\n'.\text{format}(i+1, \text{result}=
     self. print F i transformed denormed(i))
                           for i in range(self. solution.Y.shape[1])]
     return '\n'.join(psi strings + phi strings + f strings + f strings transformed + f strings transformed denormed)
  def plot graphs(self):
     fig, axes = plt.subplots(2, self._solution.Y.shape[1])
     if self. solution.Y.shape[1] == 1:
       axes[0] = [axes[0]]
       axes[1] = [axes[1]]
     for index in range(self. solution.Y.shape[1]):
       ax = axes[0][index] # real and estimated graphs
       norm ax = axes[1][index] # abs residual graph
       ax.set xticks(np.arange(0, self. solution.n + 1, 5))
       ax.plot(np.arange(1, self. solution.n + 1), self. solution.Y [:, index],
             'g-', label='$Y_{0}$'.format(index + 1))
       ax.plot(np.arange(1, self. solution.n + 1), self. solution.F [:, index],
             'r-', label='\Phi \{0\}'.format(index + 1))
       ax.legend(loc='upper right', fontsize=16)
       ax.set title('График {0}'.format(index + 1))
       ax.grid()
       norm ax.set xticks(np.arange(0, self. solution.n + 1, 5))
       norm ax.plot(np.arange(1, self. solution.n + 1),
                abs(self._solution.Y_[:, index] - self._solution.F_[:, index]), 'k-')
       norm ax.set title('Heвязка {0}'.format(index + 1))
       norm ax.grid()
     manager = plt.get current fig manager()
     manager.set window title('Graph')
     if os name == 'posix':
       fig.show()
     else:
       plt.show()
```

```
class Polynom(object):
  def _init_(self, ar, symbol = 'x', eps = 1e-15):
     self.ar = ar
     self.symbol = symbol
     self.eps = eps
  def repr (self):
     #joinder[first, negative] = str
    joiner = {
       (True, True):'-',
       (True, False): ",
       (False, True): ' - ',
       (False, False): '+'
     result = []
     for deg, coef in reversed(list(enumerate(self.ar))):
       sign = joiner[not result, coef < 0]
       coef = abs(coef)
       if coef == 1 and deg != 0:
          coef = "
       if coef < self.eps:
          continue
       f = \{0: '\{\}\}\}', 1: '\{\}\}\}' + self.symbol\}.get(deg, '\{\}\}\}' + self.symbol + '^{\{\}'})
       result.append(f.format(sign, coef, deg))
     return ".join(result) or '0'
\#s = Polynom([3,4,0,5,0,12], 'X')._repr_()
#print(s+s)
Solve.py
  author = 'KA_41_1'
from copy import deepcopy
from scipy import special
from openpyxl import Workbook
from system solve import *
from tabulate import tabulate as tb
class Solve(object):
  def init (self,d):
     self.n = d['samples']
     self.deg = d['dimensions']
     self.filename input = d['input file']
     self.filename output = d['output file']
     self.dict = d['output file']
     self.p = list(map(lambda x:x+1,d['degrees'])) # on 1 more because include 0
     self.weights = d['weights']
     self.poly_type = d['poly_type']
     self.splitted lambdas = d['lambda multiblock']
     self.eps = 1E-6
     self.norm error=0.0
     self.error=0.0
  def define data(self):
     f = open(self.filename input, 'r')
     # all data from file input in float
     self.datas = np.matrix([list(map(lambda x:float(x),f.readline().split()))) for i in range(self.n)])
```

