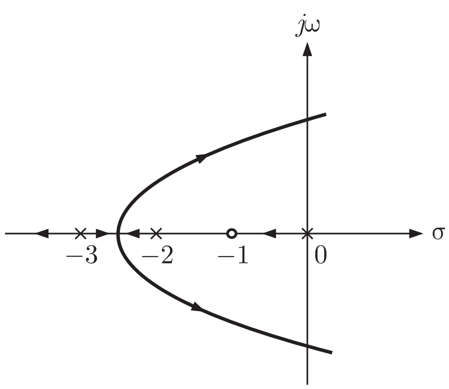
UNIT 7
Control Systems

2011 ONE MARK

MCQ 7.1

The root locus plot for a system is given below. The open loop transfer function corresponding to this plot is given by



(A) $G(s)H(s) = k\dfrac{s(s+1)}{(s+2)(s+3)}$

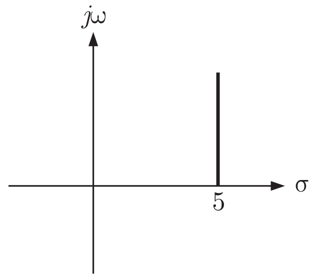
(B) $G(s)H(s) = k\dfrac{(s+1)}{s(s+2)(s+3)^2}$

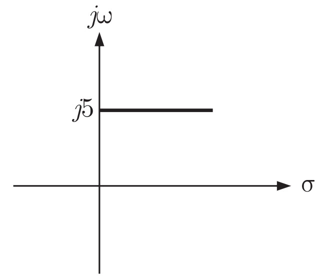
(C) $G(s)H(s) = k\dfrac{1}{s(s-1)(s+2)(s+3)}$

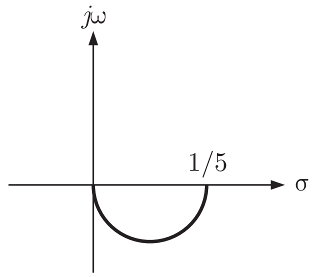
(D) $G(s)H(s) = k\dfrac{(s+1)}{s(s+2)(s+3)}$

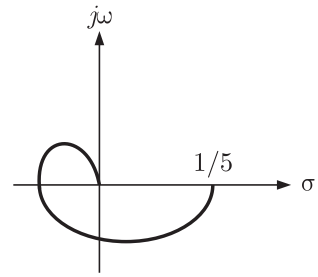
MCQ 7.2

For the transfer function $G(j\omega) = 5 + j\omega$, the corresponding Nyquist plot for positive frequency has the form

(A)


(B)


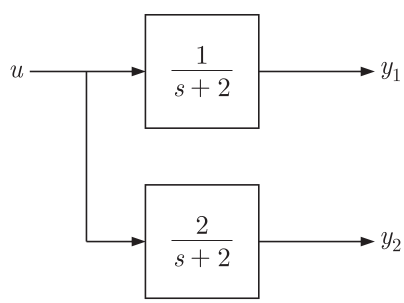
(C)


(D)


2011 **TWO MARKS**

MCQ 7.3

The block diagram of a system with one input u and two outputs y_1 and y_2 is given below.



A state space model of the above system in terms of the state vector $\underline{x}$ and the output vector $\underline{y} = [y_1 \;\; y_2]^T$ is

(A) $\dot{\underline{x}} = [2]\,\underline{x} + [1]\,u\,;\quad \underline{y} = [1 \;\; 2]\,\underline{x}$

(B) $\dot{\underline{x}} = [-2]\,\underline{x} + [1]\,u;\quad \underline{y} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\underline{x}$

(C) $\dot{\underline{x}} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}\underline{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u\,;\quad \underline{y} = \begin{bmatrix} 1 & 2 \end{bmatrix}\underline{x}$

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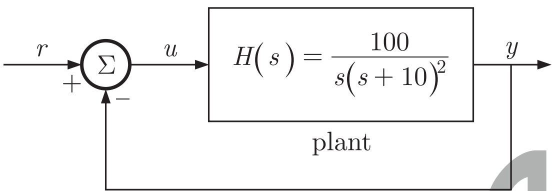
Control Systems.indd 426 10/6/2012 2:11:42 PM

(D) $\dot{\underline{x}} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\underline{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u; \quad \underline{y} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\underline{x}$

Common Data Questions: 7.4 & 7.5

The input-output transfer function of a plant $H(s) = \dfrac{100}{s(s+10)^2}$. The plant is placed in a unity negative feedback configuration as shown in the figure below.



MCQ 7.4

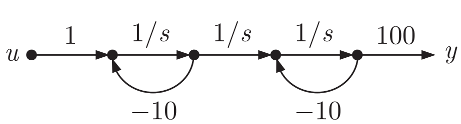
The gain margin of the system under closed loop unity negative feedback is

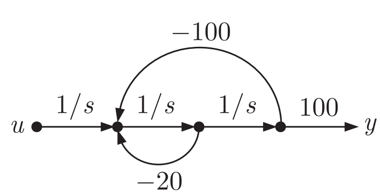
(A) 0 dB (B) 20 dB

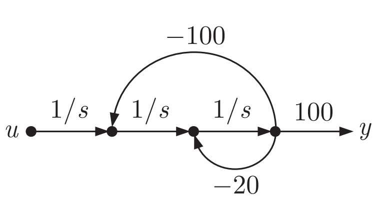
(C) 26 dB (D) 46 dB

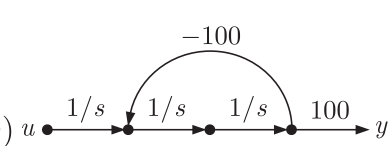
MCQ 7.5

The signal flow graph that DOES NOT model the plant transfer function $H(s)$ is

(A) 

(B) 

(C) 

(D) 

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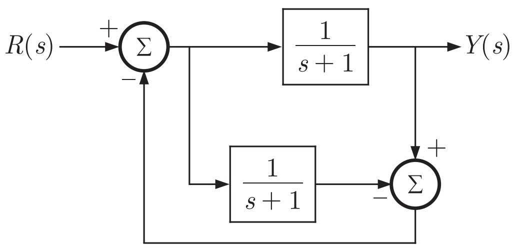
2010 ONE MARK

MCQ 7.6

The transfer function $Y(s)/R(s)$ of the system shown is



(A) 0

(B) $\frac{1}{s+1}$

(C) $\frac{2}{s+1}$

(D) $\frac{2}{s+3}$

MCQ 7.7

A system with transfer function $\frac{Y(s)}{X(s)} = \frac{s}{s+p}$ has an output $y(t) = \cos\left(2t - \frac{\pi}{3}\right)$ for the input signal $x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$. Then, the system parameter p is

(A) $\sqrt{3}$

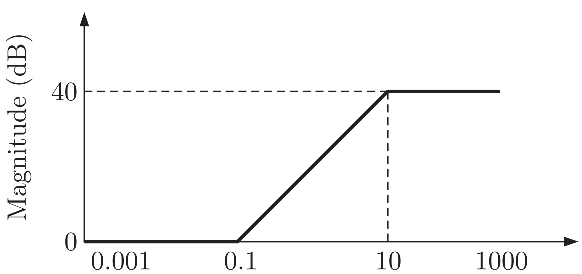
(B) $2/\sqrt{3}$

(C) 1

(D) $\sqrt{3}/2$

MCQ 7.8

For the asymptotic Bode magnitude plot shown below, the system transfer function can be



(A) $\frac{10s+1}{0.1s+1}$

(B) $\frac{100s+1}{0.1s+1}$

(C) $\frac{100s}{10s+1}$

(D) $\frac{0.1s+1}{10s+1}$

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2010 **TWO MARKS**

MCQ 7.9

A unity negative feedback closed loop system has a plant with the transfer function $G(s) = \frac{1}{s^2 + 2s + 2}$ and a controller $G_c(s)$ in the feed forward path. For a unit set input, the transfer function of the controller that gives minimum steady state error is

(A) $G_c(s) = \frac{s+1}{s+2}$

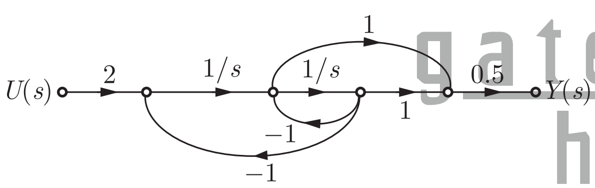
(B) $G_c(s) = \frac{s+2}{s+1}$

(C) $G_c(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)}$

(D) $G_c(s) = 1 + \frac{2}{s} + 3s$

Common Data Question : 7.10 & 7.11 :

The signal flow graph of a system is shown below:



MCQ 7.10

The state variable representation of the system can be

(A) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = [0 \;\; 0.5]\, x$

(B) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = \begin{bmatrix} 0 & 0.5 \end{bmatrix} x$

(C) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} x$

(D) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} x$

MCQ 7.11

The transfer function of the system is

(A) $\frac{s+1}{s^2+1}$

(B) $\frac{s-1}{s^2+1}$

(C) $\frac{s+1}{s^2+s+1}$

(D) $\frac{s-1}{s^2+s+1}$

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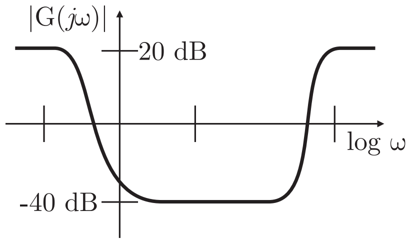
2009 **ONE MARK**

MCQ 7.12

The magnitude plot of a rational transfer function $G(s)$ with real coefficients is shown below. Which of the following compensators has such a magnitude plot ?



(A) Lead compensator
(B) Lag compensator
(C) PID compensator
(D) Lead-lag compensator

MCQ 7.13

Consider the system

$$\frac{dx}{dt} = Ax + Bu \quad \text{with} \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} p \\ q \end{bmatrix}$$

where p and q are arbitrary real numbers. Which of the following statements about the controllability of the system is true ?

(A) The system is completely state controllable for any nonzero values of p and q
(B) Only $p = 0$ and $q = 0$ result in controllability
(C) The system is uncontrollable for all values of p and q
(D) We cannot conclude about controllability from the given data

2009 **TWO MARKS**

MCQ 7.14

The feedback configuration and the pole-zero locations of

$$G(s) = \frac{s^2 - 2s + 2}{s^2 + 2s + 2}$$

are shown below. The root locus for negative values of k, i.e. for $-\infty < k < 0$, has breakaway/break-in points and angle of departure at pole P (with respect to the positive real axis) equal to

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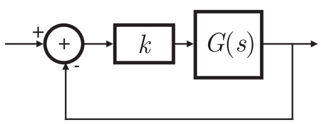
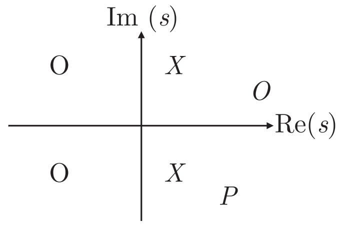




(A) $\pm\sqrt{2}$ and $0°$ (B) $\pm\sqrt{2}$ and $45°$
(C) $\pm\sqrt{3}$ and $0°$ (D) $\pm\sqrt{3}$ and $45°$

MCQ 7.15

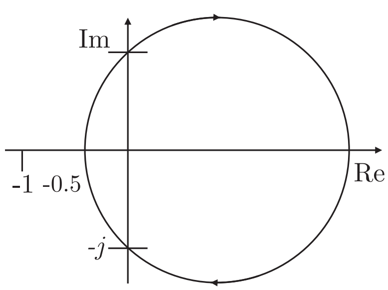
The unit step response of an under-damped second order system has steady state value of -2. Which one of the following transfer functions has theses properties ?

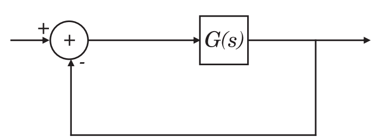
(A) $\dfrac{-2.24}{s^2+2.59s+1.12}$ (B) $\dfrac{-3.82}{s^2+1.91s+1.91}$

(C) $\dfrac{-2.24}{s^2-2.59s+1.12}$ (D) $\dfrac{-382}{s^2-1.91s+1.91}$

Common Data for Questions 7.16 and 7.17 :

The Nyquist plot of a stable transfer function $G(s)$ is shown in the figure are interested in the stability of the closed loop system in the feedback configuration shown.





MCQ 7.16

Which of the following statements is true ?

(A) $G(s)$ is an all-pass filter

(B) $G(s)$ has a zero in the right-half plane

(C) $G(s)$ is the impedance of a passive network

(D) $G(s)$ is marginally stable

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MCQ 7.17

The gain and phase margins of $G(s)$ for closed loop stability are

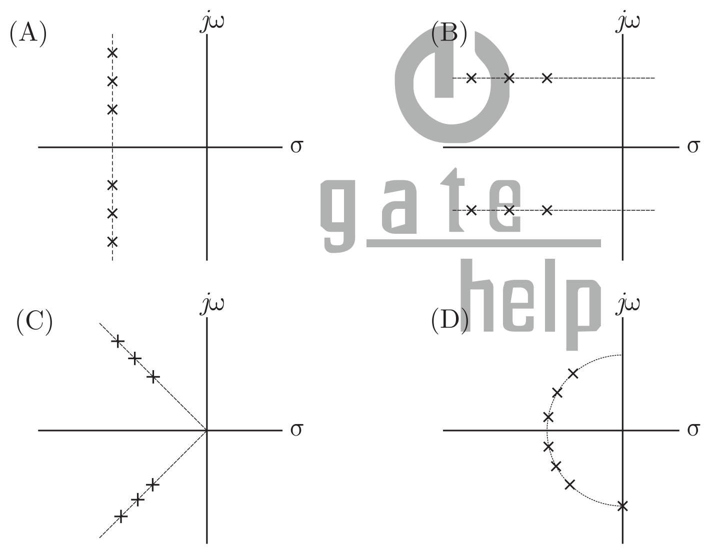
(A) 6 dB and 180° (B) 3 dB and 180°

(C) 6 dB and 90° (D) 3 dB and 90°

2008 ONE MARKS

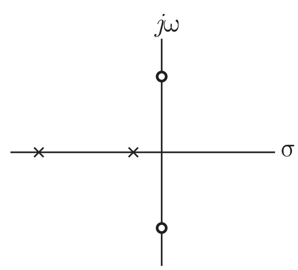
MCQ 7.18

Step responses of a set of three second-order underdamped systems all have the same percentage overshoot. Which of the following diagrams represents the poles of the three systems ?



MCQ 7.19

The pole-zero given below correspond to a



(A) Law pass filter (B) High pass filter

(C) Band filter (D) Notch filter

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2008 **TWO MARKS**

MCQ 7.20

Group I lists a set of four transfer functions. Group II gives a list of possible step response $y(t)$. Match the step responses with the corresponding transfer functions.

