Bank of America Stock Price Research*

Pancheng Qu

(Affiliation): UCLA Economic Department, Los Angeles, CA, USA
Email: kevinqpch@gmail.com

Abstract:

The stock price is always an interesting topic. On the one hand, from intuition, the stock price of bank is relatively stable since bank usually do not take risk behavior. From 2007 to 2014, the price of stock of Bank of American is relatively stable by checking the RRV (relative realized volatility). There is only one day has relatively high RRVs during 2007 to 2014. The date May 6^{th} 2010, which called crash day is a special day that need to analyze separately. On the other hand, to find the pattern of frequency of trading, there are different sample size tested and compared with Poisson distribution. The result is that we can use Poisson distribution to predict probability of no arrival trade when seconds gap is relatively small.

In addition, when plotting the daily 100 seconds accumulated RV(realized volatility) and daily average RV, there was found strong linear relationship between these two variables. In the end, Using the Heston model to verify if there exist linear relationship between daily average and mean reversion rate. Then Comparing trend of alpha with weekly VIX from Yahoo Finance. When using 5 days as a period to calculate the daily average RV and mean reversion rate, the significance of linear relationship is stronger. It proved the statistic intuition that larger sample size tends to decrease the volatility. The overall trend of VIX from Yahoo Finance is similar to the shape of five-day period alpha.

Key Words: Stability, Heston Model, Linear Regression

1. Introduction:

First of all, based on the definition of RV(realized volatility), after we calculate all the 100 seconds RVs, we find all the RVs are scatter in a wide range. In order to normalize RVs and better analyze the jump effect in RV100s, we decide to use RRVs which is divide RVs by median RV. After calculating the RRVs, we divide it into different intervals. For example, in interval RRV greater than 1000, there may be one day. In interval RRV between 500 and 1000, there may be 50 days. Then we check how many days in each interval and check if the days are consecutive. The goal is to analyze the stability of stock price of Bank of American.

Secondly, since the original data have gaps between each trade, I try to use different seconds to calculate the number of gaps daily and sum them up to the whole year. Trying to predict the relationship between seconds unit and gaps, I choose Poisson distribution to calculate the number of gaps.

Thirdly, the daily average RV and daily accumulated RV is another direction to research the stability of stock price. We can check if there are outliers when we plot daily average RV and daily accumulated RV since they have strong correlation.

In the end, building Heston model can help us to discover more about daily average realized volatility. From the formula of Heston Model. I calculate the daily average RV for each day and also 5 days average RV. Preferring 5 days average RV is because 5 day have larger sample size therefore the variance may be smaller. The overall shape of significant alphas are similar to weekly VIX from Yahoo Finance.

2. Body of the paper:

2.1. Stability of Stock Price

To get RRV(Relative Realized Volatility), the first step to calculate log return and then calculate realized volatility by using the following formulas:

$$\text{Log return} = \log(t)-\log(t-1) \tag{1}$$

$$RV_t = \sum_{i=1}^{N} r_t^2 \tag{2}$$

In this experiment, the 100 second period realized volatility is used to calculate RRV. By using the equation below, it is straight forward to get RRV:

$$\text{RRV} = \frac{\text{RV}}{\text{Daily Median RV}} \tag{3}$$

From table 1 and table 2, there is only 1 day that has RRV greater than 1000. The day is May 6th, 2010. From the output we can see that the interval 0 to 2.5 has 95.1% days and it means that most of days do not have large realized volatility. The interval 0 to 2.5 and 2.5 to 5 contains more than 99% of days. It indicates the stock prices of Bank of American are relatively stable and will not change too much in a day.

Table 1. Daily RRV from 2007 to 2014

```
> ALLRRV=RRVreport(20070103,20141231,100)
  NO.Observations Min      Max    Median     X1stQ    X3rdQ
1          371499   0 1755.393 0.9999737 0.6474082 1.416034
   Intervals Percentage
1     [0,2.5)      95.1%
2     [2.5,5)      4.12%
3      [5,10)     0.552%
4    [10,100)     0.176%
5 [100,1000)     0.0151%
6      >=1000  0.000808%
```

Table 2. Number of RRV greater than 1000 and number of NNR less than 500

```
> ddply(table7, .(RRVS), summarise, Pool.number  = sum(Pool.number))
       RRVS Pool.number
1   RRV<500        1595
2 RRV>=1000           1
```

2.2 Probability of No Trade Arrive and Poisson Distribution

Secondly, to test if Poisson distribution can predict the probability of no trade arrive, the first step is to divide the gaps by different interval such as 5 second, 10 seconds. Then using the average gap as λ in Poisson Model and calculate the Poisson model probability. For the empirical probability, it is just simply use number of average gaps divided by total gaps. The outputs from 2007 to 2014 indicates that if choose small seconds to count gaps, the result will be much closer to the Poisson distribution probability. However, when choose larger seconds to count gaps, the difference between empirical probability and Poisson model probability will be relatively large. This trend can be clearly seen in Figure 1 in every year.

Figure 1. Compare Empirical Probability and Poisson Model Probability

```
> pro2007
  Seconds   average total.gaps empirical.probability poisson.model.probability
1      X5 4402.9402  4680.0000             0.9407992                 0.9425175
2     X10 2079.7291  2340.0000             0.8887731                 0.8947357
3     X15 1313.6892  1560.0000             0.8421085                 0.8539424
4     X20  934.7171  1170.0000             0.7989035                 0.8178335
5     X25  710.4741   936.0000             0.7590535                 0.7858837
6     X30  562.9442   780.0000             0.7217234                 0.7570874
7     X35  459.0598   668.5714             0.6866278                 0.7309778
8     X40  383.2908   585.0000             0.6551980                 0.7083606
>
```

```
> pro2008
  Seconds   average total.gaps empirical.probability poisson.model.probability
1      X5 3458.2806  4680.0000             0.7389489                 0.7702415
2     X10 1342.6680  2340.0000             0.5737897                 0.6529790
3     X15  723.2846  1560.0000             0.4636440                 0.5848756
4     X20  447.2213  1170.0000             0.3822405                 0.5391510
5     X25  300.0198   936.0000             0.3205339                 0.5068876
6     X30  212.8142   780.0000             0.2728388                 0.4832790
7     X35  156.1937   668.5714             0.2336230                 0.4646936
8     X40  117.9763   585.0000             0.2016689                 0.4500795
>
```

```
> pro2009
  Seconds   average total.gaps empirical.probability poisson.model.probability
1      X5 3329.3373  4680.0000             0.7113969                 0.7493095
2     X10 1231.0437  2340.0000             0.5260870                 0.6225614
3     X15  636.2063  1560.0000             0.4078246                 0.5531227
4     X20  380.1190  1170.0000             0.3248881                 0.5090994
5     X25  248.6984   936.0000             0.2657034                 0.4798429
6     X30  173.0238   780.0000             0.2218254                 0.4592435
7     X35  124.5238   668.5714             0.1862536                 0.4431946
8     X40   92.2619   585.0000             0.1577127                 0.4307242
>
```

```
> pro2010
  Seconds   average total.gaps empirical.probability poisson.model.probability
1      X5 4027.9643  4680.0000             0.8606761                 0.8699462
2     X10 1715.1151  2340.0000             0.7329552                 0.7656387
3     X15  981.1905  1560.0000             0.6289683                 0.6900220
4     X20  638.4444  1170.0000             0.5456790                 0.6348789
5     X25  446.4206   936.0000             0.4769451                 0.5927071
6     X30  326.3810   780.0000             0.4184371                 0.5590240
7     X35  245.9008   668.5714             0.3678003                 0.5314216
8     X40  190.2500   585.0000             0.3252137                 0.5092652
>
```

```
> pro2011
  Seconds   average total.gaps empirical.probability poisson.model.probability
1      X5 4034.2421  4680.0000             0.8620175                 0.8711140
2     X10 1723.4563  2340.0000             0.7365198                 0.7683728
3     X15  992.9524  1560.0000             0.6365079                 0.6952442
4     X20  650.7698  1170.0000             0.5562135                 0.6416024
5     X25  457.5198   936.0000             0.4888032                 0.5997774
6     X30  336.6786   780.0000             0.4316392                 0.5664532
7     X35  255.8532   668.5714             0.3826864                 0.5393915
8     X40  200.7302   585.0000             0.3431285                 0.5184708
>
```

