1.

$f'(x) = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

所以切线为 $y = \frac{1}{\cos^2 \frac{\pi}{4}}\left(x - \frac{\pi}{4}\right) + 1 = 2x + 1 - \frac{\pi}{2}$

2.

$g'(x) = 2(\cos(2x+1) + \tan x)$

若 $x \le \frac{\pi}{4} - \frac{1}{2}$ 则 $\cos(2x+1) \ge 0 \wedge \tan x > 0 \implies g'(x) > 0$

若 $x > \frac{\pi}{4} - \frac{1}{2}$ 则

由1且 $f(x)$ 是凸函数可知 $\tan x \ge 2x + 1 - \frac{\pi}{2},\ \forall x \in \left(0, \frac{\pi}{2}\right)$

则 $g'(x) \ge 2\left(-\sin\left(2x + 1 - \frac{\pi}{2}\right) + 2x + 1 - \frac{\pi}{2}\right) > 0$

所以 $g'(x) > 0,\ \forall x \in \left(0, \frac{\pi}{2}\right)$

而 $g(0) = \sin 1$，又 $x \to \frac{\pi}{2} \implies g(x) \to +\infty$

故 $x \in \left(0, \frac{\pi}{2}\right)$ 时 $g(x)$ 的值域为 $(\sin 1, +\infty)$