COM1006/9/Semester 1 2009/10

UNIVERSITY OF SURREY ©

Faculty of Engineering and Physical Sciences

Department of Computing

Undergraduate Programmes in Computing

Level 1 Examination

Module COM1006: 10 Credits

Foundations of Computing

Time allowed: 2 hours   Semester 1 2009/10

Answer ALL questions.

Calculators are NOT permitted.

Solutions

1. Let:

- T denote ‘I play tennis’
- C denote ‘I go cycling’
- G denote ‘I go to the gym’
- W denote ‘I lose weight’
- S denote ‘I sleep well’

(a) Express each of the following propositions in English: [2 marks]

(i). $(G \Rightarrow W) \wedge (C \Rightarrow S)$

(ii). $(G \wedge (\neg C \vee \neg T) \wedge (W \vee S))$

(i). If I go to the gym then I lose weight, and if I go cycling then I sleep well.

(ii). I go to the gym, and I don’t go cycling or I don’t play tennis, and I lose weight or sleep well.

(b) Express each of the following as formal propositions. Identify any ambiguities and for ambiguous statements give both interpretations. [6 marks]

(i). If I play tennis then I lose weight, and if I don’t go to the gym then I don’t sleep well.

(ii). If I either play tennis or don’t sleep well then if I don’t lose weight then I don’t go to the gym.

(iii). I play tennis and I go cycling or I lose weight.

(iv). I don’t lose weight unless I go to the gym.

(v). I sleep well only if I go cycling.

(i). $(T \Rightarrow W) \wedge (\neg G \Rightarrow \neg S)$

(ii). $(T \vee \neg S) \Rightarrow (\neg W \Rightarrow \neg G)$

(iii). Ambiguous: $(T \wedge C) \vee W$ or $T \wedge (C \vee W)$

(iv). $W \Rightarrow G$

(v). $S \Rightarrow C$

(c) Explain what a *logical argument* is in propositional logic, and what it means for a logical argument to be *valid*. [2 marks]

A logical argument is a sequence or collection of propositions, one of which is the *conclusion* and the others are *premises*. The argument is valid if the conclusion is true whenever all of the premises are true.

(d) Use truth tables to establish whether the following argument is valid. Explain your answer. [5 marks]

If there's a fire then the alarm sounds and the sprinklers are on. If the alarm does not sound, then if the sprinklers are not on, then there is no fire. The sprinklers are on. Therefore if the alarm is on and the sprinklers are on then there is a fire.

F	A	S	$F \Rightarrow (A \wedge S)$	$\neg A \Rightarrow (\neg S \Rightarrow \neg F)$	$f(A \wedge S) \Rightarrow F$
1	1	1	1	1	1
1	1	0	0	1	1
1	0	1	0	1	1
1	0	0	0	0	1
0	1	1	1	1	0 $\Longleftarrow$
0	1	0	1	1	1
0	0	1	1	1	1
0	0	0	1	1	1

This is not valid: the fifth line has all the premises true but the conclusion false

(e) Let

- $S(x, y)$ denote 'store x supplies store y'
- $L(x, y)$ denote 'store x is located in location y'

(i). Express 'every location has a store' in predicate logic. [2 marks]

$$\forall\, y \bullet \exists\, x \bullet L(x, y)$$

(ii). Express 'every store supplies some store' in predicate logic. [2 marks]

$$\forall\, x \bullet \exists\, y \bullet S(x, y)$$

(iii). Express the following in English: [6 marks]

(A) $\neg \exists\, x \bullet S(x, x)$

No store supplies itself.

(B) $\forall\, x \bullet \forall\, y \bullet S(x, y) \Rightarrow \neg S(y, x)$

Stores do not supply their suppliers (stores that supply them).

(C) $\forall\, x \bullet \forall\, y \bullet \forall\, z \bullet (L(x, z) \wedge S(x, y)) \Rightarrow \neg L(y, z)$

Stores do not supply stores at the same location

Total marks: 25

2. Let

- $M = \{al, bob, chas, dot\}$
- $N = \{al, chas, eve, george, hal\}$
- $L = \{Java, Haskell, Fortran\}$

(a) Give set enumerations for the following: [6 marks]

(i). $M \cup N$

$$\{al, bob, chas, dot, eve, george, hal\}$$

(ii). $\mathbb{P}\, L$

$$\{\{\}, \{Java\}, \{Haskell\}, \{Fortran\}, \{Java, Haskell\},$$
$$\{Java, Fortran\}, \{Haskell, Fortran\}, \{Java, Haskell, Fortran\}\}$$

