有 $a_1=rac{3}{2}
eq 0$

若 $a_k
eq 0$ 则 $a_{k+1} = rac{3(k+1)a_k}{2a_k+k}
eq 0$

归纳可得 $\forall i \in \mathbb{N}^*, a_i \neq 0$

所以

$$a_n = rac{3na_{n-1}}{2a_{n-1}+n-1} \implies rac{a_n}{n} = rac{3a_{n-1}}{2a_{n-1}+n-1} \implies rac{n}{a_n} = rac{2}{3} + rac{1}{3} \cdot rac{n-1}{a_{n-1}}$$

令
$$b_n = \frac{n}{a_n}$$
 则 $b_1 = \frac{2}{3}$

又

$$b_n = rac{2}{3} + rac{b_{n-1}}{3} \implies b_n - 1 = rac{b_{n-1} - 1}{3} \implies b_n - 1 = rac{1}{3^{n-1}} \cdot \left(-rac{1}{3}
ight)$$

所以 $b_n=1-rac{1}{3^n}$ 所以 $a_n=rac{n}{1-rac{1}{3^n}}$

$$\prod_{i=1}^n a_i = rac{n!}{\prod_{i=1}^n \left(1-rac{1}{3^i}
ight)} < 2n! \iff \prod_{i=1}^n \left(1-rac{1}{3^i}
ight) > rac{1}{2}$$

由伯努利不等式

$$\prod_{i=1}^n \left(1 - \frac{1}{3^i}\right) > 1 - \sum_{i=1}^n \frac{1}{3^i} = 1 - \frac{1}{3} \cdot \frac{1 - \frac{1}{3^n}}{1 - \frac{1}{3}} = \frac{1 + \frac{1}{3^n}}{2} > \frac{1}{2}$$

即证