Group I

$P = \frac{25}{s^2 + 25}$

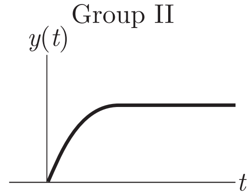
$Q = \frac{36}{s^2 + 20s + 36}$

$R = \frac{36}{s^2 + 12s + 36}$

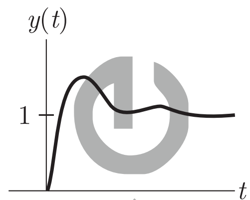
$S = \frac{49}{s^2 + 7s + 49}$

Group II

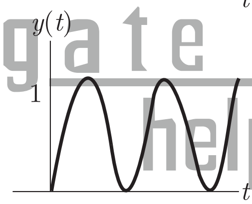
(1)



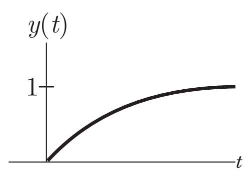
(2)



(3)



(4)

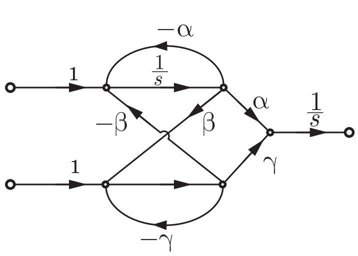


(A) $P-3, Q-1, R-4, S-2$ (B) $P-3, Q-2, R-4, S-1$

(C) $P-2, Q-1, R-4, S-2$ (D) $P-3, Q-4, R-1, S-2$

MCQ 7.21

A signal flow graph of a system is given below



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The set of equalities that corresponds to this signal flow graph is

(A) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} \beta & -\gamma & 0 \\ \gamma & \alpha & 0 \\ -\alpha & \beta & 0 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(B) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 & \alpha & \gamma \\ 0 & -\alpha & -\gamma \\ 0 & \beta & -\beta \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(C) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -\alpha & \beta & 0 \\ -\beta & -\gamma & 0 \\ \alpha & \gamma & 0 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

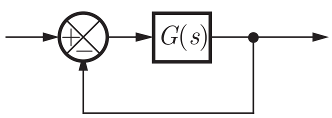
(D) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -\alpha & 0 & \beta \\ \gamma & 0 & \alpha \\ -\beta & 0 & -\alpha \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

MCQ 7.22

A certain system has transfer function

$$G(s) = \frac{s+8}{s^2 + \alpha s - 4}$$

where α is a parameter. Consider the standard negative unity feedback configuration as shown below



Which of the following statements is true?

(A) The closed loop systems is never stable for any value of α

(B) For some positive value of α, the closed loop system is stable, but not for all positive values.

(C) For all positive values of α, the closed loop system is stable.

(D) The closed loop system stable for all values of α, both positive and negative.

MCQ 7.23

The number of open right half plane of

$$G(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3} \text{ is}$$

(A) 0

(B) 1

(C) 2

(D) 3

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MCQ 7.24

The magnitude of frequency responses of an underdamped second order system is 5 at 0 rad/sec and peaks to $\frac{10}{\sqrt{3}}$ at $5\sqrt{2}$ rad/sec. The transfer function of the system is

(A) $\frac{500}{s^2 + 10s + 100}$

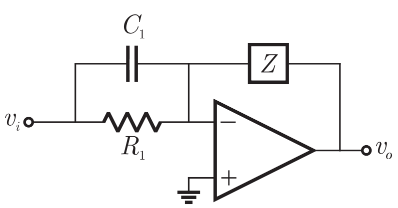
(B) $\frac{375}{s^2 + 5s + 75}$

(C) $\frac{720}{s^2 + 12s + 144}$

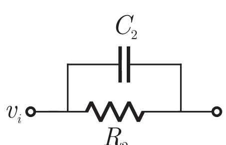
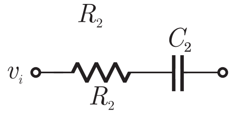
(D) $\frac{1125}{s^2 + 25s + 225}$

MCQ 7.25

Group I gives two possible choices for the impedance Z in the diagram. The circuit elements in Z satisfy the conditions $R_2 C_2 > R_1 C_1$. The transfer functions $\frac{V_0}{V_i}$ represents a kind of controller.



Match the impedances in Group I with the type of controllers in Group II

Group I	Group I
Q 	1. PID controller
	2. Lead Compensator
R 	3. Lag Compensator

(A) $Q-1, R-2$

(B) $Q-1, R-3$

(C) $Q-2, R-3$

(D) $Q-3, R-2$

2007 **ONE MARK**

MCQ 7.26

If the closed-loop transfer function of a control system is given as $T(s)\frac{s-5}{(s+2)(s+3)}$, then It is

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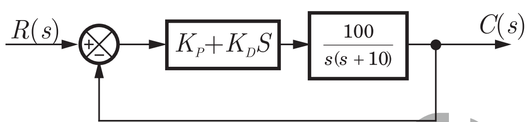
(A) an unstable system
(B) an uncontrollable system
(C) a minimum phase system
(D) a non-minimum phase system

2007 **TWO MARKS**

MCQ 7.27

A control system with PD controller is shown in the figure. If the velocity error constant $K_V = 1000$ and the damping ratio $\zeta = 0.5$, then the value of K_P and K_D are



(A) $K_P = 100,\ K_D = 0.09$
(B) $K_P = 100,\ K_D = 0.9$
(C) $K_P = 10,\ K_D = 0.09$
(D) $K_P = 10,\ K_D = 0.9$

MCQ 7.28

The transfer function of a plant is

$$T(s) = \frac{5}{(s+5)(s^2+s+1)}$$

The second-order approximation of $T(s)$ using dominant pole concept is

(A) $\frac{1}{(s+5)(s+1)}$
(B) $\frac{5}{(s+5)(s+1)}$
(C) $\frac{5}{s^2+s+1}$
(D) $\frac{1}{s^2+s+1}$

MCQ 7.29

The open-loop transfer function of a plant is given as $G(s) = \frac{1}{s^2-1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that an stabilize this control system is

(A) $\frac{10(s-1)}{s+2}$
(B) $\frac{10(s+4)}{s+2}$
(C) $\frac{10(s+2)}{s+10}$
(D) $\frac{2(s+2)}{s+10}$

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MCQ 7.30

A unity feedback control system has an open-loop transfer function

$$G(s) = \frac{K}{s(s^2 + 7s + 12)}$$

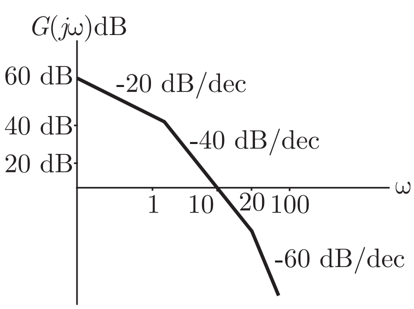
The gain K for which $s = 1 + j1$ will lie on the root locus of this system is

(A) 4 (B) 5.5

(C) 6.5 (D) 10

MCQ 7.31

The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function $G(s)$ corresponding to this Bode plot is



(A) $\frac{1}{(s+1)(s+20)}$ (B) $\frac{1}{s(s+1)(s+20)}$

(C) $\frac{100}{s(s+1)(s+20)}$ (D) $\frac{100}{s(s+1)(1+0.05s)}$

MCQ 7.32

The state space representation of a separately excited DC servo motor dynamics is given as

$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{di_o}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix} \begin{bmatrix} \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

wher ω is the speed of the motor, i_a is the armature current and u is the armature voltage. The transfer function $\frac{\omega(s)}{U(s)}$ of the motor is

(A) $\frac{10}{s^2 + 11s + 11}$ (B) $\frac{1}{s^2 + 11s + 11}$

(C) $\frac{10s + 10}{s^2 + 11s + 11}$ (D) $\frac{1}{s^2 + s + 11}$

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Statement for linked Answer Question 8.33 & 8.34 :

Consider a linear system whose state space representation is $x(t) = Ax(t)$. If the initial state vector of the system is $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response is $x(t) = \begin{bmatrix} e^{-2x} \\ -2e^{-2t} \end{bmatrix}$. If the itial state vector of the system changes to $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response becomes $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

MCQ 7.33

The eigenvalue and eigenvector pairs $(\lambda_i v_i)$ for the system are

(A) $\left(-1\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(B) $\left(-1,\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(2,\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(C) $\left(-1,\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2,\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(D) $\left(-2\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(1,\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

MCQ 7.34

The system matrix A is

(A) $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

2006 ONE MARK

MCQ 7.35

The open-loop function of a unity-gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)}$$

The gain margin of the system in dB is given by

(A) 0

(B) 1

(C) 20

(D) ∞

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2006 **TWO MARKS**

MCQ 7.36

Consider two transfer functions $G_1(s) = \dfrac{1}{s^2 + as + b}$ and $G_2(s) = \dfrac{s}{s^2 + as + b}$ The 3-dB bandwidths of their frequency responses are, respectively

(A) $\sqrt{a^2 - 4b}, \sqrt{a^2 + 4b}$

(B) $\sqrt{a^2 + 4b}, \sqrt{a^2 - 4b}$

(C) $\sqrt{a^2 - 4b}, \sqrt{a^2 - 4b}$

(D) $\sqrt{a^2 + 4b}, \sqrt{a^2 + 4b}$

MCQ 7.37

The Nyquist plot of $G(j\omega)H(j\omega)$for a closed loop control system, passes through $(-1, j0)$ point in the GH plane. The gain margin of the system in dB is equal to

(A) infinite

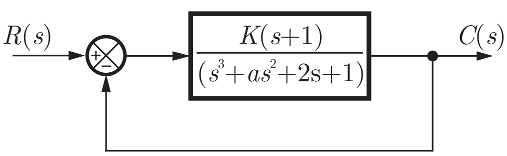
(B) greater than zero

(C) less than zero

(D) zero

MCQ 7.38

The positive values of K and a so that the system shown in the figures below oscillates at a frequency of 2 rad/sec respectively are



(A) 1, 0.75

(B) 2, 0.75

(C) 1, 1

(D) 2, 2

MCQ 7.39

The transfer function of a phase lead compensator is given by $G_c(s) = \dfrac{1 + 3Ts}{1 + Ts}$ where $T > 0$ The maximum phase shift provide by such a compensator is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

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MCQ 7.40

A linear system is described by the following state equation

$$\dot{X}(t) = AX(t) + BU(t), A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The state transition matrix of the system is

(A) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$

(B) $\begin{bmatrix} -\cos t & \sin t \\ -\sin t & -\cos t \end{bmatrix}$

(C) $\begin{bmatrix} -\cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix}$

(D) $\begin{bmatrix} \cos t & -\sin t \\ \cos t & \sin t \end{bmatrix}$

Statement for Linked Answer Questions 7.41 & 7.42 :

Consider a unity - gain feedback control system whose open - loop transfer function is : $G(s) = \frac{as+1}{s^2}$

MCQ 7.41

The value of a so that the system has a phase - margin equal to $\frac{\pi}{4}$ is approximately equal to

(A) 2.40

(B) 1.40

(C) 0.84

(D) 0.74

MCQ 7.42

With the value of a set for a phase - margin of $\frac{\pi}{4}$, the value of unit - impulse response of the open - loop system at $t = 1$ second is equal to

(A) 3.40

(B) 2.40

(C) 1.84

(D) 1.74

2005 ONE MARK

MCQ 7.43

A linear system is equivalently represented by two sets of state equations :

$$\dot{X} = AX + BU \text{ and } \dot{W} = CW + DU$$

The eigenvalues of the representations are also computed as $[\lambda]$ and $[\mu]$. Which one of the following statements is true ?

(A) $[\lambda] = [\mu]$ and $X = W$

(B) $[\lambda] = [\mu]$ and $X \neq W$

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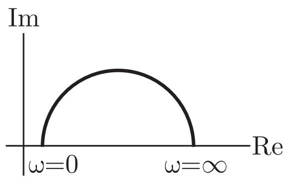
(C) $[\lambda] \neq [\mu]$ and $X = W$

(D) $[\lambda] = [\mu]$ and $X \neq W$

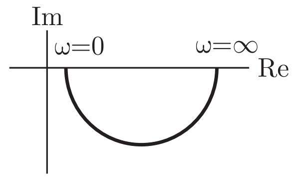
MCQ 7.44

Which one of the following polar diagrams corresponds to a lag network ?

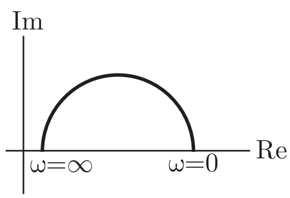
(A)



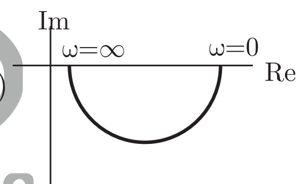
(B)



(C)



(D)



MCQ 7.45

Despite the presence of negative feedback, control systems still have problems of instability because the

(A) Components used have non- linearities

(B) Dynamic equations of the subsystem are not known exactly.

(C) Mathematical analysis involves approximations.

(D) System has large negative phase angle at high frequencies.

2005 TWO MARKS

MCQ 7.46

The polar diagram of a conditionally stable system for open loop gain $K = 1$ is shown in the figure. The open loop transfer function of the system is known to be stable. The closed loop system is stable for

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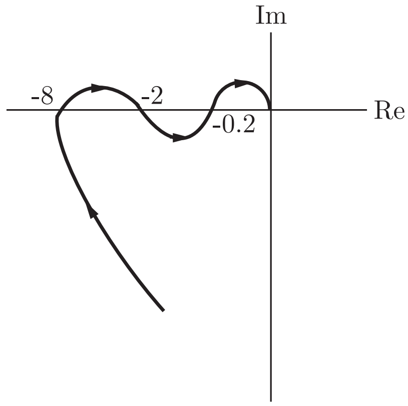

(A) $K < 5$ and $\frac{1}{2} < K < \frac{1}{8}$

(B) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$

(C) $K < \frac{1}{8}$ and $5 < K$

(D) $K > \frac{1}{8}$ and $5 > K$

MCQ 7.47

In the derivation of expression for peak percent overshoot

$$M_p = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)\times 100\%$$

Which one of the following conditions is NOT required ?

(A) System is linear and time invariant

(B) The system transfer function has a pair of complex conjugate poles and no zeroes.

(C) There is no transportation delay in the system.

(D) The system has zero initial conditions.

MCQ 7.48

A ramp input applied to an unity feedback system results in 5% steady state error. The type number and zero frequency gain of the system are respectively

(A) 1 and 20

(B) 0 and 20

(C) 0 and $\frac{1}{20}$

(D) 1 and $\frac{1}{20}$

MCQ 7.49

A double integrator plant $G(s) = K/s^2, H(s) = 1$ is to be compensated to achieve the damping ratio $\zeta = 0.5$ and an undamped natural frequency, $\omega_n = 5$ rad/sec which one of the following compensator $G_e(s)$ will be suitable ?

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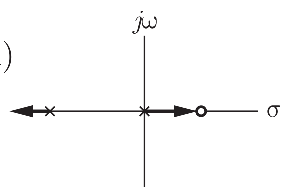
(A) $\dfrac{s+3}{s+99}$ (B) $\dfrac{s+99}{s+3}$

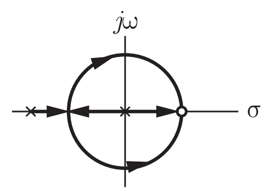
(C) $\dfrac{s-6}{s+8.33}$ (D) $\dfrac{s-6}{s}$

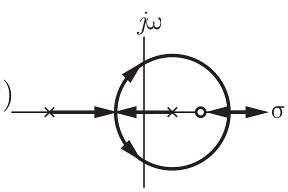
MCQ 7.50

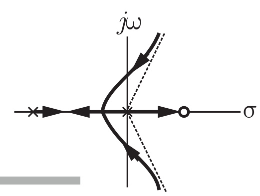
An unity feedback system is given as $G(s) = \dfrac{K(1-s)}{s(s+3)}$.

