作业一

课件 2 Exercise 6

（a）（0，-1）

（b）（1，-1）

课件 3 Exercise 4 (a)

$$\begin{pmatrix}1\\0\\2\\0\end{pmatrix}+x_2\begin{pmatrix}-2\\1\\0\\0\end{pmatrix}+x_4\begin{pmatrix}-1\\0\\-1\\1\end{pmatrix}$$

作业二

（a） $P=\begin{pmatrix}0&1&0&0\\0&0&0&1\\1&0&0&0\\0&0&1&0\end{pmatrix}$

$$L=\begin{pmatrix}1&0&0&0\\0&1&0&0\\1/3&0&1&0\\2/3&-1/2&1/2&1\end{pmatrix}\qquad U=\begin{pmatrix}3&6&-12&3\\0&2&-2&6\\0&0&8&16\\0&0&0&-5\end{pmatrix}$$

（b） $x=\begin{pmatrix}2&-1&0&1\end{pmatrix}^T$

作业四

课件 5 中第 9 题

直线拟合： $y=9.64+0.182x$ 误差： $\sum_i \varepsilon_i^2=162.9$

曲线拟合： $y=13.97+0.1818x-0.4336x^2$ 误差： $\sum_i \varepsilon_i^2=1.622$

作业五：

课件 6 中 Exercise 4

$$\text{(a)}\quad [\mathbf{A}]_{\mathcal{S}} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 7 \end{pmatrix}$$

$$\text{(b)}\quad [\mathbf{A}]_{\mathcal{S}'} = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & -9 \\ 1 & 1 & 8 \end{pmatrix} \text{ and } \mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

课件 6 中 Exercise 7

(a) 对于任意 $x \in \chi$， 可以表示为 $x = (\alpha \quad \beta \quad 0 \quad 0)$， 那么

$$T(x) = T(\alpha, \beta, 0, 0) = (\alpha + \beta, \beta, 0, 0) \in \chi$$

$$\text{(b)}\quad \left[\mathbf{T}_{/\mathcal{X}}\right]_{\{\mathbf{e}_1, \mathbf{e}_2\}} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\text{(c)}\quad [\mathbf{T}]_{\mathcal{B}} = \left(\begin{array}{cc|cc} 1 & 1 & * & * \\ 0 & 1 & * & * \\ \hline 0 & 0 & * & * \\ 0 & 0 & * & * \end{array}\right)$$

作业六

Exercise 3

(a) $\|A\|_1 = 4, \|A\|_2 = \sqrt{10}, \|A\|_\infty = 3$

(b) $\|B\|_1 = \|B\|_2 = \|B\|_\infty = 1$

(c) $\|C\|_1 = \|C\|_\infty = 10, \|C\|_2 = 9$

Exercise 12

(a)

$$\mathbf{Q} = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 0 & 1/\sqrt{3} & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{R} = \begin{pmatrix} \sqrt{3} & \sqrt{3} & -\sqrt{3} \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{6} \end{pmatrix}.$$

(b)解 $Rx = Q^T b$ 得 $x = (2/3 \quad 1/3 \quad 0)^T$

Exercise 16

(a) $$\frac{\mathbf{v}\mathbf{v}^T}{\mathbf{v}^T\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{v}^T\mathbf{u}}{\mathbf{v}^T\mathbf{v}}\right)\mathbf{v} = \frac{1}{6}\mathbf{v} = \frac{1}{6}\begin{pmatrix} 1 \\ 4 \\ 0 \\ -1 \end{pmatrix}$$

(b) $$\frac{\mathbf{u}\mathbf{u}^T}{\mathbf{u}^T\mathbf{u}}\mathbf{v} = \left(\frac{\mathbf{u}^T\mathbf{v}}{\mathbf{u}^T\mathbf{u}}\right)\mathbf{u} = \frac{1}{5}\mathbf{u} = \frac{1}{5}\begin{pmatrix} -2 \\ 1 \\ 3 \\ -1 \end{pmatrix}$$

(c) $$\left(\mathbf{I} - \frac{\mathbf{v}\mathbf{v}^T}{\mathbf{v}^T\mathbf{v}}\right)\mathbf{u} = \mathbf{u} - \left(\frac{\mathbf{v}^T\mathbf{u}}{\mathbf{v}^T\mathbf{v}}\right)\mathbf{v} = \mathbf{u} - \frac{1}{6}\mathbf{v} = \frac{1}{6}\begin{pmatrix} -13 \\ 2 \\ 18 \\ -5 \end{pmatrix}$$

(d) $$\left(\mathbf{I} - \frac{\mathbf{u}\mathbf{u}^T}{\mathbf{u}^T\mathbf{u}}\right)\mathbf{v} = \mathbf{v} - \left(\frac{\mathbf{u}^T\mathbf{v}}{\mathbf{u}^T\mathbf{u}}\right)\mathbf{u} = \mathbf{v} - \frac{1}{5}\mathbf{u} = \frac{1}{5}\begin{pmatrix} 7 \\ 19 \\ -3 \\ -4 \end{pmatrix}$$

作业七

Exercise 1

By Householder reduction

$$\begin{aligned}\mathbf{R}_2\mathbf{R}_1\mathbf{A} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3/5 & 4/5 \\ 0 & 4/5 & 3/5 \end{pmatrix}\begin{pmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \end{pmatrix}\begin{pmatrix} 1 & 19 & -34 \\ -2 & -5 & 20 \\ 2 & 8 & 37 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 15 & 0 \\ 0 & 15 & -30 \\ 0 & 0 & 45 \end{pmatrix} = \mathbf{R},\end{aligned}$$

so

$$\mathbf{Q} = (\mathbf{R}_2\mathbf{R}_1)^T = \begin{pmatrix} 1/3 & 14/15 & -2/15 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & -2/15 & 11/15 \end{pmatrix}.$$