```
# list of sum degrees [3,1,2] \rightarrow [3,4,6]
  self.degf = [sum(self.deg[:i+1]) for i in range(len(self.deg))]
def _minimize_equation(self, A, b, type='cjg'):
  Finds such vector x that |Ax-b|->min.
  :param A: Matrix A
  :param b: Vector b
  :return: Vector x
  if type == 'lsq':
     return np.linalg.lstsq(A,b)[0]
  elif type == 'cjg':
     return conjugate_gradient_method(A.T*A, A.T*b, self.eps)
def norm_data(self):
  norm vectors value to value in [0,1]
  :return: float number in [0,1]
  n,m = self.datas.shape
  vec = np.ndarray(shape=(n,m),dtype=float)
  for j in range(m):
     minv = np.min(self.datas[:,j])
     maxv = np.max(self.datas[:,j])
     for i in range(n):
        vec[i,j] = (self.datas[i,j] - minv)/(maxv - minv)
  self.data = np.matrix(vec)
def define_norm_vectors(self):
  build matrix X and Y
  :return:
  X1 = self.data[:, :self.degf[0]]
  X2 = self.data[:, self.degf[0]:self.degf[1]]
  X3 = self.data[:, self.degf[1]:self.degf[2]]
  #matrix of vectors i.e.X = [[X11,X12],[X21],...]
  self.X = [X1, X2, X3]
  #number columns in matrix X
  self.mX = self.degf[2]
  # matrix, that consists of i.e. Y1,Y2
  self.Y = self.data[:, self.degf[2]:self.degf[3]]
  self.Y_ = self.datas[:, self.degf[2]:self.degf[3]]
  self.X_=[self.datas[:,:self.degf[0]], self.datas[:,self.degf[0]:self.degf[1]],
         self.datas[:, self.degf[1]:self.degf[2]]]
def built B(self):
  def B average():
     Vector B as avarage of max and min in Y. B[i] =max Y[i,:]
     :return:
     b = \text{np.tile}((\text{self.Y.max}(\text{axis}=1) + \text{self.Y.min}(\text{axis}=1))/2, (1, \text{self.deg}[3]))
     return b
  def B scaled():
     Vector B = Y
     :return:
     return deepcopy(self.Y)
```

```
if self.weights == 'average':
     self.B = B average()
  elif self.weights == 'scaled':
     self.B = B_scaled()
  else:
     exit('B not definded')
def poly_func(self):
  Define function to polynoms
  :return: function
  if self.poly_type =='chebyshev1':
     self.poly_f = special.eval_sh\_chebyt
  elif self.poly_type =='chebyshev2':
     self.poly_f = special.eval_sh_chebyu
  elif self.poly type == 'legendre':
     self.poly f = special.eval sh legendre
  elif self.poly_type == 'laguerre':
     self.poly_f = special.eval_laguerre
  elif self.poly_type == 'hermit':
    self.poly f = \text{special.eval} hermite
def built_A(self):
  built matrix A on shifted polynomys Chebysheva
  :param self.p:mas of deg for vector X1,X2,X3 i.e.
  :param self.X: it is matrix that has vectors X1 - X3 for example
  :return: matrix A as ndarray
  def mA():
     :param X: [X1, X2, X3]
     :param p: [p1,p2,p3]
     :return: m = m1*p1+m2*p2+...
     m = 0
     for i in range(len(self.X)):
       m+= self.X[i].shape[1]*(self.p[i]+1)
    return m
  def coordinate(v,deg):
     :param v: vector
     :param deg: chebyshev degree polynom
     :return:column with chebyshev value of coordiate vector
     c = np.ndarray(shape=(self.n,1), dtype = float)
     for i in range(self.n):
       c[i,0] = self.poly_f(deg, v[i])
    return c
  def vector(vec, p):
     :param vec: it is X that consist of X11, X12, ... vectors
     :param p: max degree for chebyshev polynom
     :return: part of matrix A for vector X1
     n, m = vec.shape
     a = np.ndarray(shape=(n,0),dtype = float)
     for j in range(m):
       for i in range(p):
```