Indicate the correct root locus diagram.

(A)


(B)


(C)


(D)


Statement for Linked Answer Question 40 and 41 :

The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

MCQ 7.51

The gain and phase crossover frequencies in rad/sec are, respectively

(A) 0.632 and 1.26 (B) 0.632 and 0.485

(C) 0.485 and 0.632 (D) 1.26 and 0.632

MCQ 7.52

Based on the above results, the gain and phase margins of the system will be

(A) -7.09 dB and 87.5° (B) 7.09 dB and 87.5°

(C) 7.09 dB and -87.5° (D) -7.09 and -87.5°

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2004 **ONE MARK**

MCQ 7.53

The gain margin for the system with open-loop transfer function

$$G(s)H(s) = \frac{2(1+s)}{s^2}, \text{ is}$$

(A) ∞ (B) 0

(C) 1 (D) $-\infty$

MCQ 7.54

Given $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$.The point of intersection of the asymptotes of the root loci with the real axis is

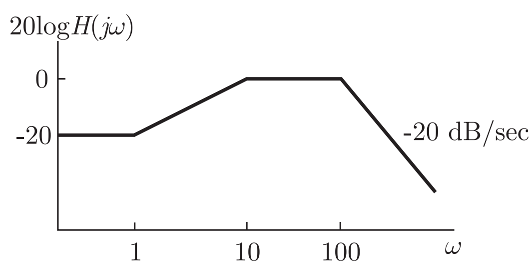
(A) -4 (B) 1.33

(C) -1.33 (D) 4

2004 **TWO MARKS**

MCQ 7.55

Consider the Bode magnitude plot shown in the fig. The transfer function $H(s)$ is



(A) $\frac{(s+10)}{(s+1)(s+100)}$ (B) $\frac{10(s+1)}{(s+10)(s+100)}$

(C) $\frac{10^2(s+1)}{(s+10)(s+100)}$ (D) $\frac{10^3(s+100)}{(s+1)(s+10)}$

MCQ 7.56

A causal system having the transfer function $H(s) = 1/(s+2)$ is excited with $10u(t)$. The time at which the output reaches 99% of its steady state value is

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(A) 2.7 sec
(B) 2.5 sec
(C) 2.3 sec
(D) 2.1 sec

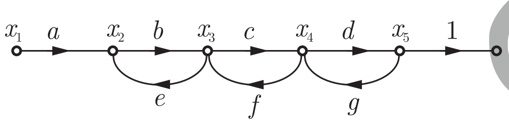
MCQ 7.57

A system has poles at 0.1 Hz, 1 Hz and 80 Hz; zeros at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is

(A) $-90°$
(B) $0°$
(C) $90°$
(D) $-180°$

MCQ 7.58

Consider the signal flow graph shown in Fig. The gain $\frac{x_5}{x_1}$ is



(A) $\frac{1-(be+cf+dg)}{abcd}$
(B) $\frac{bedg}{1-(be+cf+dg)}$
(C) $\frac{abcd}{1-(be+cf+dg)+bedg}$
(D) $\frac{1-(be+cf+dg)+bedg}{abcd}$

MCQ 7.59

If $A = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix}$, then $\sin At$ is

(A) $\frac{1}{3}\begin{bmatrix} \sin(-4t)+2\sin(-t) & -2\sin(-4t)+2\sin(-t) \\ -\sin(-4t)+\sin(-t) & 2\sin(-4t)+\sin(-t) \end{bmatrix}$

(B) $\begin{bmatrix} \sin(-2t) & \sin(2t) \\ \sin(t) & \sin(-3t) \end{bmatrix}$

(C) $\frac{1}{3}\begin{bmatrix} \sin(4t)+2\sin(t) & 2\sin(-4t)-2\sin(-t) \\ -\sin(-4t)+\sin(t) & 2\sin(4t)+\sin(t) \end{bmatrix}$

(D) $\frac{1}{3}\begin{bmatrix} \cos(-t)+2\cos(t) & 2\cos(-4t)+2\cos(-t) \\ -\cos(-4t)+\cos(-t) & -2\cos(-4t)+\cos(t) \end{bmatrix}$

MCQ 7.60

The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s^2+s+2)(s+3)}$$

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The range of K for which the system is stable is

(A) $\frac{21}{4} > K > 0$ (B) $13 > K > 0$

(C) $\frac{21}{4} < K < \infty$ (D) $-6 < K < \infty$

MCQ 7.61

For the polynomial $P(s) = s^2 + s^4 + 2s^3 + 2s^2 + 3s + 15$ the number of roots which lie in the right half of the $s-$plane is

(A) 4 (B) 2

(C) 3 (D) 1

MCQ 7.62

The state variable equations of a system are: $\dot{x}_1 = -3x_1 - x_2 = u, \dot{x}_2 = 2x_1$ and $y = x_1 + u$. The system is

(A) controllable but not observable

(B) observable but not controllable

(C) neither controllable nor observable

(D) controllable and observable

MCQ 7.63

Given $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the state transition matrix e^{At} is given by

(A) $\begin{bmatrix} 0 & e^{-t} \\ e^{-t} & 0 \end{bmatrix}$ (B) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$

(C) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$ (D) $\begin{bmatrix} 0 & e^{t} \\ e^{t} & 0 \end{bmatrix}$

2003 ONE MARK

MCQ 7.64

Fig. shows the Nyquist plot of the open-loop transfer function $G(s)H(s)$ of a system. If $G(s)H(s)$ has one right-hand pole, the closed-loop system is

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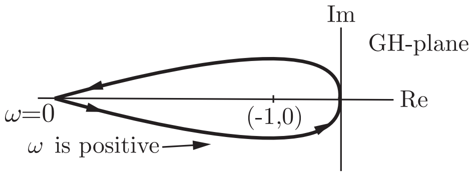


(A) always stable

(B) unstable with one closed-loop right hand pole

(C) unstable with two closed-loop right hand poles

(D) unstable with three closed-loop right hand poles

MCQ 7.65

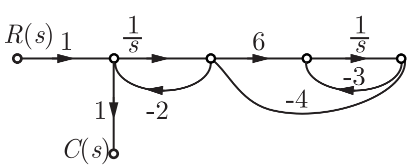
A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has

(A) a higher type number
(B) reduced damping
(C) higher noise amplification
(D) larger transient overshoot

2003 **TWO MARKS**

MCQ 7.66

The signal flow graph of a system is shown in Fig. below. The transfer function $C(s)/R(s)$ of the system is



(A) $\dfrac{6}{s^2+29s+6}$
(B) $\dfrac{6s}{s^2+29s+6}$
(C) $\dfrac{s(s+2)}{s^2+29s+6}$
(D) $\dfrac{s(s+27)}{s^2+29s+6}$

MCQ 7.67

The root locus of system $G(s)H(s) = \dfrac{K}{s(s+2)(s+3)}$ has the break-away point located at

(A) $(-0.5,0)$
(B) $(-2.548,0)$
(C) $(-4,0)$
(D) $(-0.784,0)$

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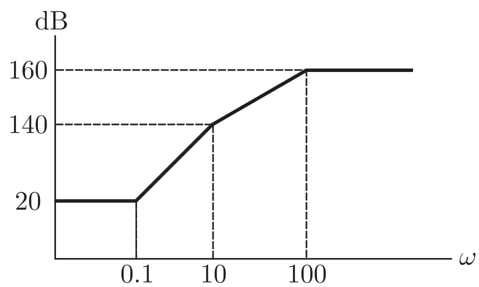
MCQ 7.68

The approximate Bode magnitude plot of a minimum phase system is shown in Fig. below. The transfer function of the system is



(A) $10^8\dfrac{(s+0.1)^3}{(s+10)^2(s+100)}$

(B) $10^7\dfrac{(s+0.1)^3}{(s+10)(s+100)}$

(C) $\dfrac{(s+0.1)^2}{(s+10)^2(s+100)}$

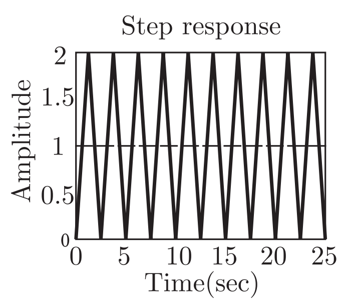
(D) $\dfrac{(s+0.1)^3}{(s+10)(s+100)^2}$

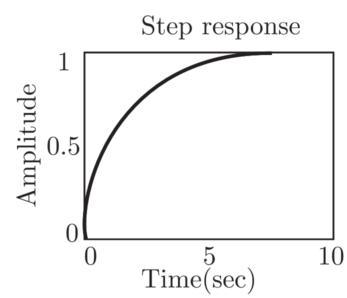
MCQ 7.69

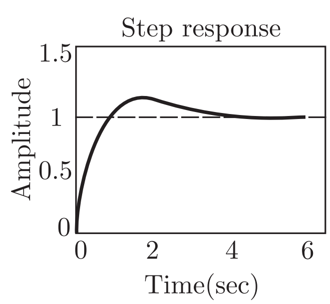
A second-order system has the transfer function

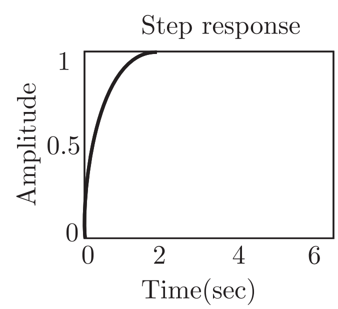
$$\frac{C(s)}{R(s)} = \frac{4}{s^2+4s+4}$$

With $r(t)$ as the unit-step function, the response $c(t)$ of the system is represented by

(A) 

(B) 

(C) 

(D) 

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MCQ 7.70

The gain margin and the phase margin of feedback system with

$$G(s)H(s) = \frac{8}{(s+100)^3} \text{ are}$$

(A) dB,0° (B) ∞, ∞

(C) ∞,0° (D) 88.5 dB, ∞

MCQ 7.71

The zero-input response of a system given by the state-space equation

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is

(A) $\begin{bmatrix} te^t \\ t \end{bmatrix}$ (B) $\begin{bmatrix} e^t \\ t \end{bmatrix}$

(C) $\begin{bmatrix} e^t \\ te^t \end{bmatrix}$ (D) $\begin{bmatrix} t \\ te^t \end{bmatrix}$

2002 **ONE MARK**

MCQ 7.72

Consider a system with transfer function $G(s) = \frac{s+6}{ks^2+s+6}$. Its damping ratio will be 0.5 when the value of k is

(A) $\frac{2}{6}$ (B) 3

(C) $\frac{1}{6}$ (D) 6

MCQ 7.73

Which of the following points is NOT on the root locus of a system with the open-loop transfer function $G(s)H(s) = \frac{k}{s(s+1)(s+3)}$

(A) $s = -j\sqrt{3}$ (B) $s = -1.5$

(C) $s = -3$ (D) $s = -\infty$

MCQ 7.74

The phase margin of a system with the open - loop transfer function

$$G(s)H(s) = \frac{(1-s)}{(1+s)(2+s)}$$

(A) 0° (B) 63.4°

(C) 90° (D) ∞

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MCQ 7.75

The transfer function $Y(s)/U(s)$ of system described by the state equation $\dot{x}(t) = -2x(t) + 2u(t)$ and $y(t) = 0.5x(t)$ is

(A) $\dfrac{0.5}{(s-2)}$ (B) $\dfrac{1}{(s-2)}$

(C) $\dfrac{0.5}{(s+2)}$ (D) $\dfrac{1}{(s+2)}$

2002 **TWO MARKS**

MCQ 7.76

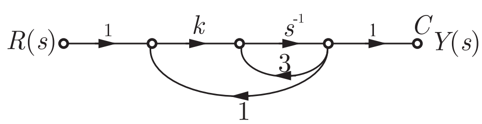
The system shown in the figure remains stable when

(A) $k < -1$ (B) $-1 < k < 3$

(C) $1 < k < 3$ (D) $k > 3$

MCQ 7.77

The transfer function of a system is $G(s) = \dfrac{100}{(s+1)(s+100)}$. For a unit - step input to the system the approximate settling time for 2% criterion is



(A) 100 sec (B) 4 sec

(C) 1 sec (D) 0.01 sec

MCQ 7.78

The characteristic polynomial of a system is

$$q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$$

The system is

(A) stable (B) marginally stable

(C) unstable (D) oscillatory

MCQ 7.79

The system with the open loop transfer function $G(s)H(s) = \dfrac{1}{s(s^2+s+1)}$ has a gain margin of

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(A) -6 db
(B) 0 db
(C) 35 db
(D) 6 db

2001 ONE MARK

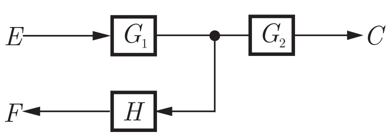
MCQ 7.80

The Nyquist plot for the open-loop transfer function $G(s)$ of a unity negative feedback system is shown in the figure, if $G(s)$ has no pole in the right-half of $s-$plane, the number of roots of the system characteristic equation in the right-half of $s-$plane is

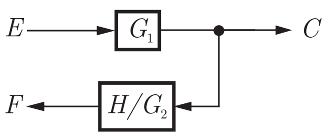
(A) 0
(B) 1
(C) 2
(D) 3

MCQ 7.81

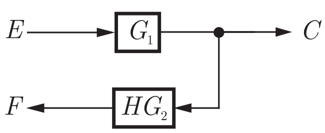
The equivalent of the block diagram in the figure is given is



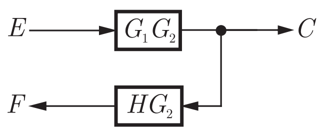
(A)



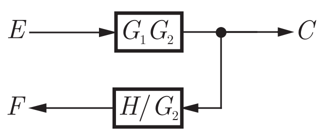
(B)



(C)



(D)



MCQ 7.82

If the characteristic equation of a closed - loop system is $s^2 + 2s + 2 = 0$, then the system is

(A) overdamped
(B) critically damped
(C) underdamped
(D) undamped

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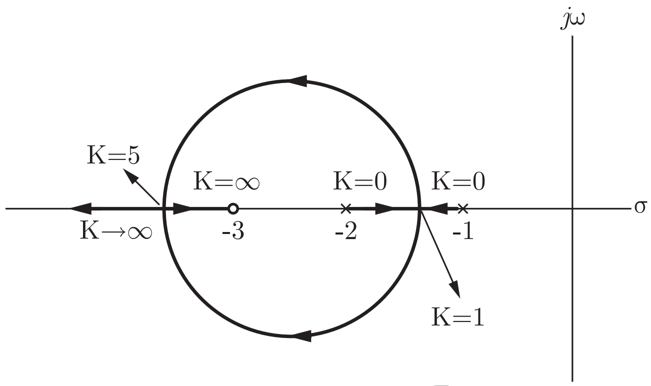
MCQ 7.83

The root-locus diagram for a closed-loop feedback system is shown in the figure. The system is overdamped.

