6. $rank(A) \leq 3$, ∵ 由 $A_{3*} = 2A_{1*}$, A_{1*}, A_{2*} 线性无关 得: $rank(A) = 2$

$$B = A\cdot A^T = \begin{pmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ 1 & 1 & 2 \\ -4 & 0 & -8 \end{pmatrix} = \begin{pmatrix} 27 & -9 & 54 \\ -9 & 11 & -18 \\ 54 & -18 & 108 \end{pmatrix}$$

$$C = A^T \cdot A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ 1 & 1 & 2 \\ -4 & 0 & -8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{pmatrix} = \begin{pmatrix} 6 & 18 & 4 & -20 \\ 18 & 54 & 12 & -60 \\ 4 & 12 & 6 & -20 \\ -20 & -60 & -20 & 80 \end{pmatrix}$$

对 B: $B_{3*} = 2B_{1*}$, B_{1*}, B_{2*} 线性无关, ∴ $rank(A\cdot A^T) = 2$

对 C: $C_{2*} = 3C_{1*}$, $C_{4*} = -2\times(C_{1*} + C_{3*})$, 而 C_{1*} 与 C_{3*} 线性无关 ∴ $rank(A^T\cdot A) = 2$

综上, 验证了: $rank(A^T\cdot A) = rank(A) = rank(A\cdot A^T)$

8. $h_1(x) = \alpha_0 + \alpha_1 x$ $\quad h_2(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$

$$J_1 = \sum_{i=1}^{11} [h_1(x_i) - y_i]^2 = (A_1X_1 - b_1)^T \cdot (A_1X_1 - b_1), \quad A_1 = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_{11} \end{pmatrix}, \quad X_1 = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}, \quad b_1 = \begin{pmatrix} y_1 \\ \vdots \\ y_{11} \end{pmatrix}$$

$$\Rightarrow \text{unique solution} = (A^T\cdot A)^{-1}\cdot A^T b = \begin{pmatrix} 9.6364 \\ 0.1818 \end{pmatrix} \quad \Rightarrow J_1 = 162.9091$$

同理可得: $J_2 = 1.6224$, 此时有 $\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} 13.9720 \\ 0.1818 \\ -0.4336 \end{pmatrix}$

∴ 2次拟合可以更好的拟合这些数据