Attrition Prediction

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1 MLE and Naïve Bayes

1.1 T1 and OT1

$$
\begin{aligned}
&\arg\max_{\alpha}\ Pr(y_N, y_{N-1}, \ldots, y_1, y_0; \alpha) \\
&= \arg\max_{\alpha}\ Pr(\bigcap_{i=0}^{N} y_i; \alpha) \\
&= \arg\max_{\alpha}\ Pr(y_N | \bigcap_{i=0}^{N-1} y_i; a) Pr(\bigcap_{i=0}^{N-1} y_i; a) \\
&= \arg\max_{\alpha}\ Pr(y_0; \alpha) \prod_{i=1}^{N} Pr(y_i | \bigcap_{j=0}^{i-1} y_j; \alpha) && \text{(Apply probabilistic Chain rule)} \\
&= \arg\max_{\alpha}\ Pr(y_0; \alpha) \prod_{i=1}^{N} Pr(y_i | y_{i-1}; \alpha) && \text{(y_i is Markov Process)} \\
&= \arg\max_{\alpha}\ Pr(y_0) \prod_{i=1}^{N} Pr(y_i | y_{i-1}; \alpha) && \text{(y_0 is independent to α)} \\
&= \arg\max_{\alpha}\ \mathcal{N}(y_0; 0, \lambda) \prod_{i=1}^{N} \mathcal{N}(y_i; \alpha y_{i-1}, \sigma^2) \\
&= \arg\max_{\alpha}\ \log \mathcal{N}(y_0; 0, \lambda) + \sum_{i=1}^{N} \log \mathcal{N}(y_i; \alpha y_{i-1}, \sigma^2) && \text{(Take log on likelihood)} \\
&= \arg\max_{\alpha}\ \log \frac{1}{\sqrt{2\pi\lambda}} + N \log \frac{1}{\sqrt{2\pi\sigma^2}} + \frac{-y_0^2}{2\lambda} + \sum_{i=1}^{N} \frac{-(y_i - \alpha y_{i-1})^2}{2\sigma^2}
\end{aligned}
$$

Take Derivative to calculate argument max

$$
\begin{aligned}
\frac{\partial}{\partial \alpha}\left(\log\frac{1}{\sqrt{2\pi\lambda}} + N\log\frac{1}{\sqrt{2\pi\sigma^2}} + \frac{-y_0^2}{2\lambda} + \sum_{i=1}^{N}\frac{-(y_i-\alpha y_{i-1})^2}{2\sigma^2}\right) &= 0 \\
\sum_{i=1}^{N}\frac{-2(y_i-\alpha y_{i-1})}{2\sigma^2}\frac{\partial}{\partial \alpha}(y_i-\alpha y_{i-1}) &= 0 \\
\sum_{i=1}^{N}\frac{-2(y_i-\alpha y_{i-1})}{2\sigma^2}(-y_{i-1}) &= 0 \\
\sum_{i=1}^{N}(y_i-\alpha y_{i-1})y_{i-1} &= 0 \\
\sum_{i=1}^{N} y_i\cdot y_{i-1} - \alpha\sum_{i=1}^{N} y_{i-1}^2 &= 0 \\
\therefore \alpha &= \frac{\sum_{i=1}^{N} y_i\cdot y_{i-1}}{\sum_{i=0}^{N-1} y_i^2}
\end{aligned}
$$

For **T1** the answer is

$$
\alpha = \frac{y_2y_1 + y_1y_0}{y_1^2 + y_0^2}
$$

1.2 T2

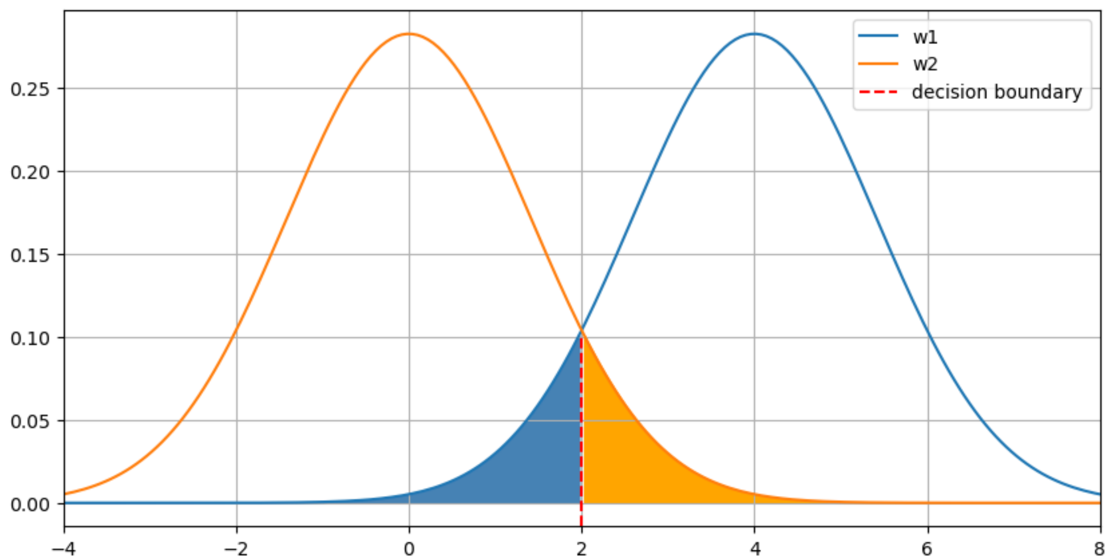
$$
\begin{aligned}
P(w_1|x) &= P(w_2|x) \\
P(x|w_1)P(w_1) &= P(x|w_2)P(w_2) \\
P(x|w_1) &= P(x|w_2) && \because P(w_1) = P(w_2) \\
\mathcal{N}(x;4,2) &= \mathcal{N}(x;0,2) \\
\frac{1}{\sqrt{2\pi\cdot 2}}\exp\left(\frac{-(x-4)^2}{2\cdot 2}\right) &= \frac{1}{\sqrt{2\pi\cdot 2}}\exp\left(\frac{-(x-0)^2}{2\cdot 2}\right) \\
(x-4)^2 &= x^2 \\
(2x-4)(-4) &= 0 \\
x &= 2
\end{aligned}
$$

The decision boundary is $x = 2$

[1]:
```python
import numpy as np
import pandas as pd
import scipy
from scipy import stats
import matplotlib.pyplot as plt
import seaborn as sns
```

[2]:
```python
from scipy.stats import norm
```

[3]:
```python
x = np.linspace(-4.0, 8.0, 200)
fig = plt.figure(figsize = (10, 5))

plt.xlim((-4, 8))
# w1 : N(4, 2)
plt.plot(x, norm.pdf(x, 4, np.sqrt(2)), label = 'w1')
plt.fill_between(x, norm.pdf(x, 4, np.sqrt(2)), where = (x<=2),␣
↪color='steelblue')
# w2: N(0, 2)
plt.plot(x, norm.pdf(x, 0, np.sqrt(2)), label = 'w2')
plt.fill_between(x, norm.pdf(x, 0, np.sqrt(2)), where = (x>=2), color='orange')


#  decision boundary x = 2
plt.axvline(x=2, ymax= norm.pdf(2, 0, np.sqrt(2)) / norm.pdf(0, 0, np.sqrt(2))␣
↪, color='r', linestyle='--', label='decision boundary')

plt.legend()
plt.grid()
plt.show()
```

