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Pattern Recognition Letters 18 (1997) 395-400

Pattern Recognition Letters

Effects of unequal focal lengths in stereo imaging

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Received 1 August 1995; revised 10 February 1997

Abstract

In stereo imaging, usually a simplified camera geometry is considered with equal focal length cameras and their axes perpendicular to the baseline of separation. In this paper, the effects of unequal focal lengths are analyzed and the epipolar line equations are derived. Depth error resulting from unequal focal lengths is calculated. This analysis is useful for designing the correspondence search space and for having the depth estimates if the focal length accuracies are known. © 1997 Elsevier Science B.V.

Keywords: Stereo imaging; Camera geometry; Stereo correspondence; Search space; Depth measurements; 3D vision

1. Introduction

In stereo imaging, one can vastly limit the computations required in determining correspondences in the two images, by constraining the geometry of the cameras during image acquisition. If two balanced, equal focal length cameras are arranged with axes parallel, then they can be conceived of sharing a single, common image plane. Any point in the scene will project to two points in that joint image plane, the connection of which produces a line parallel to the baseline between the two cameras. This line is termed as an epipolar line. If the baseline between the two cameras happens to be parallel to an axis of the cameras, then the correspondence needs to be searched only along lines parallel to that axis in the image. That is, once a pair of stereo images is rectified so that the epipolar lines are horizontal scanlines, a pair of corresponding edges should be searched only within the same horizontal scanline.

Number of researchers, such as Baker and Binford (1981), Ohta and Kanade (1985), Lloyd et al. (1987) and very recently Jones and Malik (1992) and others have assumed rectified geometry in their stereo correspondence algorithms. The problems of stereo geometry and error analysis have been addressed by Verri and Torre (1986), Blostein and Huang (1987), Alvertos et al. (1989), Weng et al. (1989) and others. All of them assumed exact matching of focal lengths of the cameras, that is difficult to obtain in practice. Effects of focal length mismatch need to be studied and this letter makes some contributions to this effect. In this paper, the effects of focal length mismatch is analyzed by considering a simple camera geometry and the equations of modified epipolar lines are derived. It is shown that the epipolar lines are no longer aligned in the direction of horizontal scanlines and the magnitudes of their slopes increase as we deviate from the middle scanline. The expression for depth error due to

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0167-8655/97/$17.00
PII S0167-8655(97)00024-X

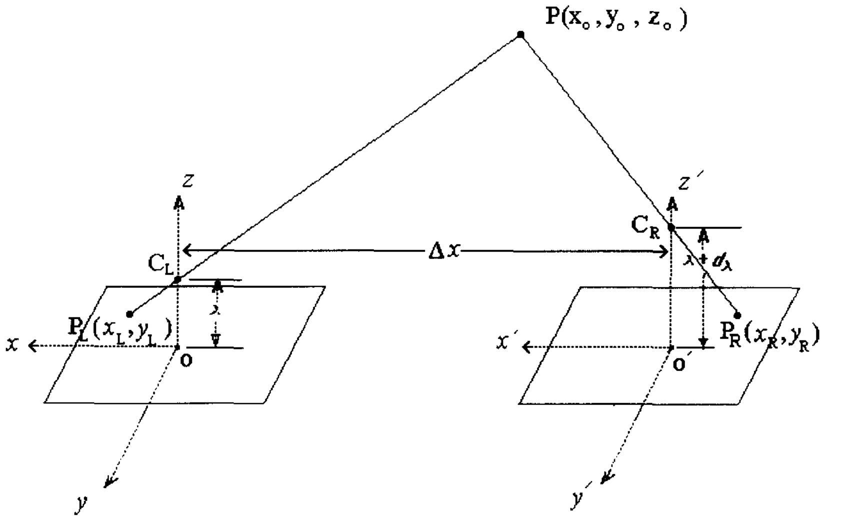

Fig. 1. Lateral stereo geometry with unequal focal lengths.

the focal length mismatch is also derived.

