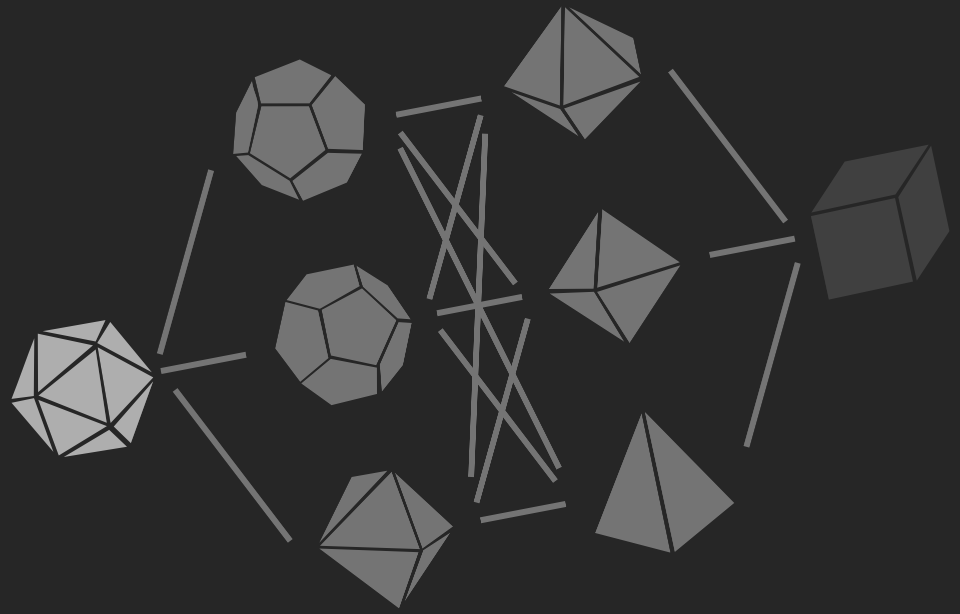
18th INFORMS Computing Society (ICS) Conference

L2O-MINLP

Learning to Optimize for Mixed-Integer Non-Linear Programming



Mechanical & Industrial Engineering
UNIVERSITY OF TORONTO

Pacific Northwest
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Presented by Bo Tang
Toronto, Mar 14, 2025

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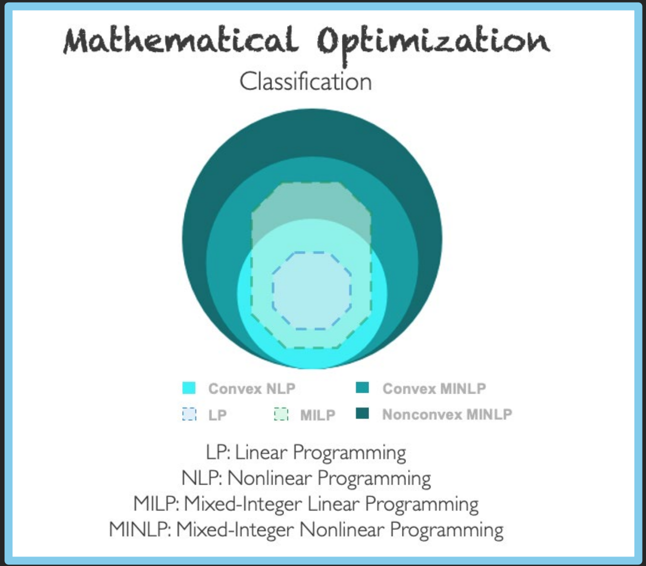
Motivation

Why does MINLP matter?

MINLP is general but very hard:

Combinatorial Complexity + Non-Convexity.



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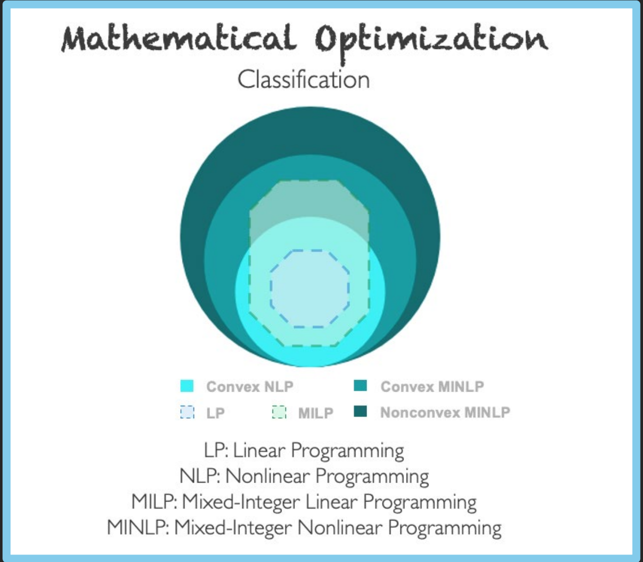

Solvers are limited:

Traditional solvers struggle with large-scale MINLP problems.

GUROBI OPTIMIZATION

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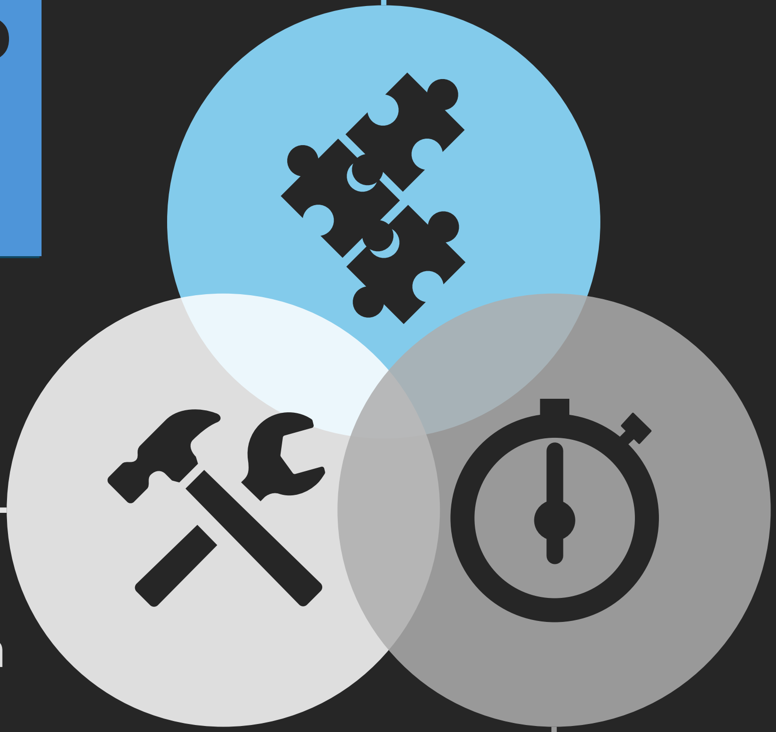

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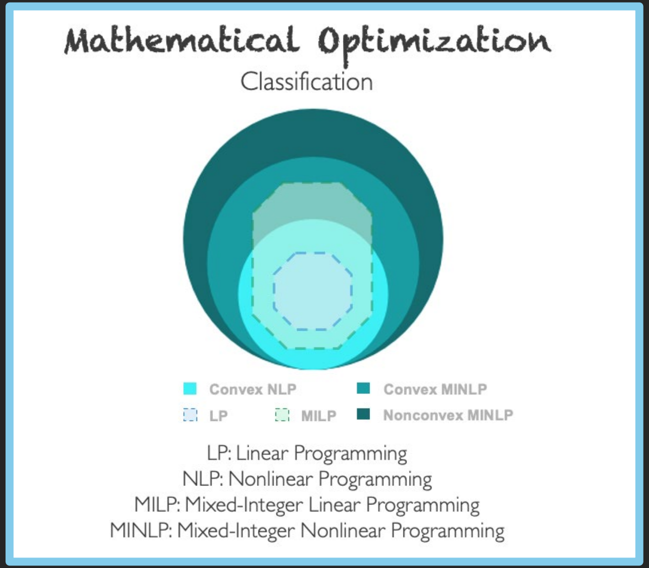

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GUROBI OPTIMIZATION

Solution time is tight:

Real-world applications often require solutions within a short time window.

e.g., Optimal Control, Energy Management, etc.

