Design and Analysis of Algorithms: Homework

Released/Deadline: Dec 17, 2020/Jan 15, 2021
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Title Format: homework - student_name

Q1. Prove that $n^5 = O(2^n)$.

Q2. Prove that $100n + \log n = O(n + (\log n)^2)$.

Q3. Consider $T(n) = \sum_{i=0}^{d} a_i \cdot n^i$, with constants $d > 0$ and $a_i > 0$ for all i. Prove that for any $k \geq d$, it holds that

$$T(n) = O(n^k).$$

Q4. Show that for any real constant $a > 0$ and $b > 0$, it holds that

$$(n + a)^b = O(n^b).$$

You can assume that a and b are all integers.

Q5. Describe the details of applying counting-sort with the following numbers,

$$1, 5, 4, 7, 2, 2, 1, 1, 4, 3, 1, 2, 9$$

which are selected from $\{0, 1, 2, \cdots, 9\}$.

Q6. Recall that given a sequence $A = a_0 a_1 \cdots a_{n-1}$ of size n, a subsequence has the form of $a_{i_0} a_{i_1} \cdots a_{i_{k-1}}$ with $i_0 < i_1 < \cdots < i_{i-1}$. Given a string $B = b_0 b_1 \cdots b_{m-1}$ with $m < n$, describe an algorithm to determine if B is a subsequence of A. Analyze the complexity of your algorithm in the big-O notation.

Q7. $T(n) = 10 \cdot T(n/3) + n^{2.5}$. Determine the order of $T(n)$, in the big-Θ notation.

Q8. Let $F(n)$ be the number of "hello" printed by algorithm $f(n)$. For example, $F(0) = F(1) = 0$, $F(2) = 1$.

Algorithm 1: $f(n)$

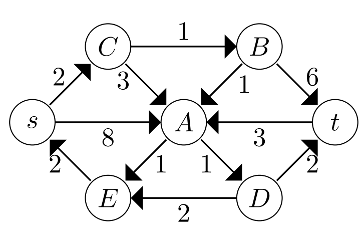
```
if n > 1 then
    print "Hello";
    f(n/2);
    f(n/2);
```

Determine the order of $F(n)$ in the big-Θ notation

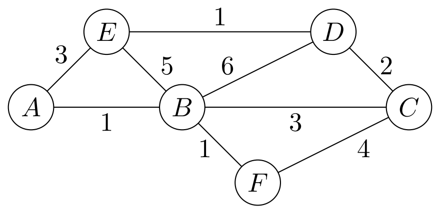
Q9. Give the pseudocode of the dynamic programming algorithm for solving the longest common subsequence problem.

Q10. Give the pseudocode of the dynamic programming algorithm for solving the sequence alignment problem.

Q11. Calculate the length of the shortest paths from s to t, using Dijkstra's algorithm (for the directed graph) step by step.



Q12. Find a minimum spanning spanning tree.



Q13. Find a maximum bipartite matching.