$$1.$$
 求极限 $\lim_{n\to+\infty} \frac{2^n}{n!}$

我们有

$$0 \leq rac{2^n}{n!} \leq rac{2^n}{2^{n-2} \cdot n} = rac{4}{n}, \quad orall n \geq 3$$

又有
$$\frac{4}{n} \rightarrow 0$$

所以
$$\lim_{n o +\infty} rac{2^n}{n!} = 0$$

$$2$$
. 求极限 $\lim_{n
ightarrow+\infty}\left(rac{1}{4n^2+1}+rac{2}{4n^2+2}+\cdots+rac{n}{4n^2+n}
ight)$

$$\diamondsuit a_n = rac{1}{4n^2+1} + rac{2}{4n^2+2} + \cdots + rac{n}{4n^2+n}$$

有

$$rac{1+1/n}{8+2/n} = rac{n^2+n}{8n^2+2n} \leq a_n \leq rac{n^2+n}{8n^2} = rac{1+1/n}{8}, \quad orall n=1,2,\dots$$

其中
$$rac{1+1/n}{8+2/n}
ightarrow rac{1}{8}$$
 且 $rac{1+1/n}{8}
ightarrow rac{1}{8}$

所以原式 =
$$\frac{1}{8}$$