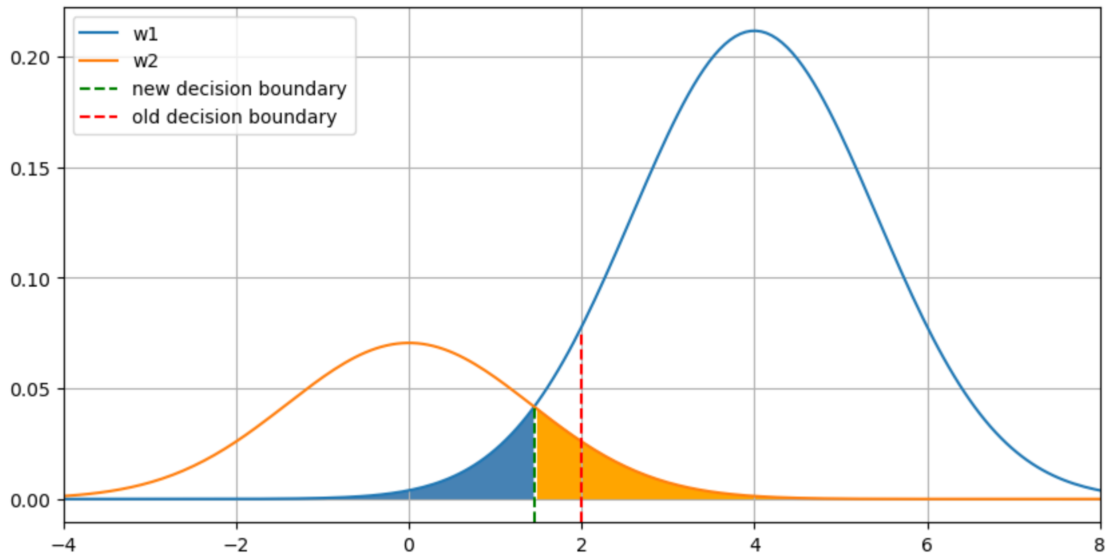


1.3 T3

$$\begin{aligned}
P(w_1|x) &= P(w_2|x) \\
P(x|w_1)P(w_1) &= P(x|w_2)P(w_2) \\
P(x|w_1)(0.75) &= P(x|w_2)(0.25) \\
3 &= \frac{\mathcal{N}(x;0,2)}{\mathcal{N}(x;4,2)} \\
3 &= \frac{\exp(-x^2/4)}{\exp(-(x-4)^2/4)} \\
4\ln 3 &= (x-4)^2 - x^2 \\
4\ln 3 &= (2x-4)(-4) \\
x &= \frac{4-\ln 3}{2}
\end{aligned}$$

$\therefore$ The decision boundary is $x = \frac{4-\ln 3}{2} \approx 1.45069$

[4]:
```
x = np.linspace(-4.0, 8.0, 200)
fig = plt.figure(figsize = (10, 5))

plt.xlim((-4, 8))

boundary = (4 - np.log(3)) / 2

# w1 : N(4, 2)
plt.plot(x, 0.75 * norm.pdf(x, 4, np.sqrt(2)), label = 'w1')
plt.fill_between(x, 0.75 * norm.pdf(x, 4, np.sqrt(2)), where = (x<=boundary),␣
↪color='steelblue')
# w2: N(0, 2)
plt.plot(x, 0.25 * norm.pdf(x, 0, np.sqrt(2)), label = 'w2')
plt.fill_between(x, 0.25 * norm.pdf(x, 0, np.sqrt(2)), where = (x>=boundary),␣
↪color='orange')
#  decision boundary x = 2

plt.axvline(x=boundary, ymax=  norm.pdf(boundary, 4, np.sqrt(2))  / norm.pdf(4,␣
↪4, np.sqrt(2)) + 0.025, color='g', linestyle='--', label='new decision␣
↪boundary')
plt.axvline(x=2, ymax=norm.pdf(2, 0, np.sqrt(2)) / norm.pdf(4, 4, np.sqrt(2)) ,␣
↪color='r', linestyle='--', label='old decision boundary')


plt.legend()
plt.grid()
plt.show()
```



1.4 OT2

$$
\begin{aligned}
\mathcal{N}(x;\mu_1,\sigma^2) &= \mathcal{N}(x;\mu_2,\sigma^2) \\
\exp\left(-\frac{(x-\mu_1)^2}{2\sigma^2}\right) &= \exp\left(-\frac{(x-\mu_2)^2}{2\sigma^2}\right) && \because \sigma \text{ is equal} \\
(x-\mu_1)^2 &= (x-\mu_2)^2 \\
(x-\mu_1)^2 - (x-\mu_2)^2 &= 0 \\
(2x-\mu_1-\mu_2)(\mu_2-\mu_1) &= 0 \\
x &= \frac{\mu_1+\mu_2}{2} && \text{where } \mu_1 \neq \mu_2
\end{aligned}
$$

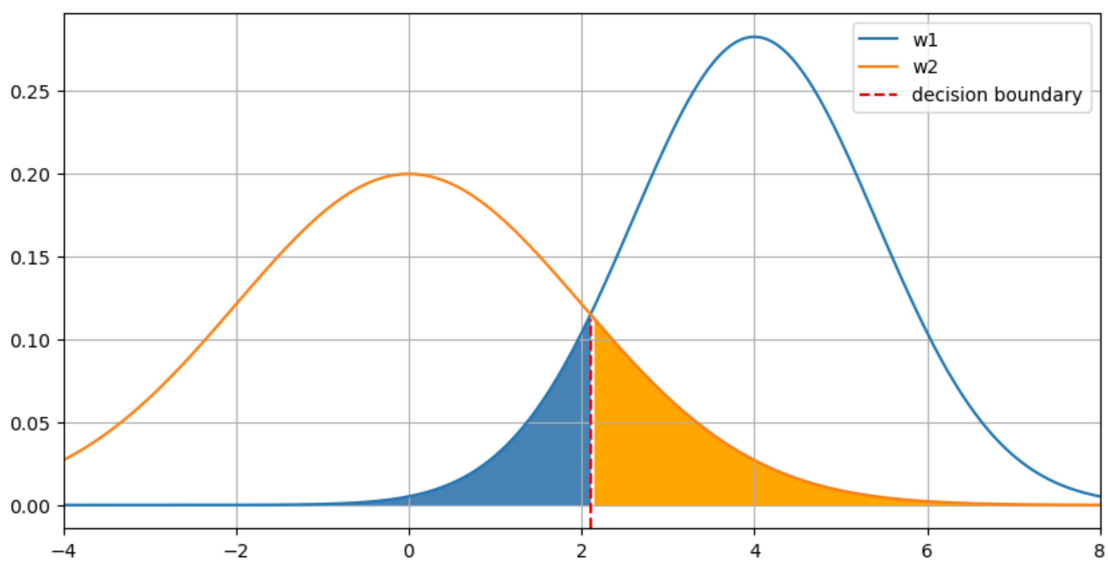
1.5 OT3

$$
\begin{aligned}
\mathcal{N}(x;4,2) &= \mathcal{N}(x;0,4) \\
\frac{1}{\sqrt{2\pi\cdot 2}}\exp\left(\frac{-(x-4)^2}{2\cdot 2}\right) &= \frac{1}{\sqrt{2\pi\cdot 4}}\exp\left(\frac{-(x-0)^2}{2\cdot 4}\right) \\
\sqrt{2} &= \exp\left(-\frac{x^2}{8}+\frac{(x-4)^2}{4}\right) \\
\frac{1}{2}\ln 2 &= -\frac{x^2}{8}+\frac{(x-4)^2}{4} \\
4\ln 2 &= 2(x-4)^2 - x^2 \\
x^2 - 16x + (32 - 4\ln 2) &= 0 \\
x &\approx 2.10317, 13.8968 && \because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}
\end{aligned}
$$

Choose $x = 2.10317$ because $0 \leq 2.10317 \leq 4$

[5]:

```
x = np.linspace(-4.0, 8.0, 200)
fig = plt.figure(figsize = (10, 5))

plt.xlim((-4, 8))

boundary = 2.10317
# w1 : N(4, 2)
plt.plot(x, stats.norm.pdf(x, 4, np.sqrt(2)), label = 'w1')
plt.fill_between(x, norm.pdf(x, 4, np.sqrt(2)), where = (x<=boundary),␣
 ↪color='steelblue')
# w2: N(0, 4)
plt.plot(x, stats.norm.pdf(x, 0, np.sqrt(4)), label = 'w2')
plt.fill_between(x, norm.pdf(x, 0, np.sqrt(4)), where = (x>=boundary),␣
 ↪color='orange')
#  decision boundary x = 2

plt.axvline(x=boundary, ymax=norm.pdf(boundary, 4, np.sqrt(2)) / norm.pdf(4, 4,␣
 ↪np.sqrt(2)) , color='r', linestyle='--', label='decision boundary')


plt.legend()
plt.grid()
```



