Design and Analysis of Algorithms

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Algorithms on Directed Graphs

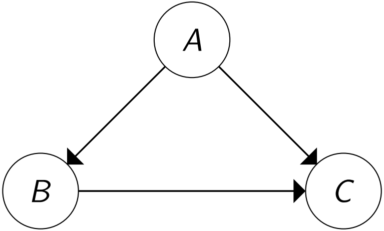
Directed Graph

Directed Graph

Directed Graph: a set of **nodes** connected by the **directed edges**.

$G = (V, E)$

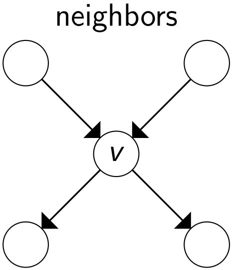
- V: the set of nodes
  - A, B, and C
- E: the set of edges
  - (A, B), (B, C), and (A, C)



Neighbors

Given a graph $G = (V, E)$, for node $v \in V$

- in-neighbors: $u \in V$, s.t. $(u, v) \in E$
- out-neighbors: $u \in V$, s.t. $(v, u) \in E$



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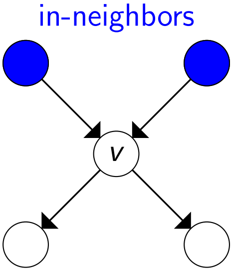

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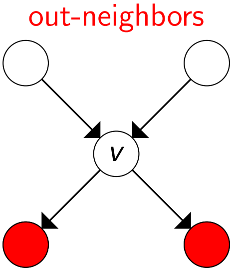


Traversal on Directed Graphs

DFS: Visit node v

- For every **out-neighbor** u of v
  - If u is not visited
    - DFS on u

BFS: pop node v from the queue

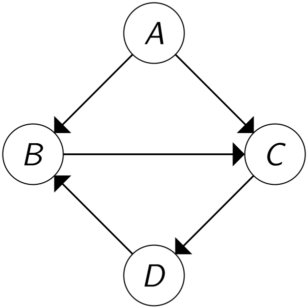
- For every **out-neighbor** u of v
  - If u is not visited
    - Visit u
    - add u to the queue

Connectivity

Connected: two nodes u and v are connected, iff.

- there is a path from u to v;
- there is a path from v to u.

Strongly Connected: all pairs of nodes are connected, i.e. for any pair of nodes u and v, there is a directed path from u to v.



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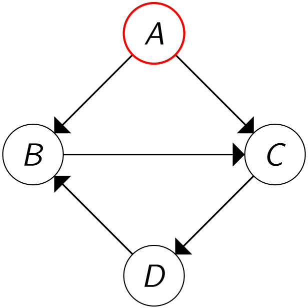


Connectivity

"**Connected**" is an equivalence relation.

- Reflexive: v is connected to v, for all $v \in V$.
- Symmetric: v is connected to $u \Rightarrow u$ is connected to v
- Transitive: if
    - v is connected to u
    - u is connected to w

  then v is connected to w

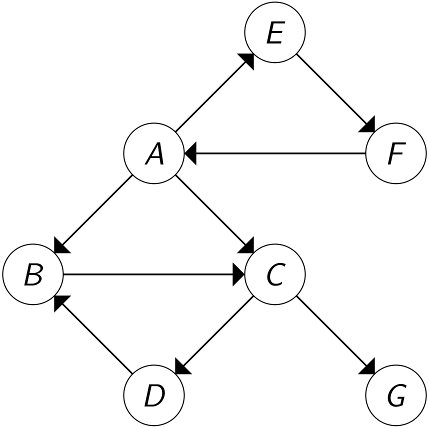
Connectivity

With an equivalence relation, the set can be divided into disjoint parts: in each part, the elements are related.

Strongly connected components: V is divided into several disjoint parts $V_0, V_1, \ldots$, such that

- all nodes in V_i are connected
- for any pair of nodes $u \in V_i$ and $v \in V_j$ with $i \neq j$, u is not connected to v.

Connectivity



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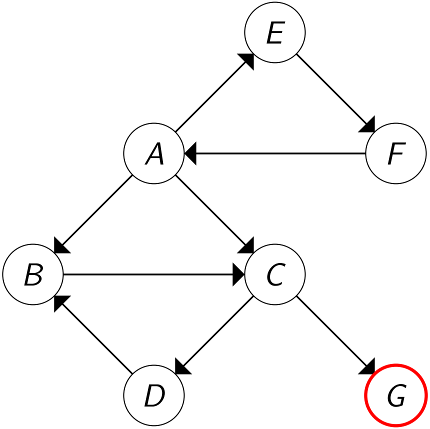

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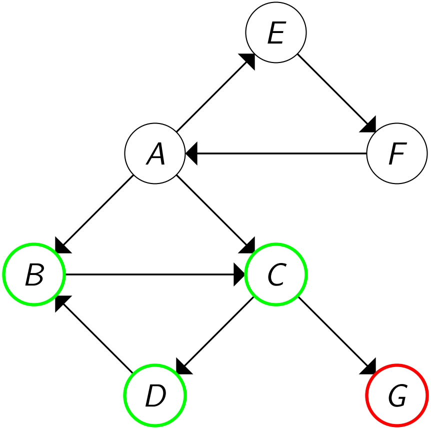

