有

$$f'(x) = -a\sin ax + \frac{2x}{1-x^2}$$

其中 $f'(0)=0$

又有

$$f''(x) = -a^2\cos ax + \frac{2+2x^2}{(1-x^2)^2}$$

其中 $f''(0) = -a^2+2$

则

$$\begin{aligned} x=0 \text{ 是 } f \text{ 的极大值点} &\Longleftarrow \quad f''(0)<0 \\ &\Longleftrightarrow \quad -a^2+2<0 \\ &\Longleftrightarrow \quad a \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty) \end{aligned}$$

而又

$$\begin{aligned} a \in [-\sqrt{2}, \sqrt{2}] \Longrightarrow \ f'(x) &= -a\sin ax + \frac{2x}{1-x^2} \\ &= -|a|\sin|a|x + \frac{2x}{1-x^2} \\ &\geq -a^2x + \frac{2x}{1-x^2} \\ &= \frac{a^2x^3+(2-a^2)x}{1-x^2} \\ &\geq \frac{a^2x^3}{1-x^2} \geq 0, \quad 0 \leq x < 1 \\ \Longrightarrow \ & f(x) \text{ 在 } [0,1) \text{ 单调增} \\ \Longrightarrow \ & x=0 \text{ 不是 } f \text{ 的极大值点} \end{aligned}$$

故而，a 的取值范围是 $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$