This paper is organized as follows. In Section 2, the camera geometry model with unequal focal lengths is considered. In Section 3, the equations of epipolar lines are derived and the search space is analyzed. This is followed by the depth error analysis in Section 4. Section 5 concludes this paper.

2. Camera geometry

In this analysis, each camera is modeled as though it were an ideal *pinhole camera* and consequently, *perspective projection* is adopted as the model of the image formation process. Actually, a camera consists of a system of lenses, but the pinhole assumption is justified for large object distances relative to the focal length. There the distance between the lens center and the image plane can be approximated as the focal length. The lateral stereo model of Alvertos et al. (1989) is adopted here. Two cameras C_L and C_R are separated by a translation of Δx in the x-direction, as indicated in Fig. 1 and I_L and I_R are the centers of the two images. Let the projection of a point $P(x_0, y_0, z_0)$ be $P_L(x_L, y_L)$ and $P_R(x_R, y_R)$ on the left and the right image planes, respectively. All point coordinates are expressed in the x-y-z coordinate system with I_L as the origin. The focal lengths of the left and the right cameras are taken as λ and $\lambda + d\lambda$, respectively.

From the similar triangles, we have for the left camera

$$\frac{x_0}{x_L} = \frac{y_0}{y_L} = \frac{\lambda - z_0}{\lambda}, \tag{1}$$

and for the right camera

$$\frac{x_0 + \Delta x}{x_R + \Delta x} = \frac{y_0}{y_R} = \frac{\lambda + d\lambda - z_0}{\lambda + d\lambda}. \tag{2}$$

Using Eqs. (1) and (2), we have

$$y_L = y_0 \frac{\lambda}{\lambda - z_0} \tag{3}$$

and

$$y_R = y_0 \frac{\lambda + d\lambda}{\lambda + d\lambda - z_0}. \tag{4}$$

Hence, in general, $y_L \neq y_R$ and, thus, the corresponding points will not lie in the same scanline, unlike the equal focal length case, (i.e., $d\lambda = 0$). The displacement e_y in y-values for the corresponding points in the left and the right images may be calculated using Eqs. (3) and (4) as

$$\begin{aligned} e_y &= y_L - y_R \\ &= y_0 \left(\frac{\lambda}{\lambda - z_0} - \frac{\lambda + d\lambda}{\lambda + d\lambda - z_0} \right) \end{aligned}$$

$$= \frac{y_L d\lambda}{\lambda(\lambda/z_0 + d\lambda/z_0 - 1)}. \tag{5}$$

To calculate the disparity in the x-direction, we use Eqs. (1) and (2) and obtain

$$x_L = \frac{\lambda}{\lambda - z_0} x_0, \tag{6}$$

$$x_R + \Delta x = \frac{\lambda + d\lambda}{\lambda + d\lambda - z_0}(x_0 + \Delta x). \tag{7}$$

The horizontal disparity is given by

$$\begin{aligned} e_x &= x_L - (x_R + \Delta x) \\ &= \frac{x_L d\lambda}{\lambda((\lambda + d\lambda)/z_0 - 1)} \\ &\quad - \left(\frac{1}{(\lambda + d\lambda)/z_0 - 1} + 1\right)\Delta x. \end{aligned} \tag{8}$$

Considering the above stereo geometry with unequal focal length cameras, two important results may be presented.