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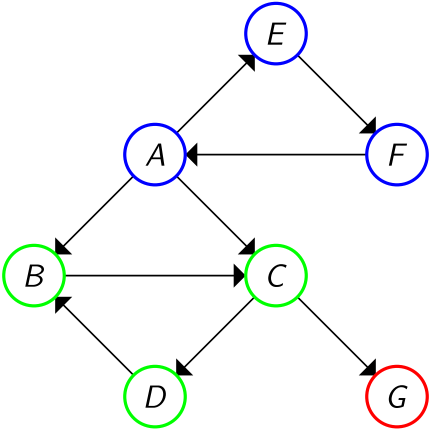

Strongly Connected Component

Strongly Connected Component

problem: given a directed graph $g = (v, e)$, find the strongly connected components, i.e. dividing v into $v_0, v_1, \ldots$, such that

- all nodes in v_i are connected
- for any pair of nodes $u \in v_i$ and $v \in v_j$ with $i \neq j$, u is not connected to v.

Component Graph

Component Graph: given a directed graph $G = (V, E)$, the components graph is defined as $G^* = (V^*, E^*)$ where

- V^*: one node for each strongly connected component
- E^*: there is an edge from node V_i to node V_j iff. there exists $u \in V_i$ and $v \in V_j$ with $(u, v) \in E$

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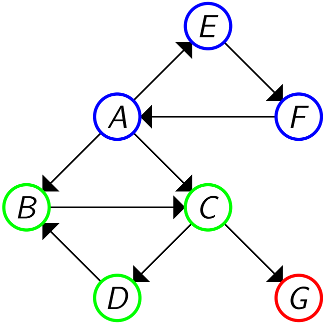

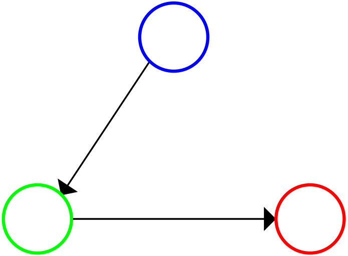

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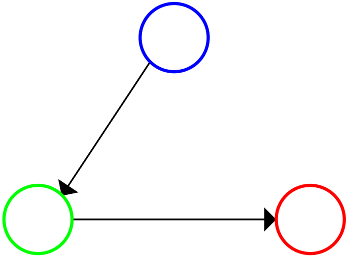

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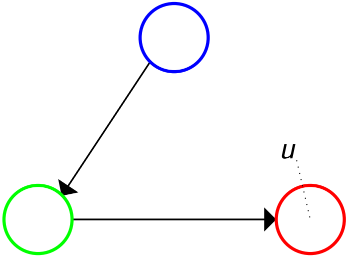
Component Graph



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- For a node u in the sink component, all nodes v connected from u are all in the sink component.

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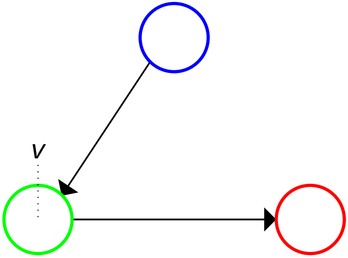

- The component graph has no cycles.
- For a node u in the sink component, all nodes v connected from u are all in the sink component.
- Then we consider the component previous to the sink component.

Component Graph

Sink: the component that has no outgoing edge.

Algorithm: Components(G)

$S = [\,]$;
while $\cup_{C \in S} C \neq V$ **do**
    let s be any node in the sink component;
    run BFS from s;
    let C be the set of visited nodes;
    append C to S;
    remove nodes and edges adjacent to C from G;
end
Return S;

Find the Sink

clock $= 0$

Modified DFS

- Visit node v
  - $pre(v) =$ **clock**
  - **clock** $+= 1$
  - For every out-neighbor u of v
    - If u is not visited: Modified DFS on u
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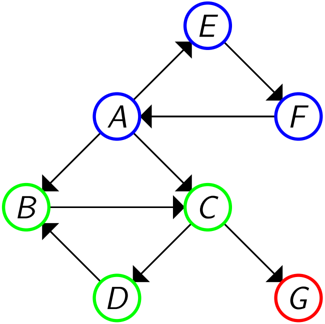

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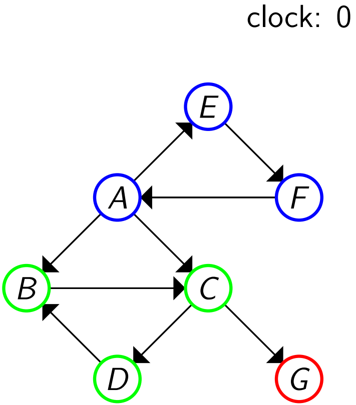

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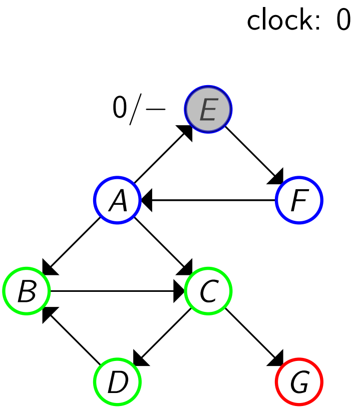

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clock: 1

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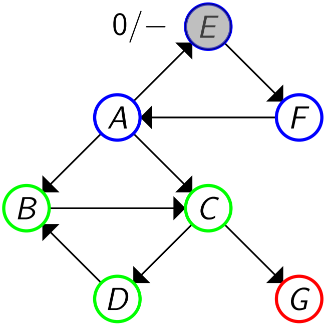