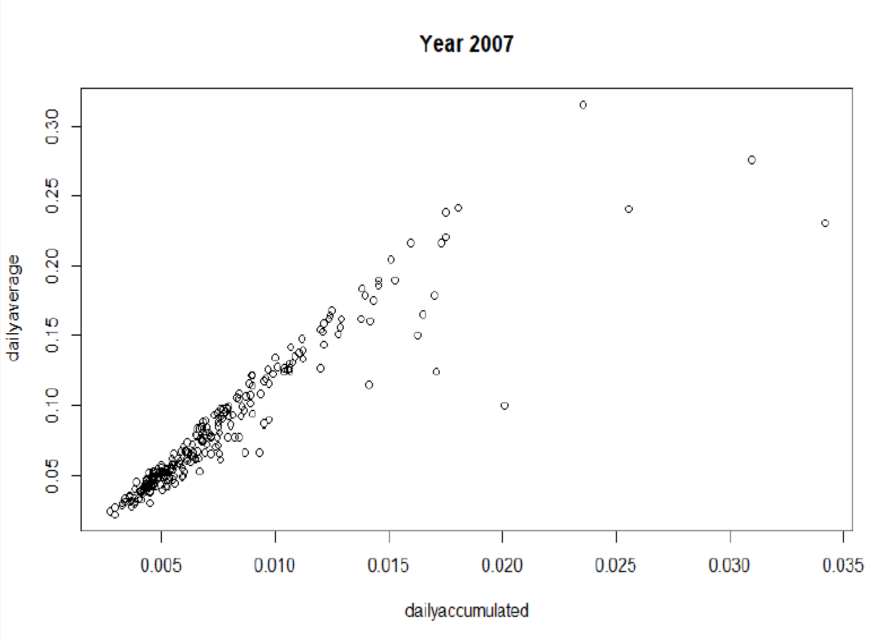
```
> pro2012
  Seconds   average total.gaps empirical.probability poisson.model.probability
1      X5 4163.2440  4680.0000             0.8895821                 0.8954598
2     X10 1830.1340  2340.0000             0.7821085                 0.8042127
3     X15 1079.0766  1560.0000             0.6917157                 0.7347064
4     X20  717.4833  1170.0000             0.6132336                 0.6792497
5     X25  512.3636   936.0000             0.5473970                 0.6359706
6     X30  383.0144   780.0000             0.4910440                 0.6011228
7     X35  294.1244   668.5714             0.4399297                 0.5711689
8     X40  231.3206   585.0000             0.3954198                 0.5463037
>
```

```
> pro2014
  Seconds   average total.gaps empirical.probability poisson.model.probability
1      X5 4109.6535  4680.0000             0.8781311                 0.8852644
2     X10 1783.0472  2340.0000             0.7619860                 0.7881917
3     X15 1037.8740  1560.0000             0.6653039                 0.7155555
4     X20  684.4646  1170.0000             0.5850125                 0.6603485
5     X25  482.0079   936.0000             0.5149657                 0.6156761
6     X30  354.9921   780.0000             0.4551181                 0.5799103
7     X35  269.3228   668.5714             0.4028333                 0.5503688
8     X40  209.8346   585.0000             0.3586917                 0.5266030
>
```

2.3 Linear Relationship Between Daily Accumulated RV and Daily Average RV

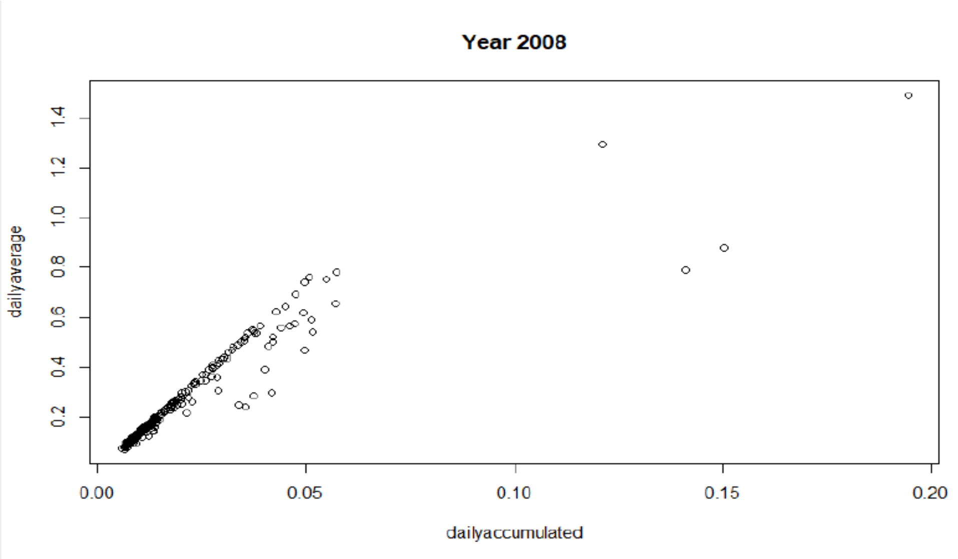
Thirdly, the daily accumulated RV and daily average RV are really close to each other and they have approximately linear relationship. Except just a few outliers, many points are approximately in linear relationship. The followings are the graphs for each year.

