Homework 4

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T1

$$x_1 = \max(1.0 * 0.3 + 0.1, 0) = 0.4$$

 $y_1 = 0.4 * (-0.2) - 0.3 = -0.38$
 $z = \max(-0.38 + 1.0, 0) = 0.62$

$$\begin{aligned} \frac{\partial z}{\partial y_1} &= 1\\ \frac{\partial z}{\partial x_1} &= \frac{\partial z}{\partial y_1} \frac{\partial y_1}{\partial x_1} &= 1 * w_1 \end{aligned}$$

∴.

$$\begin{split} \frac{\partial z}{\partial w_0} &= \frac{\partial z}{\partial x_1} \frac{\partial x_1}{\partial w_0} = w_1 * x_0 = -0.2 \\ \frac{\partial z}{\partial w_1} &= \frac{\partial z}{\partial y_1} \frac{\partial y_1}{\partial w_1} = 1 * x_1 = 0.4 \\ \frac{\partial z}{\partial b_0} &= \frac{\partial z}{\partial x_1} \frac{\partial x_1}{\partial b_0} = w_1 * 1 = -0.2 \\ \frac{\partial z}{\partial b_1} &= \frac{\partial z}{\partial y_1} \frac{\partial y_1}{\partial b_1} = 1 * 1 = 1 \end{split}$$

T2

Input: (32,30)A: (32,1024)B: (32,512)C: (32,1)

T3

1024(30+1) + 512(1024+1) + 1(512+1) = 557,057 parameters

T4

Proof.

$$L = -\sum_{j} y_j \log P(y = j)$$

$$L = -\sum_{j} y_j (h_j - \log \sum_{k} \exp(h_k))$$

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$$\frac{\partial L}{\partial h_i} = -\sum_j y_j \left(\frac{\partial}{\partial h_i} h_j - \frac{\frac{\partial}{\partial h_i} \sum_k \exp(h_k)}{\sum_k \exp(h_k)} \right)$$

$$\frac{\partial L}{\partial h_i} = -\sum_j y_j \left(\frac{\partial}{\partial h_i} h_j - \frac{\exp(h_i)}{\sum_k \exp(h_k)} \right)$$

$$\frac{\partial L}{\partial h_i} = -y_i + \sum_j y_j \frac{\exp(h_i)}{\sum_k \exp(h_k)}$$

$$\frac{\partial L}{\partial h_i} = -y_i + \frac{\exp(h_i)}{\sum_k \exp(h_k)}$$

$$\therefore \sum_j y_j = 1$$

$$\frac{\partial L}{\partial h_i} = P(y = i) - y_i$$

Simple Neural Network Lab

March 7, 2024

1 Two-Layer Neural Networks

In this part of the homework, we will work on building a simple Neural Network to classify digits using MNIST dataset. There are many powerful neural network frameworks nowadays that are relatively straightforward to use. However, we believe that in order to understand the theory behind neural networks, and to have the right intuitions in order to build, use, and debug more complex architectures, one must first start from the basics.

Your task is to complete the missing codes needed for training and testing of a simple fully-connected neural network.

The missing parts will be marked with TODO#N, where N represents the task number.

Many parts in this homework are modified from Stanford's cs231n assignments.

```
[1]: import numpy as np
import matplotlib.pyplot as plt

from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

from cattern.neural_net import TwoLayerNet

%load_ext autoreload
%autoreload 2
```

```
[2]: # Just a function that verifies if your answers are correct

def rel_error(x, y):
    """ returns relative error """
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

We will use the class TwoLayerNet in the file cattern/neural_net.py to represent instances of our network. The network parameters (weights and biases) are stored in the instance variable self.params where the keys are parameter names and values are numpy arrays. Below, we initialize some toy data and a toy model that will guide you with your implementation.