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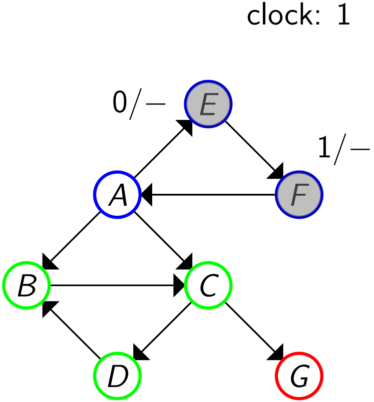

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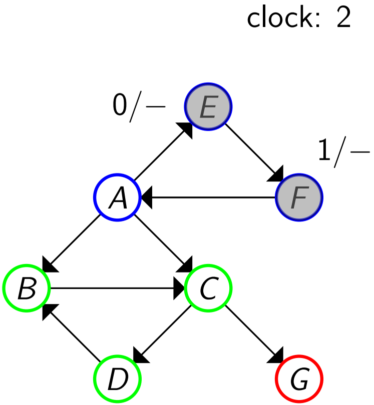

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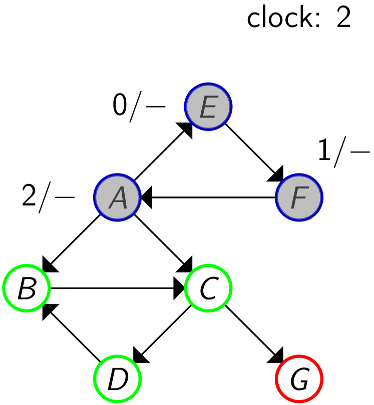

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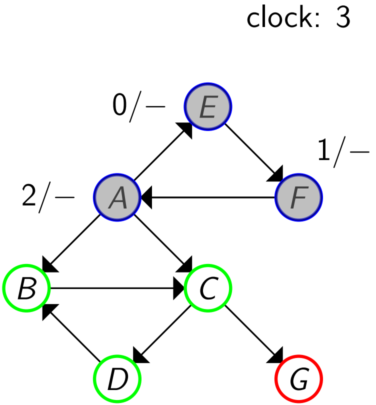

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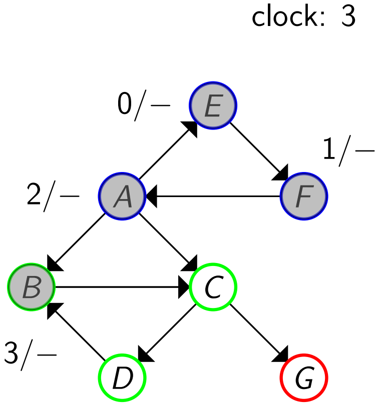

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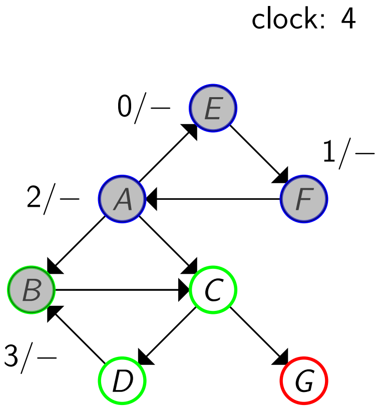

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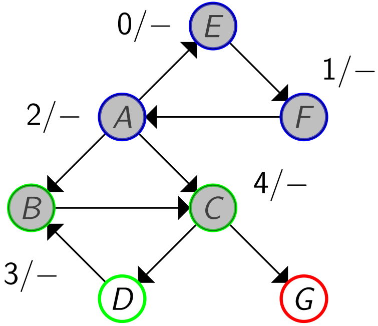
clock: 4



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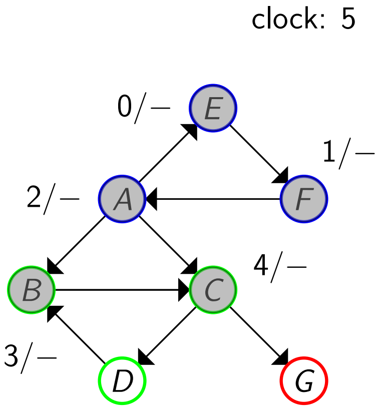

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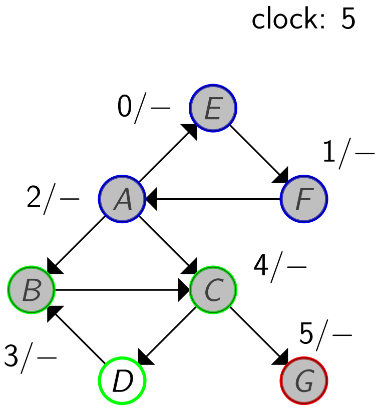

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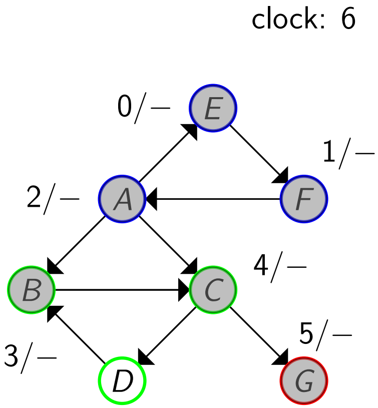