```
ch = coordinate(vec[:,j],i)
          a = np.append(a, ch, 1)
     return a
  \#k = mA()
  A = np.ndarray(shape = (self.n,0),dtype = float)
  for i in range(len(self.X)):
     vec = vector(self.X[i],self.p[i])
     A = np.append(A, vec, 1)
  self.A = np.matrix(A)
def lamb(self):
  lamb = np.ndarray(shape = (self.A.shape[1],0), dtype = float)
  for i in range(self.deg[3]):
     if self.splitted_lambdas:
       boundary_1 = self.p[0] * self.deg[0]
       boundary 2 = self.p[1] * self.deg[1] + boundary 1
       lamb1 = self. minimize equation(self.A[:, :boundary 1], self.B[:, i])
       lamb2 = self._minimize_equation(self.A[:, boundary_1:boundary_2], self.B[:, i])
       lamb3 = self. minimize equation(self.A[:, boundary 2:], self.B[:, i])
       lamb = np.append(lamb, np.concatenate((lamb1, lamb2, lamb3)), axis=1)
     else:
       lamb = np.append(lamb, self. minimize equation(self.A, self.B[:, i]), axis=1)
  self.Lamb = np.matrix(lamb) #Lamb in full events
def psi(self):
  def built psi(lamb):
     return matrix xi1 for b1 as matrix
     :param A:
     :param lamb:
     :param p:
     :return: matrix psi, for each Y
     psi = np.ndarray(shape=(self.n, self.mX), dtype = float)
     q = 0 #iterator in lamb and A
     1 = 0 #iterator in columns psi
     for k in range(len(self.X)): # choose X1 or X2 or X3
       for s in range(self.X[k].shape[1]):# choose X11 or X12 or X13
          for i in range(self.X[k].shape[0]):
               psi[i,l] = self.A[i,q:q+self.p[k]]*lamb[q:q+self.p[k], 0]
          q = self.p[k]
          1+=1
     return np.matrix(psi)
  self.Psi = [] #as list because psi[i] is matrix(not vector)
  for i in range(self.deg[3]):
     self.Psi.append(built psi(self.Lamb[:,i]))
def built a(self):
  self.a = np.ndarray(shape=(self.mX,0), dtype=float)
  for i in range(self.deg[3]):
     a1 = self._minimize_equation(self.Psi[i][:, :self.degf[0]], self.Y[:, i])
     a2 = self._minimize_equation(self.Psi[i][:, self.degf[0]:self.degf[1]], self.Y[:, i])
     a3 = self. minimize equation(self.Psi[i][:, self.degf[1]:], self.Y[:, i])
     # temp = self. minimize equation(self.Psi[i], self.Y[:, i])
     \# self.a = np.append(self.a, temp, axis=1)
     self.a = np.append(self.a, np.vstack((a1, a2, a3)),axis = 1)
def built Fli(self, psi, a):
     not use; it used in next function
     :param psi: matrix psi (only one
```

```
:param a: vector with shape = (6,1)
     :param degf: = [3,4,6]//fibonachi of deg
     :return: matrix of (three) components with F1 F2 and F3
     m = len(self.X) \# m = 3
     F1i = np.ndarray(shape = (self.n,m),dtype = float)
     k = 0 #point of begining columnt to multipy
     for j in range(m): \# 0 - 2
       for i in range(self.n): #0 - 49
          F1i[i,j] = psi[i,k:self.degf[j]]*a[k:self.degf[j],0]
       k = self.degf[j]
     return np.matrix(F1i)
def built Fi(self):
  self.Fi = []
  for i in range(self.deg[3]):
     self.Fi.append(self.built F1i(self.Psi[i],self.a[:,i]))
def built c(self):
  self.c = np.ndarray(shape = (len(self.X), 0), dtype = float)
  for i in range(self.deg[3]):
     self.c = np.append(self.c, conjugate gradient method(self.Fi[i].T*self.Fi[i], self.Fi[i].T*self.Y[:,i],self.eps),\
             axis = 1)
def built F(self):
  F = np.ndarray(self.Y.shape, dtype = float)
  for j in range(F.shape[1]):#2
     for i in range(F.shape[0]): #50
       F[i,j] = self.Fi[j][i,:]*self.c[:,j]
  self.F = np.matrix(F)
  self.norm_error = []
  for i in range(self.Y.shape[1]):
     self.norm_error.append(np.linalg.norm(self.Y[:,i] - self.F[:,i],np.inf))
def built_F_(self):
  minY = self.Y\_.min(axis=0)
  maxY = self.Y .max(axis=0)
  self.F = np.multiply(self.F,maxY - minY) + minY
  self.error = []
  for i in range(self.Y .shape[1]):
     self.error.append(np.linalg.norm(self.Y [:,i] - self.F [:,i],np.inf))
def save to file(self):
  wb = Workbook()
  #get active worksheet
  ws = wb.active
  1 = [None]
  ws.append(['Введенные данные: X'])
  for i in range(self.n):
      ws.append(l+self.datas[i,:self.degf[3]].tolist()[0])
  ws.append([])
  ws.append(['Введенные данные: Y'])
  for i in range(self.n):
      ws.append(l+self.datas[i,self.degf[2]:self.degf[3]].tolist()[0])
  ws.append([])
  ws.append(['X нормализованные:'])
  for i in range(self.n):
     ws.append(l+self.data[i,:self.degf[2]].tolist()[0])
  ws.append([])
```