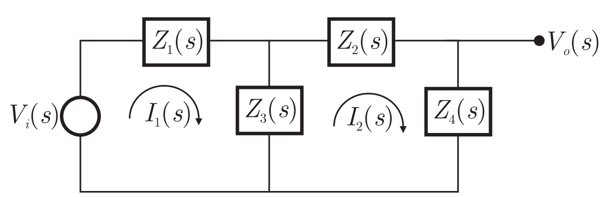


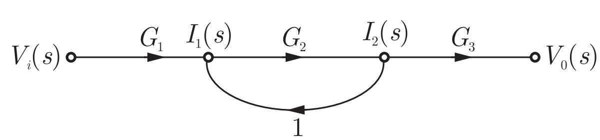
(A) only if $0 \le k \le 1$
(B) only if $1 < k < 5$
(C) only if $k > 5$
(D) if $0 \le k < 1$ or $k > 5$

2001 **TWO MARK**

MCQ 7.84

An electrical system and its signal-flow graph representations are shown the figure (A) and (B) respectively. The values of G_2 and H, respectively are





(A) $\dfrac{Z_3(s)}{Z_1(s)+Z_3(s)+Z_4(s)}, \dfrac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

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(B) $\frac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \frac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(C) $\frac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \frac{Z_3(s)}{Z_1(s)+Z_3(s)}$

(D) $\frac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \frac{Z_3(s)}{Z_1(s)+Z_3(s)}$

MCQ 7.85

The open-loop DC gain of a unity negative feedback system with closed-loop transfer function $\frac{s+4}{s^2+7s+13}$ is

(A) $\frac{4}{13}$

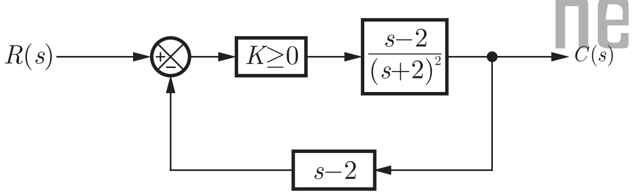
(B) $\frac{4}{9}$

(C) 4

(D) 13

MCQ 7.86

The feedback control system in the figure is stable



(A) for all $K \geq 0$

(B) only if $K \geq 0$

(C) only if $0 \leq K < 1$

(D) only if $0 \leq K \leq 1$

2000 ONE MARK

MCQ 7.87

An amplifier with resistive negative feedback has tow left half plane poles in its open-loop transfer function. The amplifier

(A) will always be unstable at high frequency

(B) will be stable for all frequency

(C) may be unstable, depending on the feedback factor

(D) will oscillate at low frequency.

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2000 **TWO MARKS**

MCQ 7.88

A system described by the transfer function $H(s) = \frac{1}{s^3 + \alpha s^2 + ks + 3}$ is stable. The constraints on α and k are.

(A) $\alpha > 0, \alpha k < 3$

(B) $\alpha > 0, \alpha k > 3$

(C) $\alpha < 0, \alpha k > 3$

(D) $\alpha > 0, \alpha k < 3$

1999 **ONE MARK**

MCQ 7.89

For a second order system with the closed-loop transfer function

$$T(s) = \frac{9}{s^2 + 4s + 9}$$

the settling time for 2-percent band, in seconds, is

(A) 1.5

(B) 2.0

(C) 3.0

(D) 4.0

MCQ 7.90

The gain margin (in dB) of a system a having the loop transfer function

$$G(s)H(s) = \frac{\sqrt{2}}{s(s+1)} \text{ is}$$

(A) 0

(B) 3

(C) 6

(D) ∞

MCQ 7.91

The system modeled described by the state equations is

$$X = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$$

(A) controllable and observable

(B) controllable, but not observable

(C) observable, but not controllable

(D) neither controllable nor observable

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MCQ 7.92

The phase margin (in degrees) of a system having the loop transfer function $G(s)H(s) = \frac{2\sqrt{3}}{s(s+1)}$ is

(A) 45° (B) -30°

(C) 60° (D) 30°

1999 TWO MARKS

MCQ 7.93

An amplifier is assumed to have a single-pole high-frequency transfer function. The rise time of its output response to a step function input is 35 n sec. The upper 3 dB frequency (in MHz) for the amplifier to as sinusoidal input is approximately at

(A) 4.55 (B) 10

(C) 20 (D) 28.6

MCQ 7.94

If the closed - loop transfer function $T(s)$ of a unity negative feedback system is given by

$$T(s) = \frac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_n}$$

then the steady state error for a unit ramp input is

(A) $\frac{a_n}{a_{n-1}}$ (B) $\frac{a_n}{a_{n-2}}$

(C) $\frac{a_{n-2}}{a_{n-2}}$ (D) zero

MCQ 7.95

Consider the points $s_1 = -3 + j4$ and $s_2 = -3 - j2$ in the s-plane. Then, for a system with the open-loop transfer function

$$G(s)H(s) = \frac{K}{(s+1)^4}$$

(A) s_1 is on the root locus, but not s_2

(B) s_2 is on the root locus, but not s_1

(C) both s_1 and s_2 are on the root locus

(D) neither s_1 nor s_2 is on the root locus

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MCQ 7.96

For the system described by the state equation

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

If the control signal u is given by $u = [-0.5 - 3 - 5]\,x + v$, then the eigen values of the closed-loop system will be

(A) $0, -1, -2$ (B) $0, -1, -3$

(C) $-1, -1, -2$ (D) $0, -1, -1$

1998 **ONE MARK**

MCQ 7.97

The number of roots of $s^3 + 5s^2 + 7s + 3 = 0$ in the left half of the s-plane is

(A) zero (B) one

(C) two (D) three

MCQ 7.98

The transfer function of a tachometer is of the form

(A) Ks (B) $\dfrac{K}{s}$

(C) $\dfrac{K}{(s+1)}$ (D) $\dfrac{K}{s(s+1)}$

MCQ 7.99

Consider a unity feedback control system with open-loop transfer function $G(s) = \dfrac{K}{s(s+1)}$.

The steady state error of the system due to unit step input is

(A) zero (B) K

(C) $1/K$ (D) infinite

MCQ 7.100

The transfer function of a zero-order-hold system is

(A) $(1/s)(1 + e^{-sT})$ (B) $(1/s)(1 - e^{-sT})$

(C) $1 - (1/s)\,e^{-sT}$ (D) $1 + (1/s)\,e^{-sT}$

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MCQ 7.101

In the Bode-plot of a unity feedback control system, the value of phase of $G(j\omega)$ at the gain cross over frequency is $-125°$. The phase margin of the system is

(A) $-125°$ (B) $-55°$

(C) $55°$ (D) $125°$

MCQ 7.102

Consider a feedback control system with loop transfer function

$$G(s)H(s) = \frac{K(1+0.5s)}{s(1+s)(1+2s)}$$

The type of the closed loop system is

(A) zero (B) one

(C) two (D) three

MCQ 7.103

The transfer function of a phase lead controller is $\frac{1+3Ts}{1+Ts}$. The maximum value of phase provided by this controller is

(A) $90°$ (B) $60°$

(C) $45°$ (D) $30°$

MCQ 7.104

The Nyquist plot of a phase transfer function $g(j\omega)H(j\omega)$ of a system encloses the (–1, 0) point. The gain margin of the system is

(A) less than zero (B) zero

(C) greater than zero (D) infinity

MCQ 7.105

The transfer function of a system is $\frac{2s^2+6s+5}{(s+1)^2(s+2)}$

The characteristic equation of the system is

(A) $2s^2+6s+5=0$

(B) $(s+1)^2(s+2)=0$

(C) $2s^2+6s+5+(s+1)^2(s+2)=0$

(D) $2s^2+6s+5-(s+1)^2(s+2)=0$

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MCQ 7.106

In a synchro error detector, the output voltage is proportional to $[\omega(t)]^n$, where $\omega(t)$ is the rotor velocity and n equals

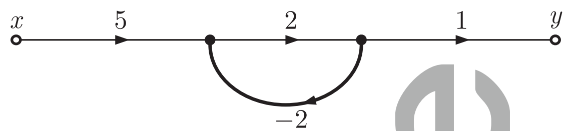
(A) –2 (B) –1

(C) 1 (D) 2

1997 ONE MARK

MCQ 7.107

In the signal flow graph of the figure is y/x equals



(A) 3 (B) $\frac{5}{2}$

(C) 2 (D) None of the above

MCQ 7.108

A certain linear time invariant system has the state and the output equations given below

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

If $X_1(0) = 1, X_2(0) = -1, u(0) = 0,$ then $\left.\frac{dy}{dt}\right|_{t=0}$ is

(A) 1 (B) –1

(C) 0 (D) None of the above

****

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SOLUTIONS

SOL 7.1

For given plot root locus exists from -3 to ∞, So there must be odd number of poles and zeros. There is a double pole at $s = -3$

Now $\quad$ poles $= 0, -2, -3, -3$

$\quad$ zeros $= -1$

Thus transfer function $\quad G(s)H(s) = \dfrac{k(s+1)}{s(s+2)(s+3)^2}$

Hence (B) is correct option.

SOL 7.2

We have $\quad G(j\omega) = 5 + j\omega$

Here $\sigma = 5$. Thus $G(j\omega)$ is a straight line parallel to $j\omega$ axis.

Hence (A) is correct option.

SOL 7.3

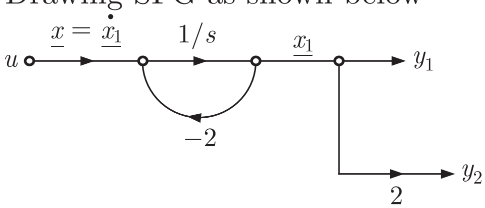
Here $\quad x = y_1$ and $\dot{x} = \dfrac{dy_1}{dx}$

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x \\ 2x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\underline{x}$$

Now $\quad y_1 = \dfrac{1}{s+2}u$

$$\begin{aligned} y_1(s+2) &= u \\ \dot{y}_1 + 2y_1 &= u \\ \dot{x} + 2x &= u \\ \dot{x} &= -2x + u \\ \underline{\dot{x}} &= [-2]\,\underline{x} + [1]\,u \end{aligned}$$

Drawing SFG as shown below



Thus $\quad \dot{x}_1 = [-2]\,\underline{x}_1 + [1]\,u$

$\quad y_1 = \underline{x}_1;\ y_2 = 2\underline{x}_1$

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$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \underline{x_1}$$

Here $\underline{x_1} = \underline{x}$

Hence (B) is correct option.

SOL 7.4

We have $G(s)H(s) = \dfrac{100}{s(s+10)^2}$

Now $G(j\omega)H(j\omega) = \dfrac{100}{j\omega(j\omega+10)^2}$

If ω_p is phase cross over frequency $\angle G(j\omega)H(j\omega) = 180^\circ$

Thus $-180^\circ = 100\tan^{-1}0 - \tan^{-1}\infty - 2\tan^{-1}\left(\dfrac{\omega_p}{10}\right)$

or $-180^\circ = -90 - 2\tan^{-1}(0.1\omega_p)$

or $45^\circ = \tan^{-1}(0.1\omega_p)$

or $\tan 45^\circ \; 0.1\omega_p = 1$

or $\omega_p = 10$ rad/sec

Now $|G(j\omega)H(j\omega)| = \dfrac{100}{\omega(\omega^2+100)}$

At $\omega = \omega_p$

$$|G(j\omega)H(j\omega)| = \frac{100}{10(100+100)} = \frac{1}{20}$$

$$\begin{aligned} \text{Gain Margin} &= -20\log_{10}|G(j\omega)H(j\omega)| \\ &= -20\log_{10}\left(\frac{1}{20}\right) \\ &= 26 \text{ dB} \end{aligned}$$

Hence (C) is correct option.

SOL 7.5

From option (D) $TF = H(s)$

$$= \frac{100}{s(s^2+100)} \neq \frac{100}{s(s+10)^2}$$

Hence (D) is correct option.

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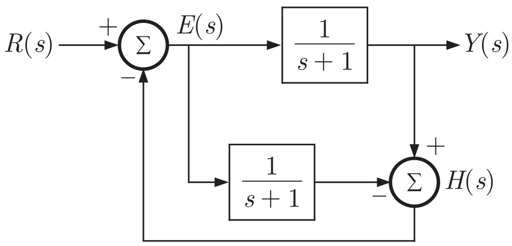
SOL 7.6

From the given block diagram



$$H(s) = Y(s) - E(s) \cdot \frac{1}{s+1}$$

$$E(s) = R(s) - H(s)$$

$$= R(s) - Y(s) + \frac{E(s)}{(s+1)}$$

$$E(s)\left[1 - \frac{1}{s+1}\right] = R(s) - Y(s)$$

$$\frac{sE(s)}{(s+1)} = R(s) - Y(s) \qquad ...(1)$$

$$Y(s) = \frac{E(s)}{s+1} \qquad ...(2)$$

From (1) and (2) $\quad sY(s) = R(s) - Y(s)$

$$(s+1)\,Y(s) = R(s)$$

Transfer function

$$\frac{Y(s)}{R(s)} = \frac{1}{s+1}$$

Hence (B) is correct option.

SOL 7.7

Transfer function is given as

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s+p}$$

$$H(j\omega) = \frac{j\omega}{j\omega + p}$$

Amplitude Response

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + p^2}}$$

Phase Response $\qquad \theta_h(\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega}{p}\right)$

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Input $$x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$$

Output $$y(t) = \left|H(j\omega)\right| x(t - \theta_h) = \cos\left(2t - \frac{\pi}{3}\right)$$

$$\left|H(j\omega)\right| = p = \frac{\omega}{\sqrt{\omega^2 + p^2}}$$

$$\frac{1}{p} = \frac{2}{\sqrt{4 + p^2}}, \qquad (\omega = 2\,\text{rad/sec})$$

or $$4p^2 = 4 + p^2 \Rightarrow 3p^2 = 4$$

or $$p = 2/\sqrt{3}$$

Alternative :

$$\theta_h = \left[-\frac{\pi}{3} - \left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{6}$$

So, $$\frac{\pi}{6} = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{p}\right)$$

$$\tan^{-1}\left(\frac{\omega}{p}\right) = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$\frac{\omega}{p} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\frac{2}{p} = \sqrt{3}, \qquad (\omega = 2\,\text{rad/sec})$$

or $$p = 2/\sqrt{3}$$

Hence (B) is correct option.

SOL 7.8

Initial slope is zero, so $K = 1$

At corner frequency $\omega_1 = 0.5\,\text{rad/sec}$, slope increases by $+20$ dB/decade, so there is a zero in the transfer function at ω_1

At corner frequency $\omega_2 = 10\,\text{rad/sec}$, slope decreases by -20 dB/decade and becomes zero, so there is a pole in transfer function at ω_2

Transfer function $$H(s) = \frac{K\left(1 + \frac{s}{\omega_1}\right)}{\left(1 + \frac{s}{\omega_2}\right)}$$

$$= \frac{1\left(1 + \frac{s}{0.1}\right)}{\left(1 + \frac{s}{0.1}\right)} = \frac{(1 + 10s)}{(1 + 0.1s)}$$

Hence (A) is correct option.

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SOL 7.9

Steady state error is given as

$$e_{SS} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)\, G_C(s)}$$

$$R(s) = \frac{1}{s} \qquad \text{(unit step unit)}$$

$$e_{SS} = \lim_{s \to 0} \frac{1}{1 + G(s)\, G_C(s)}$$

$$= \lim_{s \to 0} \frac{1}{1 + \dfrac{G_C(s)}{s^2 + 2s + 2}}$$

e_{SS} will be minimum if $\lim_{s \to 0} G_C(s)$ is maximum

In option (D)

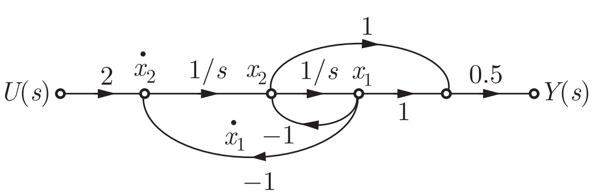
$$\lim_{s \to 0} G_C(s) = \lim_{s \to 0} 1 + \frac{2}{s} + 3s = \infty$$

So, $$e_{SS} = \lim_{s \to 0} \frac{1}{\infty} = 0 \text{ (minimum)}$$

Hence (D) is correct option.