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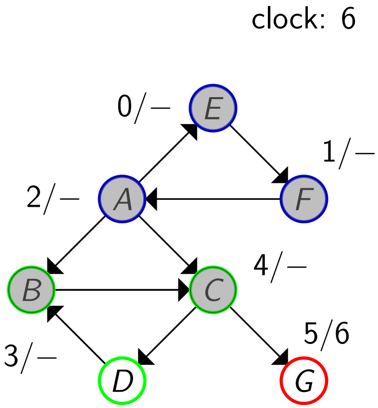

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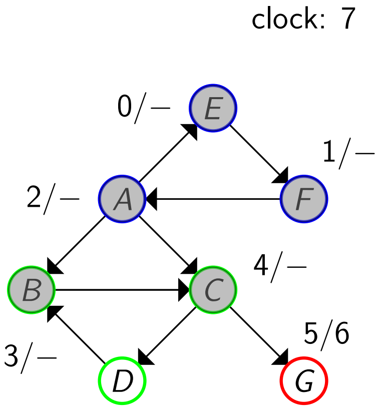

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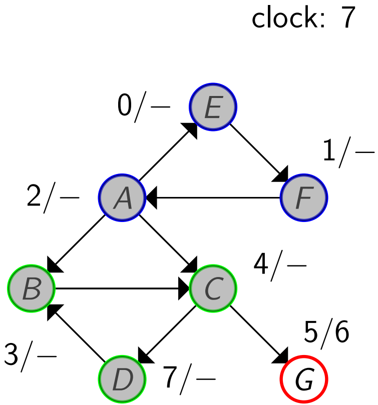

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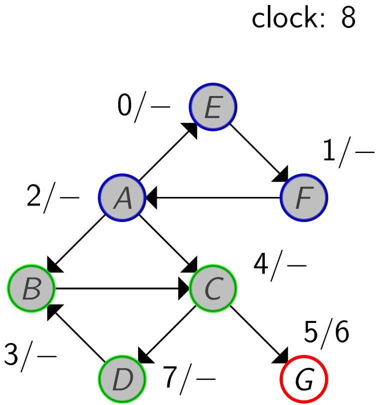

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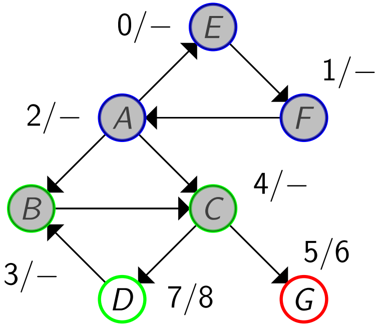
clock: 8



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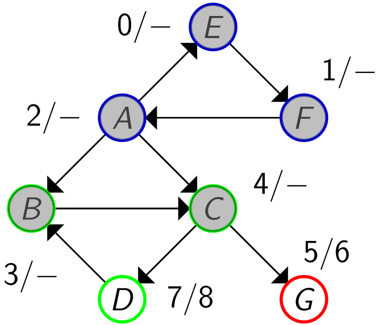

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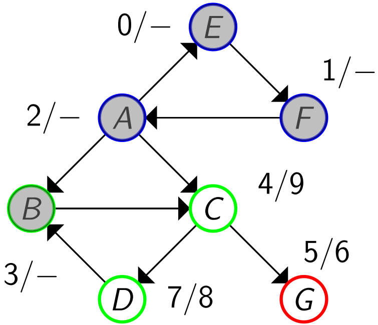

Find the Sink

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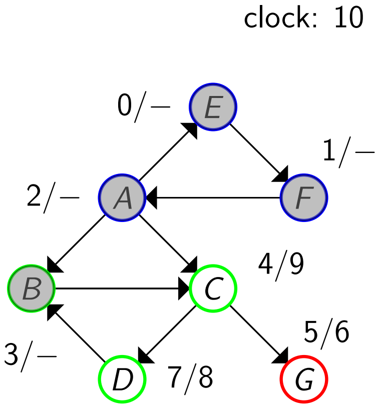

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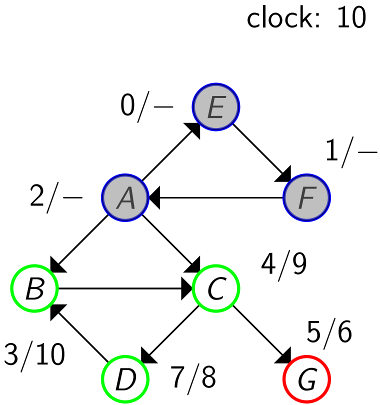

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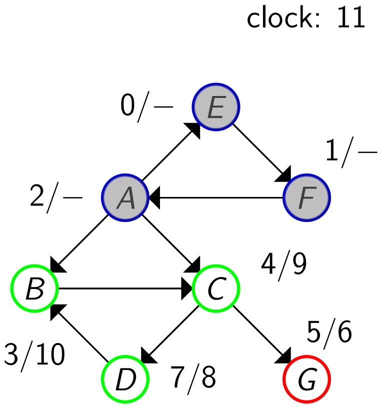

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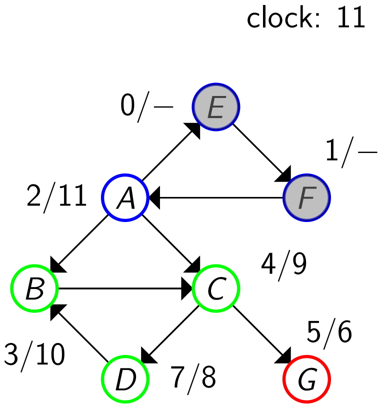