```
ws.append(['Y нормализованные:'])
for i in range(self.n):
   ws.append(l+self.data[i,self.degf[2]:self.degf[3]].tolist()[0])
ws.append([])
ws.append(['matp B:'])
for i in range(self.n):
   ws.append(l+self.B[i].tolist()[0])
ws.append([])
ws.append(['матр A:'])
for i in range(self.A.shape[0]):
   ws.append(l+self.A[i].tolist()[0])
ws.append([])
ws.append(['matp Lambda:'])
for i in range(self.Lamb.shape[0]):
   ws.append(l+self.Lamb[i].tolist()[0])
ws.append([])
for j in range(len(self.Psi)):
   s = 'Matp Psi\%i:' \%(j+1)
   ws.append([s])
   for i in range(self.n):
      ws.append(l+self.Psi[j][i].tolist()[0])
   ws.append([])
ws.append(['матр a:'])
for i in range(self.mX):
   ws.append(l+self.a[i].tolist()[0])
ws.append([])
for j in range(len(self.Fi)):
   s = 'Marp \Phi\%i:' \%(j+1)
   ws.append([s])
   for i in range(self.Fi[i].shape[0]):
      ws.append(l+self.Fi[j][i].tolist()[0])
   ws.append([])
ws.append(['матр c:'])
for i in range(len(self.X)):
   ws.append(l+self.c[i].tolist()[0])
ws.append([])
ws.append(['Y перестроенное нормализованное :'])
for i in range(self.n):
   ws.append(l+self.F[i].tolist()[0])
ws.append([])
ws.append(['Y перестроенное :'])
for i in range(self.n):
   ws.append(l+self.F_[i].tolist()[0])
ws.append([])
ws.append(['Нормализовная невязка (Y -Ф)'])
ws.append(l + self.norm error)
ws.append(['невязка (Y_ - Ф_))'])
ws.append(l+self.error)
wb.save(self.filename_output)
```

```
def show(self):
  text = []
  text.append('Вводные данные: X')
  text.append(tb(np.array(self.datas[:, :self.degf[2]])))
  text.append('\nВводные данные: Y')
  text.append(tb(np.array(self.datas[:,self.degf[2]:self.degf[3]])))
  text.append('\nX нормализованный:')
  text.append(tb(np.array(self.data[:,:self.degf[2]])))
  text.append('\nY нормализованный:')
  text.append(tb(np.array(self.data[:,self.degf[2]:self.degf[3]])))
  text.append('\nматрица В:')
  text.append(tb(np.array(self.B)))
  text.append('\nматрица A:')
  text.append(tb(np.array(self.A)))
  text.append('\nматрица Lambda:')
  text.append(tb(np.array(self.Lamb)))
  for j in range(len(self.Psi)):
     s = ' n m a т p и ц a P s i % i :' % (j+1)
     text.append(s)
     text.append(tb(np.array(self.Psi[j])))
  text.append('\nматрица a:')
  text.append(tb(self.a.tolist()))
  for j in range(len(self.Fi)):
     s = \normalfont{'}nматрица Ф%i:' %(j+1)
     text.append(s)
     text.append(tb(np.array(self.Fi[i])))
  text.append('\nматрица c:')
  text.append(tb(np.array(self.c)))
  text.append('\nY перестроенное нормализованное :')
  text.append(tb(np.array(self.F)))
  text.append('\nY перестроенное :')
  text.append(tb(self.F .tolist()))
  text.append('\nHормализованная невязка (Y - Φ)')
  text.append(tb([self.norm error]))
  text.append('\nНевязка (Y_ - \Phi_))')
  text.append(tb([self.error]))
  return '\n'.join(text)
def prepare(self):
  self.define data()
  self.norm data()
  self.define norm vectors()
  self.built B()
  self.poly func()
  self.built A()
  self.lamb()
```

```
self.psi()
    self.built a()
    self.built Fi()
    self.built_c()
    self.built F()
    self.built F ()
    self.save to file()
system_solve.py
 _{author} = 'KA_{41_{1}}'
import numpy as np
def conjugate_gradient_method(A, b, eps):
  Conjugate Gradient Method that solve equation Ax = b with given accuracy
  :param A:matrix A
  :param b:vector b
  :param eps: accuracy
  :return: solution x
  n = len(A.T) # number column
  xi1 = xi = np.zeros(shape=(n,1), dtype = float)
  vi = ri = b \# start condition
  i = 0 #loop for number iteration
  while True:
    try:
       i+=1
       ai = float(vi.T*ri)/float(vi.T*A*vi) # alpha i
       xi1 = xi+ai*vi # x i+1
       ri1 = ri-ai*A*vi # r i+1
       betai = -float(vi.T*A*ri1)/float(vi.T*A*vi) # beta i
       vi1 = ri1 + betai*vi
       if (np.linalg.norm(ri1) \le eps) or i \ge 10 * n:
         break
       else:
         xi,vi,ri = xi1,vi1,ri1
    except Exception:
       print("problem with minimization")
  return np.matrix(xi1)
main window.ui
<?xml version="1.0" encoding="UTF-8"?>
<ui version="4.0">
<class>Form</class>
<widget class="QWidget" name="Form">
 cproperty name="geometry">
 <rect>
  < x > 0 < /x >
  <y>0</y>
  <width>650</width>
  <height>733</height>
 </rect>
 </property>
 property name="windowTitle">
 <string>Form1</string>
 <layout class="QGridLayout" name="gridLayout 2">
 <item row="2" column="2">
  <widget class="QGroupBox" name="groupBox 2">
   property name="sizePolicy">
   <sizepolicy hsizetype="Fixed" vsizetype="Preferred">
```