SOL 7.10

Assign output of each integrator by a state variable



$$\dot{x}_1 = -x_1 + x_2$$

$$\dot{x}_2 = -x_1 + 2u$$

$$y = 0.5x_1 + 0.5x_2$$

State variable representation

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$\dot{y} = [0.5 \;\; 0.5]\, x$$

Hence (D) is correct option.

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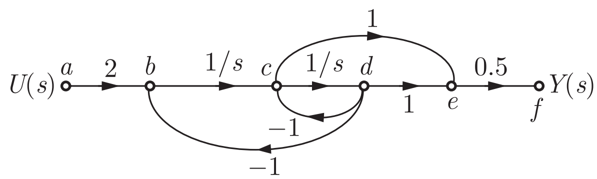
SOL 7.11

By masson's gain formula



Transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\sum P_K \Delta_K}{\Delta}$$

Forward path given

$$P_1(abcdef) = 2 \times \frac{1}{s} \times \frac{1}{s} \times 0.5 = \frac{1}{s^2}$$

$$P_2(abcdef) = 2 \times \frac{1}{3} \times 1 \times 0.5$$

Loop gain $\quad L_1(cdc) = -\frac{1}{s}$

$$L_2(bcdb) = \frac{1}{s} \times \frac{1}{s} \times -1 = \frac{-1}{s^2}$$

$$\Delta = 1 - [L_1 + L_2] = 1 - \left[-\frac{1}{s} - \frac{1}{s^2}\right] = 1 + \frac{1}{s} + \frac{1}{s^2}$$

$$\Delta_1 = 1,\ \Delta_2 = 2$$

So, $\quad H(s) = \frac{Y(s)}{U(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$$= \frac{\frac{1}{s^2} \cdot 1 + \frac{1}{s} \cdot 1}{1 + \frac{1}{s} + \frac{1}{s^2}} = \frac{(1+s)}{(s^2+s+1)}$$

Hence (C) is correct option.

SOL 7.12

This compensator is roughly equivalent to combining lead and lad compensators in the same design and it is referred also as PID compensator.

Hence (C) is correct option.

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SOL 7.13

Here
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} p \\ q \end{bmatrix}$$
$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$
$$S = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} p & q \\ q & p \end{bmatrix}$$
$$S = pq - pq = 0$$

Since S is singular, system is completely uncontrollable for all values of p and q.

Hence (C) is correct option.

SOL 7.14

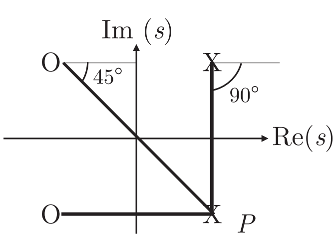
The characteristic equation is
$$1 + G(s)H(s) = 0$$
or
$$1 + \frac{K(s^2 - 2s + 2)}{s^2 + 2s + 2} = 0$$
or
$$s^2 + 2s + 2 + K(s^2 - 2s + 2) = 0$$
or
$$K = -\frac{s^2 + 2s + 2}{s^2 - 2s + 2}$$

For break away & break in point differentiating above w.r.t. s we have
$$\frac{dK}{ds} = -\frac{(s^2 - 2s + 2)(2s + 2) - (s^2 + 2s + 2)(2s - 2)}{(s^2 - 2s + 2)^2} = 0$$

Thus $\quad (s^2 - 2s + 2)(2s + 2) - (s^2 + 2s + 2)(2s - 2) = 0$

or
$$s = \pm\sqrt{2}$$

Let θ_d be the angle of departure at pole P, then



$$-\theta_d - \theta_{p1} + \theta_{z1} + \theta_{z2} = 180^\circ$$
$$-\theta_d = 180^\circ - (-\theta_{p1} + \theta_{z1} + \theta_2)$$
$$= 180^\circ - (90^\circ + 180 - 45^\circ) = -45^\circ$$

Hence (B) is correct option.

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SOL 7.15

For under-damped second order response

$$T(s) = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad \text{where } \xi < 1$$

Thus (A) or (B) may be correct

For option (A) $\quad \omega_n = 1.12$ and $2\xi\omega_n = 2.59 \to \xi = 1.12$

For option (B) $\quad \omega_n = 1.91$ and $2\xi\omega_n = 1.51 \to \xi = 0.69$

Hence (B) is correct option.

SOL 7.16

The plot has one encirclement of origin in clockwise direction. Thus $G(s)$ has a zero is in RHP.

Hence (B) is correct option.

SOL 7.17

The Nyzuist plot intersect the real axis ate - 0.5. Thus

$$\text{G. M.} = -20\log x = -20\log 0.5 = 6.020 \text{ dB}$$

And its phase margin is 90°.

Hence (C) is correct option.

SOL 7.18

Transfer function for the given pole zero plot is:

$$\frac{(s + Z_1)(s + Z_2)}{(s + P_1)(s + P_2)}$$

From the plot Re (P_1 and P_2)>(Z_1 and Z_2)

So, these are two lead compensator.

Hence both high pass filters and the system is high pass filter.

Hence (C) is correct option.

SOL 7.19

Percent overshoot depends only on damping ratio, ξ.

$$M_p = e^{-\xi\pi\sqrt{1-\xi^2}}$$

If M_p is same then ξ is also same and we get

$$\xi = \cos\theta$$

Thus $\qquad \theta = \text{constant}$

The option (C) only have same angle.

Hence (C) is correct option.

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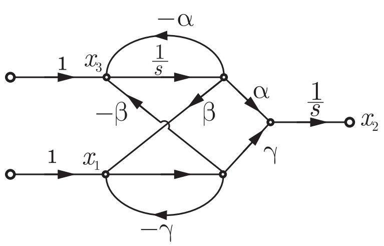
SOL 7.20

$P = \dfrac{25}{s^2 + 25}$	$2\xi\omega_n = 0, \xi = 0 \rightarrow$ Undamped	Graph 3
$Q = \dfrac{6^2}{s^2 + 20s + 6^2}$	$2\xi\omega_n = 20, \xi > 1 \rightarrow$ Overdamped	Graph 4
$R = \dfrac{6^2}{s^2 + 12s + 6^2}$	$2\xi\omega_n = 12, \xi = 1 \rightarrow$ Critically	Graph 1
$S = \dfrac{7^2}{s^2 + 7s + 7^2}$	$2\xi\omega_n = 7, \xi < 1 \rightarrow$ underdamped	Graph 2

Hence (D) is correct option.

SOL 7.21

We labeled the given SFG as below :



From this SFG we have

$$\begin{aligned} \dot{x}_1 &= -\gamma x_1 + \beta x_3 + \mu_1 \\ \dot{x}_2 &= \gamma x_1 + \alpha x_3 \\ \dot{x}_3 &= -\beta x_1 - \alpha x_3 + u_2 \end{aligned}$$

Thus
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & \beta \\ \gamma & 0 & \alpha \\ -\beta & 0 & -\alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Hence (C) is correct option.

SOL 7.22

The characteristic equation of closed lop transfer function is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{s+8}{s^2 + \alpha s - 4} = 0$$

or
$$s^2 + \alpha s - 4 + s + 8 = 0$$

or
$$s^2 + (\alpha + 1)s + 4 = 0$$

This will be stable if $(\alpha + 1) > 0 \rightarrow \alpha > -1$. Thus system is stable for all positive value of α.

Hence (C) is correct option.

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SOL 7.23

The characteristic equation is

$$1 + G(s) = 0$$

or $$s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 = 0$$

Substituting $s = \frac{1}{z}$ we have

$$3z^5 + 5z^4 + 6z^3 + 3z^2 + 2z + 1 = 0$$

The routh table is shown below. As there are tow sign change in first column, there are two RHS poles.

z^5	3	6	2
z^4	5	3	1
z^3	$\frac{21}{5}$	$\frac{7}{5}$	
z^2	$\frac{4}{3}$	3	
z^1	$-\frac{7}{4}$		
z^0	1		

Hence (C) is correct option.

SOL 7.24

For underdamped second order system the transfer function is

$$T(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

It peaks at resonant frequency. Therefore

Resonant frequency $\omega_r = \omega_n\sqrt{1 - 2\xi^2}$

and peak at this frequency

$$\mu_r = \frac{5}{2\xi\sqrt{1-\xi^2}}$$

We have $\omega_r = 5\sqrt{2}$, and $\mu_r = \frac{10}{\sqrt{3}}$. Only options (A) satisfy these values.

$$\omega_n = 10,\ \xi = \frac{1}{2}$$

where $$\omega_r = 10\sqrt{1 - 2\left(\frac{1}{4}\right)} = 5\sqrt{2}$$

and $$\mu_r = \frac{5}{2\frac{1}{2}\sqrt{1-\frac{1}{4}}} = \frac{10}{\sqrt{3}}$$ Hence satisfied

Hence (C) is correct option.

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SOL 7.25

The given circuit is a inverting amplifier and transfer function is

$$\frac{V_o}{V_i} = \frac{-Z}{\frac{R_1}{sC_1R_1+1}} = \frac{-Z(sC_1R_1+1)}{R_1}$$

For Q,
$$Z = \frac{(sC_2R_2+1)}{sC_2}$$

$$\frac{V_o}{V_i} = -\frac{(sC_2R_2+1)}{sC_2} \times \frac{(sC_1R_1+1)}{R_1} \quad \text{PID Controller}$$

For R,
$$Z = \frac{R_2}{(sC_2R_2+1)}$$

$$\frac{V_o}{V_i} = -\frac{R_2}{(sC_2R_2+1)} \times \frac{(sC_1R_1+1)}{R_1}$$

Since $R_2C_2 > R_1C_1$, it is lag compensator.
Hence (B) is correct option.

SOL 7.26

In a minimum phase system, all the poles as well as zeros are on the left half of the s-plane. In given system as there is right half zero $(s=5)$, the system is a non-minimum phase system.
Hence (D) is correct option.

SOL 7.27

We have
$$K_v = \lim_{s \to 0} sG(s)H(s)$$

or
$$1000 = \lim_{s \to 0} s\frac{(K_p + K_Ds)100}{s(s+100)} = K_p$$

Now characteristics equations is

$$1 + G(s)H(s) = 0$$

$$1000 = \lim_{s \to 0} s\frac{(K_p + K_Ds)100}{s(s+100)} = K_p$$

Now characteristics equation is

$$1 + G(s)H(s) = 0$$

or
$$1 + \frac{(100 + K_Ds)100}{s(s+10)} = 0 \qquad K_p = 100$$

or
$$s^2 + (10 + 100K_D)s + 10^4 = 0$$

Comparing with $s^2 + 2\xi\omega_n + \omega_n^2 = 0$ we get

$$2\xi\omega_n = 10 + 100K_D$$

or
$$K_D = 0.9$$

Hence (B) is correct option.

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SOL 7.28

We have
$$T(s) = \frac{5}{(s+5)(s^2+s+1)}$$
$$= \frac{5}{5\left(1+\frac{s}{5}\right)(s^2+s+1)} = \frac{1}{s^2+s+1}$$

In given transfer function denominator is $(s+5)[(s+0.5)^2+\frac{3}{4}]$. We can see easily that pole at $s=-0.5\pm j\frac{\sqrt{3}}{2}$ is dominant then pole at $s=-5$. Thus we have approximated it.
Hence (D) is correct option.

SOL 7.29

$$G(s) = \frac{1}{s^2-1} = \frac{1}{(s+1)(s-1)}$$

The lead compensator $C(s)$ should first stabilize the plant i.e. remove $\frac{1}{(s-1)}$ term. From only options (A), $C(s)$ can remove this term

Thus
$$G(s)\,C(s) = \frac{1}{(s+1)(s-1)}\times\frac{10(s-1)}{(s+2)}$$
$$= \frac{10}{(s+1)(s+2)}$$ Only option (A) satisfies.

Hence (A) is correct option.

SOL 7.30

For ufb system the characteristics equation is
$$1+G(s) = 0$$
or
$$1+\frac{K}{s(s^2+7s+12)} = 0$$
or
$$s(s^2+7s+12)+K = 0$$
Point $s=-1+j$ lie on root locus if it satisfy above equation i.e
$$(-1+j)[(-1+j)^2+7(-1+j)+12)+K] = 0$$
or
$$K = +10$$
Hence (D) is correct option.

SOL 7.31

At every corner frequency there is change of -20 db/decade in slope which indicate pole at every corner frequency. Thus
$$G(s) = \frac{K}{s(1+s)\left(1+\frac{s}{20}\right)}$$

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Bode plot is in $(1+sT)$ form

$$20\log\frac{K}{\omega}\Big|_{\omega=0.1} = 60 \text{ dB} = 1000$$

Thus $\quad K = 5$

Hence $\quad G(s) = \dfrac{100}{s(s+1)(1+.05s)}$

Hence (D) is correct option.

SOL 7.32

We have
$$\begin{bmatrix}\frac{d\omega}{dt}\\ \frac{di_a}{dt}\end{bmatrix} = \begin{bmatrix}-1 & 1\\ -1 & -10\end{bmatrix}\begin{bmatrix}\omega\\ i_n\end{bmatrix} + \begin{bmatrix}0\\ 10\end{bmatrix}u$$

or $$\frac{d\omega}{dt} = -\omega + i_n \qquad ...(1)$$

and $$\frac{di_a}{dt} = -\omega - 10i_a + 10u \qquad ...(2)$$

Taking laplace transform (i) we get

$$s\omega(s) = -\omega(s) = I_a(s)$$

or $$(s+1)\omega(s) = I_a(s) \qquad ...(3)$$

Taking laplace transform (ii) we get

$$sI_a(s) = -\omega(s) - 10I_a(s) + 10U(s)$$

or
$$\begin{aligned}\omega(s) &= (-10-s)\,I_a(s) + 10U(s)\\ &= (-10-s)(s+1)\omega(s) + 10U(s) \qquad \text{From (3)}\end{aligned}$$

or $$\omega(s) = -[s^2+11s+10]\omega(s) + 10U(s)$$

or $$(s^2+11s+11)\omega(s) = 10U(s)$$

or $$\frac{\omega(s)}{U(s)} = \frac{10}{(s^2+11s+11)}$$

Hence (A) is correct option.