2 Employee Attrition Prediction

[6]:
```
df = pd.read_csv('hr-employee-attrition-with-null.csv')
df.head()
```

[6]:
```
   Unnamed: 0   Age Attrition     BusinessTravel  DailyRate  \
0           0  41.0       Yes      Travel_Rarely        NaN
1           1   NaN        No                NaN      279.0
2           2  37.0       Yes                NaN     1373.0
3           3   NaN        No  Travel_Frequently     1392.0
4           4  27.0        No      Travel_Rarely      591.0

               Department  DistanceFromHome  Education EducationField  \
0                     NaN               1.0        NaN  Life Sciences
1  Research & Development               NaN        NaN  Life Sciences
2                     NaN               2.0        2.0            NaN
3  Research & Development               3.0        4.0  Life Sciences
4  Research & Development               2.0        1.0        Medical

   EmployeeCount  …  RelationshipSatisfaction  StandardHours  \
0            1.0  …                       1.0           80.0
1            1.0  …                       4.0            NaN
2            1.0  …                       NaN           80.0
3            NaN  …                       3.0            NaN
4            1.0  …                       4.0           80.0

  StockOptionLevel  TotalWorkingYears  TrainingTimesLastYear  WorkLifeBalance  \
0              0.0                8.0                    0.0              NaN
1              1.0               10.0                    NaN              3.0
2              0.0                7.0                    3.0              NaN
3              NaN                8.0                    3.0              NaN
4              1.0                6.0                    NaN              3.0

  YearsAtCompany  YearsInCurrentRole YearsSinceLastPromotion  \
0            6.0                 NaN                     0.0
1           10.0                 NaN                     NaN
2            NaN                 0.0                     NaN
3            8.0                 NaN                     3.0
4            2.0                 2.0                     2.0

   YearsWithCurrManager
0                   NaN
1                   7.0
2                   0.0
3                   0.0
4                   NaN
```

```
[5 rows x 36 columns]
```

[7]:
```python
df.loc[df["Attrition"] == "no", "Attrition"] = 0.0
df.loc[df["Attrition"] == "yes", "Attrition"] = 1.0

cat_cols = ['Department', 'Attrition', 'BusinessTravel', 'EducationField',␣
 ↪'Gender', 'JobRole',
                              'MaritalStatus', 'OverTime']
for col in cat_cols:
    df[col] = pd.Categorical(df[col]).codes
    df.loc[df[col] == -1, col] = np.nan

df = df.loc[:, ~df.columns.isin(['EmployeeNumber', 'Unnamed: 0',␣
 ↪'EmployeeCount', 'StandardHours', 'Over18'])]
df.head()
```

[7]:
```
    Age  Attrition  BusinessTravel  DailyRate  Department  DistanceFromHome  \
0  41.0        1.0             2.0        NaN         NaN               1.0
1   NaN        0.0             NaN      279.0         1.0               NaN
2  37.0        1.0             NaN     1373.0         NaN               2.0
3   NaN        0.0             1.0     1392.0         1.0               3.0
4  27.0        0.0             2.0      591.0         1.0               2.0

   Education  EducationField  EnvironmentSatisfaction  Gender  …  \
0        NaN             1.0                      2.0     0.0  …
1        NaN             1.0                      3.0     1.0  …
2        2.0             NaN                      NaN     1.0  …
3        4.0             1.0                      NaN     0.0  …
4        1.0             3.0                      1.0     1.0  …

   PerformanceRating  RelationshipSatisfaction  StockOptionLevel  \
0                NaN                       1.0               0.0
1                NaN                       4.0               1.0
2                3.0                       NaN               0.0
3                3.0                       3.0               NaN
4                3.0                       4.0               1.0

   TotalWorkingYears  TrainingTimesLastYear  WorkLifeBalance  YearsAtCompany  \
0                8.0                    0.0              NaN             6.0
1               10.0                    NaN              3.0            10.0
2                7.0                    3.0              NaN             NaN
3                8.0                    3.0              NaN             8.0
4                6.0                    NaN              3.0             2.0

   YearsInCurrentRole  YearsSinceLastPromotion  YearsWithCurrManager
0                 NaN                      0.0                   NaN
1                 NaN                      NaN                   7.0
```

```
2                  0.0                        NaN                     0.0
3                  NaN                        3.0                     0.0
4                  2.0                        2.0                     NaN

[5 rows x 31 columns]
```

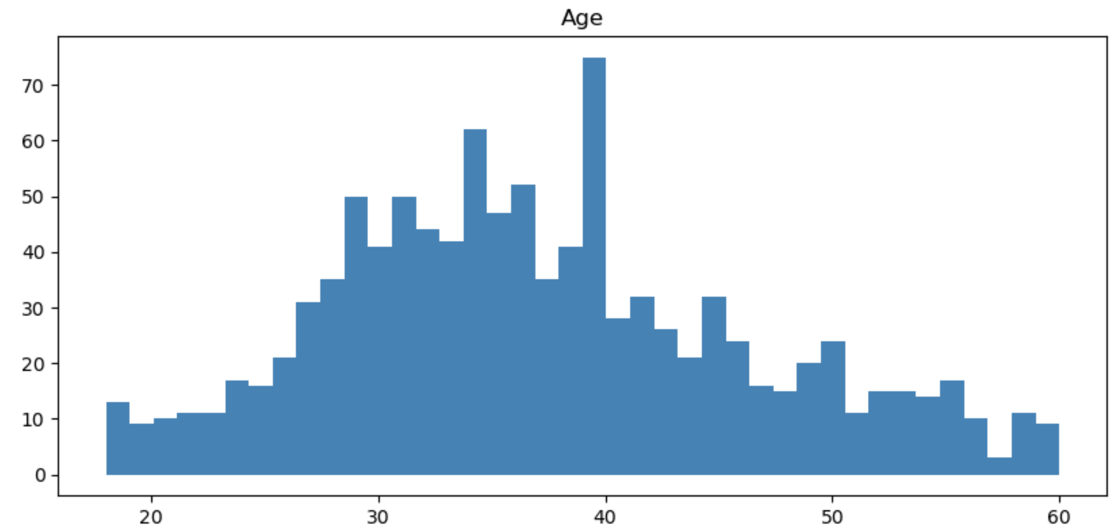
[8]:
```
from sklearn.model_selection import train_test_split
```

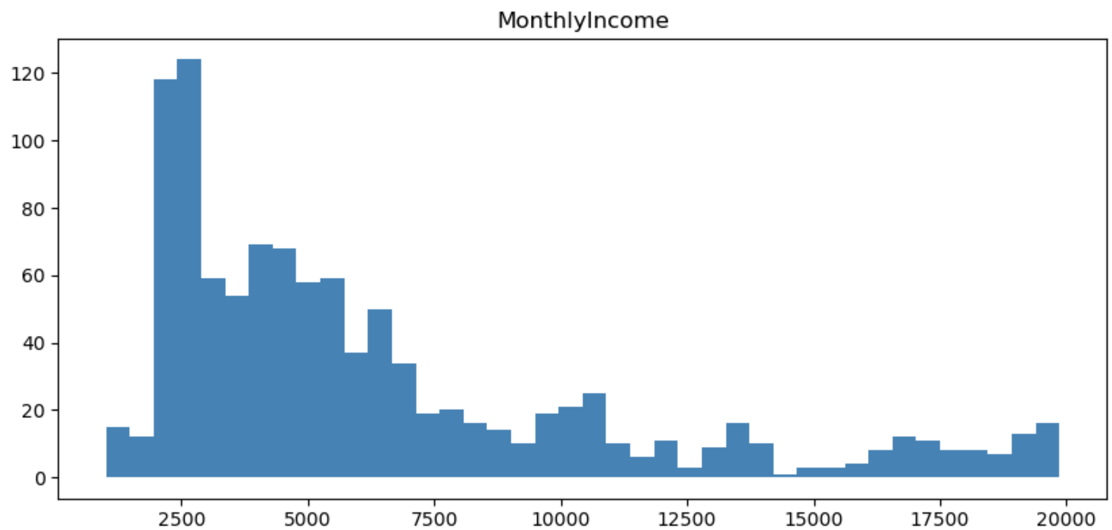
[9]:
```
df_train, df_test = train_test_split(df, test_size = 0.1, random_state = 42,␣
↪stratify = df['Attrition'])
```

