Graph Optimization

What is graph?

A graph is a pair $G = (V, E)$, where V is a set of nodes, each of which contains some parameters to be optimized. E is a set of connected information, whose elements are denotes the constraint relationship between two nodes.
Many robotics and computer vision problems can be represented by a graph problem.

How to solve graph problem?

A graph problem can be defined as a nonlinear least squares problems. $f_{ij}(v_i, v_j; e_{ij})$ shows the constraint relationship between node v_i and v_j e_{ij} is the prior error of v_i and v_j.

$$F(V) = \sum_{\{i,j\}\in E} f_{ij}(v_i, v_j; e_{ij})^2$$

We need to find a optimal set of nodes (i.e. V) to minimize the overall cost. According to guass_newton_method.md, as soon as we can compute the hessian matrix H and gradient g, we can solve this graph optimization problem.

The hessian matrix H

We note that the size of the hessian matrix will be very large, since there are many parameters for F.
The hessian matrix of f_{ij} can be show as:

$$H_{ij} = \begin{bmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & J_i^T J_i & \dots & J_i^T J_j & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & J_j^T J_i & \dots & J_j^T J_j & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

The $J_i^T J_i$ is located in row i column i of H_{ij}
The $J_j^T J_j$ is located in row j column j of H_{ij}
The $J_i^T J_j$ is located in row i column j of H_{ij}
The $J_j^T J_i$ is located in row j column i of H_{ij}

The overall hessian matrix of F is:

$$H = \sum_{\{i,j\}\in E} H_{ij}$$

The gradient g

The gradient vector of f_{ij} can be show as:

$$g_{ij} = \begin{bmatrix} \cdots \\ J_i^T r_i \\ \cdots \\ J_j^T r_n \\ \cdots \end{bmatrix}$$

The $J_i^T r_i$ is located in row i of g_{ij}
The $J_j^T J_j$ is located in row j of g_{ij}

The overall gradient vector of F is:

$$g = \sum_{\{i,j\} \in E} g_{ij}$$