SOL 7.33

We have $\quad \dot{x}(t) = Ax(t)$

Let $\quad A = \begin{bmatrix}p & q\\ r & s\end{bmatrix}$

For initial state vector $x(0) = \begin{bmatrix}1\\ -2\end{bmatrix}$ the system response is $x(t) = \begin{bmatrix}e^{-2t}\\ -2e^{-2t}\end{bmatrix}$

Thus $$\begin{bmatrix}\frac{d}{dt}e^{-2t}\\ \frac{d}{dt}(-2e^{-2t})\end{bmatrix}_{t=0} = \begin{bmatrix}p & q\\ r & s\end{bmatrix}\begin{bmatrix}1\\ -2\end{bmatrix}$$

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or $$\begin{bmatrix} -2e^{-2(0)} \\ 4e^{-2(0)} \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} p-2q \\ r-2s \end{bmatrix}$$

We get $\quad p - 2q = -2$ and $r - 2s = 4 \quad$ ...(i)

For initial state vector $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ the system response is $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

Thus $$\begin{bmatrix} \frac{d}{dt} e^{-t} \\ \frac{d}{dt}(-e^{-t}) \end{bmatrix}_{t=0} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -e^{-(0)} \\ e^{-(0)} \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} p-q \\ r-s \end{bmatrix}$$

We get $\quad p - q = -1$ and $r - s = 1 \quad$ ...(2)

Solving (1) and (2) set of equations we get

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

The characteristic equation

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & -1 \\ 2 & \lambda+3 \end{vmatrix} = 0$$

or $$\lambda(\lambda+3) + 2 = 0$$

or $$\lambda = -1, -2$$

Thus Eigen values are -1 and -2

Eigen vectors for $\lambda_1 = -1$

$$(\lambda_1 I - A) X_1 = 0$$

or $$\begin{bmatrix} \lambda_1 & -1 \\ 2 & \lambda_1+3 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$$

or $$-x_{11} - x_{21} = 0$$

or $$x_{11} + x_{21} = 0$$

We have only one independent equation $x_{11} = -x_{21}$.

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Let $x_{11} = K$, then $x_{21} =- K$, the Eigen vector will be

$$\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} K \\ -K \end{bmatrix} = K\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Now Eigen vector for $\lambda_2 =- 2$

$$(\lambda_2 I - A) X_2 = 0$$

or
$$\begin{bmatrix} \lambda_2 & -1 \\ 2 & \lambda_2 + 3 \end{bmatrix}\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = 0$$

or
$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$$

or
$$-x_{11} - x_{21} = 0$$

or
$$x_{11} + x_{21} = 0$$

We have only one independent equation $x_{11} =- x_{21}$.

Let $x_{11} = K$, then $x_{21} =- K$, the Eigen vector will be

$$\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} K \\ -2K \end{bmatrix} = K\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Hence (A) is correct option.

SOL 7.34

As shown in previous solution the system matrix is

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Hence (D) is correct option.

SOL 7.35

Given system is 2nd order and for 2nd order system G.M. is infinite.
Hence (D) is correct option.

SOL 7.36

Hence (D) is correct option.

SOL 7.37

If the Nyquist polt of $G(j\omega)H(j\omega)$ for a closed loop system pass through $(-1, j0)$ point, the gain margin is 1 and in dB

$$GM =- 20\log 1$$
$$= 0 \text{ dB}$$

Hence (D) is correct option.

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SOL 7.38

The characteristics equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$s^3 + as^2 + (2+K)s + K + 1 = 0$$

The Routh Table is shown below. For system to be oscillatory stable

$$\frac{a(2+K) - (K+1)}{a} = 0$$

or $$a = \frac{K+1}{K+2} \qquad ...(1)$$

Then we have

$$as^2 + K + 1 = 0$$

At 2 rad/sec we have

$$s = j\omega \rightarrow s^2 = -\omega^2 = -4,$$

Thus $\quad -4a + K + 1 = 0 \qquad ...(2)$

Solving (i) and (ii) we get $K = 2$ and $a = 0.75$.

s^3	1	$2+K$
s^2	a	$1+K$
s^1	$\frac{(1+K)a-(1+K)}{a}$	
s^0	$1+K$	

Hence (B) is correct option.

SOL 7.39

The transfer function of given compensator is

$$G_c(s) = \frac{1+3Ts}{1+Ts} \qquad T > 0$$

Comparing with

$$G_c(s) = \frac{1+aTs}{1+Ts} \text{ we get } a = 3$$

The maximum phase sift is

$$\phi_{max} = \tan^{-1}\frac{a-1}{2\sqrt{a}}$$

$$= \tan^{-1}\frac{3-1}{2\sqrt{3}} = \tan^{-1}\frac{1}{\sqrt{3}}$$

or $$\phi_{max} = \frac{\pi}{6}$$

Hence (D) is correct option.

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SOL 7.40

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2+1}\begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2+1} & \frac{1}{s^2+1} \\ \frac{-1}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix}$$

$$\phi(t) = e^{At} = L^{-1}[(sI - A)]^{-1} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$$

Hence (A) is correct option.

SOL 7.41

We have $$G(s) = \frac{as+1}{s^2}$$

$$\angle G(j\omega) = \tan^{-1}(\omega a) - \pi$$

Since PM is $\frac{\pi}{4}$ i.e. 45°, thus

$$\frac{\pi}{4} = \pi + \angle G(j\omega_g)\, \omega_g \rightarrow \text{Gain cross over Frequency}$$

or $$\frac{\pi}{4} = \pi + \tan^{-1}(\omega_g a) - \pi$$

or $$\frac{\pi}{4} = \tan^{-1}(\omega_g a)$$

or $$a\omega_g = 1$$

At gain crossover frequency $|G(j\omega_g)| = 1$

Thus $$\frac{\sqrt{1 + a^2\omega_g^2}}{\omega_g^2} = 1$$

or $$\sqrt{1+1} = \omega_g^2 \qquad (\text{as } a\omega_g = 1)$$

or $$\omega_g = (2)^{\frac{1}{4}}$$

Hence (C) is correct option.

SOL 7.42

For $a = 0.84$ we have

$$G(s) = \frac{0.84s+1}{s^2}$$

Due to ufb system $H(s) = 1$ and due to unit impulse response $R(s) = 1$, thus

$$C(s) = G(s)\, R(s) = G(s)$$

$$= \frac{0.84s+1}{s^2} = \frac{1}{s^2} + \frac{0.84}{s}$$

Taking inverse laplace transform

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$$c(t) = (t + 0.84)\,u(t)$$

At $t = 1$, $\quad c(1\,\text{sec}) = 1 + 0.84 = 1.84$

Hence (C) is correct option.

SOL 7.43

We have $\quad \dot{X} = AX + BU \quad$ where λ is set of Eigen values

and $\quad \dot{W} = CW + DU \quad$ where μ is set of Eigen values

If a liner system is equivalently represented by two sets of state equations, then for both sets, states will be same but their sets of Eigne values will not be same i.e.

$$X = W \text{ but } \lambda \neq \mu$$

Hence (C) is correct option.

SOL 7.44

The transfer function of a lag network is

$$T(s) = \frac{1 + sT}{1 + s\beta T} \qquad \beta > 1;\, T > 0$$

$$|T(j\omega)| = \frac{\sqrt{1 + \omega^2 T^2}}{\sqrt{1 + \omega^2 \beta^2 T^2}}$$

and $\quad \angle T(j\omega) = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T)$

At $\omega = 0$, $\quad |T(j\omega)| = 1$

At $\omega = 0$, $\quad \angle T(j\omega) = -\tan^{-1} 0 = 0$

At $\omega = \infty$, $\quad |T(j\omega)| = \dfrac{1}{\beta}$

At $\omega = \infty$, $\quad \angle T(j\omega) = 0$

Hence (D) is correct option.

SOL 7.45

Despite the presence of negative feedback, control systems still have problems of instability because components used have nonlinearity. There are always some variation as compared to ideal characteristics.

Hence (A) is correct option.

SOL 7.46

Hence (B) is correct option.

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SOL 7.47

The peak percent overshoot is determined for LTI second order closed loop system with zero initial condition. It's transfer function is

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Transfer function has a pair of complex conjugate poles and zeroes.
Hence (C) is correct option.

SOL 7.48

For ramp input we have $R(s) = \frac{1}{s^2}$

Now $$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} s\frac{R(s)}{1 + G(s)} = \lim_{s \to 0} \frac{1}{s + sG(s)}$$

or $$e_{ss} = \lim_{s \to 0} \frac{1}{sG(s)} = 5\% = \frac{1}{20}$$ Finite

But $$k_v = \frac{1}{e_{ss}} = \lim_{s \to 0} sG(s) = 20$$

k_v is finite for type 1 system having ramp input.
Hence (A) is correct option.

SOL 7.49

Hence (A) is correct option.

SOL 7.50

Any point on real axis of $s-$ is part of root locus if number of OL poles and zeros to right of that point is even. Thus (B) and (C) are possible option.
The characteristics equation is

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K(1-s)}{s(s+3)} = 0$$

or $$K = \frac{s^2 + 3s}{1 - s}$$

For break away & break in point

$$\frac{dK}{ds} = (1-s)(2s+3) + s^2 + 3s = 0$$

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or $\qquad -s^2 + 2s + 3 = 0$

which gives $s = 3, -1$

Here -1 must be the break away point and 3 must be the break in point.

Hence (C) is correct option.

SOL 7.51

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

or
$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3}{\omega\sqrt{\omega^2+4}}$$

Let at frequency ω_g the gain is 1. Thus

$$\frac{3}{\omega_g\sqrt{(\omega_g^2+4)}} = 1$$

or $\qquad \omega_g^4 + 4\omega_g^2 - 9 = 0$

or $\qquad \omega_g^2 = 1.606$

or $\qquad \omega_g = 1.26 \text{ rad/sec}$

Now $\qquad \angle G(j\omega) = -2\omega - \frac{\pi}{2} - \tan^{-1}\frac{\omega}{2}$

Let at frequency ω_ϕ we have $\angle GH = -180°$

$$-\pi = -2\omega_\phi - \frac{\pi}{2} - \tan^{-1}\frac{\omega_\phi}{2}$$

or
$$2\omega_\phi + \tan^{-1}\frac{\omega_\phi}{2} = \frac{\pi}{2}$$

or
$$2\omega_\phi + \left(\frac{\omega_\phi}{2} - \frac{1}{3}\left(\frac{\omega_\phi}{2}\right)^3\right) = \frac{\pi}{2}$$

or
$$\frac{5\omega_\phi}{2} - \frac{\omega_\phi^3}{24} = \frac{\pi}{2}$$

$$\frac{5\omega_\phi}{2} \approx \frac{\pi}{2}$$

or $\qquad \omega_\phi = 0.63 \text{ rad}$

Hence (D) is correct option.

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SOL 7.52

The gain at phase crossover frequency ω_ϕ is

$$|G(j\omega_g)| = \frac{3}{\omega_\phi\sqrt{(\omega_\phi^2+4)}} = \frac{3}{0.63\,(0.63^2+4)^{\frac{1}{2}}}$$

or $$|G(j\omega_g)| = 2.27$$

$$\text{G.M.} = -20\log|G(j\omega_g)|$$

$$-20\log 2.26 = -7.08 \text{ dB}$$

Since G.M. is negative system is unstable.

The phase at gain corss over frequency is

$$\angle G(j\omega_g) = -2\omega_g - \frac{\pi}{2} - \tan^{-1}\frac{\omega_g}{2}$$

$$= -2\times 1.26 - \frac{\pi}{2} - \tan^{-1}\frac{1.26}{2}$$

or $$= -4.65 \text{ rad or } -266.5^\circ$$

$$\text{PM} = 180^\circ + \angle G(j\omega_g) = 180^\circ - 266.5^\circ = -86.5^\circ$$

Hence (D) is correct option.

SOL 7.53

The open loop transfer function is

$$G(s)H(s) = \frac{2(1+s)}{s^2}$$

Substituting $s = j\omega$ we have

$$G(j\omega)H(j\omega) = \frac{2(1+j\omega)}{-\omega^2} \qquad \text{...(1)}$$

$$\angle G(j\omega)H(j\omega) = -180^\circ + \tan^{-1}\omega$$

The frequency at which phase becomes -180°, is called phase crossover frequency.

Thus $$-180 = -180^\circ + \tan^{-1}\omega_\phi$$

or $$\tan^{-1}\omega_\phi = 0$$

or $$\omega_\phi = 0$$

The gain at $\omega_\phi = 0$ is

$$|G(j\omega)H(j\omega)| = \frac{2\sqrt{1+\omega^2}}{\omega^2} = \infty$$

Thus gain margin is $= \frac{1}{\infty} = 0$ and in dB this is $-\infty$.

Hence (D) is correct option.

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SOL 7.54

Centroid is the point where all asymptotes intersects.

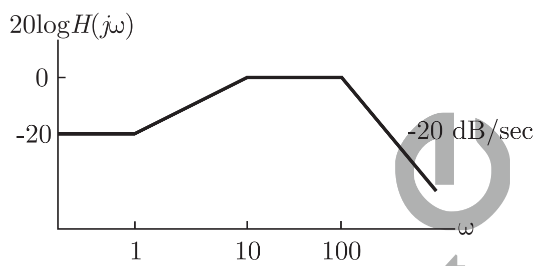
$$\sigma = \frac{\Sigma\text{Real of Open Loop Pole} - \Sigma\text{Real Part of Open Loop Pole}}{\Sigma\text{No.of Open Loop Pole} - \Sigma\text{No.of Open Loop zero}}$$

$$= \frac{-1-3}{3} = -1.33$$

Hence (C) is correct option.

SOL 7.55

The given bode plot is shown below



At $\omega = 1$ change in slope is $+20$ dB $\rightarrow$ 1 zero at $\omega = 1$
At $\omega = 10$ change in slope is -20 dB $\rightarrow$ 1 poles at $\omega = 10$
At $\omega = 100$ change in slope is -20 dB $\rightarrow$ 1 poles at $\omega = 100$

Thus $$T(s) = \frac{K(s+1)}{(\frac{s}{10}+1)(\frac{s}{100}+1)}$$

Now $$20\log_{10}K = -20 \rightarrow K = 0.1$$

Thus $$T(s) = \frac{0.1(s+1)}{(\frac{s}{10}+1)(\frac{s}{100}+1)} = \frac{100(s+1)}{(s+10)(s+100)}$$

Hence (C) is correct option.

SOL 7.56

We have $$r(t) = 10u(t)$$

or $$R(s) = \frac{10}{s}$$

Now $$H(s) = \frac{1}{s+2}$$

$$C(s) = H(s)\cdot R(s) = \frac{1}{s+2}\cdot\frac{10}{s}\frac{10}{s(s+2)}$$

or $$C(s) = \frac{5}{s} - \frac{5}{s+2}$$

$$c(t) = 5[1 - e^{-2t}]$$

The steady state value of $c(t)$ is 5. It will reach 99% of steady state

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value reaches at t, where

$$5[1-e^{-2t}] = 0.99\times 5$$

or $$1-e^{-2t} = 0.99$$

$$e^{-2t} = 0.1$$

or $$-2t = \ln 0.1$$

or $$t = 2.3\,\text{sec}$$

Hence (C) is correct option.

SOL 7.57

Approximate (comparable to 90°) phase shift are

Due to pole at 0.01 Hz $\rightarrow -90^\circ$

Due to pole at 80 Hz $\rightarrow -90^\circ$

Due to pole at 80 Hz $\rightarrow 0$

Due to zero at 5 Hz $\rightarrow 90^\circ$

Due to zero at 100 Hz $\rightarrow 0$

Due to zero at 200 Hz $\rightarrow 0$

Thus approximate total -90° phase shift is provided.

Hence (A) is correct option.

SOL 7.58

Mason Gain Formula

$$T(s) = \frac{\Sigma p_k \Delta_k}{\Delta}$$

In given SFG there is only one forward path and 3 possible loop.

$$p_1 = abcd$$

$$\Delta_1 = 1$$

$\Delta = 1-$ (sum of indivudual loops) - (Sum of two non touching loops)

$$= 1-(L_1+L_2+L_3)+(L_1L_3)$$

Non touching loop are L_1 and L_3 where

$$L_1L_2 = bedg$$

Thus $$\frac{C(s)}{R(s)} = \frac{p_1\Delta_1}{1-(be+cf+dg)+bedg}$$

$$= \frac{abcd}{1-(be+cf+dg)+bedg}$$

Hence (C) is correct option.