Lemma 1. *If a left image candidate point belongs to the scanline $y_L = 0$, the corresponding point in the right image lies along the scanline $y_R = 0$.*

Proof. The proof directly follows by substituting $y_L = 0$ in Eq. (5). □

Lemma 2. *If an object point is at infinity and the coordinates of its projection in the left image are (x_L, y_L), then the coordinates of the corresponding point in the right image, referenced to the right image coordinate system are given by*

$$\left[x_L\left(1 + \frac{d\lambda}{\lambda}\right), y_L\left(1 + \frac{d\lambda}{\lambda}\right)\right].$$

Proof. From Eqs. (5) and (8), as z_0 approaches ∞, we have

$$y_{R\infty} = y_L\left(1 + \frac{d\lambda}{\lambda}\right) \tag{9}$$

and

$$x_{R\infty} + \Delta x = x_L\left(1 + \frac{d\lambda}{\lambda}\right). \tag{10}$$

In the imaging system considered, the corresponding image point in the right image, referenced to I_R as origin (cf. Fig. 1) is $(x_R + \Delta x, y_R)$. Hence the proof. □

3. Search space analysis

Lemma 3. *The magnitude of the slope of the epipolar line is directly related to the focal length mismatch as*

$$|\alpha| = \frac{|y_L||d\lambda/\lambda|}{|x_L(d\lambda/\lambda) - \Delta x|}. \tag{11}$$

Proof. To determine the correspondence search space in the right image for a candidate left image point, we derive the equation of the epipolar lines. Without loss of generality, we consider any object point whose z-coordinates is given by $z_0 = p(\lambda + d\lambda)$ (p is a real number greater than unity). Considering (x_L, y_L) as the candidate left image point, the corresponding point in the right image, referenced to I_R can be computed using Eqs. (5) and (8) as

$$y_{Rp} = y_L\left(1 + \frac{pd\lambda}{(p-1)\lambda}\right), \tag{12}$$

$$x_{Rp} + \Delta x = x_L\left(1 + \frac{pd\lambda}{(p-1)\lambda}\right) - \frac{1}{p-1}\Delta x. \tag{13}$$

As p is changed, the point corresponding to left image candidate (x_L, y_L) moves along its epipolar line in the right image. In the limit, as p tends to infinity, the corresponding right image point is given by Eqs. (10) and (9). The epipolar line thus passes through $(x_{Rp} + \Delta x, y_{Rp})$ and $(x_{R\infty} + \Delta x, y_{R\infty})$ and its slope is given by

$$\begin{aligned} \alpha &= \frac{y_{Rp} - y_{R\infty}}{x_{Rp} - x_{R\infty}} \\ &= \frac{y_L\left(1 + \frac{pd\lambda}{(p-1)\lambda}\right) - y_L\left(1 + \frac{d\lambda}{\lambda}\right)}{x_L\left(1 + \frac{pd\lambda}{(p-1)\lambda}\right) - \frac{\Delta x}{p-1} - x_L\left(1 + \frac{d\lambda}{\lambda}\right)} \\ &= \frac{y_L(d\lambda/\lambda)}{x_L(d\lambda/\lambda) - \Delta x}. \end{aligned} \tag{14}$$

Hence the proof. □

3.1. Equation of epipolar lines

The equation of the epipolar line may be expressed in the form

$$y - y_{R\infty} = \alpha(x - (x_{R\infty} + \Delta x)), \tag{15}$$

where the right image center I_R is taken as origin for x and y and the coordinate $(x_{R\infty}, y_{R\infty})$ is referenced to

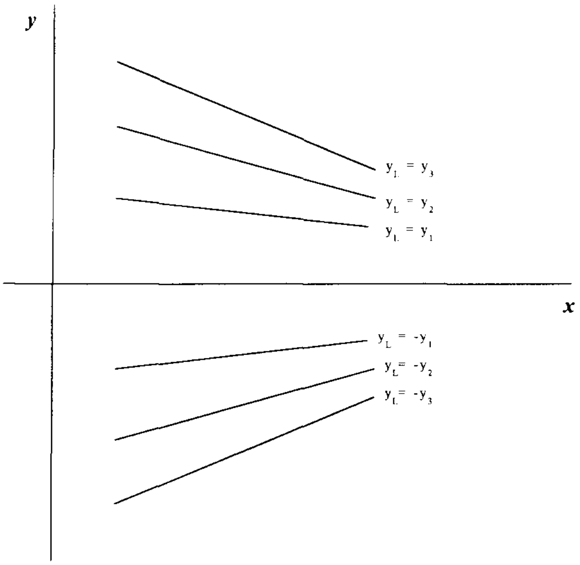

Fig. 2. Epipolar lines for unequal focal lengths.