```
<horstretch>0</horstretch>
<verstretch>0</verstretch>
</sizepolicy>
cproperty name="title">
<string>Dimensions</string>
</property>
<layout class="QGridLayout" name="gridLayout 3">
<item row="1" column="1">
<widget class="QSpinBox" name="x1 dim">
 cproperty name="value">
  <number>2</number>
 </property>
</widget>
</item>
<item row="1" column="0">
<widget class="QLabel" name="label 2">
 cproperty name="text">
  <string>X1</string>
 property name="alignment">
  <set>Qt::AlignRight|Qt::AlignTrailing|Qt::AlignVCenter</set>
 </property>
</widget>
</item>
<item row="2" column="1">
<widget class="QSpinBox" name="x2 dim">
 cproperty name="value">
 <number>2</number>
 </property>
 </widget>
</item>
<item row="2" column="0">
<widget class="QLabel" name="label 3">
 property name="text">
 <string>X2</string>
 property name="alignment">
  <set>Qt::AlignRight|Qt::AlignTrailing|Qt::AlignVCenter</set>
 </property>
</widget>
</item>
<item row="2" column="2">
<widget class="QLabel" name="label 5">
 cproperty name="text">
 <string>X3</string>
 </property>
 property name="alignment">
  <set>Qt::AlignRight|Qt::AlignTrailing|Qt::AlignVCenter</set>
 </property>
 </widget>
</item>
<item row="1" column="2">
<widget class="QLabel" name="label_4">
 property name="text">
  <string>Y</string>
 </property>
 property name="alignment">
  <set>Qt::AlignRight|Qt::AlignTrailing|Qt::AlignVCenter</set>
 </property>
</widget>
</item>
<item row="2" column="3">
```

```
<widget class="QSpinBox" name="x3 dim">
    cproperty name="value">
    <number>3</number>
    </property>
   </widget>
   </item>
   <item row="1" column="3">
   <widget class="QSpinBox" name="y dim">
    cproperty name="value">
    <number>4</number>
    </widget>
   </item>
  </layout>
  </widget>
 </item>
 <item row="2" column="3">
  <widget class="QGroupBox" name="groupBox 6">
  cproperty name="title">
   <string>Processing</string>
  </property>
  <layout class="QHBoxLayout" name="horizontalLayout 2">
   <item>
   <widget class="QPushButton" name="plot button">
    cproperty name="text">
    <string>Plot</string>
    </property>
   </widget>
   </item>
   <item>
   <widget class="QPushButton" name="exec button">
    property name="text">
    <string>Execute</string>
    </property>
    property name="default">
    <bool>true</bool>
    </property>
    property name="flat">
    <bool>false</bool>
    </property>
   </widget>
   </item>
  </layout>
  </widget>
 </item>
 <item row="6" column="0" rowspan="2" colspan="6">
  <widget class="QTextBrowser" name="results field">
  property name="lineWrapMode">
   <enum>QTextEdit::NoWrap</enum>
  property name="readOnly">
   <bool>true</bool>
  </property>
  property name="html">
   <string>&lt;!DOCTYPE
                            HTML
                                       PUBLIC
                                                    "-//W3C//DTD
                                                                          HTML
                                                                                      4.0//EN"
"http://www.w3.org/TR/REC-html40/strict.dtd">
<html&gt;&lt;head&gt;&lt;meta
                                 name="qrichtext"
                                                              content="1"
                                                                                      /><style
type="text/css">
p, li { white-space: pre-wrap; }
</style&gt;&lt;/head&gt;&lt;body style=&quot; font-family:'Ubuntu'; font-size:11pt; font-weight:400; font-
style:normal;">
```

<p style="-qt-paragraph-type:empty; margin-top:0px; margin-bottom:0px; margin-left:0px; margin-left:

