

$$\begin{aligned}
& \sum_{i=1}^n \frac{1}{\sqrt{i(i+1)}} > \ln(n+1) \\
\iff & \frac{1}{\sqrt{x(x+1)}} > \ln\left(\frac{x+1}{x}\right), \quad \forall x \in \mathbb{N}_+ \\
\iff & \frac{1}{x\sqrt{\lambda}} > \frac{\ln \lambda}{(\lambda-1)x}, \quad \forall x \in \mathbb{N}_+, \lambda = \frac{x+1}{x} \\
\iff & \frac{1}{\sqrt{\lambda}} > \frac{\ln \lambda}{(\lambda-1)}, \quad \forall \lambda \in (1, +\infty) \\
\iff & g(\lambda) := \frac{\lambda-1}{\sqrt{\lambda}} - \ln \lambda > 0, \quad \forall \lambda \in [1, +\infty) \\
\iff & \begin{cases} g(1) = 0 \\ g'(\lambda) \geq 0, \quad \forall \lambda \in [1, +\infty) \end{cases}
\end{aligned}$$

其中

$$g'(\lambda) = \frac{(\sqrt{\lambda}-1)^2}{2\lambda\sqrt{\lambda}} \geq 0$$

而 $g(1) = 0$ 是显然的，故得证