Figure 2. The Daily Accumulated Realized Volatility versus Daily Average Realized Volatility in 2007



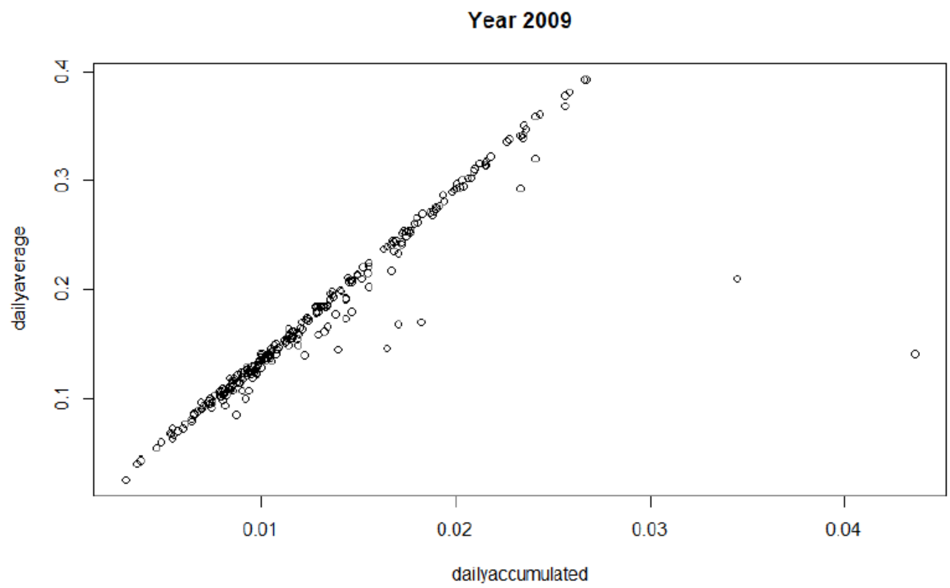
The plot of 2007 is closed to linear relationship, but it does have several obvious outliers. Those outliers are on 20070227,20070228, 20070301 and 20070816. Overall the shape is approximately linear.

Figure 3. The Daily Accumulated Realized Volatility versus Daily Average Realized Volatility in 2008



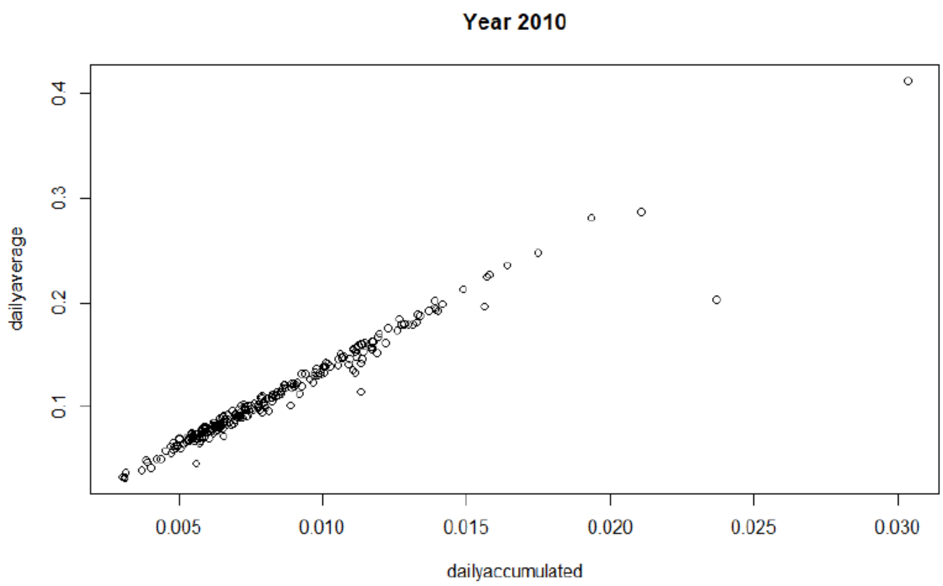
The plot of 2008 looks like that it has two separate parts. The first part which looks closed to linear and the second part is 4 distinct outlier points. These outlier points are 20081010, 20081016, 20080919 and 20080122.

Figure 4. The Daily Accumulated Realized Volatility versus Daily Average Realized Volatility in 2009



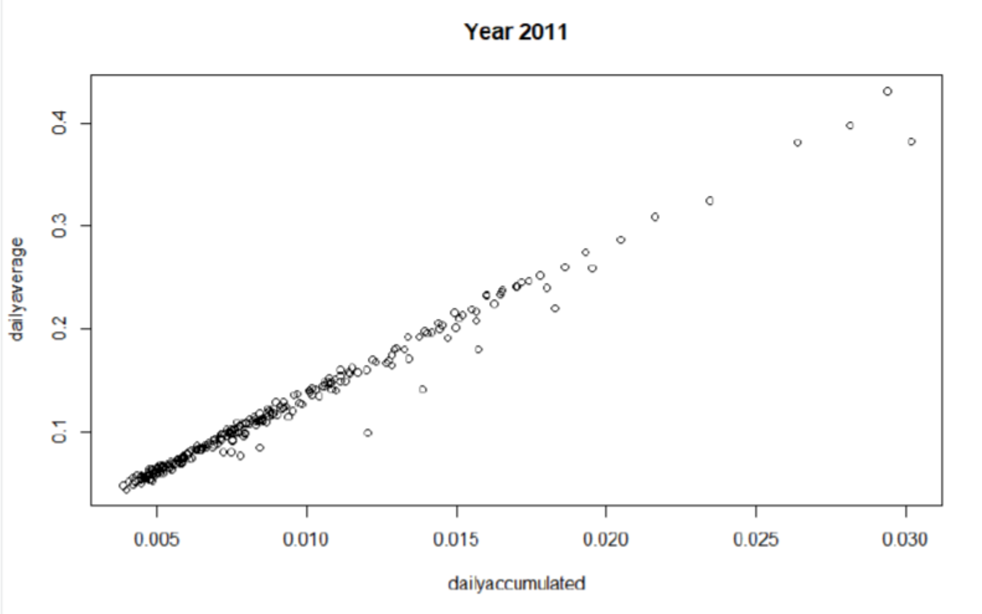
The plot of 2009 has major points remaining on linear relationship and two outlier points on the right side. These outlier points' dates are 20091207 and 20090916.

Figure 5. The Daily Accumulated Realized Volatility versus Daily Average Realized Volatility in 2010



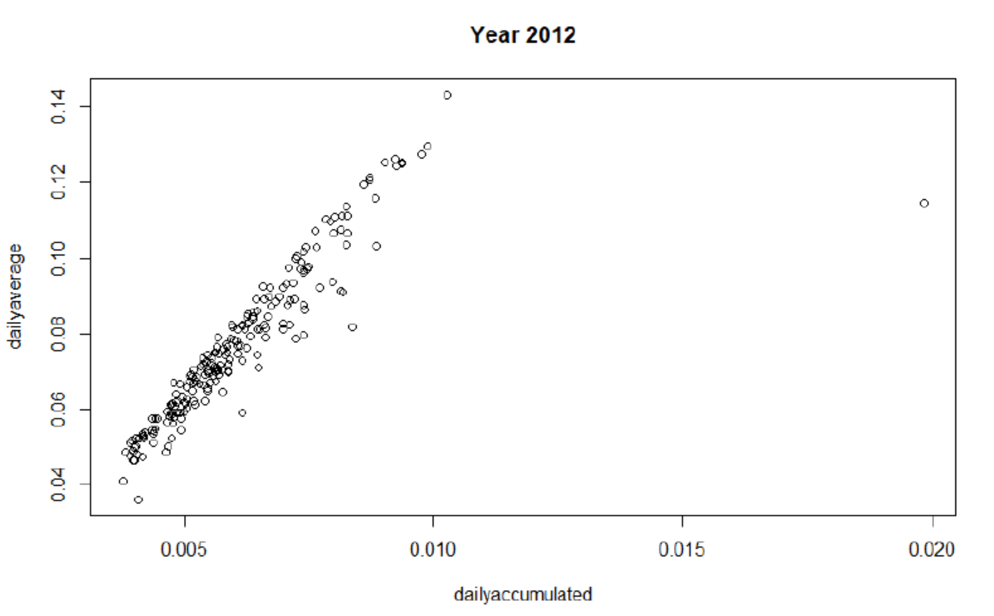
The plot of 2010 does not have obvious outlier point and is closed to linear relationship.

