Θέμα 3ο: $\min -2x_1 - 4x_2 + x_3$

μ.π. $2x_1 - x_2 + 2x_4 + x_5 = 10$

$x_1 + 4x_2 + x_3 + 2x_4 + x_6 = 4$

$x_j \geq 0, \ (j = 1, \dots, 6)$

Βέλτιστη βασική διαμέριση $B = [5, 1]$, $N = [2\ 6\ 3\ 4]$

$$A_B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, \quad X_B = B^{-1} b = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$w^T = (C_B)^T \cdot B^{-1} = \begin{bmatrix} 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \end{bmatrix}$$

$$S_N = (c_N)^T - w^T \underline{A_N} = \begin{bmatrix} -4 & 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 2 \\ 4 & 1 & 1 & 2 \end{bmatrix}$$

$$S_N = \begin{bmatrix} -4 & 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} -8 & -2 & -2 & -4 \end{bmatrix}$$

$$S_N = \begin{bmatrix} 4 & 2 & 3 & 4 \end{bmatrix}$$

Συντελεστής	c_1	c_2	c_3	c_4	c_5	c_6	b_1	b_2
Αριστερό Άκρο	$-\infty$	-8	-2	-4	$-4/9$	-2	8	0
Δεξιό Άκρο	-1	∞	∞	∞	∞	∞	∞	5
Σκιερή τιμή	—	—	—	—	—	—	0	-2
Ελαχιστικό κόστος	0	4	3	4	0	2	—	—

$C_2: C_2 - S_2 = -4 - 4 = -8$

$C_3: C_3 - S_3 = 1 - 3 = -2$

$C_4: C_4 - S_4 = 0 - 4 = -4$

$C_6: C_6 - S_6 = 0 - 2 = -2$

$C_1: \quad H_2 N = B_2^{-1} A_N = \underset{1\times 2}{\begin{bmatrix} 0 & 1 \end{bmatrix}} \underset{2\times 4}{\begin{bmatrix} -1 & 0 & 0 & 2 \\ 4 & 1 & 1 & 2 \end{bmatrix}} = \begin{bmatrix} 4 & 1 & 1 & 2 \end{bmatrix}$

$$\left.\begin{aligned} \alpha_1 &= \max\{\emptyset\} = -\infty \\ \beta_1 &= \min\left\{\frac{4}{4}, \frac{2}{1}, \frac{3}{1}, \frac{4}{2}\right\} = 1 \end{aligned}\right\} \begin{aligned} &\Rightarrow (-\infty, \overset{(c_j + \beta_j)}{-2+1}) \\ &\Rightarrow (-\infty, -1] \end{aligned}$$

c_s : $r = 1$, $H_{1.N} = B^{-1}_{1.} \cdot A_N = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 2 \\ 4 & 1 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -9 & -2 & -2 & -2 \end{bmatrix}$

$\alpha_s = \max\left\{ \frac{4}{-9}, \frac{2}{-2}, \frac{3}{-2}, \frac{4}{-4} \right\} = -\frac{4}{9}$ $\Rightarrow$ $\big[\overset{c_s+\alpha_s}{0 - \tfrac{4}{9}}, +\infty\big)$

$\beta_s = \min\{\emptyset\} = +\infty$

<u>b_1 : $i = 1$</u>

$\alpha_1 = \max\left\{ -\frac{2}{1} \right\} = -2$ $\Rightarrow$ $\big[\overset{b_1+\alpha_1}{10 - 2}, \overset{b_1+\beta_1}{+\infty}\big)$

$\beta_1 = \min\{\emptyset\} = +\infty$

$b_2 : i = 2 \quad , \quad X_B = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad , \quad B^{-1}_{\cdot 2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$$\left. \begin{aligned} \gamma_2 &= \max\left\{ -\frac{4}{1} \right\} = -4 \\ \delta_2 &= \min\left\{ -\frac{2}{(-2)} \right\} = 1 \end{aligned} \right\} \Rightarrow \begin{array}{c} b_2 + \gamma_2 \, , \; b_2 + \delta_2 \\ \left[4 - 4 \, , \; 4 + 1 \right] \end{array}$$