```
[3]: # Create a small net and some toy data to check your implementations.
     # Note that we set the random seed for repeatable experiments.
     input_size = 4
     hidden_size = 10
     num_classes = 3
     num_inputs = 5
     def init_toy_model():
         np.random.seed(0)
         return TwoLayerNet(input size, hidden size, num classes, std=1e-1)
     def init_toy_data():
         np.random.seed(1)
         X = 10 * np.random.randn(num_inputs, input_size)
         y = np.array([0, 1, 2, 2, 1])
         return X, y
     net = init_toy_model()
     X, y = init_toy_data()
```

2 Forward pass: compute scores

Open the file cattern/neural_net.py and look at the method TwoLayerNet.loss. This function takes the data and weights and computes the class scores, the loss, and the gradients on the parameters.

Complete TODO#1 in TwoLayerNet.loss, by implementing the first part of the forward pass which uses the weights and biases to compute the scores for all inputs. The scores refer to the output of the network just before the softmax layer.

```
[4]: scores = net.loss(X)
    print('Your scores:')
    print(scores)
    print()
    print('correct scores:')
    correct_scores = np.asarray([
        [-0.81233741, -1.27654624, -0.70335995],
        [-0.17129677, -1.18803311, -0.47310444],
        [-0.51590475, -1.01354314, -0.8504215],
        [-0.15419291, -0.48629638, -0.52901952],
        [-0.00618733, -0.12435261, -0.15226949]])
    print(correct_scores)
    print()

# The difference should be very small. We get < 1e-7
    print('Difference between your scores and correct scores:')</pre>
```

3 Forward pass: compute loss

In the same function, complete TODO#2 by implementing the second part that computes the data and regularization loss.

```
[5]: loss, _ = net.loss(X, y, reg=0.1)
    correct_loss = 1.30378789133

# should be very small, we get < 1e-12
    print('Difference between your loss and correct loss:')
    print(np.sum(np.abs(loss - correct_loss)))
    print(loss, correct_loss)</pre>
```

Difference between your loss and correct loss:

- 1.7985612998927536e-13
- 1.3037878913298202 1.30378789133

4 Backward pass

Implement the rest of the function by completing TODO#3. This will compute the gradient of the loss with respect to the variables W1, b1, W2, and b2. Now that you (hopefully!) have a correctly implemented forward pass, you can debug your backward pass using a numeric gradient check:

```
[6]: from cattern.gradient_check import eval_numerical_gradient

# Use numeric gradient checking to check your implementation of the backward_
pass.

# If your implementation is correct, the difference between the numeric and
```

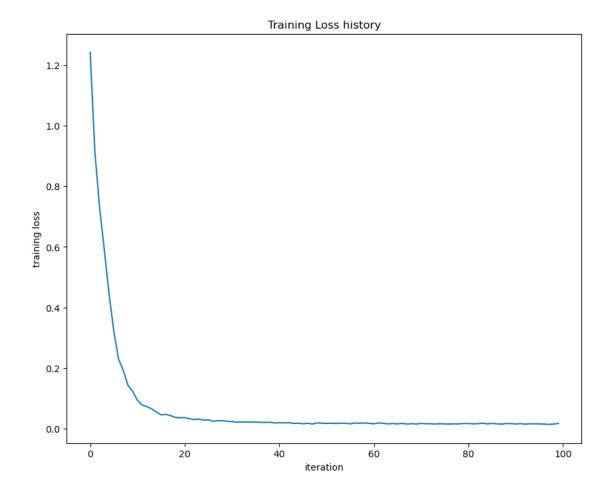
```
W1 max relative error: 4.090897e-09
W2 max relative error: 3.440708e-09
b1 max relative error: 2.738421e-09
b2 max relative error: 3.865080e-11
```

5 Train the network

To train the network we will use stochastic gradient descent (SGD). Look at the function TwoLayerNet.train and fill in the missing sections to implement the training procedure (TODO#4-6). You will also have to implement TwoLayerNet.predict (TODO#7), as the training process periodically performs prediction to keep track of accuracy over time while the network trains.

Once you have implemented the method, run the code below to train a two-layer network on toy data. You should achieve a training loss less than 0.05.

Final training loss: 0.017143643532923757



6 Load the data

Now that you have implemented a two-layer network that passes gradient checks and works on toy data, it's time to load up MNIST data so we can use it to train a classifier on a real dataset.

