11. $A = \begin{pmatrix} 1 & 2 & 4 & 17 \\ 3 & 6 & -12 & 3 \\ 2 & 3 & -3 & 2 \\ 0 & 2 & -2 & 6 \end{pmatrix}$ $b = \begin{pmatrix} 17 \\ 3 \\ 3 \\ 4 \end{pmatrix}$

(a) $[A|p] = \left(\begin{array}{cccc|c} 1 & 2 & 4 & 17 & 1 \\ 3 & 6 & -12 & 3 & 2 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ 1 & 2 & 4 & 17 & 1 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right)$

$\rightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ \frac{1}{3} & 0 & 8 & 16 & 1 \\ \frac{2}{3} & -1 & 5 & 0 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ 0 & 2 & -2 & 6 & 4 \\ \frac{1}{3} & 0 & 8 & 16 & 1 \\ \frac{2}{3} & -1 & 5 & 0 & 3 \end{array}\right)$

$\rightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ 0 & 2 & -2 & 6 & 4 \\ \frac{1}{3} & 0 & 8 & 16 & 1 \\ \frac{2}{3} & -\frac{1}{2} & 4 & 3 & 3 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ 0 & 2 & -2 & 6 & 4 \\ \frac{1}{3} & 0 & 8 & 16 & 1 \\ \frac{2}{3} & -\frac{1}{2} & \frac{1}{2} & -5 & 3 \end{array}\right)$

$\therefore L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & 1 & 0 \\ \frac{2}{3} & -\frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$ $U = \begin{pmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{pmatrix}$ $p = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

(b) $Ax=b \rightarrow PAx=Pb$

由 $PA=LU$,

$LUx=Pb$ 设 $Ux=y$，则有 $Ly=Pb$

即 $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & 1 & 0 \\ \frac{2}{3} & -\frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 17 \\ 3 \\ 3 \\ 4 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 16 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 17 \\ 3 \end{pmatrix}$$

又 $Ux=y$，即：$\begin{pmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 16 \\ -5 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$