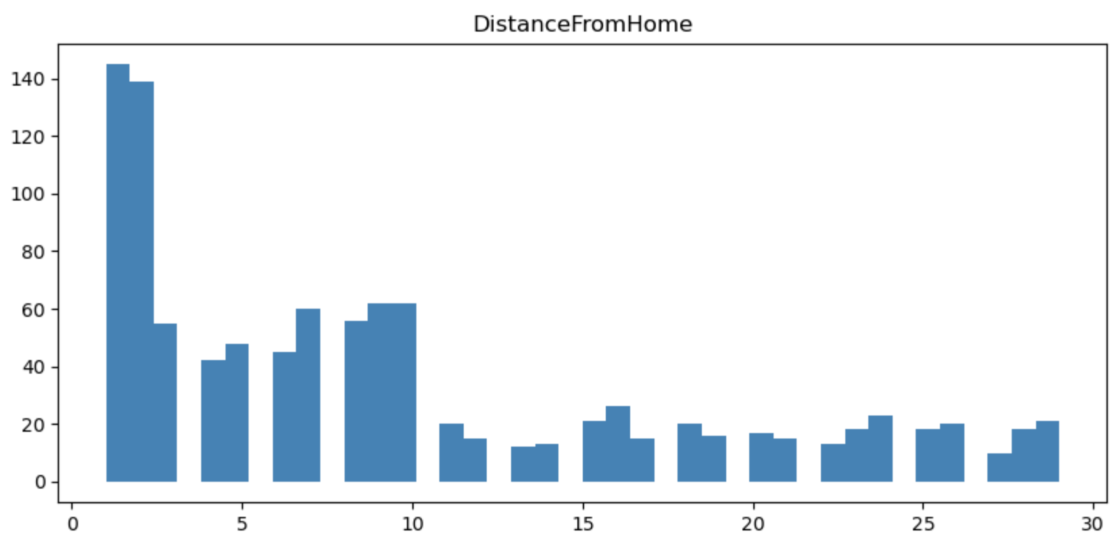
2.1 T4

[10]:
```
cols = ['Age', 'MonthlyIncome', 'DistanceFromHome']
fig, ax = plt.subplots(3, figsize=(10, 15))

for i, col in enumerate(cols):
    train_col_no_nan = df_train[~df_train[col].isna()][col]
    hist, bin_edge = np.histogram(train_col_no_nan, 40)

    print(f'{col} column has {(hist == 0).sum()} bins zero counts ')

    ax[i].set_title(col)
    ax[i].fill_between(bin_edge.repeat(2)[1:-1], hist.repeat(2),␣
↪facecolor='steelblue')

plt.show()
```

```
Age column has 0 bins zero counts
MonthlyIncome column has 0 bins zero counts
DistanceFromHome column has 11 bins zero counts
```


Age
0
10
20
30
40
50
60
70
20
30
40
50
60


MonthlyIncome
0
20
40
60
80
100
120
2500
5000
7500
10000
12500
15000
17500
20000


DistanceFromHome
0
20
40
60
80
100
120
140
0
5
10
15
20
25
30

Age and **MonthlyIncome** have a good discretization because there is no sparse in data, On the other hand, **DistanceFromHome** has total 11 empty bins the sparse of data in this feature. The test data have a chance to appear in the empty bins so that will make the probability value zero therefore the DistanceFromHome feature is bad discretization.

2.2 T5

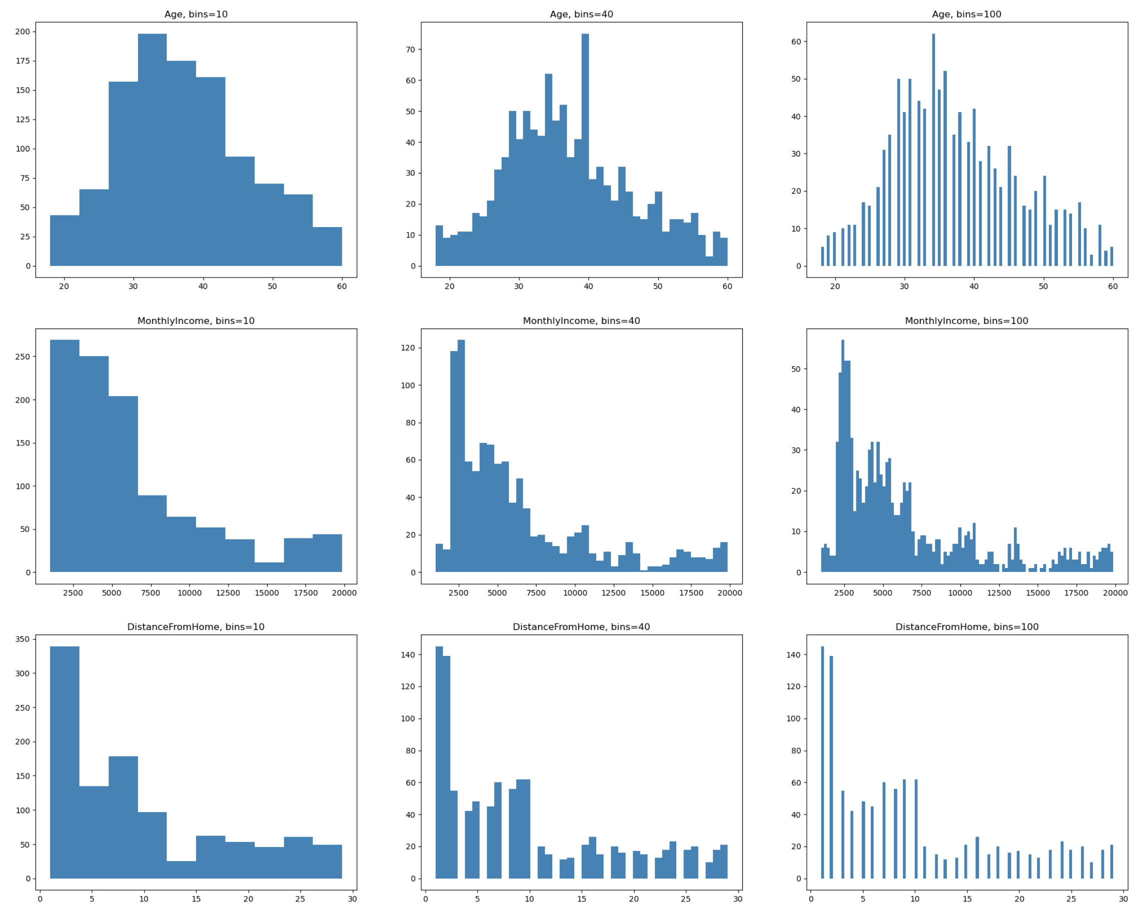
We can you Gaussian estimate for an Age feature because the histogram looks like it is a Gaussian distribution. The Monthly Income and DistanceFromHome have the right skewness so it is not good to eliminate with the Gaussian.

the **Gaussian Mixture Model (GMM)** can estimate all feature include **Monthly Income** and **DistanceFromHome** because it handles the skewness by divide the histogram and make each of divided histogram to Gaussian Distribution.

2.3 T6

[11]:
```
def plot_discretize_hist(df, col, bins, ax):
    train_col_no_nan = df[~df[col].isna()][col]
    hist, bin_edge = np.histogram(train_col_no_nan, bins=bins)

    ax.set_title(f'{col}, bins={bins}')
    ax.fill_between(bin_edge.repeat(2)[1:-1], hist.repeat(2),␣
 ↪facecolor='steelblue')

    discretized_col = np.digitize(train_col_no_nan, bin_edge)
    return discretized_col
```

[12]:
```
all_bins = [10, 40, 100]

fig, ax = plt.subplots(3, 3, figsize=(25, 20))
for i, col in enumerate(cols):
    for j, bins in enumerate(all_bins):
        plot_discretize_hist(df_train, col, bins, ax[i, j])

plt.show()
```



Considering the bins size of each feature by the sparseness of histogram

Age - bins = 40

MonthlyIncome - bins = 40 (The bins=100 look good but has a little sparse)

DistanceFromHome - bins = 10

2.4 T7

[13]:
```
num_cols = np.setdiff1d(df_train.columns, np.array(cat_cols))
for col in num_cols:
    train_col_no_nan = df_train[~df_train[col].isna()][col]
    hist, bin_edge = np.histogram(train_col_no_nan, bins=10)
    if (zero_cnt := (hist == 0).sum()) == 0:
        print(f'{col} column has {zero_cnt} bins zero counts ')
```

```
Age column has 0 bins zero counts
DailyRate column has 0 bins zero counts
DistanceFromHome column has 0 bins zero counts
HourlyRate column has 0 bins zero counts
```

```
MonthlyIncome column has 0 bins zero counts
MonthlyRate column has 0 bins zero counts
NumCompaniesWorked column has 0 bins zero counts
PercentSalaryHike column has 0 bins zero counts
TotalWorkingYears column has 0 bins zero counts
YearsAtCompany column has 0 bins zero counts
YearsInCurrentRole column has 0 bins zero counts
YearsSinceLastPromotion column has 0 bins zero counts
YearsWithCurrManager column has 0 bins zero counts
```