```
[8]: from mnist_data import load_mnist

def get_mnist_data(num_training=55000, num_validation=5000, num_test=10000):
    """

    Load the MNIST dataset from disk and perform preprocessing to prepare
    it for the two-layer neural net classifier.
    """

# Load the raw MNIST data
X_train, y_train, X_val, y_val, X_test, y_test = load_mnist.
    Gread_data_sets('mnist_data')

# Normalize the data: subtract the mean image
```

```
mean_image = np.mean(X_train, axis=0)
        X_train = X_train - mean_image
        X_val = X_val - mean_image
        X_test = X_test - mean_image
         # Reshape data to rows
        X_train = X_train.reshape(num_training, -1)
        X_val = X_val.reshape(num_validation, -1)
        X_test = X_test.reshape(num_test, -1)
        return X_train, y_train, X_val, y_val, X_test, y_test
     X_train, y_train, X_val, y_val, X_test, y_test = get_mnist_data()
     print('Train data shape: ', X_train.shape)
     print('Train labels shape: ', y_train.shape)
     print('Validation data shape: ', X_val.shape)
     print('Validation labels shape: ', y_val.shape)
     print('Test data shape: ', X_test.shape)
     print('Test labels shape: ', y_test.shape)
    Extracting mnist_data/train-images-idx3-ubyte.gz
    Extracting mnist_data/train-labels-idx1-ubyte.gz
    Extracting mnist_data/t10k-images-idx3-ubyte.gz
    Extracting mnist_data/t10k-labels-idx1-ubyte.gz
    Train data shape: (55000, 784)
    Train labels shape: (55000,)
    Validation data shape: (5000, 784)
    Validation labels shape: (5000,)
    Test data shape: (10000, 784)
    Test labels shape: (10000,)
[9]: from mnist_data.vis_utils import visualize_grid
     # Visualize mnist data
     def show_mnist_image(data):
       data = data.reshape(-1, 28, 28, 1)
      plt.imshow(visualize_grid(data, padding=3).astype('uint8').squeeze(axis=2))
      plt.gca().axis('off')
      plt.show()
     show_mnist_image(X_train[:36])
```



7 Train a network

To train our network we will use SGD with momentum. We will use fixed learning rate to train this model.

```
[10]: input_size = 28 * 28 * 1
hidden_size = 50
num_classes = 10
net = TwoLayerNet(input_size, hidden_size, num_classes)

# Train the network
stats = net.train(X_train, y_train, X_val, y_val,
```

```
iteration 0 / 2000: loss 2.302592
iteration 100 / 2000: loss 2.302232
iteration 200 / 2000: loss 2.300876
iteration 300 / 2000: loss 2.291126
iteration 400 / 2000: loss 2.239348
iteration 500 / 2000: loss 2.008828
iteration 600 / 2000: loss 1.814898
iteration 700 / 2000: loss 1.425479
iteration 800 / 2000: loss 1.268225
iteration 900 / 2000: loss 0.994899
iteration 1000 / 2000: loss 0.847616
iteration 1100 / 2000: loss 0.758159
iteration 1200 / 2000: loss 0.633341
iteration 1300 / 2000: loss 0.536447
iteration 1400 / 2000: loss 0.478063
iteration 1500 / 2000: loss 0.397137
iteration 1600 / 2000: loss 0.415201
iteration 1700 / 2000: loss 0.385851
iteration 1800 / 2000: loss 0.517132
iteration 1900 / 2000: loss 0.425830
Validation accuracy: 0.8922
```

8 Learning Rate Decay

In the previous run, we used the same learning rate during the whole training process. This fix-sized learning rate disregards the benefit of larger learning rate at the beginning of the training, and it might suffer from overshooting around the minima.

Add learning rate decay to the train function, run the model again with larger starting learning rate and learning rate decay, then compare the losses.