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SOL 7.59

We have $$A = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix}$$

Characteristic equation is

$$[\lambda I - A] = 0$$

or $$\begin{vmatrix} \lambda + 2 & -2 \\ -1 & \lambda + 3 \end{vmatrix} = 0$$

or $$(\lambda + 2)(\lambda + 3) - 2 = 0$$

or $$\lambda^2 + 5\lambda + 4 = 0$$

Thus $$\lambda_1 = -4 \text{ and } \lambda_2 = -1$$

Eigen values are -4 and -1.

Eigen vectors for $\lambda_1 = -4$

$$(\lambda_1 I - A)X_1 = 0$$

or $$\begin{bmatrix} \lambda_1 + 2 & -2 \\ 1 & \lambda_1 + 3 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & -2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$$

or $$-2x_{11} - 2x_{21} = 0$$

or $$x_{11} + x_{21} = 0$$

We have only one independent equation $x_{11} = -x_{21}$.

Let $x_{21} = K$, then $x_{11} = -K$, the Eigen vector will be

$$\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} -K \\ K \end{bmatrix} = K\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Now Eigen vector for $\lambda_2 = -1$

$$(\lambda_2 I - A)X_2 = 0$$

or $$\begin{bmatrix} \lambda_2 + 2 & -2 \\ -1 & \lambda_2 + 3 \end{bmatrix}\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = 0$$

or $$\begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = 0$$

We have only one independent equation $x_{12} = 2x_{22}$

Let $x_{22} = K$, then $x_{12} = 2K$. Thus Eigen vector will be

$$\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 2K \\ K \end{bmatrix} = K\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Digonalizing matrix

$$M = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$$

Now $$M^{-1} = \left(\frac{-1}{3}\right)\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}$$

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Now Diagonal matrix of $\sin At$ is D where

$$D = \begin{bmatrix} \sin(\lambda_1 t) & 0 \\ 0 & \sin(\lambda_2 t) \end{bmatrix} = \begin{bmatrix} \sin(-4t) & 0 \\ 0 & \sin(\lambda_2 t) \end{bmatrix}$$

Now matrix $\quad B = \sin At = MDM^{-1}$

$$= -\left(\frac{1}{3}\right)\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \sin(-4t) & 0 \\ 0 & \sin(-t) \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}$$

$$= -\left(\frac{1}{3}\right)\begin{bmatrix} -\sin(-4t) - 2\sin(-t) & 2\sin(-4t) - 2\sin(-t) \\ \sin(-4t) + 2\sin(t) & -2\sin(-4t) - \sin(-t) \end{bmatrix}$$

$$= -\left(\frac{1}{3}\right)\begin{bmatrix} -\sin(-4t) - 2\sin(-t) & 2\sin(-4t) - 2\sin(-t) \\ \sin(-4t) - \sin(-t) & -2\sin(-4t) + 2\sin(-t) \end{bmatrix}$$

$$= \left(\frac{1}{3}\right)\begin{bmatrix} \sin(-4t) + 2\sin(-t) & -2\sin(-4t) + 2\sin(-t) \\ -\sin(-4t + \sin(-t) & 2\sin(-4t) + \sin(-t) \end{bmatrix} s$$

Hence (A) is correct option.

SOL 7.60

For ufb system the characteristic equation is

$$1 + G(s) = 0$$

$$1 + \frac{K^{1+G(s)}}{s(s^2 + 2s + 2)(s + 3)} = 0$$

$$s^4 + 4s^3 + 5s^2 + 6s + K = 0$$

The routh table is shown below. For system to be stable,

$$0 < K \text{ and } 0 < \frac{(21 - 4K)}{2/7}$$

This gives $\quad 0 < K < \frac{21}{4}$

s^4	1	5	K
s^3	4	6	0
s^2	$\frac{7}{2}$	K	
s^1	$\frac{21 - 4K}{7/2}$	0	
s^0	K		

Hence (A) is correct option.

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SOL 7.61

We have $$P(s) = s^5 + s^4 + 2s^3 + 3s + 15$$

The routh table is shown below.

If $\varepsilon \to 0^+$ then $\frac{2\varepsilon + 12}{\varepsilon}$ is positive and $\frac{-15\varepsilon^2 - 24\varepsilon - 144}{2\varepsilon + 12}$ is negative. Thus there are two sign change in first column. Hence system has 2 root on RHS of plane.

s^5	1	2	3
s^4	1	2	15
s^3	ε	-12	0
s^2	$\frac{2\varepsilon + 12}{\varepsilon}$	15	0
s^1	$\frac{-15\varepsilon^2 - 24\varepsilon - 144}{2\varepsilon + 12}$		
s^0	0		

Hence (B) is correct option.

SOL 7.62

We have $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

and $$Y = [1 \ \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

Here $$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } C = [1 \ \ 0]$$

The controllability matrix is

$$Q_C = [B \ \ AB] = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$$

$$\det Q_C \neq 0 \qquad \text{Thus controllable}$$

The observability matrix is

$$Q_0 = [C^T \ \ A^T C^T] = \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix} \neq 0$$

$$\det Q_0 \neq 0 \qquad \text{Thus observable}$$

Hence (D) is correct option.

SOL 7.63

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}$$

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$$(sI-A)^{-1} = \frac{1}{(s-1)^2}\begin{bmatrix}(s-1) & 0 \\ 0 & (s-1)\end{bmatrix} = \begin{bmatrix}\frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-1}\end{bmatrix}$$

$$e^{At} = L^{-1}[(sI-A)]^{-1}$$

$$= \begin{bmatrix}e^t & 0 \\ 0 & e^t\end{bmatrix}$$

Hence (B) is correct option.

SOL 7.64

$$Z = P - N$$

$N \rightarrow$ Net encirclement of $(-1+j0)$ by Nyquist plot,
$P \rightarrow$ Number of open loop poles in right hand side of $s-$ plane
$Z \rightarrow$ Number of closed loop poles in right hand side of $s-$ plane
Here $N = 1$ and $P = 1$
Thus $\quad Z = 0$
Hence there are no roots on RH of $s-$plane and system is always stable.
Hence (A) is correct option.

SOL 7.65

PD Controller may accentuate noise at higher frequency. It does not effect the type of system and it increases the damping. It also reduce the maximum overshoot.
Hence (C) is correct option.

SOL 7.66

Mason Gain Formula

$$T(s) = \frac{\Sigma p_k \Delta_k}{\Delta}$$

In given SFG there is only forward path and 3 possible loop.

$$p_1 = 1$$

$$\Delta_1 = 1 + \frac{3}{s} + \frac{24}{s} = \frac{s+27}{s}$$

$$L_1 = \frac{-2}{s}, L_2 = \frac{-24}{s} \text{ and } L_3 = \frac{-3}{s}$$

where L_1 and L_3 are non-touching

This $$\frac{C(s)}{R(s)} = \frac{p_1 \Delta_1}{1 - (\text{loop gain}) + \text{pair of non} - \text{touching loops}}$$

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$$= \frac{\left(\frac{s+27}{s}\right)}{1-\left(\frac{-3}{s}-\frac{24}{s}-\frac{2}{s}\right)+\frac{-2}{s}.\frac{-3}{s}} = \frac{\left(\frac{s+27}{s}\right)}{1+\frac{29}{s}+\frac{6}{s^2}}$$

$$= \frac{s(s+27)}{s^2+29s+6}$$

Hence (D) is correct option.

SOL 7.67

We have

$$1+G(s)H(s) = 0$$

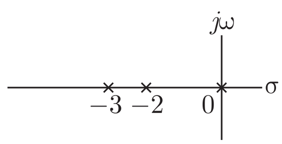
or $$1+\frac{K}{s(s+2)(s+3)} = 0$$

or $$K = -s(s^2+5s^2+6s)$$

$$\frac{dK}{ds} = -(3s^2+10s+6) = 0$$

which gives $$s = \frac{-10\pm\sqrt{100-72}}{6} = -0.784,\ -2.548$$

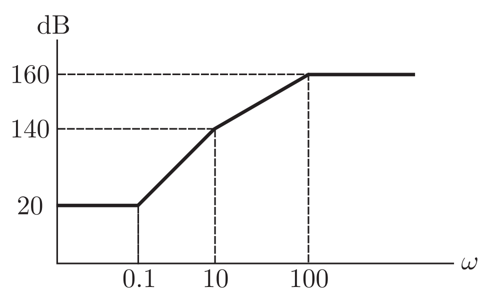
The location of poles on $s-$ plane is



Since breakpoint must lie on root locus so $s = -0.748$ is possible. Hence (D) is correct option.

SOL 7.68

The given bode plot is shown below



At $\omega = 0.1$ change in slope is $+60$ dB $\rightarrow$ 3 zeroes at $\omega = 0.1$
At $\omega = 10$ change in slope is -40 dB $\rightarrow$ 2 poles at $\omega = 10$
At $\omega = 100$ change in slope is -20 dB $\rightarrow$ 1 poles at $\omega = 100$

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Thus $$T(s) = \frac{K(\frac{s}{0.1}+1)^3}{(\frac{s}{10}+1)^2(\frac{s}{100}+1)}$$

Now $$20\log_{10}K = 20$$

or $$K = 10$$

Thus $$T(s) = \frac{10(\frac{s}{0.1}+1)^3}{(\frac{s}{10}+1)^2(\frac{s}{100}+1)} = \frac{10^8(s+0.1)^3}{(s+10)^2(s+100)}$$

Hence (A) is correct option.

SOL 7.69

The characteristics equation is

$$s^2+4s+4 = 0$$

Comparing with

$$s^2+2\xi\omega_n+\omega_n^2 = 0$$

we get $$2\xi\omega_n = 4 \text{ and } \omega_n^2 = 4$$

Thus $$\xi = 1 \qquad \text{Critically damped}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{1\times 2} = 2$$

Hence (B) is correct option.

SOL 7.70

Hence (B) is correct option.

SOL 7.71

We have

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \text{ and } \begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix}$$

$$A = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

$$(sI-A) = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}s-1 & 0\\ -1 & s-1\end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{(s-1)^2}\begin{bmatrix}(s-1) & 0\\ +1 & (s-1)\end{bmatrix} = \begin{bmatrix}\frac{1}{s-1} & 0\\ \frac{+1}{(s-1)^2} & \frac{1}{s-1}\end{bmatrix}$$

$$L^{-1}[(sI-A)^{-1}] = e^{At} = \begin{bmatrix}e^t & 0\\ te^t & e^t\end{bmatrix}$$

$$x(t) = e^{At}\times[x(t_0)] = \begin{bmatrix}e^t & 0\\ te^t & e^t\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}e^t\\ te^t\end{bmatrix}$$

Hence (C) is correct option.

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SOL 7.72

The characteristics equation is

$$ks^2 + s + 6 = 0$$

or
$$s^2 + \frac{1}{K}s + \frac{6}{K} = 0$$

Comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ we have

we get
$$2\xi\omega_n = \frac{1}{K} \text{ and } \omega_n^2 = \frac{6}{K}$$

or
$$2 \times 0.5 \times \sqrt{6}\,K\omega = \frac{1}{K} \qquad \text{Given } \xi = 0.5$$

or
$$\frac{6}{K} = \frac{1}{K^2} \Rightarrow K = \frac{1}{6}$$

Hence (C) is correct option.

SOL 7.73

Any point on real axis lies on the root locus if total number of poles and zeros to the right of that point is odd. Here $s = -1.5$ does not lie on real axis because there are total two poles and zeros (0 and -1) to the right of $s = -1.5$.
Hence (B) is correct option.

SOL 7.74

From the expression of OLTF it may be easily see that the maximum magnitude is 0.5 and does not become 1 at any frequency. Thus gain cross over frequency does not exist. When gain cross over frequency does not exist, the phase margin is infinite.
Hence (D) is correct option.

SOL 7.75

We have
$$\dot{x}(t) = -2x(t) + 2u(t) \qquad \text{...(i)}$$

Taking laplace transform we get

$$sX(s) = -2X(s) + 2U(s)$$

or
$$(s+2)X(s) = 2U(s)$$

or
$$X(s) = \frac{2U(s)}{(s+2)}$$

Now
$$y(t) = 0.5x(t)$$
$$Y(s) = 0.5X(s)$$

or
$$Y(s) = \frac{0.5 \times 2U(s)}{s+2}$$

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or $$\frac{Y(s)}{U(s)} = \frac{1}{(s+2)}$$

Hence (D) is correct option.

SOL 7.76

From Mason gain formula we can write transfer function as

$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{s}}{1-(\frac{3}{s}+\frac{-K}{s})} = \frac{K}{s-3(3-K)}$$

For system to be stable $(3-K) < 0$ i.e. $K > 3$

Hence (D) is correct option.

SOL 7.77

The characteristics equation is

$$(s+1)(s+100) = 0$$
$$s^2 + 101s + 100 = 0$$

Comparing with $s^2 + 2\xi\omega_n + \omega_n^2 = 0$ we get

$$2\xi\omega_n = 101 \text{ and } \omega_n^2 = 100$$

Thus $$\xi = \frac{101}{20} \qquad \text{Overdamped}$$

For overdamped system settling time can be determined by the dominant pole of the closed loop system. In given system dominant pole consideration is at $s = -1$. Thus

$$\frac{1}{T} = 1 \qquad \text{and} \quad T_s = \frac{4}{T} = 4\,\text{sec}$$

Hence (B) is correct option.

SOL 7.78

Routh table is shown below. Here all element in 3rd row are zero, so system is marginal stable.

s^5	2	4	2
s^4	1	2	1
s^3	0	0	0
s^2			
s^1			
s^0			

Hence (B) is correct option.

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SOL 7.79

The open loop transfer function is

$$G(s)\,H(s) = \frac{1}{s(s^2+s+1)}$$

Substituting $s = j\omega$ we have

$$G(j\omega)\,H(j\omega) = \frac{1}{j\omega(-\omega^2+j\omega+1)}$$

$$\angle G(j\omega)\,H(j\omega) = -\frac{\pi}{2} - \tan^{-1}\frac{\omega}{(1-\omega^2)}$$

The frequency at which phase becomes -180°, is called phase crossover frequency.

Thus $$-180 = -90 - \tan^{-1}\frac{\omega_\phi}{1-\omega_\phi^2}$$

or $$-90 = -\tan^{-1}\frac{\omega_\phi}{1-\omega_\phi^2}$$

or $$1-\omega_\phi^2 = 0$$

$$\omega_\phi = 1 \text{ rad/sec}$$

The gain margin at this frequency $\omega_\phi = 1$ is

$$\begin{aligned} \text{GM} &= -20\log_{10}|G(j\omega_\phi)\,H(j\omega_\phi)| \\ &= 20\log_{10}(\omega_\phi\sqrt{(1-\omega_\phi^2)^2+\omega_\phi^2} \\ &= -20\log 1 = 0 \end{aligned}$$

Hence (B) is correct option.

SOL 7.80

$$Z = P - N$$

$N \rightarrow$ Net encirclement of $(-1+j0)$ by Nyquist plot,

$P \rightarrow$ Number of open loop poles in right had side of $s-$ plane

$Z \rightarrow$ Number of closed loop poles in right hand side of $s-$ plane

Here $N = 0$ (1 encirclement in CW direction and other in CCW)

and $P = 0$

Thus $Z = 0$

Hence there are no roots on RH of $s-$ plane.