Introduction to L2O

01 What is L2O?

L2O is a data-driven approach that leverages machine learning to improve optimization processes.

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02 Why do we use L2O?

L2O enables high-quality solutions with significantly reduced computational cost.

03 How does L2O work?

L2O learns from optimal or near-optimal solutions to either generate solutions directly or guide solvers in narrowing the search space.

What we did here!

Traditional Methods vs. L2O Approaches

Traditional Methods

- **Require costly iterative procedures** (e.g., branch-and-bound, matrix inversion) that scale poorly with problem size.
- **Solve from scratch for every instance**, even if similar problems have been solved before.
- **Require manual tuning** of heuristics and parameters for good performance.
- **Optimality is theoretical guaranteed** through exact methods.

L2O Approaches

- **Bypass expensive iterations** by learning direct mappings or guiding solvers for faster convergence.
- **Leverage past patterns** to generalize and quickly generate solutions for new instances.
- **Automate tuning** by learning optimization strategies from data.
- **No guarantee** of optimality or even feasibility.

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Problem Formulation

MINLP Formulation:

$$\min_{\boldsymbol{x}} f(\boldsymbol{x})$$

$$\text{s.t.} \quad \boldsymbol{g}(\boldsymbol{x}) \leq 0$$

$$\boldsymbol{x} \in \mathbb{R}^{n_r} \times \mathbb{Z}^{n_z}$$

Problem Formulation

Parametric MINLP Formulation:

$$
\begin{aligned}
\min_{\boldsymbol{x}} \; & f(\boldsymbol{x}, \xi) \\
\text{s.t.} \; & \boldsymbol{g}(\boldsymbol{x}, \xi) \leq 0 \\
& \boldsymbol{x} \in \mathbb{R}^{n_r} \times \mathbb{Z}^{n_z} \\
& \xi \in \boldsymbol{\Xi}
\end{aligned}
$$

Parameters ξ influence the objective and constraints of optimization problems.

$$
\text{Multiple Instances} \begin{cases} \xi_1 \rightarrow \boldsymbol{x}_1^* \\ \xi_2 \rightarrow \boldsymbol{x}_2^* \\ \quad \vdots \\ \xi_m \rightarrow \boldsymbol{x}_m^* \end{cases}
$$

Problem Formulation

Prediction for Parametric MINLP Formulation:

$$\min_{\boldsymbol{\Theta}} \mathbb{E}_{\boldsymbol{\Xi}}[f(\hat{\boldsymbol{x}}_i, \boldsymbol{\xi}_i)]$$

$$\text{s. t.} \quad \boldsymbol{g}(\hat{\boldsymbol{x}}_i, \boldsymbol{\xi}_i) \leq 0$$

$$\hat{\boldsymbol{x}}_i \in \mathbb{R}^{n_r} \times \mathbb{Z}^{n_z}$$

$$\boldsymbol{\xi}_i \in \boldsymbol{\Xi}$$

$$\hat{\boldsymbol{x}}_i, = \psi_{\boldsymbol{\Theta}}(\boldsymbol{\xi}_i)$$

Expected objective function:

$$\mathbb{E}_{\boldsymbol{\Xi}}[f(\hat{\boldsymbol{x}}, \boldsymbol{\xi})] \approx \frac{1}{m} \sum_{i=1}^{m} f(\hat{\boldsymbol{x}}_i, \boldsymbol{\xi}_i)$$

Machine learning model $\psi_{\boldsymbol{\Theta}}$ with inputs $\boldsymbol{\xi}$ and weights $\boldsymbol{\Theta}$

Solution prediction

Problem Formulation

Prediction for Parametric MINLP Formulation:

$$\min_{\Theta} \mathbb{E}_{\Xi}[f(\hat{x},\xi)]$$

Minimize expected objective function:

$$\mathbb{E}[f(\hat{x},\xi)] \approx \frac{1}{m}\sum_{i=1}^{m} f(\hat{x}_i,\xi_i)$$

IT SEEMS IDEAL:

What Prevents L2O from Extending to this MINLP?

$$\hat{x} \in \mathbb{R}^{n_r} \times \mathbb{Z}^{n_z}$$

$$\xi \in \Xi$$

Machine learning model ψ_{Θ} with input ξ and weights Θ

Solution prediction

$$\hat{x} = \psi_{\Theta}(\xi)$$

Limitations of Existing L2O

01 Collecting Solutions as Training Label is Very Expensive

02 Neural Networks Cannot Directly Output Integer Values

03 It is Difficult to Ensure Feasibility, Especially in Integers

Limitations of Existing L2O

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Our Solution: We propose a Self-Supervised Approach without requiring solutions for training.