```
iteration 0 / 2000: loss 2.302581
iteration 100 / 2000: loss 2.119326
iteration 200 / 2000: loss 0.804760
iteration 300 / 2000: loss 0.492363
iteration 400 / 2000: loss 0.407158
iteration 500 / 2000: loss 0.286767
iteration 600 / 2000: loss 0.318665
iteration 700 / 2000: loss 0.363674
iteration 800 / 2000: loss 0.273462
iteration 900 / 2000: loss 0.278906
iteration 1000 / 2000: loss 0.263698
iteration 1100 / 2000: loss 0.181063
iteration 1200 / 2000: loss 0.257531
iteration 1300 / 2000: loss 0.353448
iteration 1400 / 2000: loss 0.188915
iteration 1500 / 2000: loss 0.165813
iteration 1600 / 2000: loss 0.248227
iteration 1700 / 2000: loss 0.259703
iteration 1800 / 2000: loss 0.292172
iteration 1900 / 2000: loss 0.196248
Validation accuracy: 0.9432
```

9 Debug the training

With the default parameters we provided above, you should get a validation accuracy of about 0.94 on the validation set. This isn't very good for MNIST data which has reports of up to 0.99 accuracy.

One strategy for getting insight into what's wrong is to plot the loss function and the accuracies on the training and validation sets during optimization.

Another strategy is to visualize the weights that were learned in the first layer of the network. In most neural networks trained on visual data, the first layer weights typically show some visible structure when visualized.

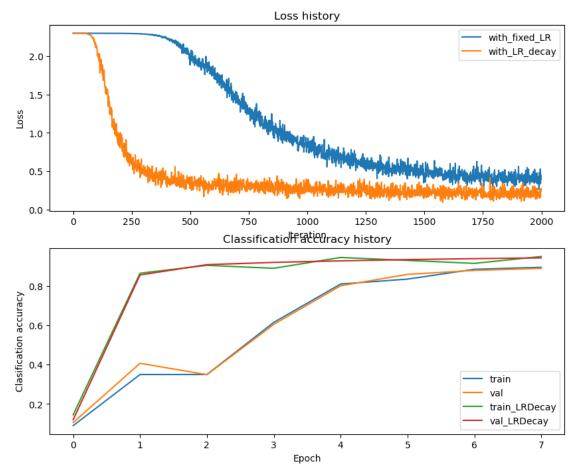
(You can think of the first layer weights as a projection W^TX. This is very similar to how we project our training data using PCA projection in our previous homework. Just like how we visualize the eigenfaces. We can also visualize the weights of the neural network in the same manner.)

Below, we will also show you lossed between two models we trained above. Do you notice the difference between the two?

```
[12]: # Plot the loss function and train / validation accuracies
plt.subplot(2, 1, 1)
plt.plot(stats['loss_history'], label='with_fixed_LR')
plt.plot(stats_LRDecay['loss_history'], label='with_LR_decay')
plt.title('Loss history')
plt.legend()
plt.xlabel('Iteration')
```

```
plt.ylabel('Loss')