the left image center I_L. By substituting the expression for slope using Eq. (14) and the expressions for $x_{R\infty}$ and $y_{R\infty}$ from Eqs. (10) and (9) we have the equation of epipolar line for the left image candidate point as

$$y = \frac{y_L(d\lambda/\lambda)}{x_L(d\lambda/\lambda) - \Delta x}\left[x - x_L\left(1 + \frac{d\lambda}{\lambda}\right)\right] + y_L\left(1 + \frac{d\lambda}{\lambda}\right). \quad (16)$$

As the distance of separation Δx is much higher than x_L and the fractional error in focal length mismatch $d\lambda/\lambda$ is much less than unity, we have $(x_L(d\lambda/\lambda) - \Delta x)$ much less than 0. If $d\lambda$ is taken as positive, it is seen from Eq. (14) that the slope is negative.

As $|x_L(d\lambda/\lambda)|$ is much less than $|\Delta x|$, the equation of the epipolar line may be approximated as

$$y \approx -y_L\frac{f_\lambda}{\Delta x}[x - x_L(1 + f_\lambda)] + y_L(1 + f_\lambda), \quad (17)$$

where $f_\lambda = d\lambda/\lambda$ is the fractional error in focal lengths.

A typical set of epipolar lines are illustrated in Fig. 2. The lines are shown for several y_L's ($y_1 < y_2 < y_3$).

A set of tabulated results are shown in Tables 1 and 2. A set of candidate points are considered from left image and their corresponding right image points

Table 1
Correspondence and epipolar line slopes for $f_\lambda = 0.02$ and $\Delta x = 500$

Left image	Correspondence in right image		
candidate	$p = \infty$	$p = 4$	Slope
(100, 120)	(102.0, 122.4)	(−64.0, 123.2)	−0.0048
(100, 100)	(102.0, 102.0)	(−64.0, 102.7)	−0.0040
(100, 80)	(102.0, 81.6)	(−64.0, 82.1)	−0.0032
(100, 60)	(102.0, 61.2)	(−64.0, 61.6)	−0.0024
(100, 40)	(102.0, 40.8)	(−64.0, 41.1)	−0.0016
(100, 20)	(102.0, 20.4)	(−64.0, 20.5)	−0.0008
(100, 0)	(102.0, 0.0)	(−64.0, 0.0)	0.0000

are computed using Eqs. (12) and (13) and the epipolar line slopes are computed using Eq. (14). Two values of p are considered. In one extreme case, the object point is assumed to be at infinity ($p = \infty$) for which the disparity is ideally zero in both x and y. The other case is considered for a near object point ($p = 4$). Two typical values of f_λ (0.02 and 0.05) are used. The distance of separation Δx is taken as 500 pixel units. In these results, the coordinates for both left and the right images are referred with respect to their image centers and the direction of positive x is as shown in Fig. 1.

4. Depth error analysis

To analyse the depth error due to unequal focal lengths, we adopt the following approach. We consider a vertical line in 3D space with a given constant depth and determine its projections in the left and the right image, using the stereo geometry shown in Fig. 1. If these projections are given to us, we can determine the correspondence along the same horizontal scan-line, without being aware of focal length mismatch and estimate the depth from the disparity. The difference between the actual depth and the estimated depth gives us the depth error.

We consider a straight line in 3D space, given by the equations

$$z = z_C, \qquad x = x_C, \quad (18)$$

where x_C and z_C are constants.