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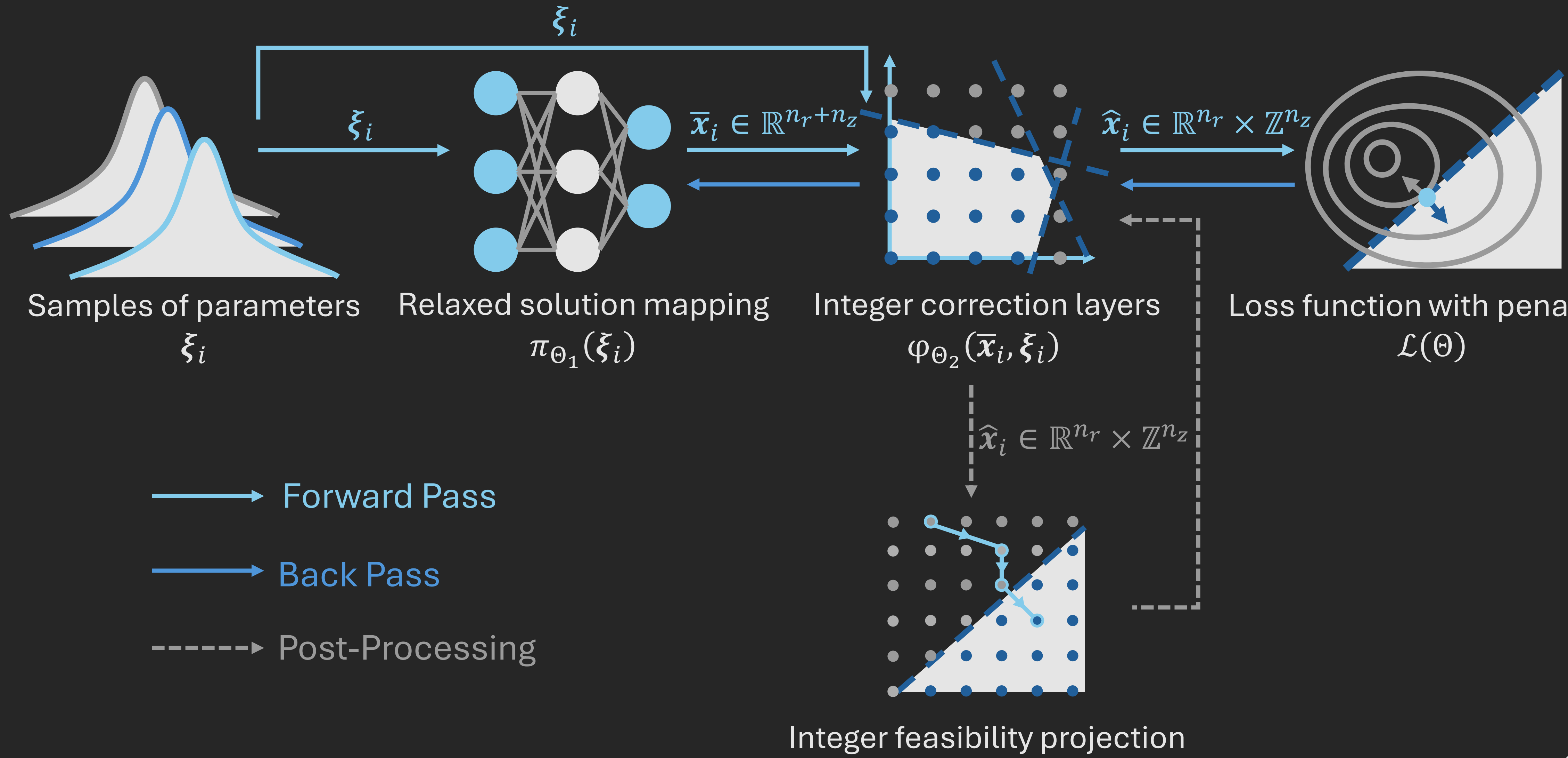
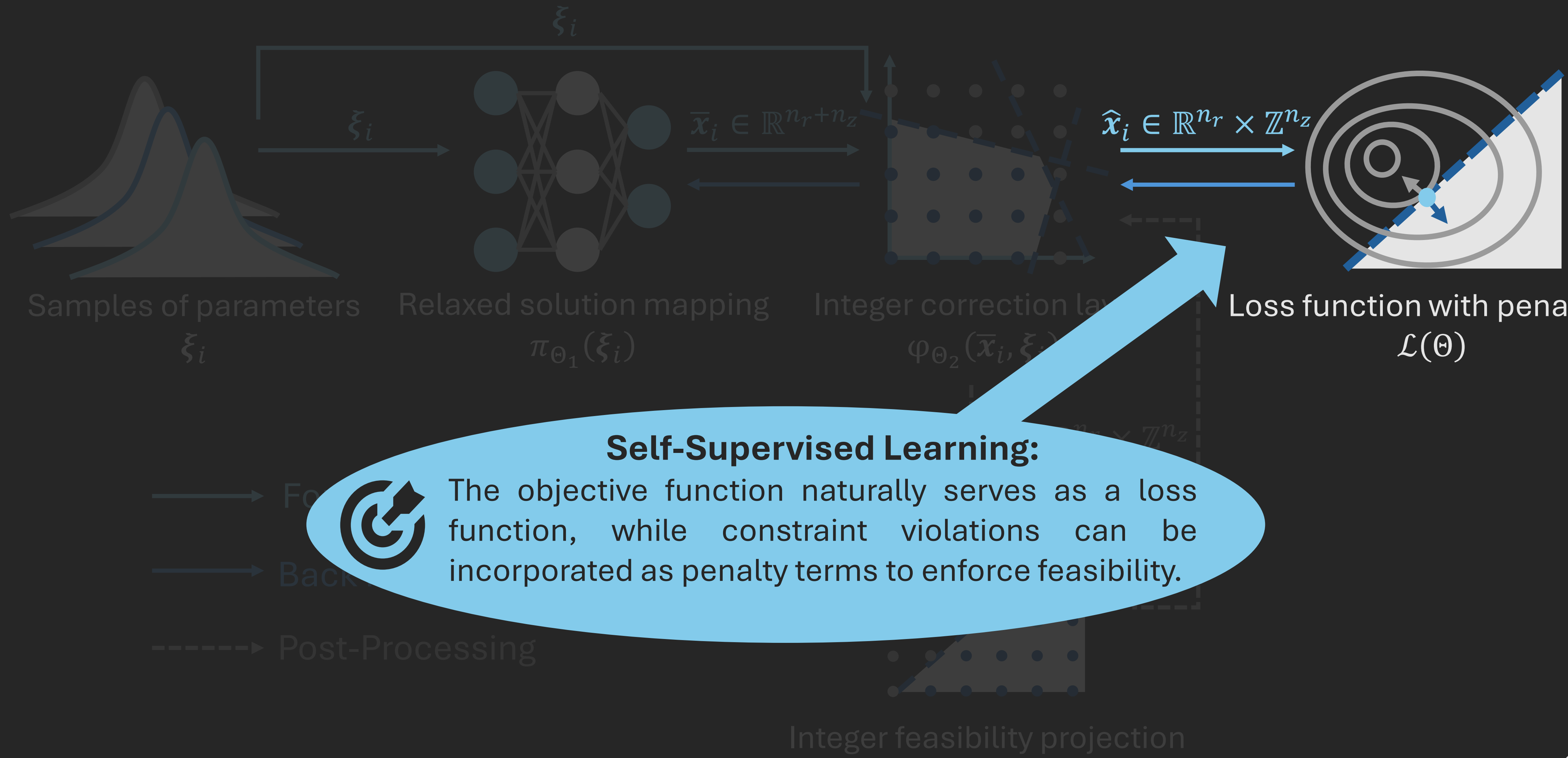
Our Solution: We introduce a gradient-based Feasibility Projection to adjust integer solutions.

Methodology



Loss Function



Self-Supervised Learning:

The objective function naturally serves as a loss function, while constraint violations can be incorporated as penalty terms to enforce feasibility.

Loss Function

Loss Function with Penalty

$$\mathcal{L}(\Theta) = \sum_{i=1}^{m} \left[f(\hat{\boldsymbol{x}}_i, \xi_i) + \lambda \cdot \|\boldsymbol{g}(\hat{\boldsymbol{x}}_i, \xi_i)_+\|_1 \right]$$

Loss Function

Loss Function with Penalty

$$\mathcal{L}(\Theta) = \sum_{i=1}^{m} [\underbrace{f(\hat{\boldsymbol{x}}_i, \xi_i)}_{\text{Objective Function}} + \lambda \cdot \|g(\hat{\boldsymbol{x}}_i, \xi_i)_+\|_1]$$

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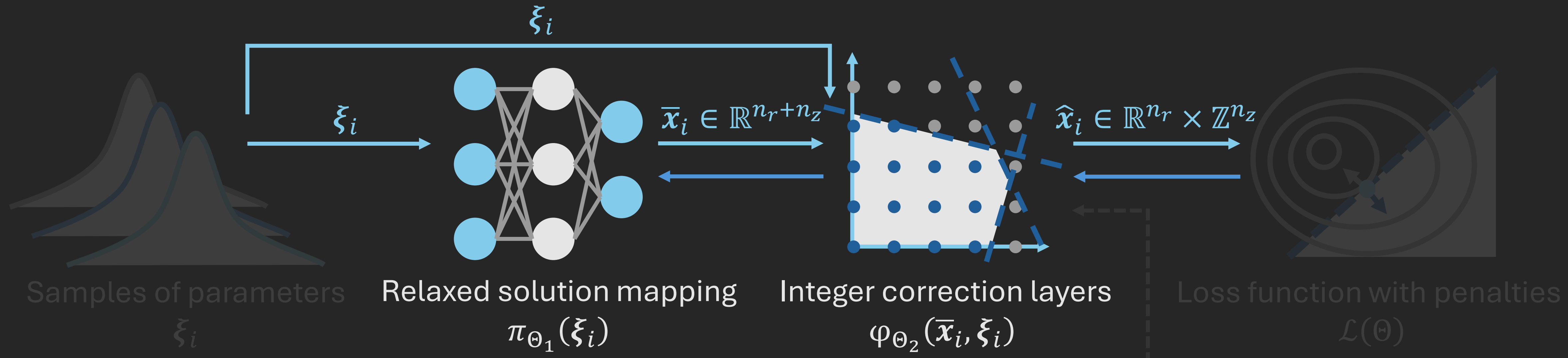
Loss Function

Loss Function with Penalty

$$\mathcal{L}(\Theta) = \sum_{i=1}^{m} \Big[\underbrace{f(\hat{x}_i, \xi_i)}_{\text{Objective Function}} + \overbrace{\lambda}^{\text{Penalty Weight}} \cdot \underbrace{\| g(\hat{x}_i, \xi_i)_+ \|_1}_{\text{Constraints Violation}} \Big]$$