2.4.1 Feature should Discretize

Age, DailyRate, DistanceFromHome, HourlyRate, JobRole, MonthlyIncome, MonthlyRate, NumCompaniesWorked, PercentSalaryHike, TotalWorkingYears, YearsAtCompany, YearsInCurrentRole, YearsSinceLastPromotion, YearsWithCurrManager

because these features have no 0 bins counts

2.5 T8

Distribution: **Multinomial Distribution**

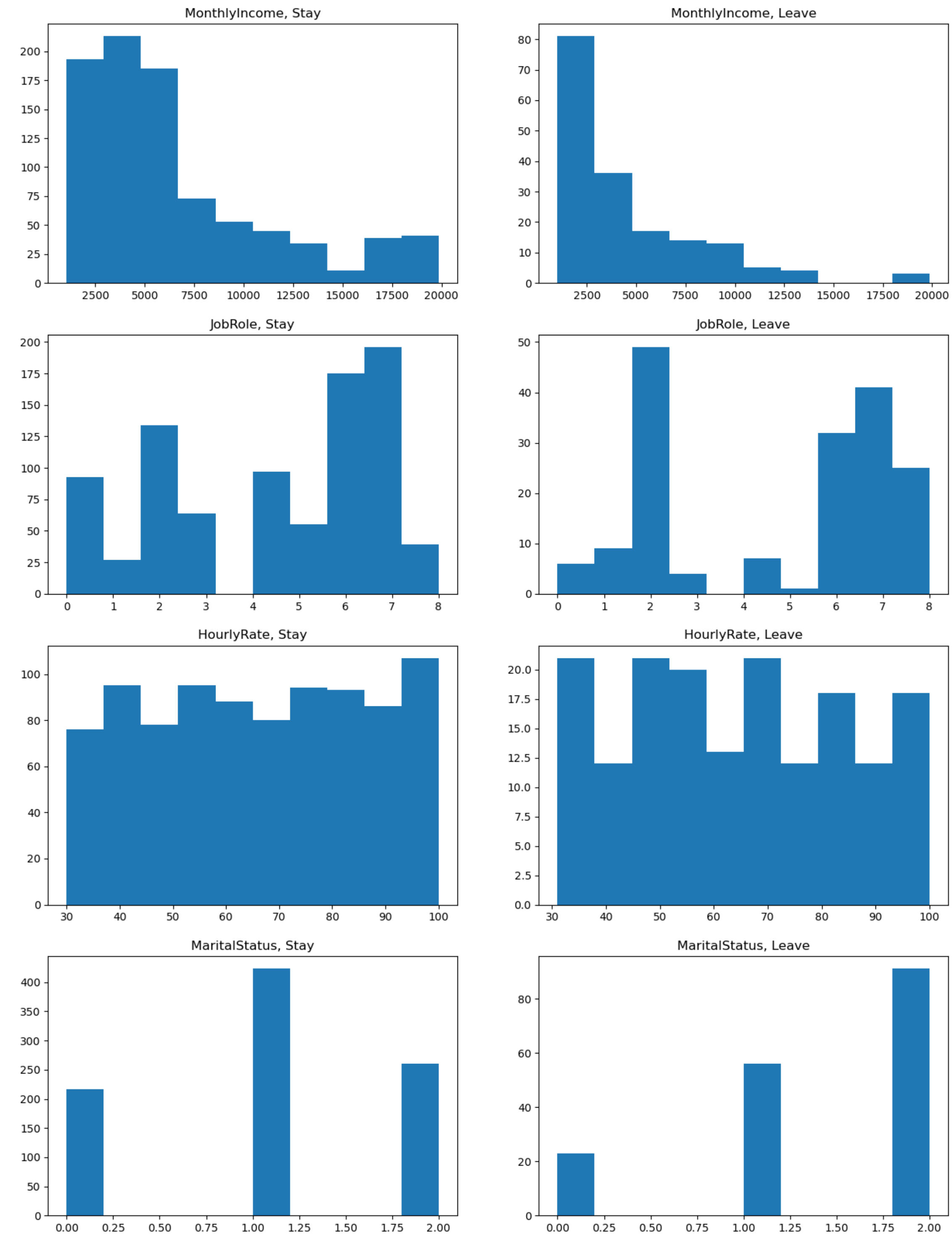
$$X \sim multinomial(\mathbf{p}, n)$$

MLE of Multinomial Distribution:

$$p_i = \frac{\text{size of bin}_i}{\#\text{samples data}}$$

[14]:
```
def plot_likelihood(df, col, cl, ax):
    attrition_match = df[df['Attrition'] == cl]
    ax.hist(attrition_match.loc[~attrition_match[col].isna(), col], bins=10)
    ax.set_title(f"{col}, { 'Stay' if cl==0 else 'Leave' }")
```

[15]:
```
cols = ['MonthlyIncome', 'JobRole', 'HourlyRate', 'MaritalStatus']
fig, ax = plt.subplots(4, 2, figsize=(15, 20))

for i,col in enumerate(cols):
    for cl in range(2):
        plot_likelihood(df_train, col, cl, ax[i, cl])

plt.show()
```



2.6 T9

Binomial Distribution because Attrition has only two classes (Stay, Leave) same as flipping coin.

2.7 T10

1. Flooring: use small value instead (1e-10, 1e-8)
2. Smoothing: smooth the values using counts from other observations (mean of adjacent bin, etc.)
3. Use priors (MAP adaptation)

2.8 T11

[16]:
```
class Simple_Binary_BayesClassifier_Hist:
    def __init__(self, bins=10, threshold=0):
        self.bins = bins
        self.threshold = threshold


    def _discretize_test(self, dat, feature_index):
        return np.digitize(dat, self.bin_edges[feature_index])


    def fit(self, X, y):
        num_samples, num_features = X.shape

        # Calculate prior
        self.priors = [np.sum(y == w) / num_samples for w in range(2)]

        bin_edges = []
        features_prob = [[], []]

        for feature_index in range(num_features):
            non_nan_mask = ~np.isnan(X[:, feature_index])

            cur_feature_no_nan = X[non_nan_mask, feature_index]
            cur_class_no_nan = y[non_nan_mask]

            _, bin_edge = np.histogram( cur_feature_no_nan, bins = self.bins )
            bin_edge[0], bin_edge[-1] = -np.inf, np.inf # Expand edge

            for w in range(2):
                current_feature_class = cur_feature_no_nan[cur_class_no_nan ==␣
↪w]
                hist, _ = np.histogram( current_feature_class , bins = bin_edge)
                bins_prob = hist / len(current_feature_class)
                bins_prob[bins_prob == 0] = 1e-6 # Flooring
                features_prob[w].append(bins_prob.tolist())

            bin_edges.append(bin_edge)

        self.bin_edges = np.array(bin_edges)
```

```
        self.features_prob = np.array(features_prob)
        return self


    def predict(self, _X, threshold = None, get_prob = False):
        if threshold == None:
            threshold = self.threshold

        X = _X.copy()
        # Discretize X
        _, num_features = X.shape
        for feature_index in range(num_features):
            non_nan_mask = ~np.isnan(X[:, feature_index])
            X[non_nan_mask, feature_index] = np.digitize(X[non_nan_mask,␣
↪feature_index], self.bin_edges[feature_index]) # Binning

        prediction = []
        for data in X:
            lH = np.log(self.priors[1]) - np.log(self.priors[0])
            lH += sum([ np.log(self.features_prob[ 1, feature_index,␣
↪int(data[feature_index]) - 1 ])
                      -np.log(self.features_prob[ 0, feature_index,␣
↪int(data[feature_index]) - 1 ])
                       if not np.isnan(data[feature_index]) else 0
                       for feature_index in range(num_features)
                     ])
            if get_prob:
                prediction.append(np.exp(lH))
            else:
                prediction.append(1 if lH > threshold else 0)

        return np.array(prediction)

    def predict_proba(self, _X):
        return self.predict(_X, None, get_prob=True)
```

[17]:
```
X_train = df_train.drop(columns='Attrition').to_numpy()
y_train = df_train['Attrition'].to_numpy()
X_test = df_test.drop(columns='Attrition').to_numpy()
y_test = df_test['Attrition'].to_numpy()

classifier_hist = Simple_Binary_BayesClassifier_Hist().fit(X_train, y_train)
y_pred = classifier_hist.predict(X_test)
```

[18]:
```
def precision_score(y_test, y_pred):
    if np.sum(y_pred == 1) == 0:
        return 0
```