Figure 6. The Daily Accumulated Realized Volatility versus Daily Average Realized Volatility in 2011



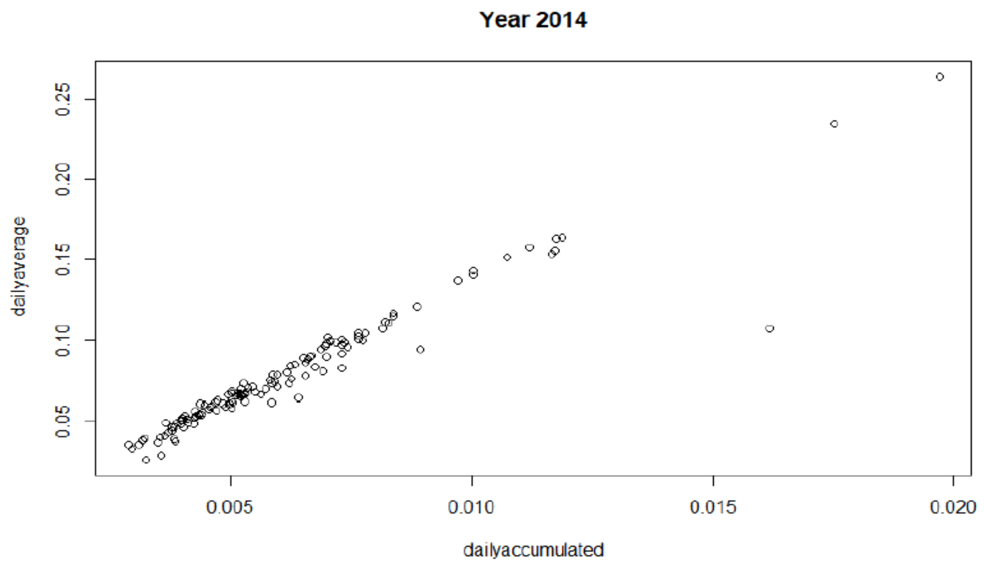
The plot of year 2011 is interesting since it is closed to linear relationship and it does not have obvious outliers.

Figure 7. The Daily Accumulated Realized Volatility versus Daily Average Realized Volatility in 2012



There is one obvious outlier point which is on the date 20120724

Figure 8. The Daily Accumulated Realized Volatility versus Daily Average Realized Volatility in 2010



The plot of 2014 is almost on the one line except for one point on the date 20141027

2.4 Heston Model

In the end, the final goal is to measure the alpha (the daily average realized volatility for 100 seconds) and Beta (the mean-reversion rate) by linear regression. To derive the linear format, the following equation is the initial equation for Heston model:

$$RV_t - RV_{t-1} = \beta \cdot (RV_{t-1} - R_{bar}) + error \tag{4}$$

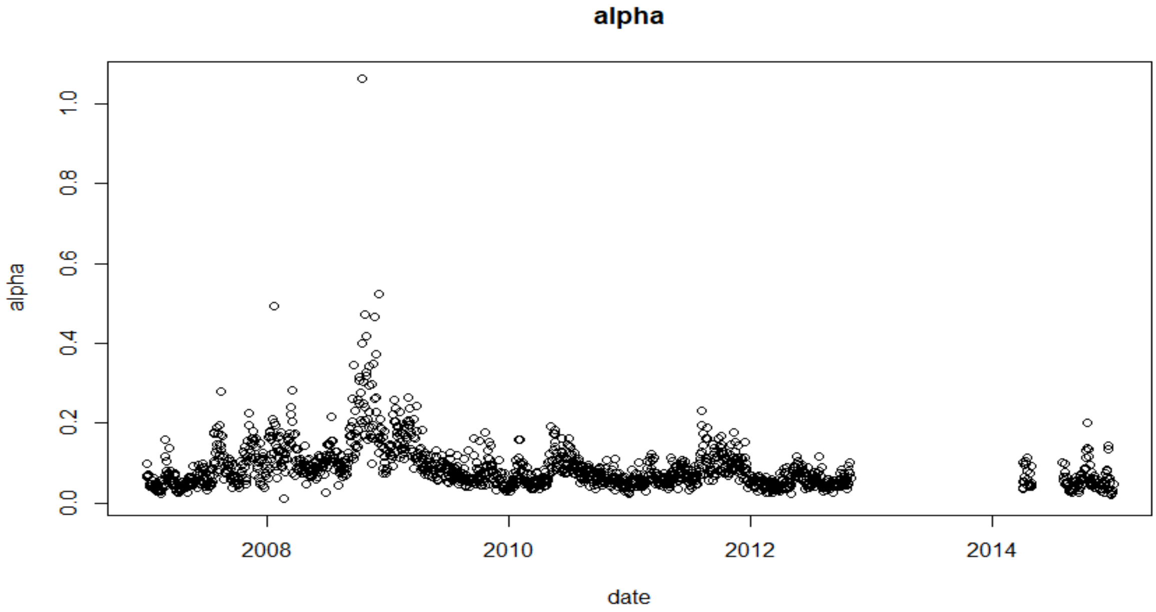
After switching terms and some calculations:

$$RV_t = \beta \cdot RV_{t-1} + \alpha + \epsilon \tag{5}$$

Here the formula looks more like a simple linear regression. After calculating one-day period and five-day period RV_t and RV_{t-1}, then using the lm function in R studio to generate alphas and betas. It was found that the five-day period obviously has smaller variance. It makes sense because larger sample size tends to make variance smaller. The most obvious is the outlier point date May 6th, 2010 which is the Crash Day. On that day, the alpha of that day is more than 4 but in five-day period, the alpha is less than 4. After computing the alpha, we will use 5-day period to compare with the VIX from yahoo finance from 2007 to 2014. In addition, since the alpha for the crash day is not significant anyway, therefore it was excluded from the alpha plots.