Integer Correction Layers



Bi-Level Neural Network Architecture

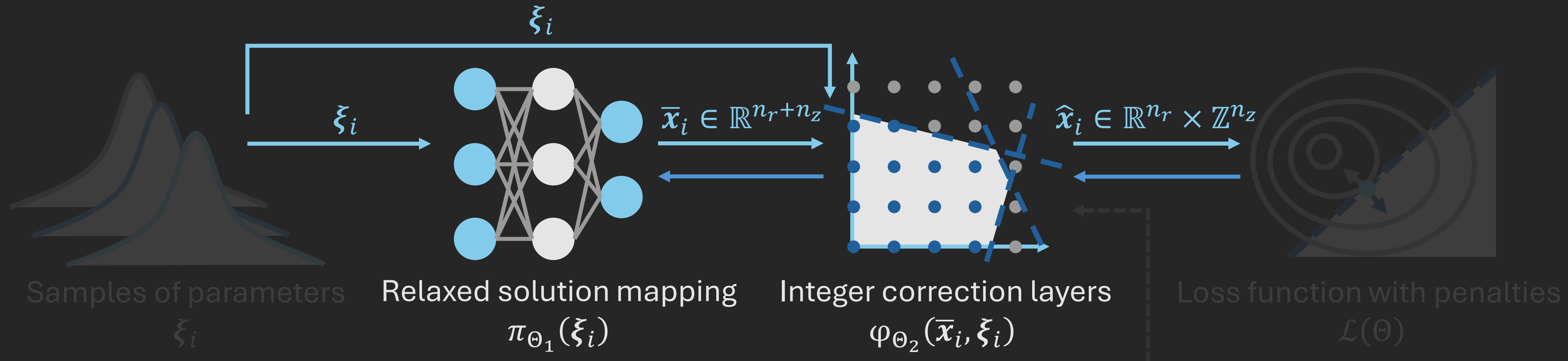
Relaxed Solution Mapping

From
Parameters ξ
To
Relaxed Solution $\bar{\boldsymbol{x}}$

Integer Correction

From
Relaxed Solution $\bar{\boldsymbol{x}}$
and
Parameters ξ
To
Mixed-Integer Solution $\hat{\boldsymbol{x}}$

Integer Correction Layers



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Relaxed Solution Mapping

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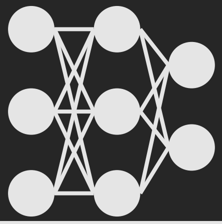
RENS (Relaxation Enforced Neighborhood Search)

Integer Correction Layers

Example

$$\min_{x\in\mathbb{R}, y\in\mathbb{Z}} (a-x)^2 + 50(y-x^2)^2$$
$$s.t.\ y \geq \frac{1}{2}b, x^2 \leq b, x \leq 0, y \geq 0$$

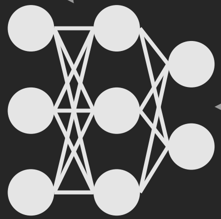
Input: $a = 3.83, b = 6.04$



Relaxed Solution Mapping
$\pi_{\Theta_1}(a, b)$

Relaxed Solution:
$\bar{x} = -1.17, \bar{y} = 2.98$

Rounding Classification
$\varphi_{\Theta_1}(a, b, \bar{x}, \bar{y})$



Neural Network $\delta_{\Theta_2}(a, b, \bar{x}, \bar{y})$

Hidden State:
$h_x = -0.68, h_y = 9.49$

Update Continuous Var:
$\hat{x} = \bar{x} + h_x = -1.85$

Round Integer Var:
$\text{Gumbel\_Sigmoid}(h_y) \geq 0$
$\rightarrow \hat{y} = \lceil \bar{y} \rceil = 3$

Mixed-Integer Solution: $\hat{x} = -1.85, \hat{y} = 3$

Integer Correction Layers

Example

$$\min_{x\in\mathbb{R},y\in\mathbb{Z}} (a-x)^2 + 50(y-x^2)^2$$
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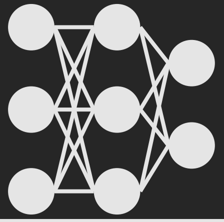

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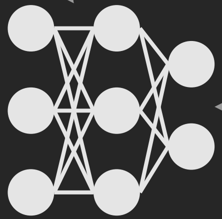

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This Operator is Not Differentiable!!!

Mixed-Integer Solution: $\hat{x} = -1.85, \hat{y} = 3$

Integer Correction Layers

Example

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Straight-Through Estimator

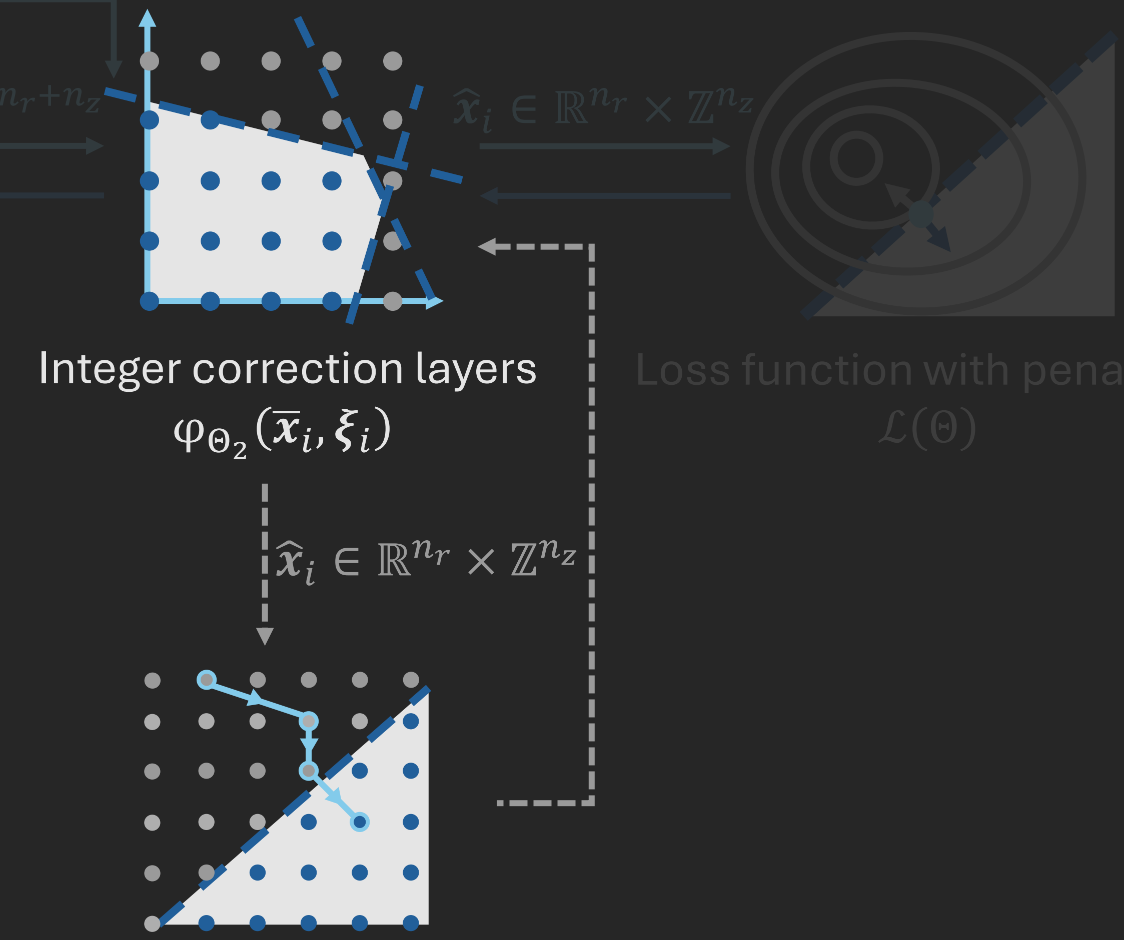
This Operator is Not Differentiable!!!

Mixed-Integer Solution: $\hat{x} = -1.85, \hat{y} = 3$

Integer Feasibility Projection

Feasibility projection is a computationally efficient <u>post-processing heuristic</u> that refines neural network outputs while preserving integer constraints.

- **Computational Efficiency:** Avoids the need for repeated projections and second-order gradient computation during training.
- **Stable Training:** Keeps projection separate from training, preventing interference with the output of model.

