$$\lim_{x\to 0}\frac{\sqrt{1+x\cos x}-\sqrt{1+x}}{x^3}$$

$$=\lim_{x\to 0}\frac{x(\cos x-1)}{\left(\sqrt{1+x\cos x}+\sqrt{1+x}\right)x^3}$$

$$=\lim_{x\to 0}\frac{x\cdot\left(-\frac{1}{2}x^2\right)}{2x^3}$$

$$=-\frac{1}{4}$$