```
</property>
property name="placeholderText">
 <string>Results are shown here</string>
</widget>
</item>
<item row="0" column="4">
<widget class="QGroupBox" name="groupBox 5">
cproperty name="title">
 <string>Polynomials</string>
</property>
<layout class="QHBoxLayout" name="horizontalLayout">
 <item>
  <widget class="QGroupBox" name="groupBox 4">
  property name="title">
   <string>Degreees</string>
  </property>
  <layout class="QGridLayout" name="gridLayout 4">
   <item row="0" column="0">
   <widget class="QLabel" name="label 6">
    cproperty name="text">
    <string>X1</string>
    </property>
    property name="alignment">
    <set>Qt::AlignRight|Qt::AlignTrailing|Qt::AlignVCenter</set>
    </property>
   </widget>
   </item>
   <item row="2" column="1">
   <widget class="QSpinBox" name="x3_deg">
    cproperty name="value">
    <number>4</number>
    </widget>
   </item>
   <item row="1" column="0">
   <widget class="QLabel" name="label 8">
    cproperty name="text">
    <string>X2</string>
    property name="alignment">
    <set>Qt::AlignRight|Qt::AlignTrailing|Qt::AlignVCenter</set>
    </property>
   </widget>
   </item>
   <item row="0" column="1">
   <widget class="QSpinBox" name="x1 deg">
    cproperty name="value">
    <number>7</number>
    </widget>
   </item>
   <item row="1" column="1">
   <widget class="QSpinBox" name="x2 deg">
    cproperty name="value">
    <number>7</number>
    </widget>
   </item>
   <item row="2" column="0">
```

```
<widget class="QLabel" name="label 7">
    property name="text">
    <string>X3</string>
    property name="alignment">
    <set>Qt::AlignRight|Qt::AlignTrailing|Qt::AlignVCenter</set>
    </widget>
   </item>
  </layout>
 </widget>
 </item>
</layout>
</widget>
</item>
<item row="0" column="0" colspan="3">
<widget class="QGroupBox" name="groupBox">
property name="autoFillBackground">
 <bool>false</bool>
cproperty name="title">
 <string>Data</string>
</property>
<layout class="QGridLayout" name="gridLayout 5">
 <item row="2" column="1">
 <widget class="QToolButton" name="select output">
  cproperty name="text">
   <string>...</string>
  </property>
  </widget>
 </item>
 <item row="0" column="1">
 <widget class="QSpinBox" name="sample_spin">
  property name="maximum">
  <number>999</number>
  </property>
  property name="value">
   <number>45</number>
  </property>
 </widget>
 </item>
 <item row="1" column="1">
 <widget class="QToolButton" name="select input">
  property name="text">
   <string>...</string>
  </widget>
 </item>
 <item row="1" column="0">
 <widget class="QLineEdit" name="line input">
  property name="placeholderText">
   <string>Input file</string>
  </widget>
 </item>
 <item row="0" column="0">
 <widget class="QLabel" name="label">
  cproperty name="text">
   <string>Number of samples:</string>
  </property>
 </widget>
 </item>
 <item row="2" column="0">
```

```
<widget class="QLineEdit" name="line output">
  property name="placeholderText">
   <string>Output file</string>
  </widget>
 </item>
</layout>
</widget>
</item>
<item row="0" column="3">
<widget class="QGroupBox" name="groupBox 3">
property name="title">
 <string>Types</string>
</property>
<layout class="QVBoxLayout" name="verticalLayout">
 <item>
 <widget class="QRadioButton" name="radio cheb1">
  property name="text">
   <string>Chebyshev</string>
  cproperty name="checked">
   <bool>true</bool>
  </widget>
 </item>
 <item>
 <widget class="QRadioButton" name="radio legend">
  cproperty name="text">
  <string>Legendre</string>
  </widget>
 </item>
 <item>
 <widget class="QRadioButton" name="radio lagg">
  property name="text">
  <string>Laguerre</string>
  </property>
 </widget>
 </item>
 <item>
 <widget class="QRadioButton" name="radio herm">
  cproperty name="text">
  <string>Hermit</string>
  </widget>
 </item>
 <item>
 <widget class="QRadioButton" name="radio cheb2">
  property name="text">
   <string>Chebyshev II</string>
  </property>
 </widget>
 </item>
</layout>
</widget>
</item>
<item row="2" column="4">
<widget class="QGroupBox" name="groupBox 7">
cproperty name="title">
 <string>Additional</string>
<layout class="QHBoxLayout" name="horizontalLayout_3">
 <item>
```