The equation of the projected straight line in the left image and the right image are obtained from Eqs. (6) and (7) as

Table 2
Correspondence and epipolar line slopes for $f_\lambda = 0.05$ and $\Delta x = 500$

Left image	Correspondence in right image		
candidate	$p = \infty$	$p = 4$	Slope
(100, 120)	(105.0, 126.0)	(−60.0, 128.0)	−0.0120
(100, 100)	(105.0, 105.0)	(−60.0, 106.7)	−0.0100
(100, 80)	(105.0, 84.0)	(−60.0, 85.3)	−0.0080
(100, 60)	(105.0, 63.0)	(−60.0, 64.0)	−0.0060
(100, 40)	(105.0, 42.0)	(−60.0, 42.7)	−0.0040
(100, 20)	(105.0, 21.0)	(−60.0, 21.3)	−0.0020
(100, 0)	(105.0, 0.0)	(−60.0, 0.0)	0.0000

$$x_L = \frac{x_C \lambda}{\lambda - z_C} \tag{19}$$

and

$$x_R + \Delta x = (x_C + \Delta x)\frac{\lambda + d\lambda}{\lambda + d\lambda - z_C}. \tag{20}$$

Combining Eqs. (19) and (20), the actual depth z_C may be expressed in terms of x_L, x_R and Δx as

$$\begin{aligned} z_C &= (\lambda + d\lambda)\frac{x_R - x_L}{x_R + \Delta x - x_L - x_L(d\lambda/\lambda)} \\ &= (\lambda + d\lambda)\left[1 + \frac{\Delta x - x_L(d\lambda/\lambda)}{x_L - (x_R + \Delta x) + x_L(d\lambda/\lambda)}\right]. \end{aligned} \tag{21}$$

The right image projection line, given by Eq. (20) is a vertical line and it intersects the horizontal scanline $y = y_L$ at $(x_R + \Delta x, y_L)$. This is the point of correspondence for the left image candidate (x_L, y_L). Disregarding the focal length mismatch, the depth may be estimated (cf. Alvertos et al., 1988) as

$$z_{CE} = \lambda\left[1 + \frac{\Delta x}{x_L - (x_R + \Delta x)}\right]. \tag{22}$$

The depth error may be determined from Eqs. (21) and (22) as

$$\begin{aligned} z_{ERROR} &= z_C - z_{CE} \\ &= d\lambda + \frac{\Delta x - x_L\frac{d\lambda}{\lambda}}{x_L - (x_R + \Delta x) + x_L\frac{d\lambda}{\lambda}} - \frac{\Delta x}{x_L - (x_R + \Delta x)} \\ &= d\lambda + \frac{\frac{d\lambda}{\lambda}(x_R - x_L)}{\left[x_L - (x_R + \Delta x)\right]\left[x_L - (x_R + \Delta x) + x_L\frac{d\lambda}{\lambda}\right]}. \end{aligned} \tag{23}$$

5. Conclusions

Most of the stereo correspondence algorithms proposed till now are based on rectified geometry assumptions, where the camera focal lengths are always assumed to be equal. This paper analyzes the effects of focal length mismatch on the correspondence search space and the estimated depth values. Our results indicate that the corresponding points are deviated significantly due to focal length mismatch as we decrease the spacing between the cameras and consider points with lower depth values. We show that the epipolar lines no longer remain horizontal and their slopes increase as we go further away from the middle scan line. The equation of epipolar lines and the expressions for depth error have been derived. Using the results shown in this paper, it is possible to estimate depths if two cameras with unequal focal lengths are used and these are known a priori.

Acknowledgements

The author wishes to thank Professors B.N. Chatterjee and M. Mukherjee of the Department of E and ECE, I.I.T. Kharagpur, India and Professor S.C. Sahasrabudhe of Electrical Engineering Department, I.I.T. Bombay, India for their valuable comments and advice on this work. The author also wishes to thank the anonymous reviewer for his very valuable suggestions on the improvement of this paper.

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