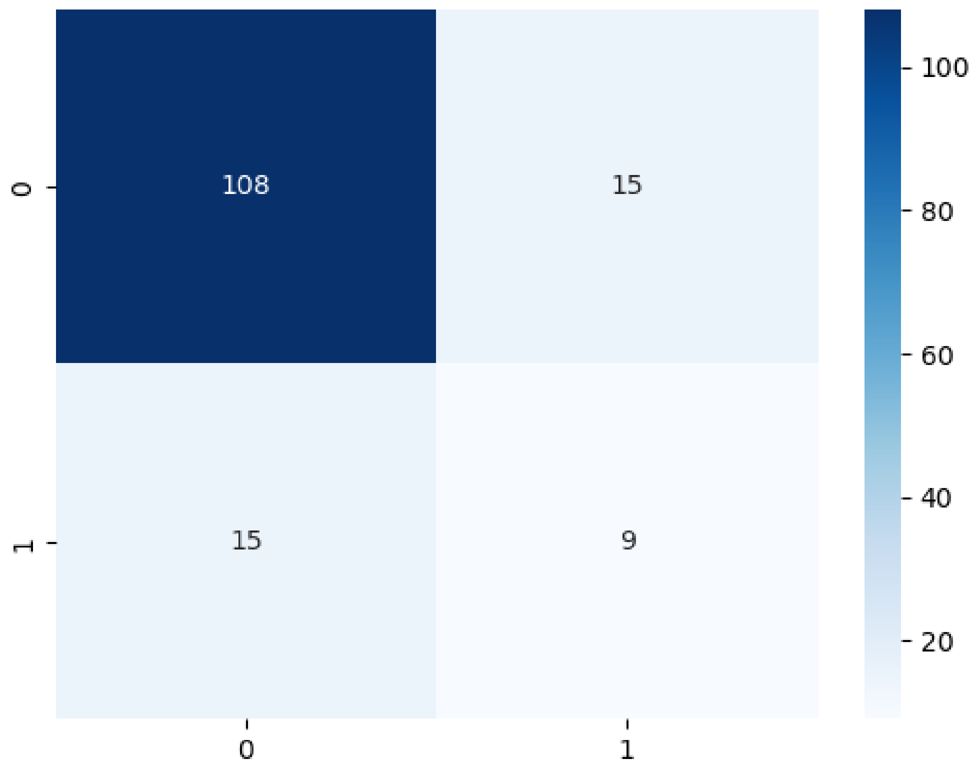
```python
    return np.sum((y_test == 1) & (y_pred == 1)) / np.sum(y_pred == 1)

def recall_score(y_test, y_pred):
    if np.sum(y_test == 1) == 0:
        return 0
    return np.sum((y_test == 1) & (y_pred == 1)) / np.sum(y_test == 1)

def f1_score(y_test, y_pred):
    prec = precision_score(y_test, y_pred)
    recall = recall_score(y_test, y_pred)
    if(prec + recall == 0):
        return 0
    return 2 * prec * recall / (prec + recall)

def accuracy_score(y_test, y_pred):
    return np.sum(y_test == y_pred) / len(y_test)

def fpr_rate(y_test, y_pred):
    return np.sum((y_test == 0) & (y_pred == 1)) / np.sum(y_test == 0)


def roc_curve(y, prob):
    thresholds = np.sort( np.block([0, np.unique(prob), 1]) )
    fpr, tpr = [], []
    for threshold in thresholds:
        y_pred = prob >= threshold
        fpr.append( fpr_rate(y, y_pred) )
        tpr.append( recall_score(y, y_pred) )
    return fpr, tpr

def confusion_matrix(y_test, y_pred):
    return np.array([
            [np.sum((y_test == 0) & (y_pred == 0)), np.sum((y_test == 0) &␣
 ↪(y_pred == 1))] ,
            [np.sum((y_test == 1) & (y_pred == 0)), np.sum((y_test == 1) &␣
 ↪(y_pred == 1))]
        ])

def classification_report(y_test, y_pred):
    s = '\tprecision\trecall\t\tf1-score\n'
    s += f'\t{precision_score(y_test, y_pred):.2f}\t\t{recall_score(y_test,␣
 ↪y_pred):.2f}\t\t{f1_score(y_test, y_pred):.2f}\n'
    s += f'accuracy: {accuracy_score(y_test, y_pred):.2f}'
    return s
```

[19]:

```python
print(classification_report(y_test, y_pred))
sns.heatmap(confusion_matrix(y_test, y_pred), annot=True ,fmt='g', cmap='Blues')
```

```
plt.show()
```

```
        precision         recall          f1-score
        0.38              0.38            0.38
accuracy: 0.80
```



2.9 T12

[20]:
```
class Simple_Binary_BayesClassifier_Gaussian:
    def __init__(self, threshold=0):
        self.threshold = threshold

    def fit(self, X, y):
        num_samples, num_features = X.shape

        # Calculate prior
        self.priors = [np.sum(y == w) / num_samples for w in range(2)]
        dists = [[], []]
        for feature_index in range(num_features):
            cur_data = X[:, feature_index]
            for w in range(2):
                cur_data_class = cur_data[y == w]
```

```
                dists[w].append( norm(np.nanmean(cur_data_class), np.
↪nanstd(cur_data_class)) )
        self.dists = np.array(dists)
        return self

    def predict(self, X, threshold=None, get_prob=False):
        if threshold == None:
            threshold = self.threshold

        _, num_features = X.shape
        prediction = []
        for data in X:
            lH = np.log(self.priors[1]) - np.log(self.priors[0])
            lH += sum([ np.log( self.dists[1, feature_index].
↪pdf(data[feature_index]) )
                       -np.log( self.dists[0, feature_index].
↪pdf(data[feature_index]) )
                       if not np.isnan(data[feature_index]) else 0
                       for feature_index in range(num_features)
                      ])

            if get_prob:
                prediction.append(np.exp(lH))
            else:
                prediction.append(1 if lH > threshold else 0)

        return np.array(prediction)

    def predict_proba(self, _X):
        return self.predict(_X, None, get_prob=True)
```

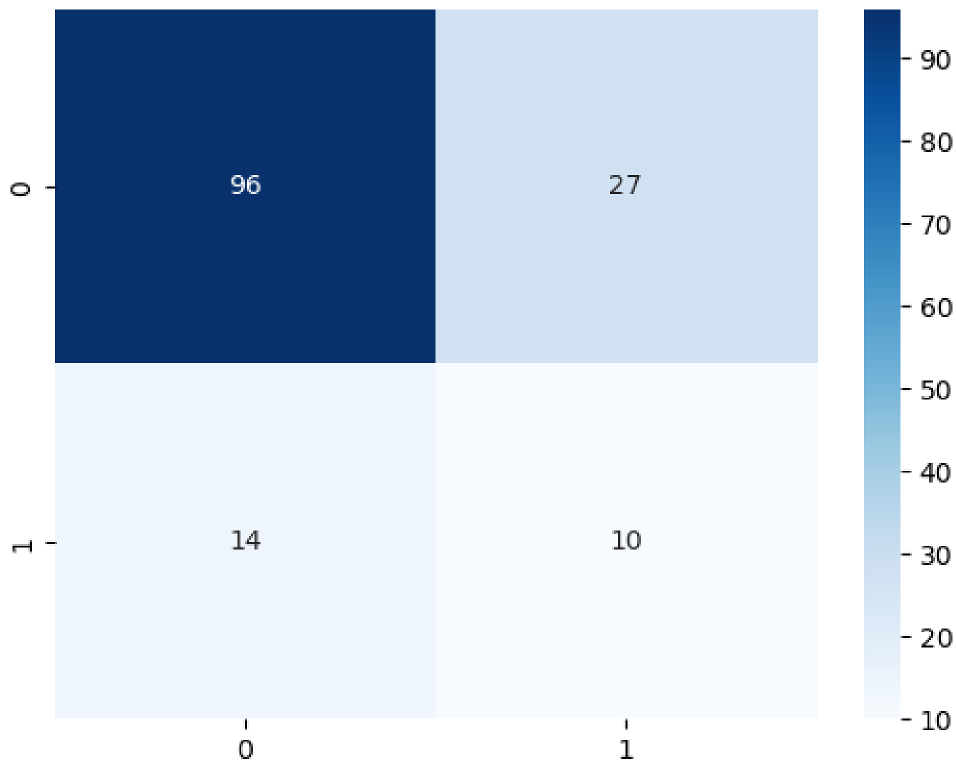
[21]:

```
classifier_gaussian = Simple_Binary_BayesClassifier_Gaussian().fit(X_train,␣
↪y_train)
y_pred = classifier_gaussian.predict(X_test)
```

[22]:

```
print(classification_report(y_test, y_pred))
sns.heatmap(confusion_matrix(y_test, y_pred), annot=True ,fmt='g', cmap='Blues')
plt.show()
```

```
       precision       recall          f1-score
       0.27            0.42            0.33
accuracy: 0.72
```