```
<widget class="QLabel" name="label 9">
   property name="text">
    <string>Weights:</string>
    </property>
   </widget>
  </item>
  <item>
   <widget class="QComboBox" name="weights box">
   <item>
    property name="text">
     <string>Average</string>
    </item>
    <item>
    property name="text">
     <string>Scaled</string>
    </item>
   </widget>
  </item>
  </layout>
 </widget>
 </item>
 <item row="1" column="4">
 <widget class="QCheckBox" name="lambda_check">
  property name="text">
  <string>Use 3-block lambda calculation</string>
  </widget>
 </item>
</layout>
</widget>
<tabstops>
<tabstop>radio_cheb1</tabstop>
<tabstop>radio_cheb2</tabstop>
<tabstop>radio_legend</tabstop>
<tabstop>radio lagg</tabstop>
<tabstop>radio herm</tabstop>
<tabstop>x1_deg</tabstop>
<tabstop>x2_deg</tabstop>
<\!tabstop\!\!>\!\!x3\_deg\!\!<\!\!/tabstop\!\!>
<tabstop>x1_dim</tabstop>
<tabstop>x2_dim</tabstop>
<tabstop>sample_spin</tabstop>
<tabstop>line_input</tabstop>
<tabstop>select input</tabstop>
<tabstop>line output</tabstop>
<tabstop>select output</tabstop>
<tabstop>exec button</tabstop>
<tabstop>results field</tabstop>
</tabstops>
<resources/>
<connections>
<connection>
 <sender>exec button</sender>
 <signal>clicked()</signal>
 <receiver>Form</receiver>
 <slot>exec clicked()</slot>
 <hints>
 <hint type="sourcelabel">
  < x > 379 < /x >
  <y>219</y>
 </hint>
```

```
<hint type="destinationlabel">
 < x > 309 < /x >
  <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>plot button</sender>
<signal>clicked()</signal>
<receiver>Form</receiver>
<slot>plot_clicked()</slot>
<hints>
 <hint type="sourcelabel">
  < x > 526 < /x >
  <y>219</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
  < y > 199 < /y >
 </hint>
</hints>
</connection>
<connection>
<sender>sample_spin</sender>
<signal>valueChanged(int)</signal>
<receiver>Form</receiver>
<slot>samples_modified(int)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 568 < / x >
  <y>53</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
  <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>select input</sender>
<signal>clicked()</signal>
<receiver>Form</receiver>
<slot>input clicked()</slot>
<hints>
 <hint type="sourcelabel">
  < x > 557 < /x >
  <y>95</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
  <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>select output</sender>
<signal>clicked()</signal>
<receiver>Form</receiver>
<slot>output clicked()</slot>
<hints>
 <hint type="sourcelabel">
 <x>557</x>
  <y>136</y>
```

```
</hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>x1 dim</sender>
<signal>valueChanged(int)</signal>
<receiver>Form</receiver>
<slot>dimension_modified(int)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 361 < /x >
 <y>44</y>
 </hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>x2_dim</sender>
<signal>valueChanged(int)</signal>
<receiver>Form</receiver>
<slot>dimension modified(int)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 361 < /x >
 <y>78</y>
 </hint>
 <hint type="destinationlabel">
 <x>309</x>
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>x3 dim</sender>
<signal>valueChanged(int)</signal>
<receiver>Form</receiver>
<slot>dimension modified(int)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 361 < / x >
 <y>112</y>
 </hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>x1 deg</sender>
<signal>valueChanged(int)</signal>
<receiver>Form</receiver>
<slot>degree modified(int)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 227 < /x >
```

```
<y>81</y>
 </hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>x2 deg</sender>
<signal>valueChanged(int)</signal>
<receiver>Form</receiver>
<slot>degree_modified(int)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 227 < /x >
 <y>131</y>
 </hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>x3 deg</sender>
<signal>valueChanged(int)</signal>
<receiver>Form</receiver>
<slot>degree modified(int)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 244 < /x >
 <y>133</y>
 </hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>radio cheb1</sender>
<signal>toggled(bool)</signal>
<receiver>Form</receiver>
<slot>type_modified(bool)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 86 < / x >
 <y>78</y>
 </hint>
 <hint type="destinationlabel">
 <x>309</x>
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>radio cheb2</sender>
<signal>toggled(bool)</signal>
<receiver>Form</receiver>
<slot>type modified(bool)</slot>
<hints>
 <hint type="sourcelabel">
```