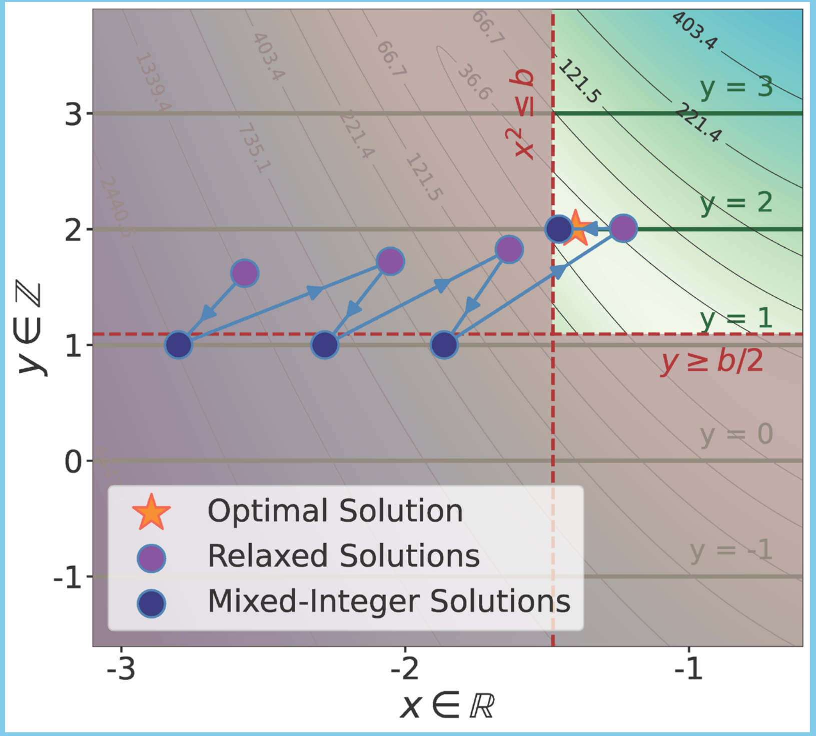


Integer feasibility projection

Integer Feasibility Projection

Algorithm 2 Integer Feasibility Projection: Inference

Input: parameters $\boldsymbol{\xi}^i$, layers $\pi_{\Theta_1}(\cdot)$ and $\varphi_{\Theta_2}(\cdot)$, step size η
Predict a continuously relaxed solution $\bar{\mathbf{x}}^i \leftarrow \pi_{\Theta_1}(\boldsymbol{\xi}^i)$
while True **do**
  Obtain a mixed-integer solution $\hat{\mathbf{x}}^i \leftarrow \varphi_{\Theta_2}(\bar{\mathbf{x}}^i, \boldsymbol{\xi}^i)$
  Compute feasibility violation $\mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i) \leftarrow \|\mathbf{g}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i)_+\|_1$
  if $\mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i) = 0$ **then**
    Break
  else
    Update relaxed solution $\bar{\mathbf{x}}^i \leftarrow \bar{\mathbf{x}}^i - \eta \nabla_{\bar{\mathbf{x}}} \mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i)$
  end if
end while
Output: a mixed-integer solution $\hat{\mathbf{x}}^i$



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Input: parameters $\boldsymbol{\xi}^i$, layers $\pi_{\Theta_1}(\cdot)$ and $\varphi_{\Theta_2}(\cdot)$, step size η
Predict a continuously relaxed solution $\bar{\mathbf{x}}^i \leftarrow \pi_{\Theta_1}(\boldsymbol{\xi}^i)$
while True **do** (1) Start from a relaxed solution
    Obtain a mixed-integer solution $\hat{\mathbf{x}}^i \leftarrow \varphi_{\Theta_2}(\bar{\mathbf{x}}^i, \boldsymbol{\xi}^i)$
    Compute feasibility violation $\mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i) \leftarrow \|\mathbf{g}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i)_+\|_1$
    if $\mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i) = 0$ **then**
        Break
    else
        Update relaxed solution $\bar{\mathbf{x}}^i \leftarrow \bar{\mathbf{x}}^i - \eta \nabla_{\bar{\mathbf{x}}} \mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i)$
    end if
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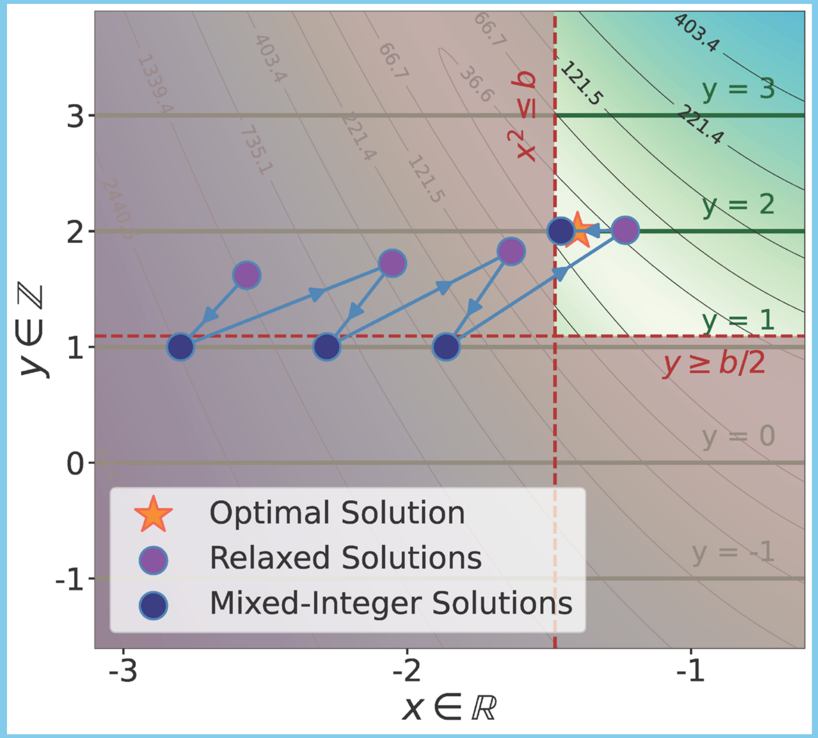

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Input: parameters $\boldsymbol{\xi}^i$, layers $\pi_{\Theta_1}(\cdot)$ and $\varphi_{\Theta_2}(\cdot)$, step size η
Predict a continuously relaxed solution $\bar{\mathbf{x}}^i \leftarrow \pi_{\Theta_1}(\boldsymbol{\xi}^i)$
while True **do** (2) Differentiable integer correction
  Obtain a mixed-integer solution $\hat{\mathbf{x}}^i \leftarrow \varphi_{\Theta_2}(\bar{\mathbf{x}}^i, \boldsymbol{\xi}^i)$
  Compute feasibility violation $\mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i) \leftarrow \|\mathbf{g}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i)_+\|_1$
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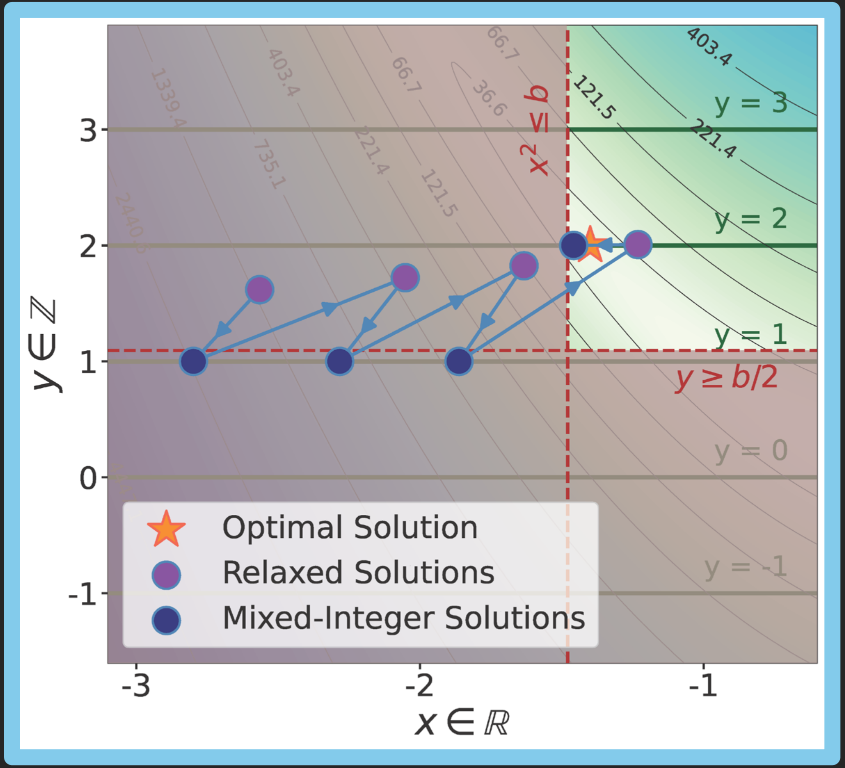