2) $\min \quad -2x_1 + 24x_2 + 4x_3 + 3x_4$

$$\text{s.t.} \quad -x_1 + 3x_2 + x_3 - x_5 = 0$$

$$-x_1 + 4x_2 + x_4 - x_6 = 1$$

$$x_j \geq 0 \; (j = 1, \dots, 6)$$

i) $B = [5\ 4]$, $N = [1\ 2\ 3\ 6]$

$A_B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $x_B = B^{-1} \cdot b = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$w^T = (c_B)^T \cdot B^{-1} = [0\ 3] \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = [0\ 3]$$

$$s_N = (c_N)^T - w^T A_N = [-2\ 24\ 4\ 0] - [0\ 3] \begin{bmatrix} -1 & 3 & 1 & 0 \\ -1 & 4 & 0 & -1 \end{bmatrix}$$

$$SN = [-2 \; 24 \; 4 \; 0] - [-3 \; 12 \; 0 \; -3]$$

$$SN = [1 \; 12 \; 4 \; 3]$$

Επειδή $SN \geq 0$ η βασική διαμέριση είναι βέλτιστη.

ii)

Συντελεστής	c_1	c_2	c_3	c_4	c_5	c_6	b_1	b_2
Α.Α	-3	12	0	2	-4	3	$-\infty$	0
Δ.Α	∞	∞	∞	6	1	∞	0	∞
Σκιερή τιμή	–	–	–	–	–	–	0	3
Ελλειμματικό κόστος	1	12	4	0	0	3	–	–

C_1 : $[c_j - S_j, +\infty) = [-3, +\infty)$

C_2 : $[12, +\infty)$

C_3 : $[0, +\infty)$

C_6 : $[-3, +\infty)$

C_4 : $r = 2$, $H_{2N} = B_2^{-1} A_N = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & 1 & 0 \\ -1 & 4 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 & 0 & -1 \end{bmatrix}$

$\alpha_4 = \max\left\{ \frac{1}{-1}, \frac{3}{-1} \right\} = -1$ $\left\{ \begin{array}{l} [C_4 + \alpha_4, C_4 + \beta_4] \\ [2, 6] \end{array} \right.$

$\beta_5 = \min\left\{ \frac{12}{4} \right\} = 3$

$$C_S: \; r = 1, \quad H_1 N = \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 & 1 & 0 \\ -1 & 4 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -1 & 0 \end{bmatrix}$$

$$\left.\begin{aligned} \alpha_S &= \max\left\{ \frac{12}{-3}, \frac{4}{-1} \right\} = -4 \\ \beta_S &= \min\left\{ \frac{1}{1} \right\} = 1 \end{aligned}\right\} \Longrightarrow \begin{array}{l} [C_S + \alpha_S, \; C_S + \beta_S] \\ {[-4, 1]} \end{array}$$

$$b_1: \; i = 1, \quad x_B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B^{-1}_{\cdot 1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} \gamma_1 &= \max\{ \emptyset \} = -\infty \\ \delta_1 &= \min\{ 0 \} = 0 \end{aligned}\right\} \quad \begin{array}{l} b_1 + \gamma_1 \quad b_1 + \delta_1 \\ {[-\infty, 0]} \end{array}$$

b_2: $i=2$ $\quad x_B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad B^{-1}_{\cdot 2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\left.\begin{aligned} \delta_2 &= \max\left\{-\frac{1}{1}\right\} = -1 \\ \delta_2 &= \min\{\emptyset\} = +\infty \end{aligned}\right\} \Rightarrow \overset{b_2+\delta_2 \quad b_2+\delta_2}{[0, +\infty)}$$

Θέμα 3

$B = [1\ 6]$, $N = [3\ 2\ 5\ 4]$

$$\min\ -x_1 + 2x_2 - 4x_3 + 2x_4$$

$$x_1 + 2x_2 + 6x_3 + 2x_4 + x_5 = 5$$
$$2x_1 - x_2 - x_3 + 4x_4 + x_6 = 12$$
$$x_j \geq 0,\ (j = 1, \dots, 6)$$

$A_B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, $w^T = (c_B)^T \cdot B^{-1} = [-1\ 0]\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$w^T = [-1\ 0]$

$s_N = (c_N)^T - w^T A_N = [-4\ 2\ 0\ 2] - [-1\ 0]\begin{bmatrix} 6 & 2 & 1 & 2 \\ -1 & -1 & 0 & 4 \end{bmatrix}$

$s_N = [-4\ 2\ 0\ 2] - [-6\ -2\ -1\ -2]$

$S_N = [2\ 4\ 1\ 4]$

Συντελεστής	C2	C3	C4	C5
Α.Α	-2	-6	-2	-1
Δ.Α	∞	∞	∞	∞
S_j	4	2	4	1

C2: $[-2, +\infty)$

C3: $[-6, +\infty)$

C4: $[-2, +\infty)$

C5: $[-1, +\infty)$