```
< x > 86 < /x >
 <y>78</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
  <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>radio herm</sender>
<signal>toggled(bool)</signal>
<receiver>Form</receiver>
<slot>type_modified(bool)</slot>
<hints>
 <hint type="sourcelabel">
  < x > 86 < / x >
 <y>186</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>radio lagg</sender>
<signal>toggled(bool)</signal>
<receiver>Form</receiver>
<slot>type_modified(bool)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 86 < /x >
  <y>150</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>radio legend</sender>
<signal>toggled(bool)</signal>
<receiver>Form</receiver>
<slot>type modified(bool)</slot>
<hints>
 <hint type="sourcelabel">
  < x > 86 < /x >
  <y>114</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
  <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>Form</sender>
<signal>output changed(QString)</signal>
<receiver>line output</receiver>
<slot>setText(QString)</slot>
<hints>
```

```
<hint type="sourcelabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
 <hint type="destinationlabel">
 < x > 483 < /x >
 <y>136</y>
 </hint>
</hints>
</connection>
<connection>
<sender>Form</sender>
<signal>input_changed(QString)</signal>
<receiver>line_input</receiver>
<slot>setText(QString)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 309 < /x >
 < y > 199 < /y >
 </hint>
 <hint type="destinationlabel">
 < x > 483 < /x >
 <y>95</y>
 </hint>
</hints>
</connection>
<connection>
<sender>line input</sender>
<signal>textChanged(QString)</signal>
<receiver>Form</receiver>
<slot>input_modified(QString)</slot>
<hints>
 <hint type="sourcelabel">
 <x>483</x>
 <y>95</y>
 </hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>line output</sender>
<signal>textChanged(QString)</signal>
<receiver>Form</receiver>
<slot>output modified(QString)</slot>
<hints>
 <hint type="sourcelabel">
 < x > 483 < /x >
 <y>136</y>
 </hint>
 <hint type="destinationlabel">
 < x > 309 < /x >
 <y>199</y>
 </hint>
</hints>
</connection>
<connection>
<sender>lambda check</sender>
<signal>toggled(bool)</signal>
<receiver>Form</receiver>
<slot>lambda_calc_method_changed(bool)</slot>
```

```
<hints>
 <hint type="sourcelabel">
  <x>301</x>
  <y>219</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
  <y>199</y>
 </hint>
 </hints>
</connection>
<connection>
 <sender>weights_box</sender>
 <signal>currentIndexChanged(QString)</signal>
 <receiver>Form</receiver>
 <slot>weights_modified(QString)</slot>
 <hints>
 <hint type="sourcelabel">
  < x > 136 < /x >
  <y>220</y>
 </hint>
 <hint type="destinationlabel">
  < x > 309 < /x >
  <y>199</y>
 </hint>
 </hints>
</connection>
<connection>
 <sender>y dim</sender>
 <signal>valueChanged(int)</signal>
 <receiver>Form</receiver>
 <slot>dimension modified(int)</slot>
 <hints>
 <hint type="sourcelabel">
  < x > 378 < /x >
  <y>146</y>
 </hint>
 <hint type="destinationlabel">
  < x > 311 < /x >
  <y>299</y>
 </hint>
 </hints>
</connection>
</connections>
<slots>
<signal>input changed(QString)</signal>
<signal>output changed(QString)</signal>
<signal>x1 dim changed(int)</signal>
<signal>x2 dim changed(int)</signal>
<signal>x3 dim changed(int)</signal>
<signal>x1_deg_changed(int)</signal>
<signal>x2 deg changed(int)</signal>
<signal>x3_deg_changed(int)</signal>
<signal>type_cheb()</signal>
<signal>type lege()</signal>
<signal>type_lagg()</signal>
<signal>type herm()</signal>
<slot>input clicked()</slot>
<slot>output clicked()</slot>
<slot>samples modified(int)</slot>
<slot>dimension modified(int)</slot>
<slot>degree modified(int)</slot>
<slot>type_modified(bool)</slot>
```

```
<slot>plot_clicked()</slot>
<slot>exec_clicked()</slot>
<slot>input_modified(QString)</slot>
<slot>output_modified(QString)</slot>
<slot>weights_modified(QString)</slot>
<slot>lambda_calc_method_changed(bool)</slot>
</slots>
</ui>
```