By Givens reduction $P_{23}P_{13}P_{12}A = R$

$$\mathbf{P}_{12} = \begin{pmatrix} 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{P}_{13} = \begin{pmatrix} \sqrt{5}/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ -2/3 & 0 & \sqrt{5}/3 \end{pmatrix}$$

$$\mathbf{P}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 11/5\sqrt{5} & -2/5\sqrt{5} \\ 0 & 2/5\sqrt{5} & 11/5\sqrt{5} \end{pmatrix}$$

Exercise 2

set $\mathbf{u} = \mathbf{A}_{*1} - \|\mathbf{A}_{*1}\|\,\mathbf{e}_1 = \begin{pmatrix} -1 \\ 2 \\ -2 \\ 1 \end{pmatrix}$, so

$$\mathbf{R}_1 = \mathbf{I} - 2\frac{\mathbf{u}\mathbf{u}^*}{\mathbf{u}^*\mathbf{u}} = \frac{1}{5}\begin{pmatrix} 4 & 2 & -2 & 1 \\ 2 & 1 & 4 & -2 \\ -2 & 4 & 1 & 2 \\ 1 & -2 & 2 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{R}_1\mathbf{A} = \begin{pmatrix} 5 & -15 & 5 \\ 0 & 10 & -5 \\ 0 & -10 & 2 \\ 0 & 5 & 14 \end{pmatrix}.$$

Next use $\mathbf{u} = \begin{pmatrix} 10 \\ -10 \\ 5 \end{pmatrix} - \begin{pmatrix} 15 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ -10 \\ 5 \end{pmatrix}$ to build

$$\hat{\mathbf{R}}_2 = \mathbf{I} - 2\frac{\mathbf{u}\mathbf{u}^*}{\mathbf{u}^*\mathbf{u}} = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix} \quad \text{and} \quad \mathbf{R}_2 = \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & -2 & 1 \\ 0 & -2 & -1 & 2 \\ 0 & 1 & 2 & 2 \end{pmatrix},$$

so

$$\mathbf{R}_2\mathbf{R}_1\mathbf{A} = \begin{pmatrix} 5 & -15 & 5 \\ 0 & 15 & 0 \\ 0 & 0 & 12 \\ 0 & 0 & 9 \end{pmatrix}.$$

Finally, with $\mathbf{u} = \begin{pmatrix} 12 \\ 9 \end{pmatrix} - \begin{pmatrix} 15 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$, build

$$\hat{\mathbf{R}}_3 = \frac{1}{5}\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix} \quad \text{and} \quad \mathbf{R}_3 = \frac{1}{5}\begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 3 & -4 \end{pmatrix},$$

so that

$$\mathbf{R}_3\mathbf{R}_2\mathbf{R}_1\mathbf{A} = \begin{pmatrix} 5 & -15 & 5 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \\ 0 & 0 & 0 \end{pmatrix}.$$

Therefore, $\mathbf{PA} = \mathbf{T} = \begin{pmatrix} \mathbf{R} \\ \mathbf{0} \end{pmatrix}$, where

$$\mathbf{P} = \mathbf{R}_3\mathbf{R}_2\mathbf{R}_1 = \frac{1}{15}\begin{pmatrix} 12 & 6 & -6 & 3 \\ 9 & -8 & 8 & -4 \\ 0 & -5 & 2 & 14 \\ 0 & -10 & -11 & -2 \end{pmatrix} \quad \text{and} \quad \mathbf{R} = \begin{pmatrix} 5 & -15 & 5 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{pmatrix}.$$

The first three columns in

$$\mathbf{P}^T = \mathbf{R}_1\mathbf{R}_2\mathbf{R}_3 = \frac{1}{15}\begin{pmatrix} 12 & 9 & 0 & 0 \\ 6 & -8 & -5 & -10 \\ -6 & 8 & 2 & -11 \\ 3 & -4 & 14 & -2 \end{pmatrix}$$

are an orthonormal basis for $R(\mathbf{A})$. Since the diagonal entries of $\mathbf{R}$ are positive,

$$\frac{1}{15}\begin{pmatrix} 12 & 9 & 0 \\ 6 & -8 & -5 \\ -6 & 8 & 2 \\ 3 & -4 & 14 \end{pmatrix}\begin{pmatrix} 5 & -15 & 5 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{pmatrix} = \mathbf{A}$$

Exercise 7

(a) The fact that

$$rank(\mathbf{B}) = rank[\mathbf{X}\,|\,\mathbf{Y}] = rank\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} = 3$$

implies $\mathcal{B}_{\mathcal{X}} \cup \mathcal{B}_{\mathcal{Y}}$ is a basis for $\Re^3$, so $\mathcal{X}$ and $\mathcal{Y}$ are complementary.

(b) the projector onto $\mathcal{X}$ along $\mathcal{Y}$ is

$$\mathbf{P} = [\mathbf{X}\,|\,\mathbf{0}][\mathbf{X}\,|\,\mathbf{Y}]^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}^{-1}$$
$$= \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{pmatrix},$$

and the complementary projector onto $\mathcal{Y}$ along $\mathcal{X}$ is

$$\mathbf{Q} = \mathbf{I} - \mathbf{P} = \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix}.$$

(c) $\mathbf{Qv} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$