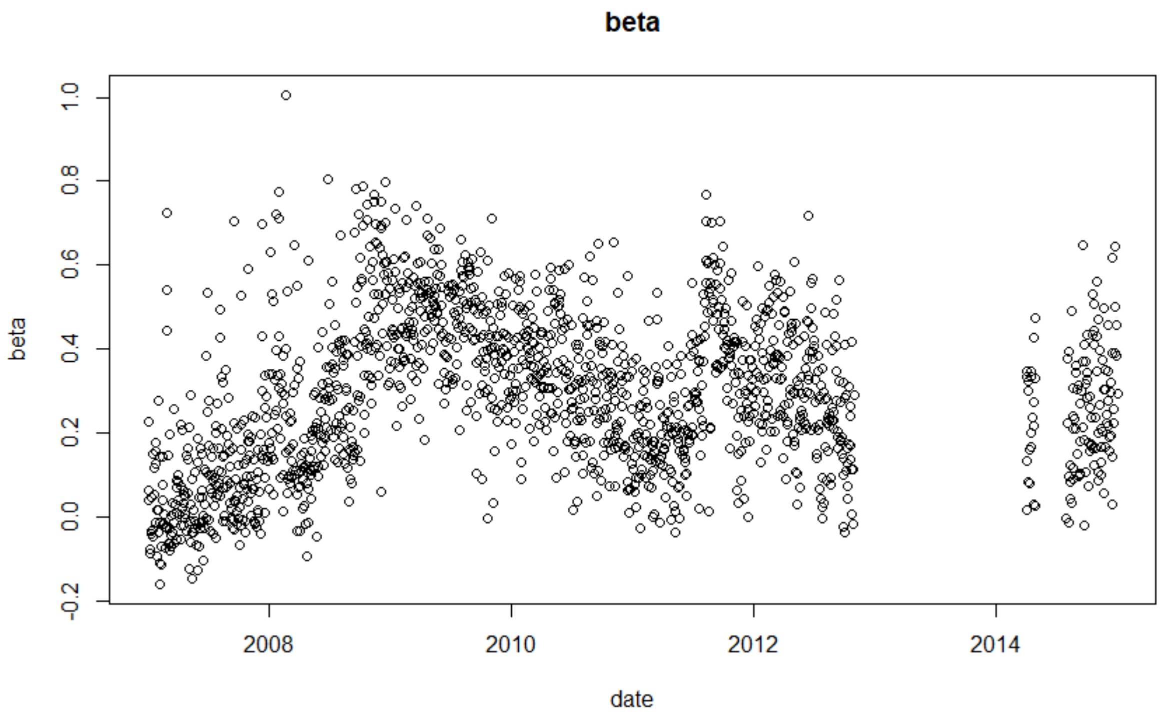
The following is one day period alphas and betas plots.

Figure 9. The one-day period alphas (Daily Average RV from 2007 to 2014)



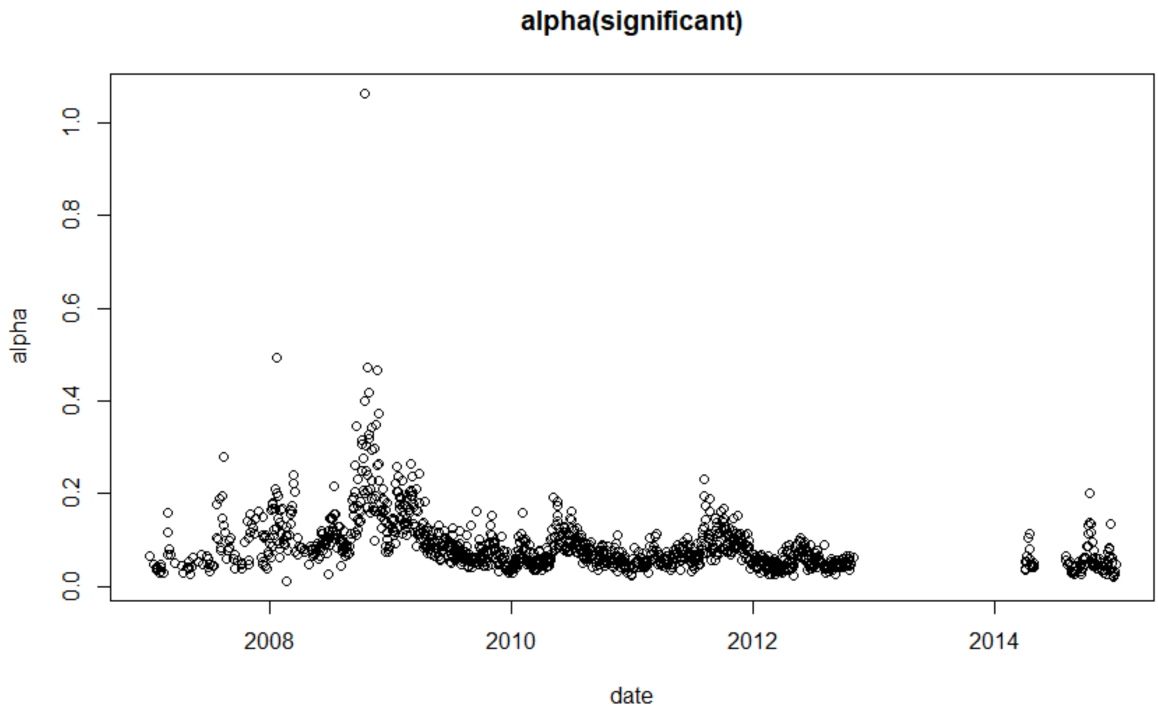
alpha without outlier 20100506

Figure 10. The one-day period beta (Mean-Reversion Rate from 2007 to 2014)



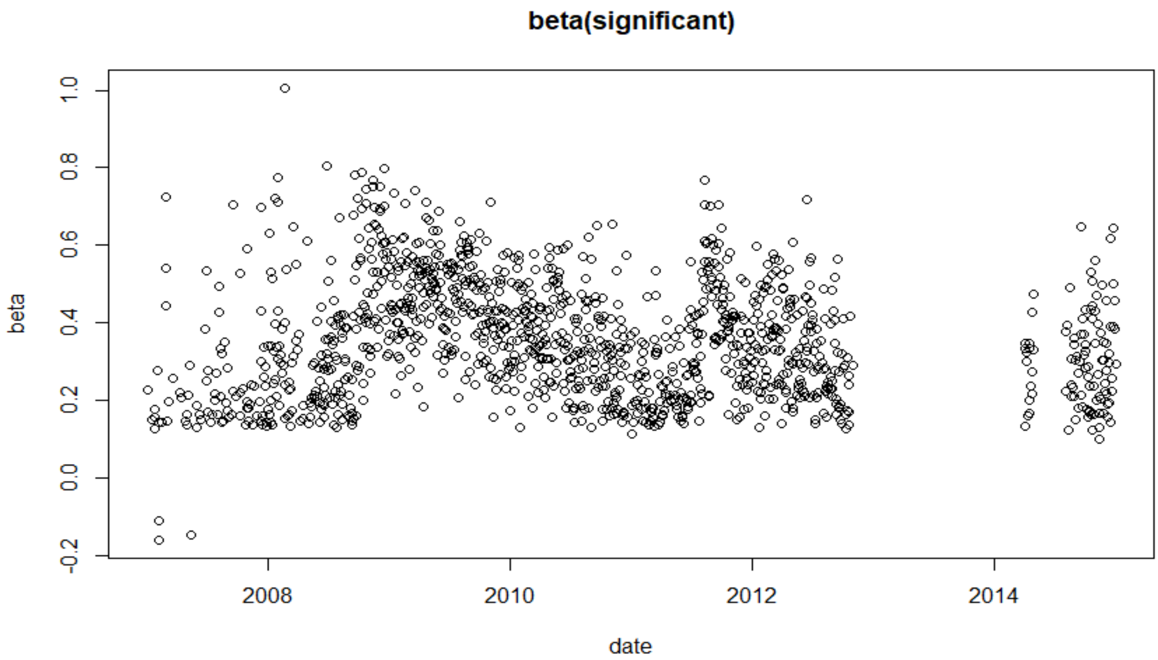
The beta plot does not have obvious or significant outliers. The highest beta is a little bit larger than 1.