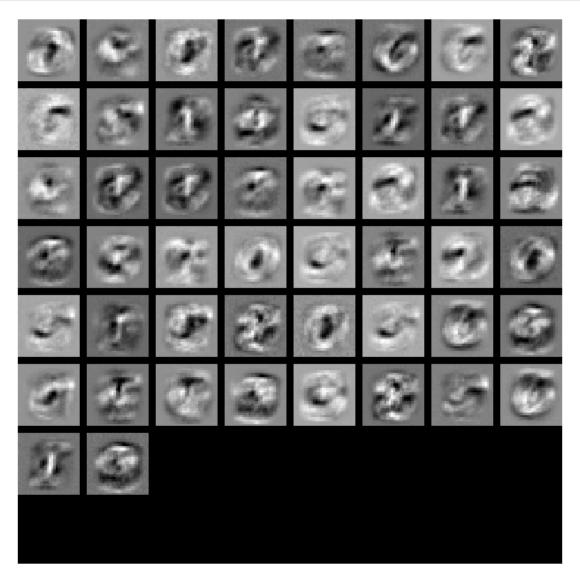
plt.subplot(2, 1, 2)
plt.plot(stats['train_acc_history'], label='train')
plt.plot(stats['val_acc_history'], label='val')
plt.plot(stats_LRDecay['train_acc_history'], label='train_LRDecay')
plt.plot(stats_LRDecay['val_acc_history'], label='val_LRDecay')
plt.title('Classification accuracy history')
plt.legend()
plt.xlabel('Epoch')
plt.ylabel('Clasification accuracy')
plt.show()
```



```
[13]: # Visualize the weights of the network

def show_net_weights(net):
    W1 = net.params['W1']
    W1 = W1.reshape(28, 28, 1, -1).transpose(3, 0, 1, 2)
```

```
plt.imshow(visualize_grid(W1, padding=3).astype('uint8').squeeze(axis=2))
plt.gca().axis('off')
plt.show()
show_net_weights(net)
```



10 Tune your hyperparameters

Tuning. Tuning the hyperparameters and developing intuition for how they affect the final performance is a large part of using Neural Networks, so we want you to get a lot of practice. Below, you should experiment with different values of the various hyperparameters, including hidden layer size, learning rate, numer of training epochs, and regularization strength. You might also consider

tuning the learning rate decay, but you should be able to get good performance using the default value.

Approximate results. You should be aim to achieve a classification accuracy of greater than 97.4% on the validation set.

Experiment: You goal in this exercise is to get as good of a result on MNIST as you can, with a fully-connected Neural Network. Feel free implement your own techniques (e.g. PCA to reduce dimensionality, or adding dropout, or adding features to the solver, etc.).

```
[15]: best_net = None # store the best model into this
     # TODO#8: Tune hyperparameters using the validation set. Store your best,
      ⇔trained#
     # model in best net.
                                                                           ш
      →#
      _#
     # To help debug your network, it may help to use visualizations similar to the \Box
     # ones we used above; these visualizations will have significant qualitative
      →#
     # differences from the ones we saw above for the poorly tuned network.
      →#
     #
     # Tweaking hyperparameters by hand can be fun, but you might find it useful to \Box
      →#
     # write code to sweep through possible combinations of hyperparameters
     # automatically like we did on the previous exercises.
                                                                           ш
     from itertools import product
     learning rates = [1e-3, 1e-4]
     regs = [0.01, 0.05]
     hidden_sizes = [10, 50, 100]
     epochs = [2000, 3000]
     batch_sizes = [200, 400, 1000]
     gcv = product(learning_rates, regs, hidden_sizes, epochs, batch_sizes)
     gcv_size = len(learning_rates) * len(regs) * len(hidden_sizes) * len(epochs) *__
      →len(batch_sizes)
     best val = 0
     for i, params in enumerate(gcv):
        print(f'CV# {i+1}/{gcv_size}', 10*'-')
```