(iii). $L \times (N \setminus M)$

$$\{(Java, eve), (Java, george), (Java, hal),$$
$$(Haskell, eve), (Haskell, george), (Haskell, hal),$$
$$(Fortran, eve), (Fortran, george), (Fortran, hal)\}$$

(b) State which of the following are true: [4 marks]

(i). $george \in M \cup N$
(ii). $bob \in M \cap N$
(iii). $Java \notin M \setminus N$
(iv). $M \subseteq N$
(v). $(chas, Haskell) \notin M \times L$
(vi). $\{(chas, eve), (dot, hal)\} \subseteq M \times N$
(vii). $\{Java\} \in \mathbb{P}\, L$
(viii). $\{(chas, eve), (dot, hal)\} \in \mathbb{P}(M \times N)$

(i). true
(ii). false
(iii). true
(iv). false
(v). false
(vi). true
(vii). true
(viii). true

(c) Which of the following laws are true for any sets A, B, and C? Justify your answers. [5 marks]

QUESTION 2 CONTINUES ON NEXT PAGE

(i). $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

true. Can be shown with a venn diagram

(ii). $A \setminus (B \setminus C) = (A \cap C) \cup (A \setminus B)$

True. Can be shown with venn diagram

(d) In a Z specification tracking registrations on programming courses, *java* denotes the set of students registered on the Java course, *fortran* denotes the set registered on the Fortran course, and *basic* denotes the set of students registered on the BASIC course. These are the only programming courses available.

(i). What do the following constraints specify:

(A) $java \cap fortran = \{\}$ [2 marks]

No-one can be registered for both java and fortran

(B) $basic \subseteq (fortran \cup java)$ [2 marks]

Anyone registered for BASIC must be registered for at least one of fortran or java

(ii). Express each of the following specifications using set notation on *java*, *fortran*, and *basic*:

(A) No-one can be registered on more than one of the Java, Fortran, and BASIC courses. [3 marks]

$java \cap fortran = \emptyset \wedge fortan \cap basic = \emptyset \wedge basic \cap java = \emptyset$

(B) Anyone registered on at least two of the programming courses must be registered on the Java course [3 marks]

Several ways of expressing this. e.g. $(fortran \cap basic) \subseteq java$, $(fortran \cap basic) \setminus java = \emptyset$

Total marks: 25

3. (a) (i). What is a *relation* $A \leftrightarrow B$? [1 mark]

A set of pairs (a, b) where $a \in A$ and $b \in B$

(ii). What is a *partial function* $A \nrightarrow B$? Give a diagram illustrating the multiplicities between A and B. [2 marks]

A subset of $A \times B$ where each $a \in A$ appears at most once.
A 0..1 — B

(iii). What is a *total function* $A \rightarrow B$? Give a diagram illustrating the multiplicities between A and B. [2 marks]

A subset of $A \times B$ where each $a \in A$ appears exactly once.
A 1 — B

(iv). What is an *injective function* $A \rightarrowtail\!\!\!\!\!\!\nrightarrow B$? Give a diagram illustrating the multiplicities between A and B. [2 marks]

A partial function $A \nrightarrow B$ where each $b \in B$ appears at most once.
A 0..1 — 0..1 B

(v). Can a partial function be a total function? Justify your answer. [2 marks]

Yes. 'at most once' includes 'exactly once', thus any total function is also a partial function.

(vi). Can a total function be injective? Justify your answer. [2 marks]

Yes: when each element in the range appears at most once. e.g. $student \rightarrowtail URN$

(b) Consider a partial function $uni : PERSON \nrightarrow UNIVERSITY$ which maps people to the Univerity at which they are registered for their undergraduate degree. Let *greyfriarspupils* be the set of pupils who attended Greyfriars school, and *rugbypupils* be the set of pupils who attended *Rugby* school.

(i). Why is it not appropriate to define *uni* as a total function? [1 mark]

That would require every person to be registered at a university, which does not reflect reality

(ii). Why is it not appropriate to define *uni* as an injective function? [1 mark]

That would allow at most one person per university, which does not reflect reality

(iii). Explain in English what are meant by the following: [8 marks]

(A) $(rugbypupils \cup greyfriarspupils) \lhd uni$

The registrations restricted to Rugby and Greyfriars schools.