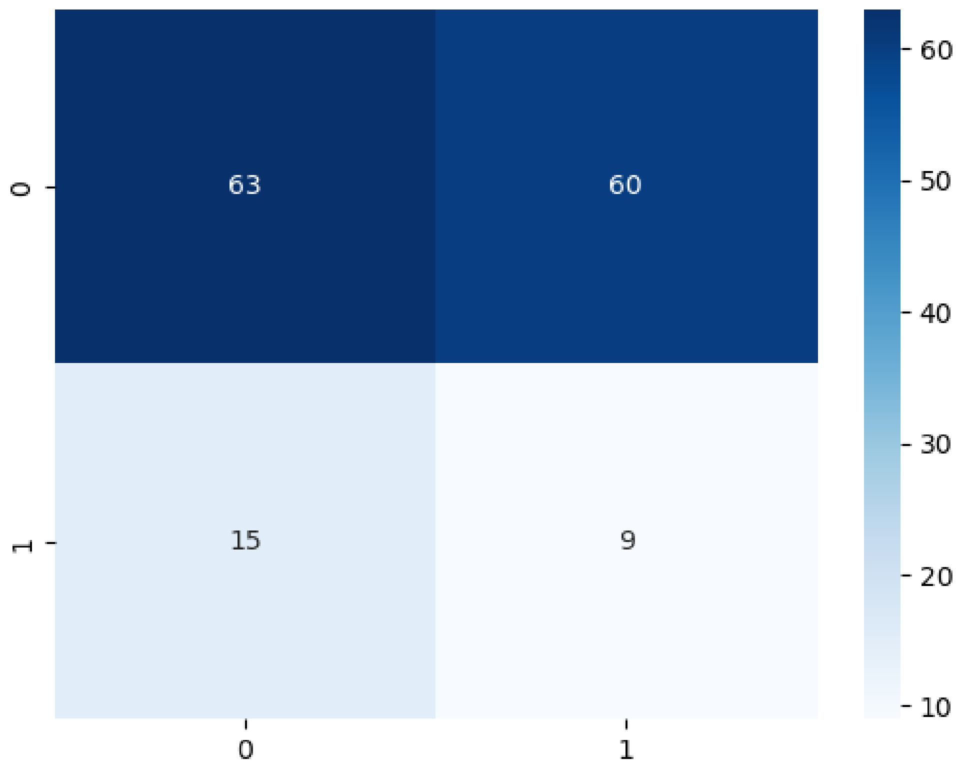


2.10 T13

[23]:
```
np.random.seed(42)
```

[24]:
```
y_pred_baseline_1 = (np.random.rand(len(y_test)) >= 0.5).astype(int)
print(classification_report(y_test, y_pred_baseline_1))
sns.heatmap(confusion_matrix(y_test, y_pred_baseline_1), annot=True ,fmt='g',␣
  ↪cmap='Blues')
plt.show()
```

```
        precision       recall          f1-score
        0.13            0.38            0.19
accuracy: 0.49
```



2.11 T14

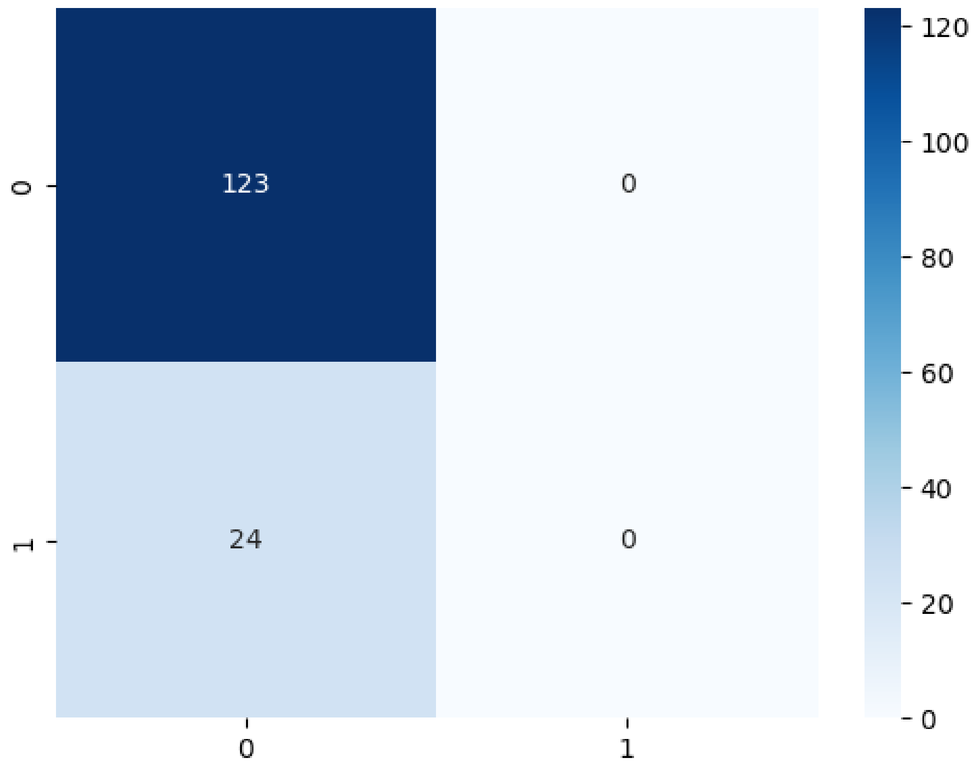
[25]:
```
unique, cnt = np.unique(y_train, return_counts = True)
print(unique, cnt, sep='\n')
frequent_class = unique[cnt.argmax()]
```

```
[0. 1.]
[1110  213]
```

[26]:
```
import warnings
warnings.filterwarnings(action='ignore')

y_pred_majority_rule = np.full(len(y_test) ,frequent_class)
print(classification_report(y_test, y_pred_majority_rule))
sns.heatmap(confusion_matrix(y_test, y_pred_majority_rule), annot=True␣
 ↪,fmt='g', cmap='Blues')

warnings.filterwarnings(action='default')

plt.show()
```

```
        precision     recall      f1-score
        0.00          0.00        0.00
```

```
accuracy: 0.84
```



2.12 T15

Model	Recall	Precision	F1	Accuracy
Naïve Bayes Histogram	.38	.38	.38	.80
Naïve Bayes Gaussian	.27	.42	.32	.72
Baseline random	.13	.38	.19	.49
Baseline majority rule	0	0	0	.84

2.13 T16

[27]:
```
t = np.arange(-5, 5, 0.05)

f1_max_hist = (-1, -1)
f1_max_gau = (-1, -1)
acc_max_hist = (-1, -1)
acc_max_gau = (-1, -1)

hist_classifier = Simple_Binary_BayesClassifier_Hist().fit(X_train, y_train)
gaussian_classifier = Simple_Binary_BayesClassifier_Gaussian().fit(X_train,␣
    ↪y_train)
```

```
for threshold in t:

    y_pred_hist = hist_classifier.predict(X_test, threshold)
    y_pred_gau = gaussian_classifier.predict(X_test, threshold)

    f1_max_hist = max( f1_max_hist, (f1_score(y_test, y_pred_hist), threshold) )
    f1_max_gau = max( f1_max_gau, (f1_score(y_test, y_pred_gau), threshold) )

    acc_max_hist = max( acc_max_hist, (accuracy_score(y_test, y_pred_hist),␣
 ↪threshold) )
    acc_max_gau = max( acc_max_gau, (accuracy_score(y_test, y_pred_gau),␣
 ↪threshold) )
```

[28]:
```
print(f"Accuracy max Histogram {acc_max_hist[0]:.2f} at threshold␣
 ↪{acc_max_hist[1]:.2f}")
print(f"Accuracy max Gaussian {acc_max_gau[0]:.2f} at threshold {acc_max_gau[1]:
 ↪.2f}")


print(f"F1 max Histogram {f1_max_hist[0]:.2f} at threshold {f1_max_hist[1]:.
 ↪2f}")
print(f"F1 max Gaussian {f1_max_gau[0]:.2f} at threshold {f1_max_gau[1]:.2f}")
```