```
net = TwoLayerNet(input_size, hidden_size, num_classes)
   lr, reg, hidden_size, epoch, batch_size = params
   net.train(X_train, y_train, X_val, y_val,
                 num_iters=epoch, batch_size=batch_size,
                 learning_rate=lr, learning_rate_decay=0.95,
                 reg=reg, verbose=False)
   train_acc = (net.predict(X_train) == y_train).mean()
   val acc = (net.predict(X val) == y val).mean()
   print(f"train accuracy: {train_acc}")
   print(f"validate accuracy: {val acc}")
   if val_acc > best_val:
       best_val = val_acc
       best_net = net
       best_param = params
print(f'best validate accuracy = {best_val} with parameters (lr, reg, hidden, ⊔
 ⇔epoch, batch)= {best_param}')
END OF TODO#8
CV# 1/72 -----
train accuracy: 0.93438181818182
validate accuracy: 0.9338
CV# 2/72 -----
train accuracy: 0.9330909090909091
validate accuracy: 0.9324
CV# 3/72 -----
train accuracy: 0.9078363636363637
validate accuracy: 0.9118
CV# 4/72 -----
train accuracy: 0.9333272727272727
validate accuracy: 0.9352
CV# 5/72 -----
train accuracy: 0.9381090909090909
validate accuracy: 0.9392
CV# 6/72 -----
train accuracy: 0.9259636363636363
validate accuracy: 0.9272
CV# 7/72 -----
train accuracy: 0.9291818181818182
validate accuracy: 0.9302
CV# 8/72 -----
```

validate accuracy: 0.9578

CV# 9/72 -----

train accuracy: 0.9451454545454545

validate accuracy: 0.9486

CV# 10/72 -----train accuracy: 0.9682
validate accuracy: 0.9648

CV# 11/72 -----

train accuracy: 0.9689090909090909

validate accuracy: 0.9656

CV# 12/72 -----

train accuracy: 0.9445272727272728

validate accuracy: 0.9468

CV# 13/72 -----train accuracy: 0.9614
validate accuracy: 0.9628

CV# 14/72 -----

train accuracy: 0.9648181818181818

validate accuracy: 0.9634

CV# 15/72 -----

train accuracy: 0.94709090909091

validate accuracy: 0.9508

CV# 16/72 -----train accuracy: 0.9724
validate accuracy: 0.9676

CV# 17/72 -----

train accuracy: 0.97232727272727

validate accuracy: 0.9696

CV# 18/72 -----

train accuracy: 0.9491636363636363

validate accuracy: 0.95 CV# 19/72 -----

train accuracy: 0.9628181818181818

validate accuracy: 0.9628

CV# 20/72 -----

train accuracy: 0.9320545454545455

validate accuracy: 0.9292

CV# 21/72 -----

train accuracy: 0.923509090909091

validate accuracy: 0.9254

CV# 22/72 -----

train accuracy: 0.9358363636363637

validate accuracy: 0.9334

CV# 23/72 -----

train accuracy: 0.93292727272727

validate accuracy: 0.9338

CV# 24/72 -----

validate accuracy: 0.9252

CV# 25/72 -----

train accuracy: 0.9312909090909091

validate accuracy: 0.9324

CV# 26/72 -----

train accuracy: 0.9621090909090909

validate accuracy: 0.961

CV# 27/72 -----train accuracy: 0.944
validate accuracy: 0.9462

CV# 28/72 -----

train accuracy: 0.9661636363636363

validate accuracy: 0.9594

CV# 29/72 -----

train accuracy: 0.9671454545454545

validate accuracy: 0.9618

CV# 30/72 -----

train accuracy: 0.9467818181818182

validate accuracy: 0.9486

CV# 31/72 -----

train accuracy: 0.9610909090909091

validate accuracy: 0.9596

CV# 32/72 -----

train accuracy: 0.9643454545454545

validate accuracy: 0.9638

CV# 33/72 -----

train accuracy: 0.94718181818182

validate accuracy: 0.95 CV# 34/72 -----

train accuracy: 0.97194545454546

validate accuracy: 0.9678

CV# 35/72 -----

train accuracy: 0.9725090909090909