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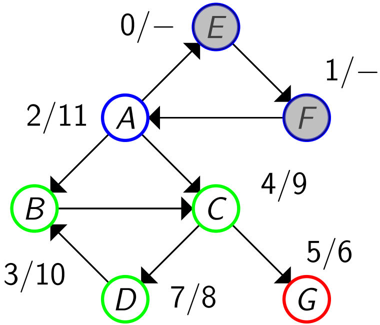
    - If u is not visited: Modified DFS on u
  - $post(v) =$ **clock**
  - **clock** $+= 1$

clock: 12



Find the Sink

clock $= 0$

Modified DFS

- Visit node v
  - $pre(v) =$ **clock**
  - **clock** $+= 1$
  - For every out-neighbor u of v
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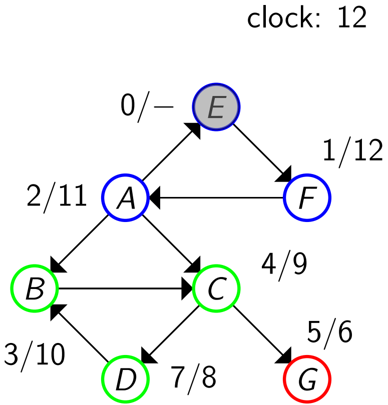
  - $post(v) =$ **clock**
  - **clock** $+= 1$



Find the Sink

clock $= 0$

Modified DFS

- Visit node v
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  - **clock** $+ = 1$
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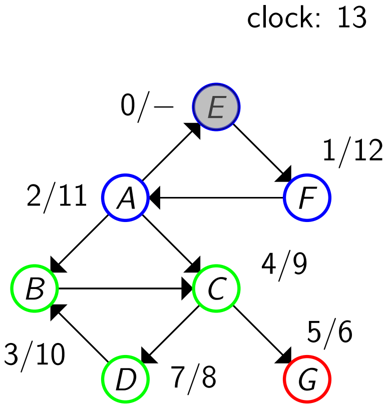

Find the Sink

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Modified DFS

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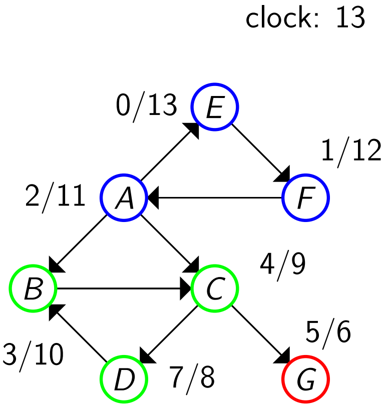
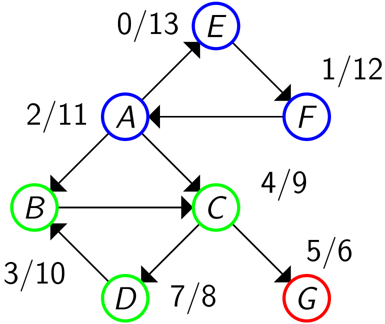


Post Clock

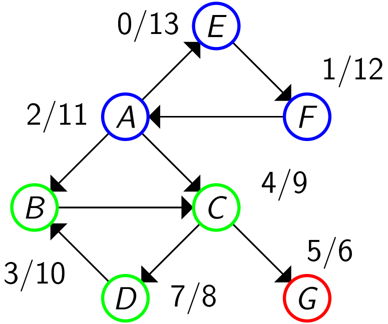
Largest *post*: node in a source component



Post Clock

Largest *post*: node in a source component

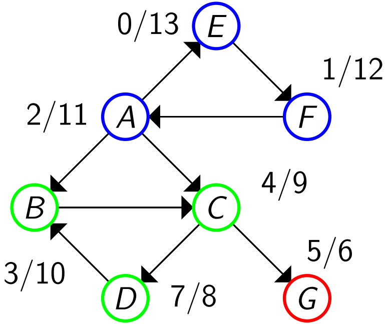
- pre(*source*)



Post Clock

Largest *post*: node in a source component

- pre(*source*)
- visit other components



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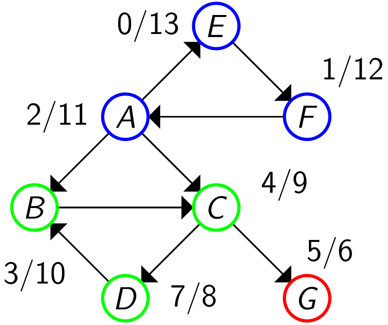

Find Strongly Connected Components

Largest *post*: node in a source component

- pre(*source*)
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Find Strongly Connected Components

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- pre(*source*)
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1. G^R : reverse all edges of G

Find Strongly Connected Components

Largest *post*: node in a source component

- pre(*source*)
- visit other components
- post(*source*)

1. G^R : reverse all edges of G
2. **Modified DFS** on G^R

Find Strongly Connected Components

Largest *post*: node in a source component

- pre(*source*)
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- post(*source*)

1. G^R : reverse all edges of G
2. **Modified DFS** on G^R
3. v : node of largest *post*

Find Strongly Connected Components

Largest *post*: node in a source component

- pre(*source*)
- visit other components
- post(*source*)