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    end if
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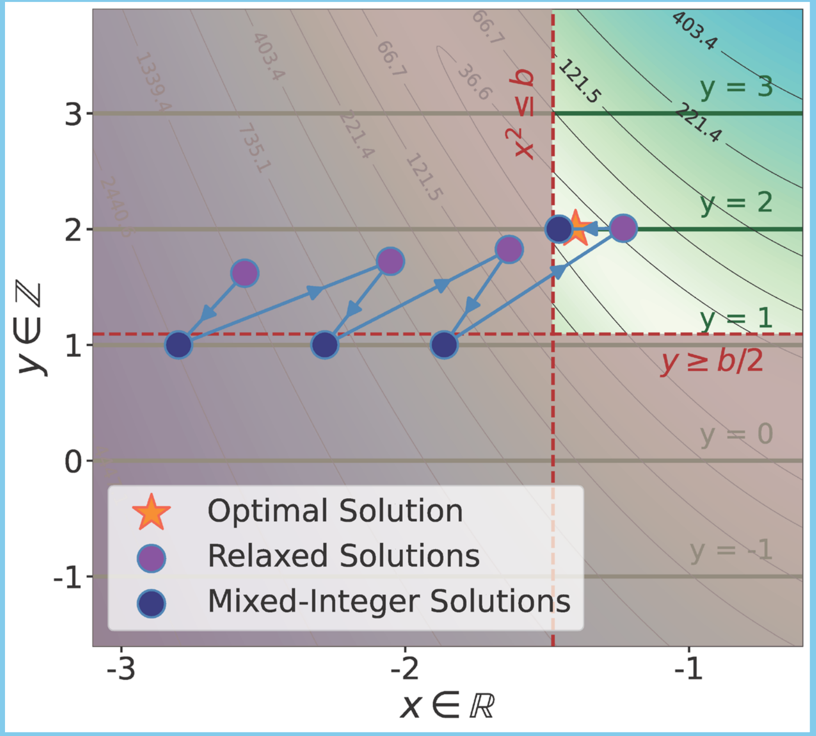
Output: a mixed-integer solution $\hat{\mathbf{x}}^i$

(3) Compute constraint violation
based on mixed-integer solution



Integer Feasibility Projection

Algorithm 2 Integer Feasibility Projection: Inference

```
Input: parameters $\boldsymbol{\xi}^i$, layers $\pi_{\Theta_1}(\cdot)$ and $\varphi_{\Theta_2}(\cdot)$, step size $\eta$
Predict a continuously relaxed solution $\bar{\mathbf{x}}^i \leftarrow \pi_{\Theta_1}(\boldsymbol{\xi}^i)$
while True do
    Obtain a mixed-integer solution $\hat{\mathbf{x}}^i \leftarrow \varphi_{\Theta_2}(\bar{\mathbf{x}}^i, \boldsymbol{\xi}^i)$
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    if $\mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i) = 0$ then
        Break
    else
        Update relaxed solution $\bar{\mathbf{x}}^i \leftarrow \bar{\mathbf{x}}^i - \eta \nabla_{\bar{\mathbf{x}}} \mathcal{V}(\hat{\mathbf{x}}^i, \boldsymbol{\xi}^i)$
    end if
end while
Output: a mixed-integer solution $\hat{\mathbf{x}}^i$
```
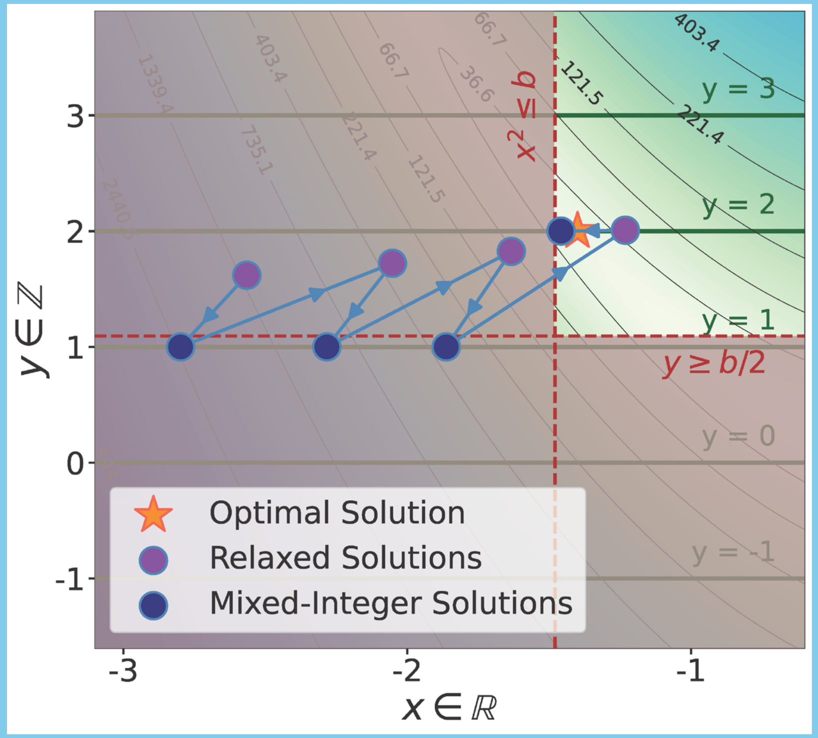
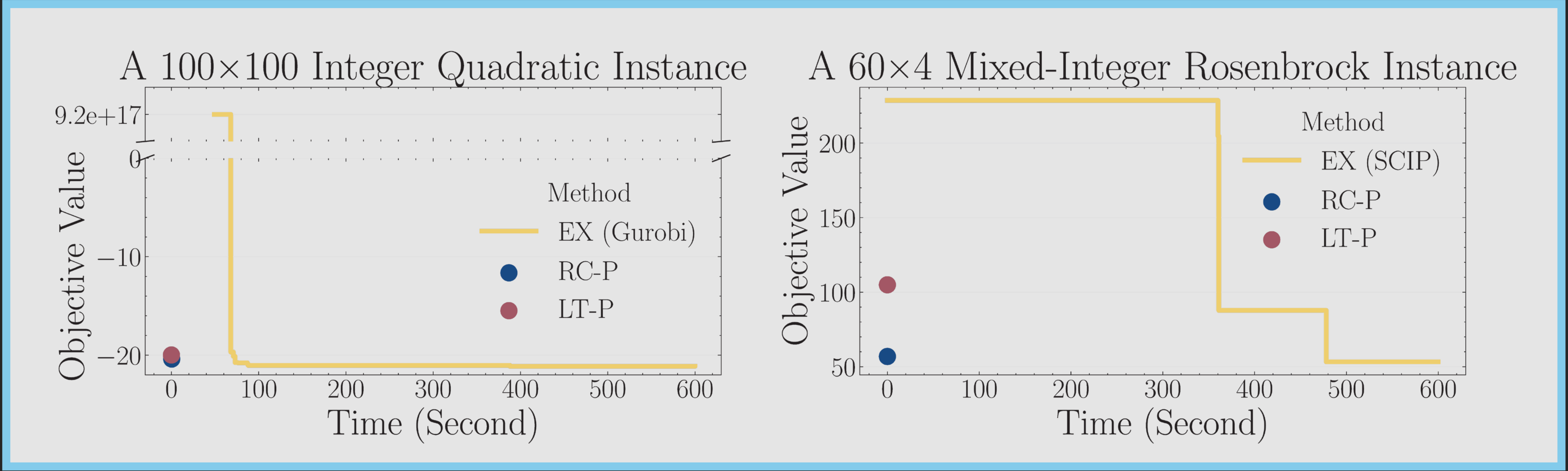