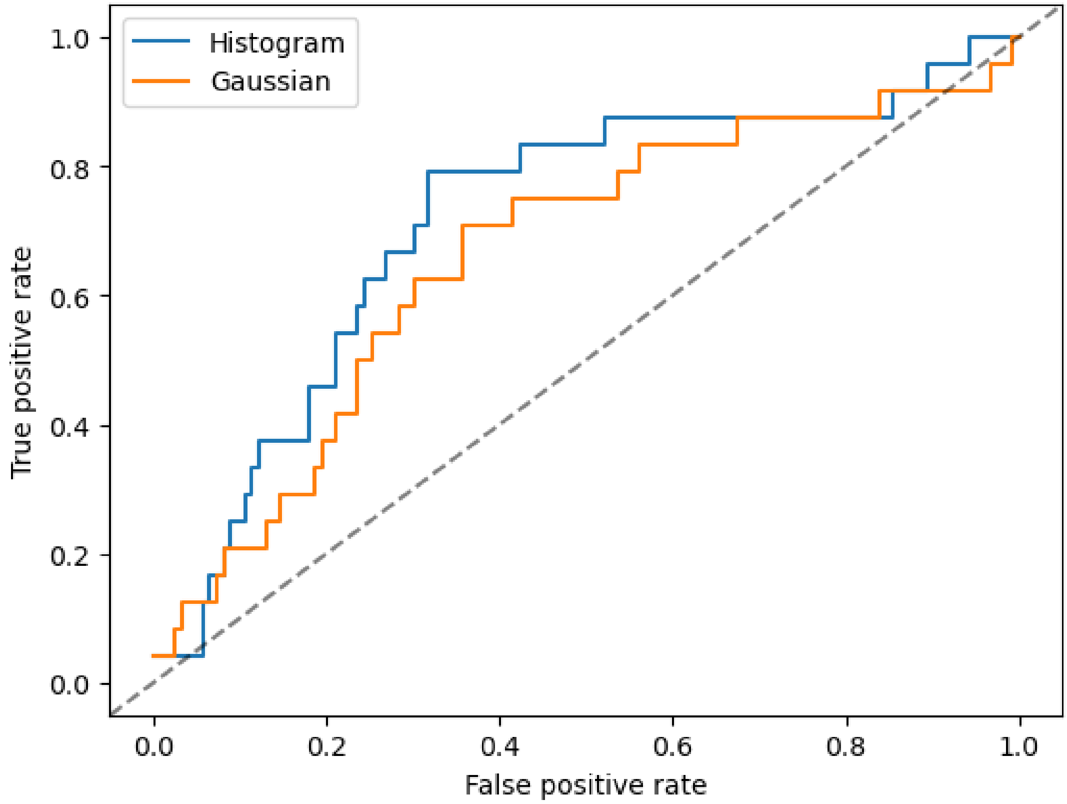
```
Accuracy max Histogram 0.84 at threshold 4.95
Accuracy max Gaussian 0.84 at threshold 4.50
F1 max Histogram 0.46 at threshold -2.10
F1 max Gaussian 0.40 at threshold -1.15
```

2.14 T17

[29]:
```
def plot_roc(model, X, y, label):
    y_pred_prob = model.predict_proba(X)
    fpr, tpr = roc_curve(y, y_pred_prob)
    plt.plot(fpr, tpr, label=label)
```

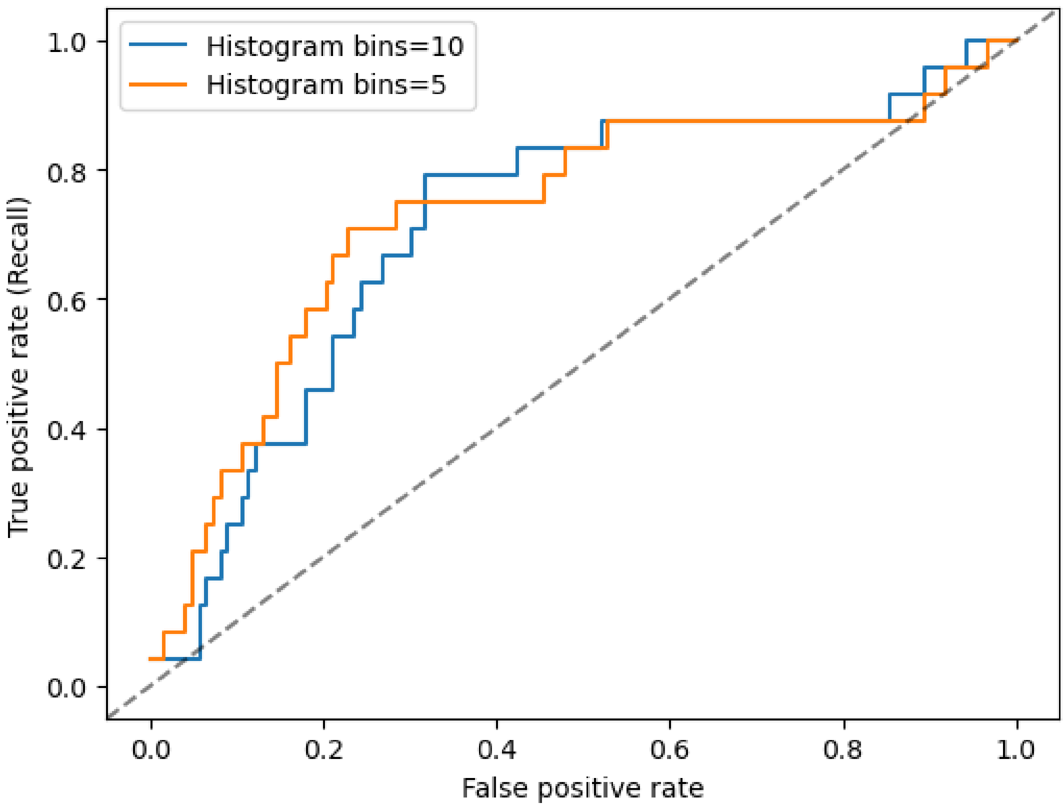
[30]:
```
hist_bins10_classifier = Simple_Binary_BayesClassifier_Hist(bins=10).
 ↪fit(X_train, y_train)
gaussian_classifier = Simple_Binary_BayesClassifier_Gaussian(threshold =␣
 ↪threshold).fit(X_train, y_train)

plot_roc(hist_bins10_classifier, X_test, y_test, 'Histogram')
plot_roc(gaussian_classifier, X_test, y_test, 'Gaussian')

plt.axline((0, 0), slope=1, c='k', linestyle = '--', alpha=0.5)
plt.xlabel('False positive rate')
plt.ylabel('True positive rate')
```

```
plt.legend()
plt.show()
```



2.15 T18

[31]:
```
hist_bins5_classifier = Simple_Binary_BayesClassifier_Hist(bins=5).fit(X_train,␣
↪y_train)

plot_roc(hist_bins10_classifier, X_test, y_test, 'Histogram bins=10')
plot_roc(hist_bins5_classifier, X_test, y_test, 'Histogram bins=5')

plt.axline((0, 0), slope=1, c='k', linestyle = '--', alpha=0.5)
plt.xlabel('False positive rate')
plt.ylabel('True positive rate (Recall)')
plt.legend()
plt.show()
```



The **Employee Attrition** has consider **recall** over precision because the employee that leave has high affect to the company.

Considering to choose bins=10 over bins = 5, at FPR ≈ 0.3 is acceptable and TPR of bins=10 is higher than TPR of bins=5.

2.16 T19

Submit your code (.py or .ipynb) on mycourseville.

2.17 OT4

[32]:
```
random_states = list(range(10))

acc_hist = []
acc_gau = []

for random_state in random_states:
    df_train, df_test = train_test_split(df, test_size = 0.1, random_state = random_state, stratify = df['Attrition'], shuffle=True)

    X_train = df_train.drop(columns='Attrition').to_numpy()
```

```
    y_train = df_train['Attrition'].to_numpy()
    X_test = df_test.drop(columns='Attrition').to_numpy()
    y_test = df_test['Attrition'].to_numpy()

    hist_classifier = Simple_Binary_BayesClassifier_Hist().fit(X_train, y_train)
    gaussian_classifier = Simple_Binary_BayesClassifier_Gaussian().fit(X_train,␣
  ↪y_train)

    y_pred_hist = hist_classifier.predict(X_test)
    y_pred_gau = gaussian_classifier.predict(X_test)

    acc_hist.append(accuracy_score(y_test, y_pred_hist))
    acc_gau.append(accuracy_score(y_test, y_pred_gau))

acc_hist = np.array(acc_hist)
acc_gau = np.array(acc_gau)

print(f"Histogram discretize Naïve Bayes mean = {acc_hist.mean()}, var =␣
  ↪{acc_hist.var()}")
print(f"Gaussian discretize Naïve Bayes mean = {acc_gau.mean()}, var = {acc_gau.
  ↪var()}")
```

```
Histogram discretize Naïve Bayes mean = 0.8068027210884352, var =
0.0011680318385857743
Gaussian discretize Naïve Bayes mean = 0.7802721088435375, var =
0.002628997177102134
```