(B) $uni(\!| \; rugbypupils \; |\!)$

The universities attended by pupils of Rugby school

(C) $uni^{\sim} \llparenthesis \{SurreyUniversity\} \rrparenthesis$

The people registered with Surrey University

(D) $uni^{\sim} \llparenthesis (uni \llparenthesis rugby \rrparenthesis) \rrparenthesis$

All the people registered with universities at which rugby people are registered

(iv). Give the set of universities that have students from both Rugby and Greyfriars [2 marks]

$uni \llparenthesis rugbypupils \rrparenthesis \cap uni \llparenthesis greyfriars \rrparenthesis$

(v). Give the set of universities that have students from neither Rugby nor Greyfriars [2 marks]

$UNIVERSITY \setminus (uni \llparenthesis rugbypupils \cup greyfriars \rrparenthesis)$

Total marks: 25

4. A system at a school is required to track the subjects that students are registered for. The set of possible subjects is given as

$$\begin{aligned} SUBJECT = \{\ & maths, english, science, ICT, history, geography, \\ & french, german, spanish, mandarin, latin, art, music, drama\} \end{aligned}$$

The basic type $STUDENT$ is used as the overall set of all possible students.

The system will track the registrations using a relation $reg : STUDENT \leftrightarrow SUBJECT$, where $s \mapsto c \in reg$ represents the fact that student s is registered for course c. The system will also track the set of students on the school roll: $roll \subseteq STUDENT$.

(a) Give a state schema that captures the following requirements: [12 marks]

- All students on the school roll should be registered for *maths*, *english*, *science*, and *ICT*.
- Only students on the school roll should be registered for any subject.
- Nobody can be registered for both *german* and *spanish*.
- Anybody registered for *latin* must also be registered for *french*.
- Anybody registered for *mandarin* must be registered for at least one other language.
- Students can be registered for at most one of *art*, *music*, and *drama*
- Students can be registered for at most 8 courses.

Registrations

$roll \subseteq STUDENT$
$reg : STUDENT \leftrightarrow SUBJECT$

$dom(reg) = roll$
$roll \times \{english, maths, science, ICT\} \subseteq reg$
$reg^{-1}(\!|\ \{german\}\ |\!) \cap reg^{-1}(\!|\ \{spanish\}\ |\!) = \emptyset$
$reg^{-1}(\!|\ \{latin\}\ |\!) \subseteq reg^{-1}(\!|\ \{french\}\ |\!)$
$reg^{-1}(\!|\ \{mandarin\}\ |\!) \subseteq reg^{-1}(\!|\ \{french, german, spanish, latin\}\ |\!)$
$\forall s : STUDENT \bullet \#(\{s\} \lhd reg \rhd \{art, music, drama\}) \leqslant 1$
$\forall s : STUDENT \bullet \#(reg(\!|\ \{s\}\ |\!)) \leqslant 8$

(b) Specify the operations below. In each case you should make explicit any inputs of the operation that are necessary to define it, its precondition, and the effect it has on the state.

(i). *SwitchGermanToSpanish*: A student switches from *german* to *spanish*. [4 marks]

QUESTION 4 CONTINUES ON NEXT PAGE

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{SwitchGermanToSpanish}} \\
\Delta Registrations \\
name? : STUDENT \\
\hline
name? \mapsto german \in reg \\
reg' = reg \cup \{name? \mapsto spanish\} - \{name? \mapsto german\} \\
\hline
\end{array}$$

(ii). *ArtRegistration*: A student registers for *art*. [4 marks]

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{ArtRegistration}} \\
\Delta Registrations \\
name? : STUDENT \\
\hline
name? \in roll \\
name? \notin reg^{-1}(\!|\ \{art, music, drama\}\ |\!) \\
\#(reg(\!|\ \{name?\}\ |\!)) < 8 \\
reg' = reg \cup \{name? \mapsto art\} \\
\hline
\end{array}$$

(iii). *NewStudent*: A new student wishes to join the school roll. [5 marks]

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{NewStudent}} \\
\Delta Registrations \\
name? : STUDENT \\
\hline
name? \notin roll \\
roll' = roll \cup \{name\} \\
reg' = reg \cup (\{name?\} \times \{english, maths, science, ICT\}) \\
\hline
\end{array}$$

Total marks: 25

INTERNAL EXAMINER: Steve Schneider