Figure 11. The one-day period significant alphas (Daily Average RV from 2007 to 2014)



The plot without outlier 20100506 and only includes alphas which have significant linear relationship

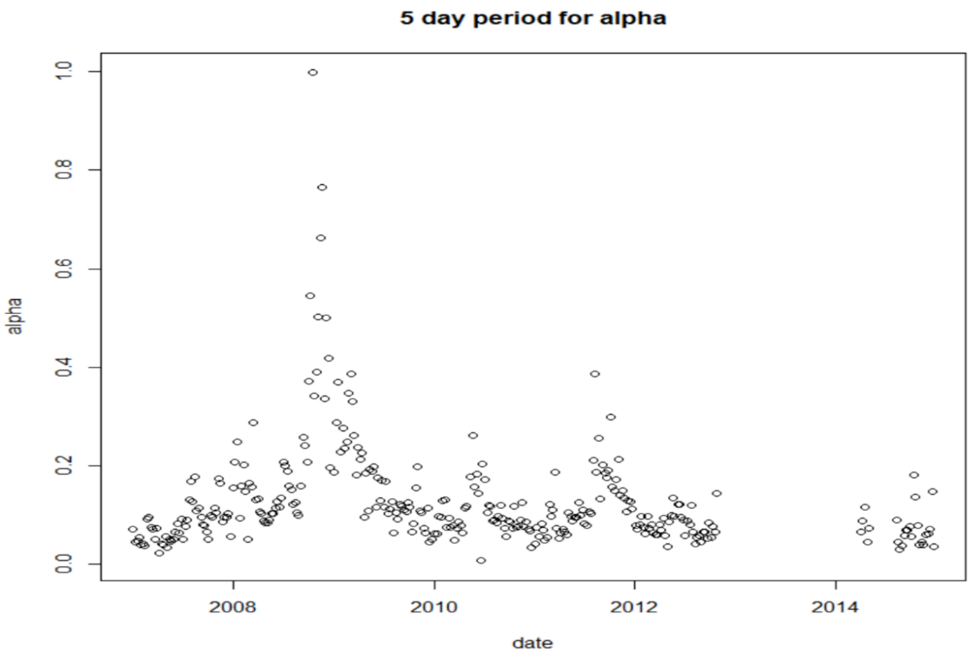
Figure 12. The one-day period significant alphas (Daily Average RV from 2007 to 2014)



Beta graph with only significant days does not have significant or obvious outlier.

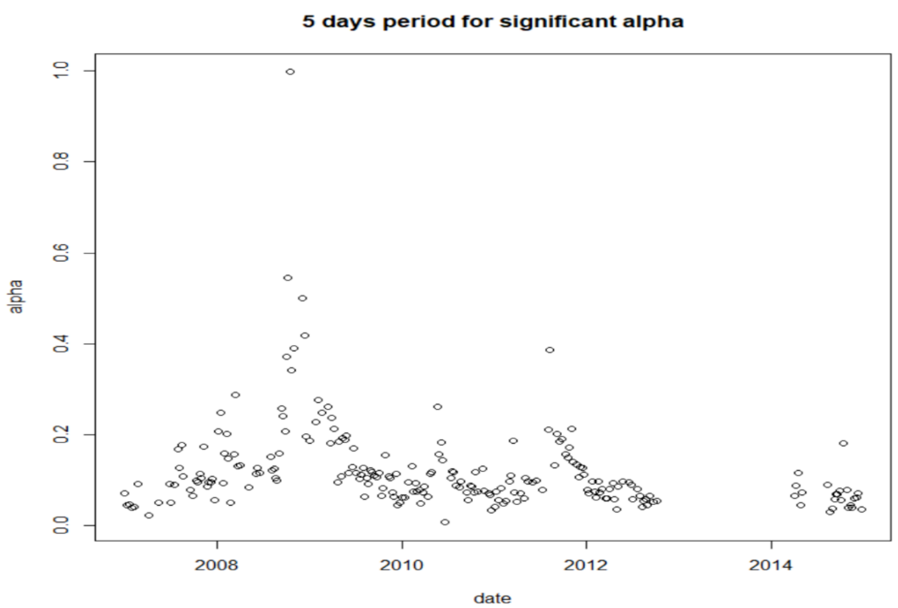
The following is five-day period:

Figure 13. The five-day period significant alphas (Daily Average RV from 2007 to 2014)



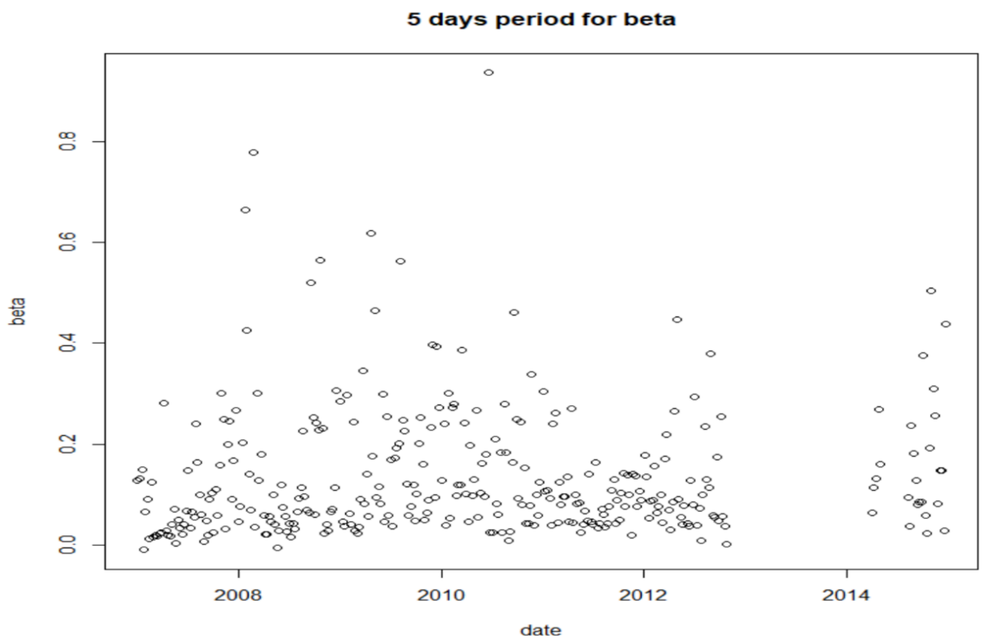
Five-day period without outlier

Figure 14. The five-day period significant alphas (Daily Average RV from 2007 to 2014)