validate accuracy: 0.9674

CV# 36/72 -----

train accuracy: 0.9488181818181818

validate accuracy: 0.9504

CV# 37/72 -----

train accuracy: 0.8785090909090909

validate accuracy: 0.8856

CV# 38/72 -----

train accuracy: 0.6809090909090909

validate accuracy: 0.6948

CV# 39/72 -----

train accuracy: 0.60258181818182

validate accuracy: 0.6114

CV# 40/72 -----

train accuracy: 0.8688 validate accuracy: 0.881

CV# 41/72 -----

train accuracy: 0.8096545454545454

validate accuracy: 0.8192

CV# 42/72 -----

train accuracy: 0.60909090909091

validate accuracy: 0.6202

CV# 43/72 -----

train accuracy: 0.79992727272727

validate accuracy: 0.8092

CV# 44/72 -----

train accuracy: 0.87614545454546

validate accuracy: 0.8844

CV# 45/72 -----

train accuracy: 0.6891454545454545

validate accuracy: 0.6934

CV# 46/72 -----

train accuracy: 0.8958363636363637

validate accuracy: 0.9024

CV# 47/72 -----

train accuracy: 0.8967636363636363

validate accuracy: 0.9032

CV# 48/72 -----

train accuracy: 0.7578181818181818

validate accuracy: 0.7622

CV# 49/72 -----

train accuracy: 0.8728727272727272

validate accuracy: 0.8826

CV# 50/72 -----

train accuracy: 0.8782363636363636

validate accuracy: 0.8852

CV# 51/72 -----

train accuracy: 0.7096727272727272

validate accuracy: 0.7172

CV# 52/72 -----

train accuracy: 0.89812727272728

validate accuracy: 0.9048

CV# 53/72 -----

train accuracy: 0.8980181818181818

validate accuracy: 0.9036

CV# 54/72 -----

train accuracy: 0.78698181818182

validate accuracy: 0.789

CV# 55/72 -----

train accuracy: 0.8795454545454545

validate accuracy: 0.8856

CV# 56/72 -----

validate accuracy: 0.7856

CV# 57/72 -----

train accuracy: 0.4999636363636364

validate accuracy: 0.4954

CV# 58/72 -----

train accuracy: 0.8602545454545455

validate accuracy: 0.8674

CV# 59/72 -----

train accuracy: 0.8806727272727273

validate accuracy: 0.8884

CV# 60/72 -----

train accuracy: 0.64918181818182

validate accuracy: 0.6594

CV# 61/72 -----

train accuracy: 0.7459636363636364

validate accuracy: 0.7458

CV# 62/72 -----

train accuracy: 0.8746727272727273

validate accuracy: 0.8802

CV# 63/72 -----

train accuracy: 0.6923090909090909

validate accuracy: 0.6994

CV# 64/72 -----

train accuracy: 0.8956909090909091

validate accuracy: 0.9032

CV# 65/72 -----

train accuracy: 0.8965636363636363

validate accuracy: 0.9036

CV# 66/72 -----

train accuracy: 0.73918181818182

validate accuracy: 0.7468

CV# 67/72 -----

train accuracy: 0.8757454545454545

validate accuracy: 0.883

CV# 68/72 -----

train accuracy: 0.8790545454545454

validate accuracy: 0.8846

CV# 69/72 -----

train accuracy: 0.70627272727273

validate accuracy: 0.7124

CV# 70/72 -----

train accuracy: 0.89798181818182

validate accuracy: 0.9034

CV# 71/72 -----

train accuracy: 0.8987636363636363

validate accuracy: 0.9052

CV# 72/72 -----

validate accuracy: 0.7896

best validate accuracy = 0.9696 with parameters (lr, reg, hidden, epoch, batch)=

(0.001, 0.01, 100, 3000, 400)

[16]: # visualize the weights of the best network

show_net_weights(best_net)



11 Run on the test set

When you are done experimenting, you should evaluate your final trained network on the test set; you should get above 96.3%.

```
[17]: test_acc = (best_net.predict(X_test) == y_test).mean()
print('Test accuracy: ', test_acc)
```

Test accuracy: 0.9656