(4) Update relaxed solution to reduce violation



Key Methods in Comparison

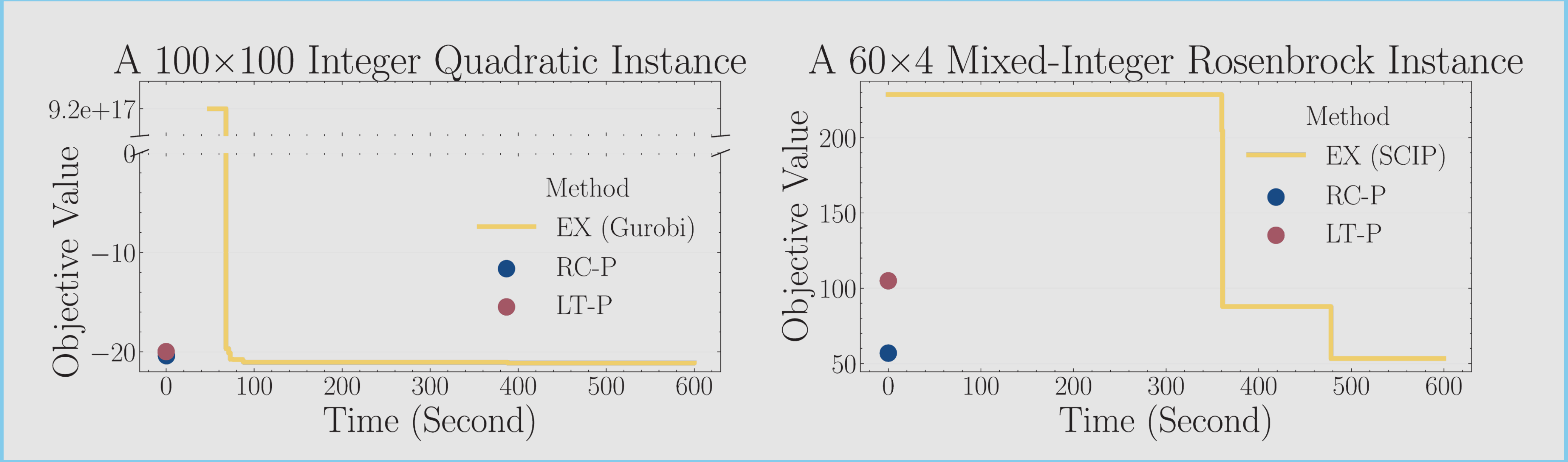
Method	Description
EX (Exact Solver)	Solves problems exactly using traditional solver with **1000-sec time-limit** as a benchmark.
N1 (Root Node Solution)	Finds the first feasible solution from the root node of the solver, combining various heuristics.
RC (Rounding Classification)	A neural network-based correction layer that learns a classification to determine how to round each integer variable.
LT (Learnable Thresholding)	A neural network-based correction layer that learns a threshold value to decide to round up or down for each integer variable.
RC-P (RC + Feasibility Projection)	RC combined with feasibility projection, which corrects infeasibilities while preserving integer constraints.
LT-P (LT + Feasibility Projection)	LT combined with feasibility projection, which corrects infeasibilities while preserving integer constraints.

Subsecond Solution



Exact solvers such as Gurobi and SCIP find better solutions over time but can be somewhat slow. In contrast, our methods (RC-P & LT-P) achieve high-quality feasible solutions within milliseconds.

Subsecond Solution



Exact solvers such as Gurobi and SCIP find better solutions over time but can be somewhat slow. In contrast, our methods (RC-P & LT-P) achieve high-quality feasible solutions within milliseconds.

Training time is about 100 second!

More Experiments

Result for IQPs. Each problem size is evaluated on a test set of 100 instance

Method	RC				RC-P				LT			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.54	-13.6	96%	0.0022	-13.54	-13.57	100%	0.0050	-13.65	-13.77	93%	0.0023
200×200	-31.62	-31.71	97%	0.0021	-31.62	-31.71	100%	0.0050	-31.34	-31.61	95%	0.0022
500×500	-73.31	-73.38	86%	0.0025	-73.31	-73.38	100%	0.0065	-72.36	-72.48	94%	0.0026
1000×1000	-142.7	-142.7	82%	0.0042	-142.7	-142.7	100%	0.009	-142.6	-142.6	100%	0.0047

Method	LT-P				EX				N1			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.65	-13.77	100%	0.0100	-20.79	-20.78	100%	1237	1.5E+18	1.4E+18	100%	104.2
200×200	-31.34	-31.61	100%	0.0064	-	-	-	-	-	-	-	-
500×500	-72.36	-72.48	100%	0.0063	-	-	-	-	-	-	-	-
1000×1000	-142.6	-142.6	100%	0.0086	-	-	-	-	-	-	-	-

More Experiments

Gurobi Fails to Find a Solution!

Result for IQPs. Each … luated on a test set of 100 instance

Method / Metric	RC Obj Mean	RC Obj Median	RC % Feasible	RC Time (Sec)	RC-P Obj Mean	RC-P Obj Median	RC-P % Feasible	RC-P Time (Sec)	LT Obj Mean	LT Obj Median	LT % Feasible	LT Time (Sec)
100×100	-13.54	-13.6	96%	0.0022	-13.54	-13.57	100%	0.0050	-13.65	-13.77	93%	0.0023
200×200	-31.62	-31.71	97%	0.0021	-31.62	-31.71	100%	0.0050	-31.34	-31.61	95%	0.0022
500×500	-73.31	-73.38	86%	0.0025	-73.31	-73.38	100%	0.0065	-72.36	-72.48	94%	0.0026
1000×1000	-142.7	-142.7	82%	0.0042	-142.7	-142.7	100%	0.009	-142.6	-142.6	100%	0.0047

Method / Metric	LT-P Obj Mean	LT-P Obj Median	LT-P % Feasible	LT-P Time (Sec)	EX Obj Mean	EX Obj Median	EX % Feasible	EX Time (Sec)	N1 Obj Mean	N1 Obj Median	N1 % Feasible	N1 Time (Sec)
100×100	-13.65	-13.77	100%	0.0100	-20.79	-20.78	100%	1237	1.5E+18	1.4E+18	100%	104.2
200×200	-31.34	-31.61	100%	0.0064	-	-	-	-	-	-	-	-
500×500	-72.36	-72.48	100%	0.0063	-	-	-	-	-	-	-	-
1000×1000	-142.6	-142.6	100%	0.0086	-	-	-	-	-	-	-	-

More Experiments

Milliseconds Inference (with 100sec Training)

Result for IQPs. Each ... ated on a test set of 100 instance

Method	RC				RC-P				LT			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.54	-13.6	96%	0.0022	-13.54	-13.57	100%	0.0050	-13.65	-13.77	93%	0.0023
200×200	-31.62	-31.71	97%	0.0021	-31.62	-31.71	100%	0.0050	-31.34	-31.61	95%	0.0022
500×500	-73.31	-73.38	86%	0.0025	-73.31	-73.38	100%	0.0065	-72.36	-72.48	94%	0.0026
1000×1000	-142.7	-142.7	82%	0.0042	-142.7	-142.7	100%	0.0090	-142.6	-142.6	100%	0.0047

Method	LT-P				EX				N1			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.65	-13.77	100%	0.0100	-20.79	-20.78	100%	1237	1.5E+18	1.4E+18	100%	104.2
200×200	-31.34	-31.61	100%	0.0064	-	-	-	-	-	-	-	-
500×500	-72.36	-72.48	100%	0.0063	-	-	-	-	-	-	-	-
1000×1000	-142.6	-142.6	100%	0.0086	-	-	-	-	-	-	-	-

More Experiments

100% Feasibility With Projection

Result for IQPs. Each [illegible] luated on a test set of 100 instance

Method	RC				RC-P				LT			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.54	-13.6	96%	0.0022	-13.54	-13.57	100%	0.0050	-13.65	-13.77	93%	0.0023
200×200	-31.62	-31.71	97%	0.0021	-31.62	-31.71	100%	0.0050	-31.34	-31.61	95%	0.0022
500×500	-73.31	-73.38	86%	0.0025	-73.31	-73.38	100%	0.0065	-72.36	-72.48	94%	0.0026
1000×1000	-142.7	-142.7	82%	0.0042	-142.7	-142.7	100%	0.0090	-142.6	-142.6	100%	0.0047

Method	LT-P				EX				N1			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.65	-13.77	100%	0.0100	-20.79	-20.78	100%	1237	1.5E+18	1.4E+18	100%	104.2
200×200	-31.34	-31.61	100%	0.0064	-	-	-	-	-	-	-	-
500×500	-72.36	-72.48	100%	0.0063	-	-	-	-	-	-	-	-
1000×1000	-142.6	-142.6	100%	0.0086	-	-	-	-	-	-	-	-

More Experiments

We can even achieve better!