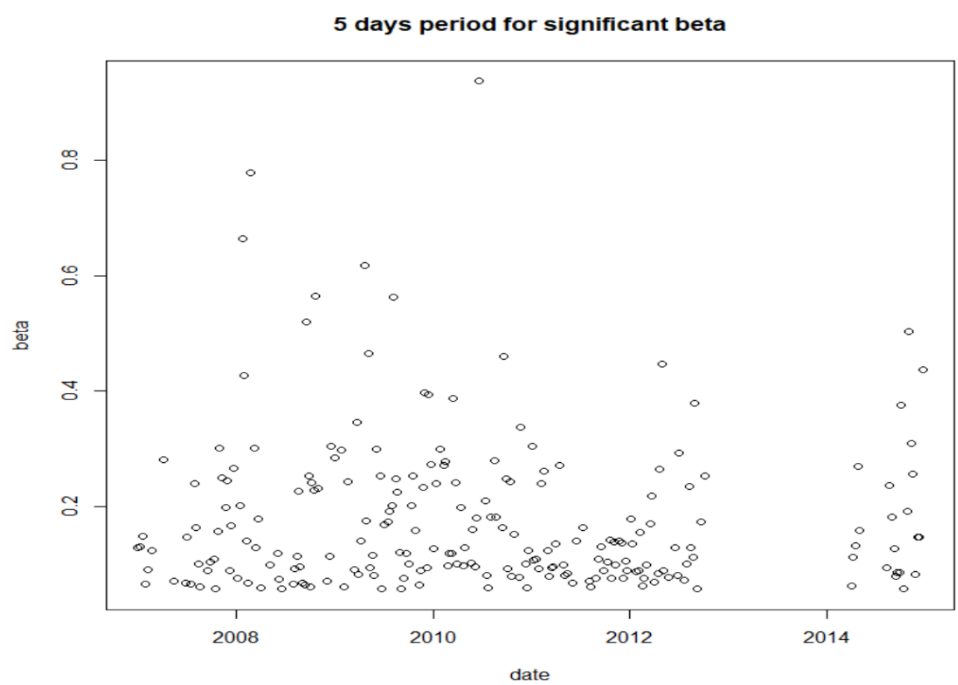
The overall shape of five day period is similar to one day period but the variance is smaller due to larger sample size.

Figure 15. The five-day period significant betas (Daily Average RV from 2007 to 2014)



There is no significant outlier in five-day period beta.

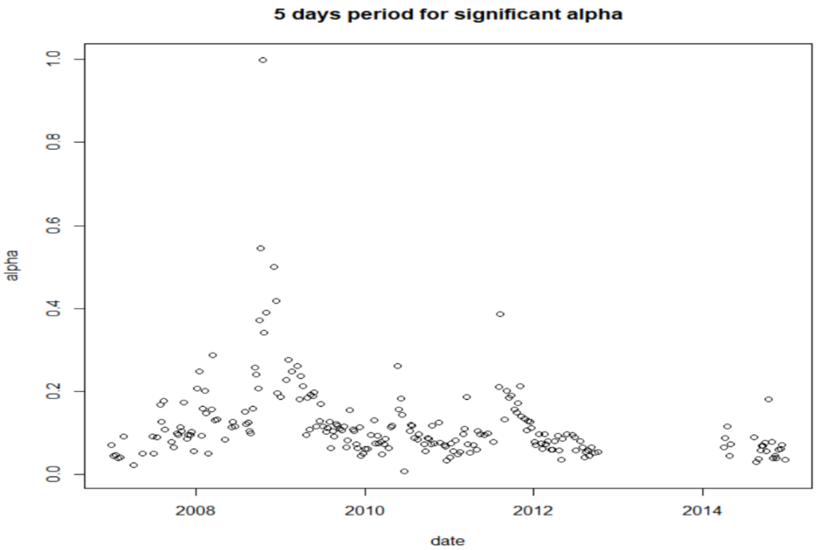
Figure 16. The five-day period significant betas (Daily Average RV from 2007 to 2014)



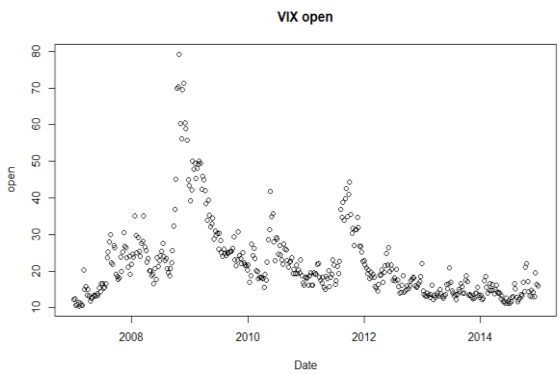
After calculating the alphas and betas from one-day period and five-day period, in order to compare with the movement of the market volatility, the resource that this paper chooses is

VIX from yahoo finance. By plotting VIX with time from 2007 to 2014, then the goal is compare VIX distribution and distribution of significant alphas. The following are the plot about VIX and plot about significant alpha.

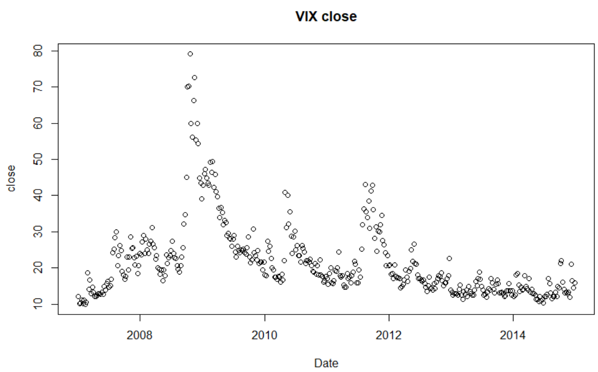
Figure 17. Comparing distribution of VIX with distribution of significant alphas



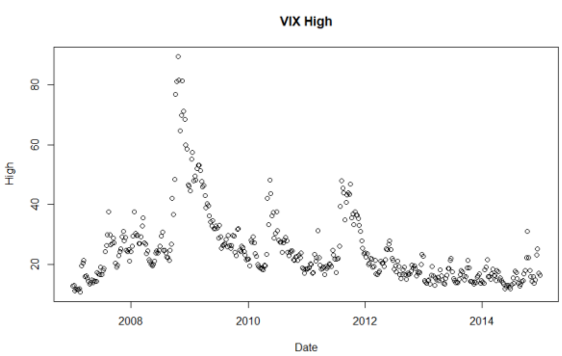
note: the data is missing year 2013



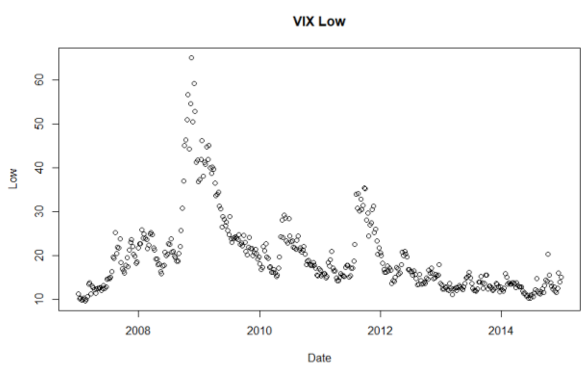
plot with open price



plot with closed price



plot with high price



plot with low price

The VIX values are calculated from the implied volatility of stock options. However, our RVs are from actual stock prices. Comparing the four plots from VIX, their shapes are approximately the same, but surprisingly, our significant alphas with five-day period have close shape to the plots from VIX by ignoring the missing values in year 2013.

3. Conclusion:

Through the research, each question in the beginning of the paper is answered. First, although the data used in this research missing the price of year 2013, the stock price of Bank of America is relatively stable and there is no abnormal variance expect for the crash day in May 6^{th}, 2010.

Secondly, if defining the gap as small number of seconds, for example, 5 seconds or 10 seconds as a gap, then Poisson distribution is a good prediction about probability of no trade arrival. However, when unit of gaps increase, Poisson distribution may not be a good choice.