Hence (A) is correct option.

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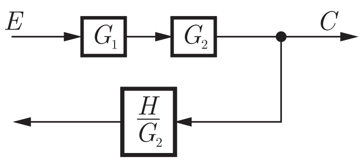
SOL 7.81

Take off point is moved after G_2 as shown below



Hence (D) is correct option.

SOL 7.82

The characteristics equation is

$$s^2 + 2s + 2 = 0$$

Comparing with $s^2 + 2\xi\omega_n + \omega_n^2 = 0$ we get

$$2\xi\omega_n = 2 \text{ and } \omega_n^2 = 2$$

$$\omega_n = \sqrt{2}$$

and

$$\xi = \frac{1}{\sqrt{2}}$$

Since $\xi < 1$ thus system is under damped

Hence (C) is correct option.

SOL 7.83

If roots of characteristics equation lie on negative axis at different positions (i.e. unequal), then system response is over damped.

From the root locus diagram we see that for $0 < K < 1$, the roots are on imaginary axis and for $1 < K < 5$ roots are on complex plain. For $K > 5$ roots are again on imaginary axis.

Thus system is over damped for $0 \le K < 1$ and $K > 5$.

Hence (D) is correct option.

SOL 7.84

From SFG we have

$$I_1(s) = G_1 V_i(s) + HI_2(s) \quad ...(1)$$

$$I_2(s) = G_2 I_1(s) \quad ...(2)$$

$$V_0(s) = G_3 I_2(s) \quad ...(3)$$

Now applying KVL in given block diagram we have

$$V_i(s) = I_1(s) Z_1(s) + [I_1(s) - I_2(s)] Z_3(s) \quad ...(4)$$

$$0 = [I_2(s) - I_1(s)] Z_3(s) + I_2(s) Z_2(s) + I_2(s) Z_4(s) \quad ...(5)$$

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From (4) we have

or $$V_i(s) = I_1(s)[Z_1(s) + Z_3(S)] - I_2(s)Z_3(S)$$

or $$I_1(s) = V_i \frac{1}{Z_1(s) + Z_3(s)} + I_2 \frac{Z_3(s)}{Z_1(s) + Z_3(s)} \qquad ...(6)$$

From (5) we have

$$I_1(s)Z_3(S) = I_2(s)[Z_2(s) + Z_3(s) + Z_4(s)] \qquad ...(7)$$

or $$I_s(s) = \frac{I_1(s)Z_3(s)}{Z_3(s) + Z_2(s) + Z_4(s)}$$

Comparing (2) and (7) we have

$$G_2 = \frac{Z_3(s)}{Z_3(s) + Z_2(s) + Z_4(s)}$$

Comparing (1) and (6) we have

$$H = \frac{Z_3(s)}{Z_1(s) + Z_3(s)}$$

Hence (C) is correct option.

SOL 7.85

For unity negative feedback system the closed loop transfer function is

$$\text{CLTF} = \frac{G(s)}{1 + G(s)} = \frac{s+4}{s^2 + 7s + 13}, \qquad G(s) \to OL \text{ Gain}$$

or $$\frac{1 + G(s)}{G(s)} = \frac{s^2 + 7s + 13}{s + 4}$$

or $$\frac{1}{G(s)} = \frac{s^2 + 7s + 13}{s + 4} - 1 = \frac{s^2 + 6s + 9}{s + 4}$$

or $$G(s) = \frac{s + 4}{s^2 + 6s + 9}$$

For DC gain $s = 0$, thus

Thus $$G(0) = \frac{4}{9}$$

Hence (B) is correct option.

SOL 7.86

From the Block diagram transfer function is

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Where $$G(s) = \frac{K(s-2)}{(s+2)}$$

and $$H(s) = (s-2)$$

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The Characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s-2)}{(s+2)^2}(s-2) = 0$$

or $$(s+2)^2 + K(s-2)^2 = 0$$

or $$(1+K)s^2 + 4(1-K)s + 4K + 4 = 0$$

Routh Table is shown below. For System to be stable $1 + k > 0$, and $4 + 4k > 0$ and $4 - 4k > 0$. This gives $-1 < K < 1$

As per question for $0 \leq K < 1$

s^2	$1+k$	$4+4k$
s^1	$4-4k$	0
s^0	$4+4k$	

Hence (C) is correct option.

SOL 7.87

It is stable at all frequencies because for resistive network feedback factor is always less than unity. Hence overall gain decreases.

Hence (B) is correct option.

SOL 7.88

The characteristics equation is $s^2 + \alpha s^2 + ks + 3 = 0$

The Routh Table is shown below

For system to be stable $\alpha > 0$ and $\frac{\alpha K - 3}{\alpha} > 0$

Thus $\alpha > 0$ and $\alpha K > 3$

s^3	1	K
s^2	α	3
s^1	$\frac{\alpha K - 3}{\alpha}$	0
s^0	3	

Hence (B) is correct option.

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SOL 7.89

Closed loop transfer function is given as

$$T(s) = \frac{9}{s^2 + 4s + 9}$$

by comparing with standard form we get natural freq.

$$\omega_A^2 = 9$$
$$\omega_n = 3$$
$$2\xi\omega_n = 4$$

damping factor $\quad \xi = \frac{4}{2 \times 3} = 2/3$

for second order system the setting time for 2-percent band is given by

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{3 \times 2/3} = \frac{4}{2} = 2$$

Hence (B) is correct option.

SOL 7.90

Given loop transfer function is

$$G(s)\,H(s) = \frac{\sqrt{2}}{s(s+1)}$$
$$G(j\omega)\,H(j\omega) = \frac{\sqrt{2}}{j\omega(j\omega+1)}$$

Phase cross over frequency can be calculated as

$$\phi(\omega)\big|_{\text{at } \omega = \omega_p} = -180^\circ$$

So here $\quad \phi(\omega) = -90^\circ - \tan^{-1}(\omega)$

$$-90^\circ - \tan^{-1}(\omega_p) = -180^\circ$$
$$\tan^{-1}(\omega_p) = 90^\circ$$
$$\omega_p = \infty$$

Gain margin

$$20\log_{10}\left[\frac{1}{|G(j\omega)\,H(j\omega)|}\right] \text{at } \omega = \omega_p$$

$$G.M. = 20\log_{10}\left(\frac{1}{|G(j\omega)\,H(j\omega_p)|}\right)$$

$$|G(j\omega_p)\,H(j\omega_p)| = \frac{\sqrt{2}}{\omega_p\sqrt{\omega_p^2 + 1}} = 0$$

so $\quad G.M. = 20\log_{10}\left(\frac{1}{0}\right) = \infty$

Hence (D) is correct option.

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SOL 7.91

Here $\quad A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C = [1 \ \ 1]$

The controllability matrix is

$$Q_C = [B \ \ AB] = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$\det Q_C \neq 0$ $\qquad$ Thus controllable

The observability matrix is

$$Q_0 = [C^T \ \ A^T C^T] = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \neq 0$$

$\det Q_0 \neq 0$ $\qquad$ Thus observable

Hence (A) is correct option.

SOL 7.92

we have $\quad G(s)H(s) = \dfrac{2\sqrt{3}}{s(s+1)}$

or $\quad G(j\omega)H(j\omega) = \dfrac{2\sqrt{3}}{j\omega(j\omega+1)}$

Gain cross over frequency

$$\left| G(j\omega)H(j\omega) \right|_{at\ \omega = \omega_g} = 1$$

or

$$\frac{2\sqrt{3}}{\omega\sqrt{\omega^2+1}} = 1$$

$$12 = \omega^2(\omega^2+1)$$

$$\omega^4 + \omega^2 - 12 = 0$$

$$(\omega^2+4)(\omega^2-3) = 0$$

$$\omega^2 = 3 \text{ and } \omega^2 = -4$$

which gives $\quad \omega_1, \omega_2 = \pm\sqrt{3}$

$$\omega_g = \sqrt{3}$$

$$\begin{aligned} \phi(\omega)\big|_{at\ \omega=\omega_g} &= -90 - \tan^{-1}(\omega_g) \\ &= -90 - \tan^{-1}\sqrt{3} \\ &= -90 - 60 = -150 \end{aligned}$$

$$\begin{aligned} \text{Phase margin} &= 180 + \phi(\omega)\big|_{at\ \omega=\omega_g} \\ &= 180 - 150 = 30^\circ \end{aligned}$$

Hence (D) is correct option.

SOL 7.93

Hence (B) is correct option.

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SOL 7.94

Closed-loop transfer function is given by

$$T(s) = \frac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n}$$

$$= \frac{\dfrac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + \dots a_{n-2}s^2}}{1 + \dfrac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + \dots a_{n-2}s^2}}$$

Thus $$G(s)H(s) = \frac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + \dots . a_{n-2}s^2}$$

For unity feed back $H(s) = 1$

Thus $$G(s) = \frac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + \dots . a_{n-2}s^2}$$

Steady state error is given by

$$E(s) = \lim_{s \to 0} R(s)\frac{1}{1 + G(s)H(s)}$$

for unity feed back $H(s) = 1$

Here input $R(s) = \frac{1}{s^2}$(unit Ramp)

so $$E(s) = \lim_{s \to 0} \frac{1}{s^2}\frac{1}{1 + G(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2}\frac{s^n + a_1 s^{n-1} + \dots . + a_{n-2}s^2}{s^n + a_1 s^{n-1} + \dots . + a_n}$$

$$= \frac{a_{n-2}}{a_n}$$

Hence (C) is correct option.

SOL 7.95

Hence (B) is correct option.

SOL 7.96

Hence (A) is correct option.

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SOL 7.97

By applying Routh's criteria

$$s^3 + 5s^2 + 7s + 3 = 0$$

s^3	1	7
s^2	5	3
s^1	$\frac{7\times 5-3}{5} = \frac{32}{5}$	0
s^0	3	

There is no sign change in the first column. Thus there is no root lying in the left-half plane.
Hence (A) is correct option.

SOL 7.98

Techometer acts like a differentiator so its transfer function is of the form ks.
Hence (A) is correct option.

SOL 7.99

Open loop transfer function is

$$G(s) = \frac{K}{s(s+1)}$$

Steady state error

$$E(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

Where $R(s) = \text{input}$ $\qquad H(s) = 1$ (unity feedback)

$$R(s) = \frac{1}{s}$$

so $$E(s) = \lim_{s\to 0} \frac{s\frac{1}{s}}{1+\frac{K}{s(s+1)}} = \lim_{s\to 0} \frac{s(s+1)}{s^2+s+K} = 0$$

Hence (A) is correct option.

SOL 7.100

Fig given below shows a unit impulse input given to a zero-order hold circuit which holds the input signal for a duration T & therefore, the output is a unit step function till duration T.

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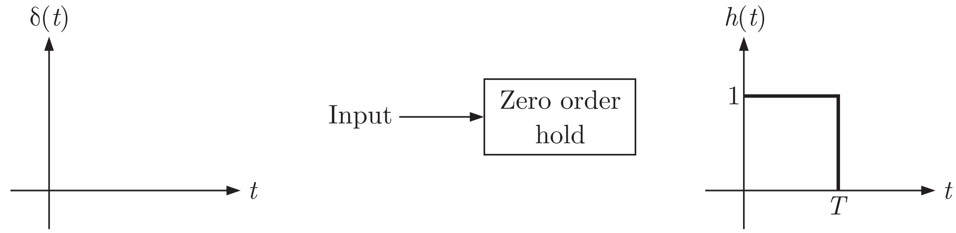

$$h(t) = u(t) - u(t - T)$$

Taking Laplace transform we have

$$H(s) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1}{s}[1 - e^{-sT}]$$

Hence (B) is correct option.

SOL 7.101

Phase margin $= 180^\circ + \theta_g$ where θ_g = value of phase at gain crossover frequency.

Here $\qquad \theta_g = -125^\circ$

so $\qquad \text{P.M} = 180^\circ - 125^\circ = 55^\circ$

Hence (C) is correct option.

SOL 7.102

Open loop transfer function is given by

$$G(s)H(s) = \frac{K(1 + 0.5s)}{s(1 + s)(1 + 2s)}$$

Close looped system is of type 1.

It must be noted that type of the system is defined as no. of poles $\frac{\text{lies}}{\text{lying}}$ at origin in OLTF.

Hence (B) is correct option.

SOL 7.103

Transfer function of the phase lead controller is

$$T.F = \frac{1 + 3Ts}{1 + s} = \frac{1 + (3T\omega)j}{1 + (T\omega)j}$$

Phase is

$$\phi(\omega) = \tan^{-1}(3T\omega) - \tan^{-1}(T\omega)$$

$$\phi(\omega) = \tan^{-1}\left[\frac{3T\omega - T\omega}{1 + 3T^2\omega^2}\right]$$

$$\phi(\omega) = \tan^{-1}\left[\frac{2T\omega}{1 + 3T^2\omega^2}\right]$$

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For maximum value of phase

$$\frac{d\phi(\omega)}{d\omega} = 0$$

or $$1 = 3T^2\omega^2$$

$$T\omega = \frac{1}{\sqrt{3}}$$

So maximum phase is

$$\phi_{\max} = \tan^{-1}\left[\frac{2T\omega}{1+3T^2\omega^2}\right] \text{ at } T\omega = \frac{1}{\sqrt{3}}$$

$$= \tan^{-1}\left[\frac{2\frac{1}{\sqrt{3}}}{1+3\times\frac{1}{3}}\right] = \tan^{-1}\left[\frac{1}{\sqrt{3}}\right] = 30^\circ$$

Hence (D) is correct option.

SOL 7.104

$G(j\omega)H(j\omega)$ enclose the $(-1, 0)$ point so here $|G(j\omega_p)H(j\omega_p)| > 1$

$$\omega_p = \text{Phase cross over frequency}$$

$$\text{Gain Margin} = 20\log_{10}\frac{1}{|G(j\omega_p)H(j\omega_p)|}$$

so gain margin will be less than zero.
Hence (A) is correct option.

SOL 7.105

The denominator of Transfer function is called the characteristic equation of the system. so here characteristic equation is

$$(s+1)^2(s+2) = 0$$

Hence (B) is correct option.

SOL 7.106

In synchro error detector, output voltage is proportional to $[\omega(t)]$, where $\omega(t)$ is the rotor velocity so here $n = 1$
Hence (C) is correct option.

SOL 7.107

By masson's gain formulae

$$\frac{y}{x} = \frac{\sum \Delta_k P_k}{\Delta}$$

Forward path gain $\quad P_1 = 5\times 2\times 1 = 10$

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$$\Delta = 1 - (2 \times -2) = 1 + 4 = 5$$

$$\Delta_1 = 1$$

so gain $\qquad \dfrac{y}{x} = \dfrac{10 \times 1}{5} = 2$

Hence (C) is correct option.

SOL 7.108

By given matrix equations we can have

$$\dot{X}_1 = \frac{dx_1}{dt} = x_1 - x_2 + 0$$

$$\dot{X}_2 = \frac{dx_2}{dt} = 0 + x_2 + \mu$$

$$y = [1 \;\; 1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 + x_2$$

$$\frac{dy}{dt} = \frac{dx_1}{dt} + \frac{dx_2}{dt}$$

$$\frac{dy}{dt} = x_1 + \mu$$

$$\left.\frac{dy}{dt}\right|_{t=0} = x_1(0) + \mu(0)$$

$$= 1 + 0 = 0$$

Hence (C) is correct option.

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