1. G^R : reverse all edges of G
2. **Modified DFS** on G^R
3. v : node of largest *post*
4. v in the sink component of G

Find Strongly Connected Components

Largest *post*: node in a source component

- pre(*source*)
- visit other components
- post(*source*)

1. G^R : reverse all edges of G
2. **Modified DFS** on G^R
3. v : node of largest *post*
4. v in the sink component of G
5. remove all nodes connected from v

Find Strongly Connected Components

Largest *post*: node in a source component

- pre(*source*)
- visit other components
- post(*source*)

1. G^R : reverse all edges of G
2. **Modified DFS** on G^R
3. v : node of largest *post*
4. v in the sink component of G
5. remove all nodes connected from v
6. next? repeat $1-5$ on the remaining graph

Find Strongly Connected Components

Largest *post*: node in a source component

- pre(*source*)
- visit other components
- post(*source*)

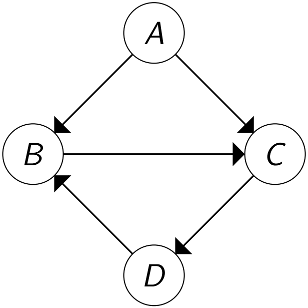
1. G^R : reverse all edges of G
2. **Modified DFS** on G^R
3. v : node of largest *post*
4. v in the sink component of G
5. remove all nodes connected from v
6. next? repeat $1-5$ on the remaining graph
7. **or**, largest *post* among the remaining nodes

Directed Cycle

Directed Cycle

Directed Cycle: a closed path consisting of directed edges.

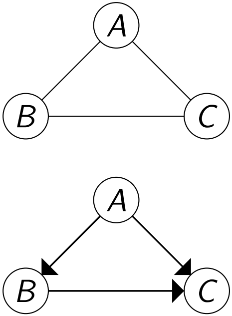
- $B - C - D - B$
  - (B, C) is in E
  - (C, D) is in E
  - (D, B) is in E



Directed Cycle

Directed Cycle: a closed path consisting of directed edges.

- Undirected: A - B - C - A
- Directed: A - B - C - A
    - (A, B) is in E
    - (B, C) is in E
    - (C, A) is not in E

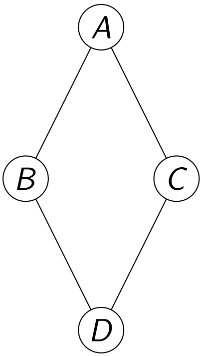


Cycle in Undirected Graphs

Problem: check if the given undirected graph contains a cycle.

DFS

- explore the children after visiting a node
- **cycle** $\Leftrightarrow$ visited child



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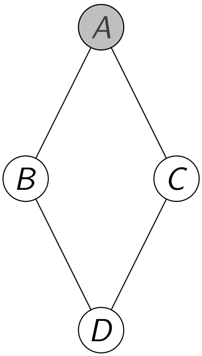

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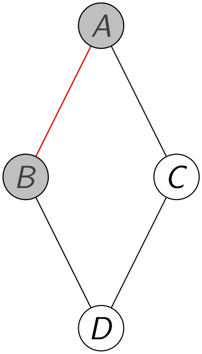

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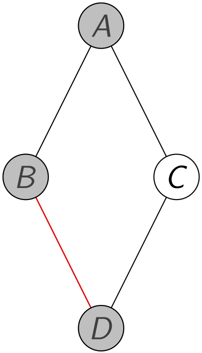

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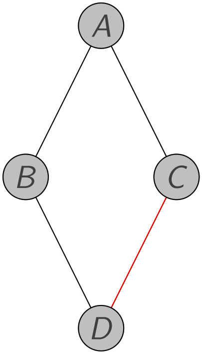

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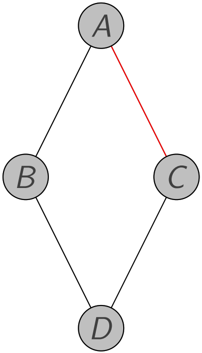

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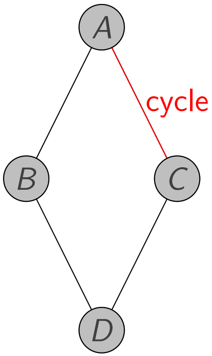

Cycle in Directed Graphs

Problem: check if the given directed graph contains a cycle.

Modified DFS

- Visit node v
  - $pre(v) =$ **clock**
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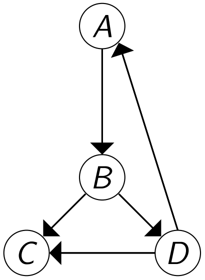

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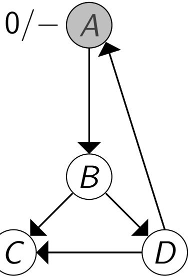

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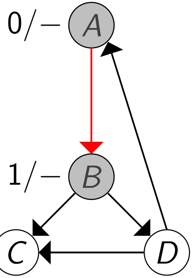

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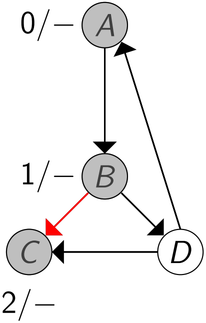

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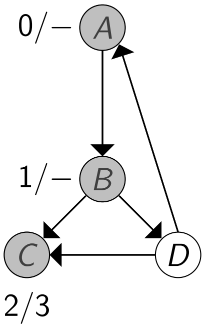