Thirdly, the linear relationship between the Daily Accumulated Realized Volatility and Daily Average Realized Volatility is strong. Through all the plots from 2007 to 2014, there are obvious linear relationship in plots.

In the end, by transformation of Heston model and linear regression function of R studio, the daily average realized volatility and mean reversion rate were calculated. Then Selecting significant the daily average realized volatility and mean reversion rate and plot them versus time. The result is that their distribution is closed to the distribution of VIX from Yahoo Finance. The Heston model can be used for stock price of Bank of America.

4. Theory:

Log return: the formula of log return is following:

$$\text{Log return} = \log(t)-\log(t-1)$$

Realized Volatility: the realized volatility is calculated by taking the sum over the past squared return

$$RV_t = \sum_{i=1}^{N} r_t^2$$

The definition and use of RRV: in order to observed pronounced volatility jumps in high-frequency data instead of in frequent data. to better analyze the jump effects in RV100s, it is helpful to calculate the relative realized volatility to normalize RV100s. RRV has the formula as follows:

$$RRV = \frac{RV}{\text{Daily Median RV}}$$

From the mean-reversion feature of Heston model, we have the following:

$$RV_t - RV_{t-1} = \beta \cdot (RV_{t-1} - R_{bar}) + error$$

By removing terms, we got the following equation:

$$RV_t = \beta \cdot RV_{t-1} + \alpha + \epsilon$$

We have the alpha which is daily average and beta as mean-reversion rate. Since the equation is similar to linear equation formula, we decide to test if there is strong linear relationship between mean reversion rate and daily average.

5. Additional section:

From the analysis of data of stock price, the most interesting values are RV in 2010/05/06. There was a significant flash crash happened on that day. It started at 14:32 pm and lasted about 36 minutes. Some research indicated that the most obvious reason is the debt crisis from Greece. The equity market began to fall rapidly and followed one 300-point drop and two 600-point drop.(Treanor Jill, 2015) It was the most significant crash on year 2010. The following are the RV versus time, log return versus time and price versus time.

Figure 18. Realized volatility versus time on the Crash day

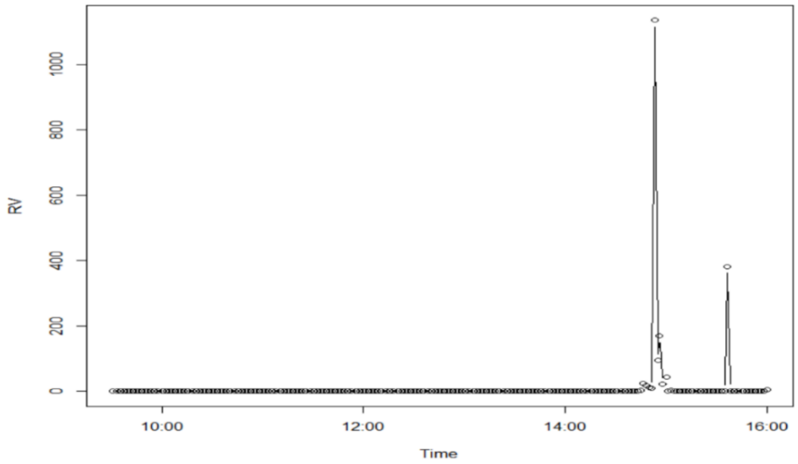


Figure 19. Log return versus time on the Crash day

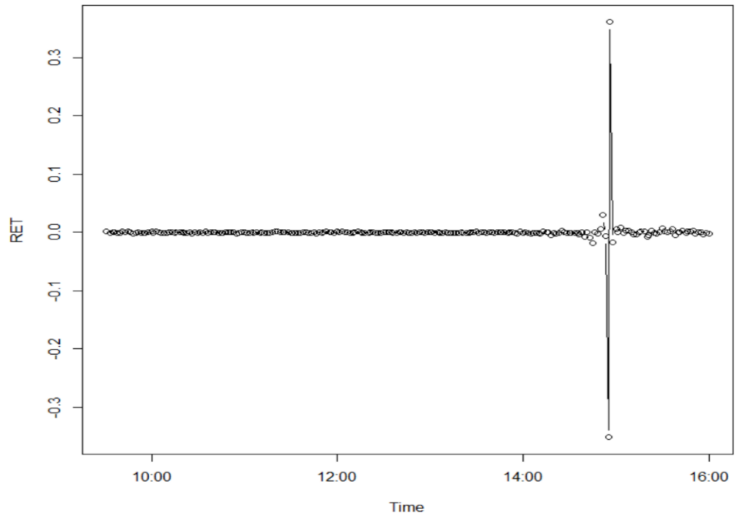
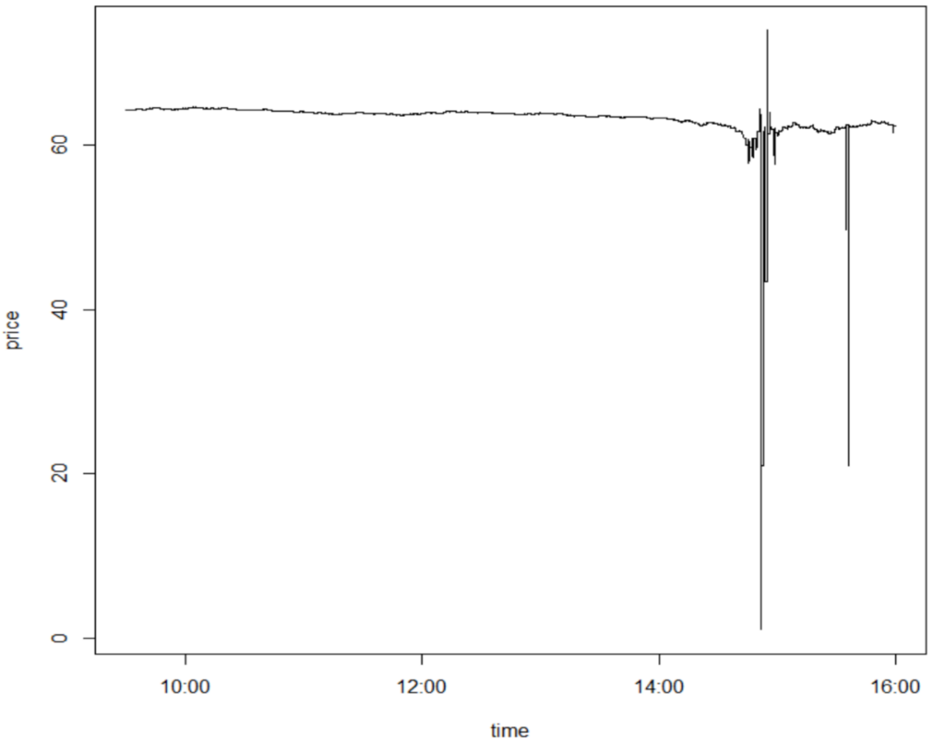


Figure 20. Stock price versus time on the Crash day



Reference:

Treanor, Jill. “The 2010 'Flash Crash': How It Unfolded.” *The Guardian*, Guardian News and Media, 22 Apr. 2015, www.theguardian.com/business/2015/apr/22/2010-flash-crash-new-york-stock-exchange-unfolded.

Shreve, Steven E. *Stochastic Calculus for Finance*. Springer, 2010.