Result for IQPs. Each … luated on a test set of 100 instance

Method	RC				RC-P				LT			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.54	-13.6	96%	0.0022	-13.54	-13.57	100%	0.0050	-13.65	-13.77	93%	0.0023
200×200	-31.62	-31.71	97%	0.0021	-31.62	-31.71	100%	0.0050	-31.34	-31.61	95%	0.0022
500×500	-73.31	-73.38	86%	0.0025	-73.31	-73.38	100%	0.0065	-72.36	-72.48	94%	0.0026
1000×1000	-142.7	-142.7	82%	0.0042	-142.7	-142.7	100%	0.009	-142.6	-142.6	100%	0.0047

Method	LT-P				EX				N1			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	-13.65	-13.77	100%	0.0100	-20.79	-20.78	100%	1237	1.5E+18	1.4E+18	100%	104.2
200×200	-31.34	-31.61	100%	0.0064	-	-	-	-	-	-	-	-
500×500	-72.36	-72.48	100%	0.0063	-	-	-	-	-	-	-	-
1000×1000	-142.6	-142.6	100%	0.0086	-	-	-	-	-	-	-	-

More Experiments

Result for INPs. Each problem size is evaluated on a test set of 100 instance

Method	RC				RC-P				LT			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	1.664	1.594	100%	0.0022	1.664	1.594	100%	0.0060	0.669	0.649	96%	0.0021
200×200	1.472	1.436	99%	0.0022	1.471	1.436	100%	0.0054	-0.356	-0.373	100%	0.0023
500×500	0.526	0.526	96%	0.0029	0.524	0.526	100%	0.0061	-1.374	-1.594	98%	0.0029
1000×1000	1.423	0.809	97%	0.0040	1.423	0.809	100%	0.0115	-3.744	-3.716	99%	0.0050

Method	LT-P				EX				N1			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
100×100	0.669	0.649	100%	0.0058	256.93	134.62	14%	1001	4411	155.2	14%	940.4
200×200	-0.356	-0.373	100%	0.0056	-	-	-	-	-	-	-	-
500×500	-1.374	-1.594	100%	0.0072	-	-	-	-	-	-	-	-
1000×1000	-3.744	-3.716	100%	0.0117	-	-	-	-	-	-	-	-

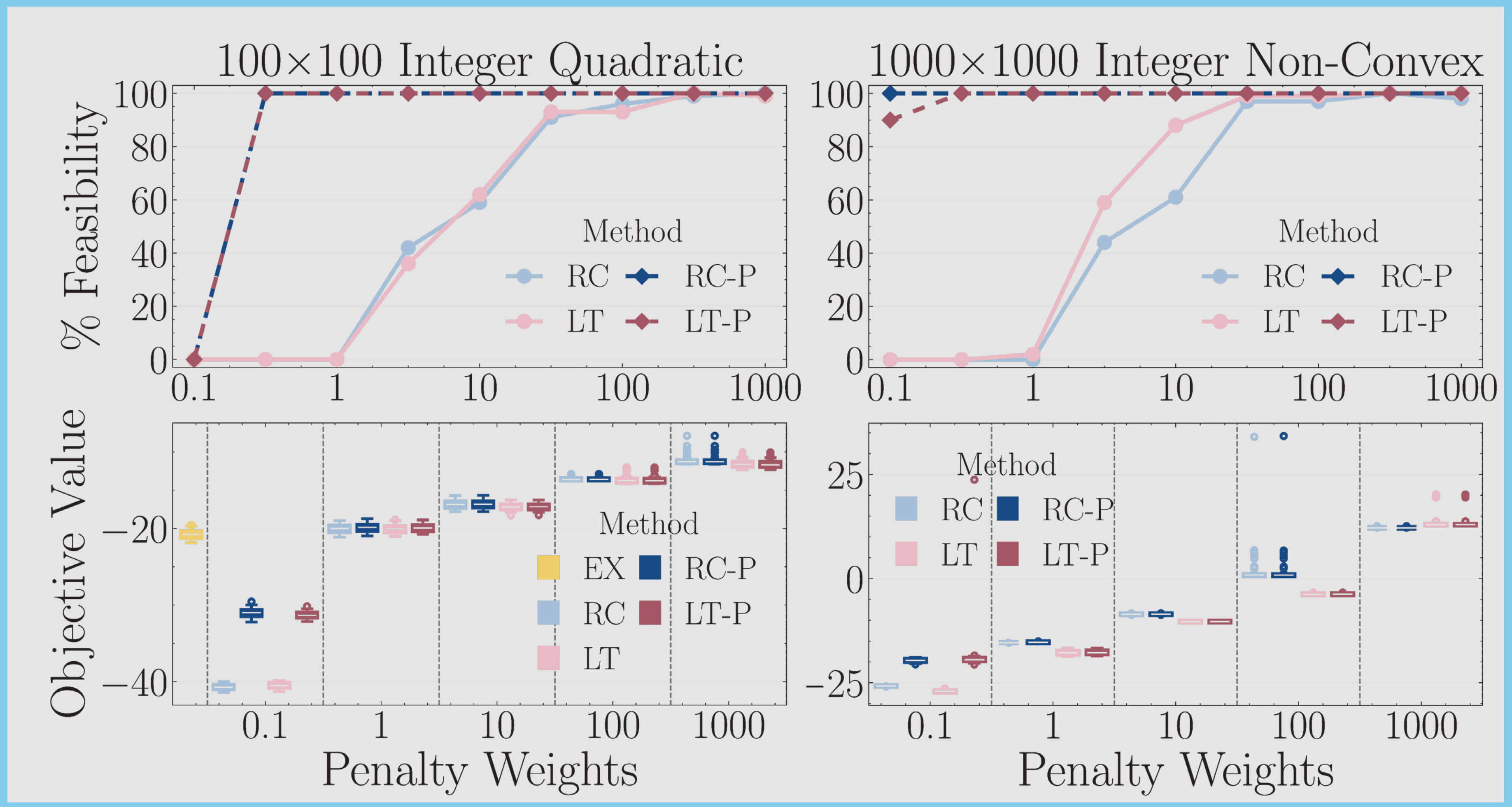
More Experiments

Result for MIRBs. Each problem size is evaluated on a test set of 100 instance

Method	RC				RC-P				LT			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
20×4	59.39	48.86	100%	0.0019	59.39	48.86	100%	0.0048	62.51	63.40	100%	0.0020
200×4	503.5	461.7	99%	0.0021	504.2	461.7	100%	0.0052	622.8	626.0	100%	0.0026
2000×4	5938	5792	99%	0.0033	5942	5792	100%	0.0070	5612	5558	97%	0.0030
20000×4	6.7E+4	6.7E+4	76%	0.0121	9.8E+4	7.3E+4	100%	0.0824	4.8E+4	3.5E+4	66%	0.0127

Method	LT-P				EX				N1			
Metric	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)	Obj Mean	Obj Median	% Feasible	Time (Sec)
20×4	62.51	63.40	100%	0.0055	64.67	59.16	100%	1005	87.83	77.34	100%	0.0813
200×4	622.8	626.0	100%	0.0062	8.4E+5	908.8	100%	1002	3.7E+8	957.4	100%	0.2608
2000×4	5615	5558	100%	0.0071	4.7E+10	9262	96%	1002	8.3E+12	9379	95%	71.91
20000×4	8.0E+4	4.5E+4	100%	0.0639	1.1E+15	1.0E5	78%	1040	1.2E+15	1.0E5	78%	78[illegible]

Effect of Penalty Weight



There is an inherent trade-off between achieving more feasible solutions and lower objective values prior to the integer feasibility projection (RC and LT).

After applying the integer feasibility projection (RC-P and LT-P), the high infeasibility rates observed even with smaller penalty weights.

Thank You

Paper

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