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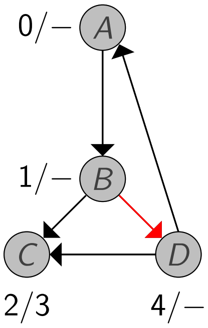

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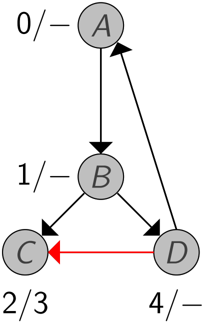

cross edge; not a cycle

Cycle in Directed Graphs

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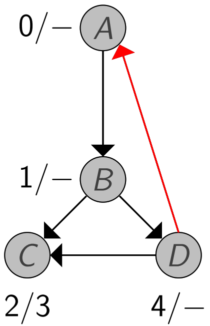


back edge; found a cycle

DAG: Directed Acyclic Graph

Conclusion: A directed graph has a cycle if and only if the DFS reveals a **back edge**.

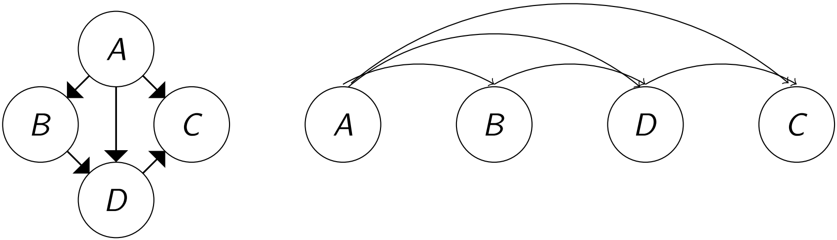
Directed Acyclic Graph: a directed graph contains no cycles.

Topological Sort

Directed Acyclic Graph

Problem: given a directed acyclic graph (DAG) $G = (V, E)$, list the nodes in a sequence, such that for any edge $(u, v) \in E$, node u precedes node v, i.e.

$$u \prec v, \forall (u, v) \in E$$



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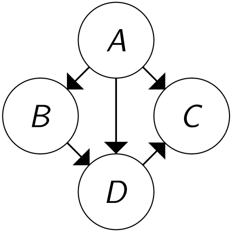

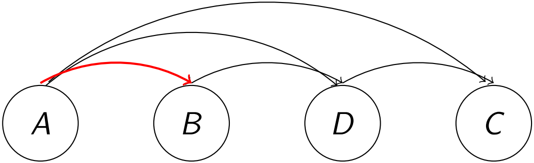

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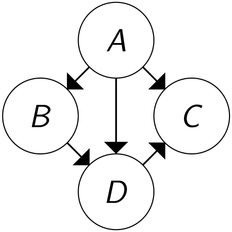

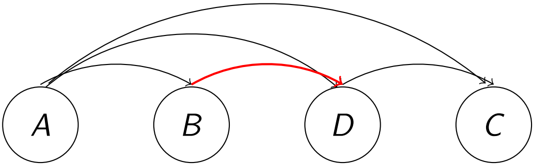

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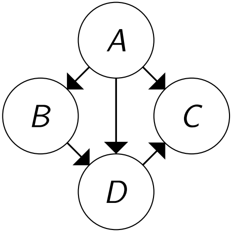

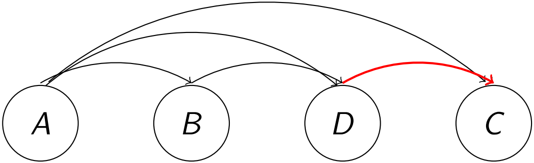

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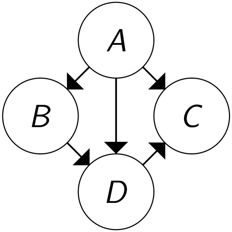

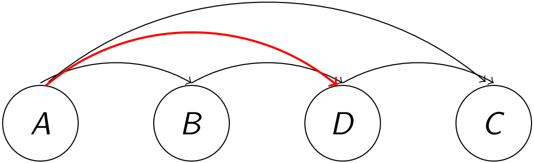

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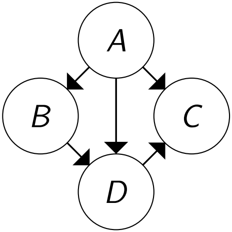
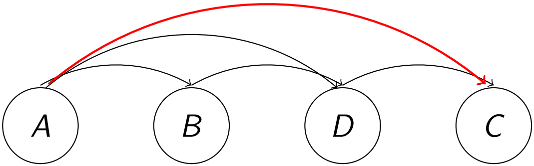
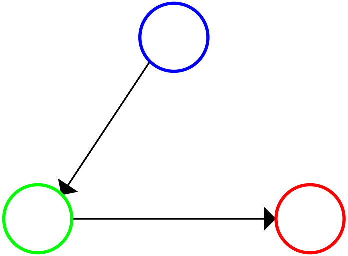




Strongly Connected Components



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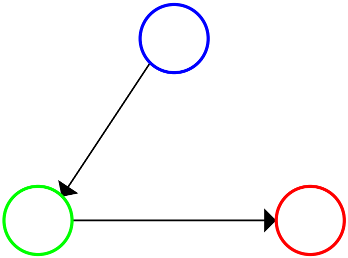
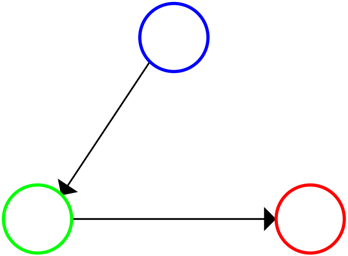


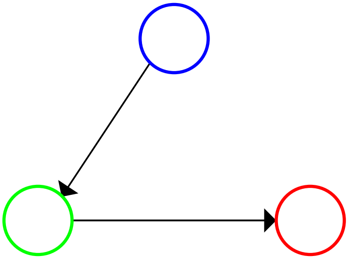
- Find the node of the largest post time.

