Q.2) In a quant test of the CAT exam, the population std. deviation is known to be 100. A sample of 25 tests taken has a mean of 520. Construct an 80% Confidence interval about the mean.

Ans: Given : $\sigma = 100$, $n = 25$, $\bar{x} = 520$

Step ① Confidence interval = 80% $\Rightarrow$ 0.8

∴ Significance value = 1 − C.I.
$$= 1 - 0.8$$
$$\boxed{\therefore \alpha = 0.2}$$

Step ② As pop. std. dev. is given, ∴ We shall use Z-test.
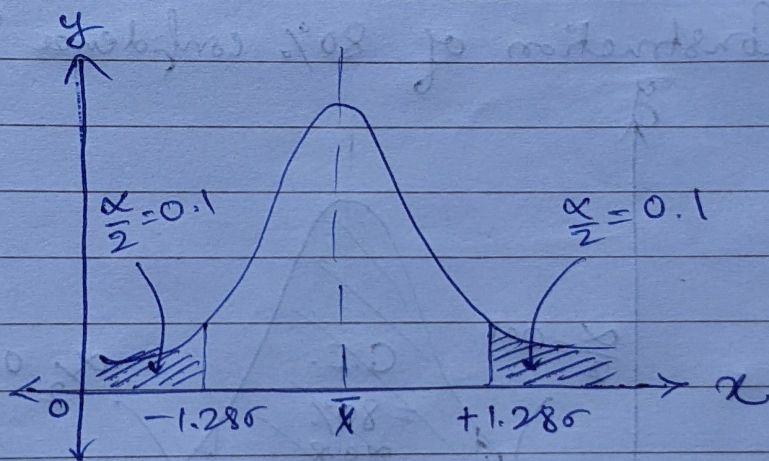
Step ③ In order to find lower fence & higher fence.

Calculate margin of error.

∴ Margin of Error $= Z_{\alpha/2} \times \dfrac{\sigma}{\sqrt{n}}$

∴ $Z_{\alpha/2} = Z_{\frac{0.2}{2}} = Z_{0.1}$

From Z-table, for area = 0.1

Std. deviation limit $= Z_{0.1} = \pm 1.28$

∴ Margin of Error $= Z_{\alpha/2} \times \dfrac{\sigma}{\sqrt{n}} = Z_{0.1} \times \dfrac{\sigma}{\sqrt{n}}$

$$= 1.28 \times \dfrac{100}{\sqrt{25}}$$

$$= 1.28 \times \dfrac{\cancel{100}\,20}{\cancel{5}}$$

∴ Error $= 25.6$

Step ④ Lower fence = Point estimate − Margin of error

$$= \bar{X} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 520 - 25.6$$

$$\boxed{\therefore \text{Lower fence} = 494.4}$$

Higher Fence = Point est. + Margin of error

$$= \bar{X} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 520 + 25.6$$

$$\boxed{\therefore \text{Higher fence} = 545.6}$$

Step ⑤ Construction of 80% confidence Interval.

$\frac{\alpha}{2} = 0.1$ C.I. = 80% = 0.8 $\frac{\alpha}{2}$ 0.1

−1.28σ $\bar{X}$ +1.28σ
= 494.4   545.6