Strongly Connected Components



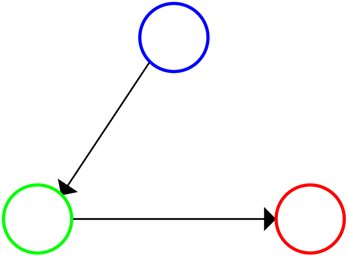
- Find the node of the largest post time.
- Remove adjacent nodes and edges?

Strongly Connected Components



- Find the node of the largest post time.
- Remove adjacent nodes and edges?
  - Not necessary!

Strongly Connected Components

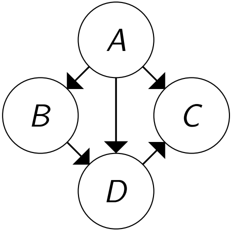


- Find the node of the largest post time.
- Remove adjacent nodes and edges?
    - Not necessary!
- Find the node of the largest post time **in the remaining ones**.

Topological Sort

DFS: Start at node r

- Visit node r
- For every **out-neighbor** v of r
  - If v is not visited
    - DFS on v



$S = [\,]$, the sorted the sequence

TopologicalSort: Start at node r

- Visit node r
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- put r at the front of S

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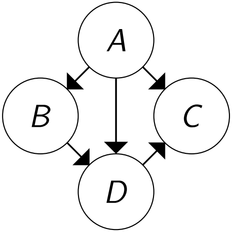

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Visit A

Topological Sort

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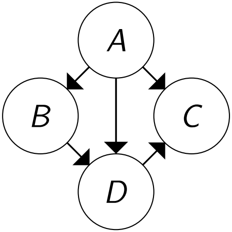

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$S = [\ ]$, the sorted the sequence

TopologicalSort: Start at node r

- Visit node r
- For every **out-neighbor** v of r
  - If v is not visited
    - TopologicalSort on v
- put r at the front of S

Visit A

- Visit B

Topological Sort

DFS: Start at node r

- Visit node r
- For every **out-neighbor** v of r
  - If v is not visited
    - DFS on v

$S = [\,]$, the sorted the sequence

TopologicalSort: Start at node r

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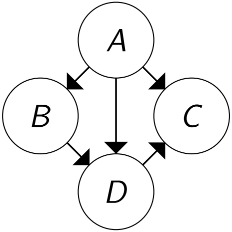


$S = [\,]$

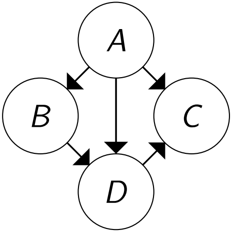
Visit A

- Visit B
  - Visit D

Topological Sort

DFS: Start at node r

- Visit node r
- For every **out-neighbor** v of r
  - If v is not visited
    - DFS on v



$S = [\,]$

$S = [\,]$, the sorted the sequence

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- put r at the front of S

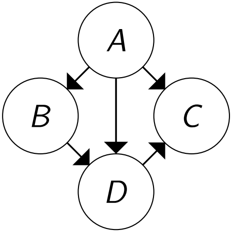
Visit A

- Visit B
  - Visit D
    - Visit C

Topological Sort

DFS: Start at node r

- Visit node r
- For every **out-neighbor** v of r
  - If v is not visited
    - DFS on v



$S = [C]$

$S = [\,]$, the sorted the sequence

TopologicalSort: Start at node r

- Visit node r
- For every **out-neighbor** v of r
  - If v is not visited
    - TopologicalSort on v
- put r at the front of S

Visit A

- Visit B
  - Visit D

Topological Sort

DFS: Start at node r

- Visit node r
- For every **out-neighbor** v of r
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$S = [\ ]$, the sorted the sequence

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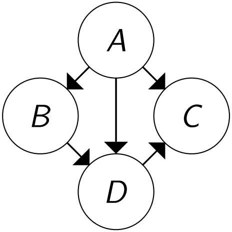


$S = [D, C]$

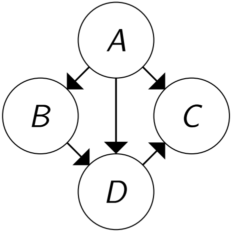
Visit A

- Visit B

Topological Sort

DFS: Start at node r

- Visit node r
- For every **out-neighbor** v of r
  - If v is not visited
    - DFS on v



$S = [B, D, C]$

$S = [\ ]$, the sorted the sequence

TopologicalSort: Start at node r

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Visit A

Topological Sort

DFS: Start at node r

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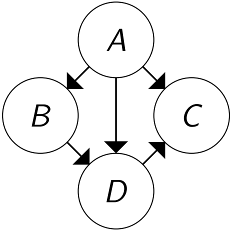

$S = [A, B, D, C]$

THANK YOU

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