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Contest (1)

genfolders.sh	6 lines
<pre>chmod +x bld* for f in {A..Z} do mkdir \$f cp main.cpp bld* \$f done</pre>	

bld	1 lines
<pre>g++ -std=c++20 -g -DLOCAL -fsanitize=address,bounds,undefined -o \$1 \$1.cpp</pre>	

bldf	1 lines
<pre>g++ -std=c++20 -g -O2 -o \$1 \$1.cpp</pre>	

hacks.sh	2 lines
<pre>UBSAN_OPTIONS=print_stacktrace=1 ./main gdb rbreak regex</pre>	

hash.sh	3 lines
<pre># Hashes a file, ignoring all whitespace and comments. # Use for verifying that code was correctly typed. cpp -dD -P -fpreprocessed tr -d '[:space:]' md5sum cut -c-6</pre>	

clion.cpp	2 lines
<pre>set (CMAKE_CXX_STANDARD 20) set (CMAKE_CXX_FLAGS "-DLOCAL")</pre>	

C++ (2)

GpHashtable.cpp	1 lines
<p>Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).</p>	

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
```

```
const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch().count();
struct hasher {
    int operator()(int x) const { return x ^ RANDOM; }
};
```

```
gp_hash_table<int, int, hasher> table;
```

OrderedSet.cpp	dff260, 37 lines
<p>Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null.type.</p> <p>Time: $\mathcal{O}(\log(n))$</p> <pre><bits/extc++.h>, <bits/stdc++.h></pre> <pre>using namespace __gnu_pbds; using namespace std;</pre>	

```
template <typename T>
using ordered_set =
    tree<T, null_type, less<>, rb_tree_tag,
        tree_order_statistics_node_update>;

int main() {
    ordered_set<int> X;
    X.insert(1);
    X.insert(2);
    X.insert(4);
    X.insert(8);
    X.insert(16);

    assert(*X.find_by_order(1) == 2);
    assert(*X.find_by_order(2) == 4);
```

```
assert(*X.find_by_order(4) == 16);
assert(X.find_by_order(6) ==X.end());

assert(X.order_of_key(-5) == 0);
assert(X.order_of_key(1) == 0);
assert(X.order_of_key(3) == 2);
assert(X.order_of_key(4) == 2);
assert(X.order_of_key(400) == 5);
// std::cout << *X.find_by_order(1) << std::endl; // 2
// std::cout << *X.find_by_order(2) << std::endl; // 4
// std::cout << *X.find_by_order(4) << std::endl; // 16
// std::cout << (end(X) == X.find_by_order(6)) << std::endl; // true

// std::cout << X.order_of_key(-5) << std::endl; // 0
// std::cout << X.order_of_key(1) << std::endl; // 0
// std::cout << X.order_of_key(3) << std::endl; // 2
// std::cout << X.order_of_key(4) << std::endl; // 2
// std::cout << X.order_of_key(400) << std::endl; // 5
return 0;
}
```

bitset.cpp	521d1f, 2 lines
<p>Description: bitset</p> <pre>bs._Find_first() bs._Find_next(idx) - returns right after</pre>	

alloc.cpp	8726b1, 11 lines
<p>Description: fastalloc</p> <pre>const int MAX_MEM = 1e8; int mpos = 0; char mem[MAX_MEM]; inline void *operator new(size_t n) { assert((mpos += n) <= MAX_MEM); return (void *) (mem + mpos - n); } void operator delete(void *) noexcept { } // must have! void operator delete(void *, size_t) noexcept { } // must have!</pre>	

fastio.cpp	79fd14, 52 lines
<pre>inline int readChar(); template <class T = int> inline T readInt(); template <class T> inline void writeInt(T x, char end = 0); inline void writeChar(int x); inline void writeWord(const char *s); static const int buf_size = 4096; inline int getChar() { static char buf[buf_size]; static int len = 0, pos = 0; if (pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin); if (pos == len) return -1; return buf[pos++]; } inline int readChar() { int c = getChar(); while (c <= 32) c = getChar(); return c; } template <class T> inline T readInt() { int s = 1, c = readChar(); T x = 0; if (c == '-') s = -1, c = getChar(); while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar(); return s == 1 ? x : -x; } static int write_pos = 0; static char write_buf[buf_size]; inline void writeChar(int x) { if (write_pos == buf_size) fwrite(write_buf, 1, buf_size, stdout), write_pos = 0; write_buf[write_pos++] = x;</pre>	

```

}
template <class T>
inline void writeInt(T x, char end) {
    if (x < 0) writeChar('-'), x = -x;
    char s[24];
    int n = 0;
    while (x || !n) s[n++] = '0' + x % 10, x /= 10;
    while (n--) writeChar(s[n]);
    if (end) writeChar(end);
}
inline void writeWord(const char *s) {
    while (*s) writeChar(*s++);
}
struct Flusher {
    ~Flusher() {
        if (write_pos) fwrite(write_buf, 1, write_pos, stdout), write_pos = 0;
    }
} flusher;
```

Strings (3)

Manacher.cpp	a6ddfb, 27 lines
<p>Description: Manacher algorithm</p> <p>Time: $\mathcal{O}(n)$</p>	

```
vector<int> manacherOdd(string s) {
    int n = s.size();
    vector<int> d1(n);
    int l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
        while (i + k < n && i - k >= 0 && s[i + k] == s[i - k])
            ++k;
        d1[i] = k;
        if (i + k - 1 > r)
            l = i - k + 1, r = i + k - 1;
    }
}
```

```
vector<int> manacherEven(string s) {
    int n = s.size();
    vector<int> d2(n);
    l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1])
            ++k;
        d2[i] = k;
        if (i + k - 1 > r)
            l = i - k, r = i + k - 1;
    }
}
```

AhoCorasick.cpp	ae5fc2, 19 lines
<p>Description: Build aho-corasick automaton.</p> <p>Time: $\mathcal{O}(n)$</p>	

```
int go(int v, char c);

int get_link(int v) {
    if (t[v].link == -1)
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    return t[v].link;
}

int go(int v, char c) {
    if (t[v].go[c] == -1)
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), c);
    return t[v].go[c];
}
```

SuffixArray.cpp

Description: Build suffix array
Time: $\mathcal{O}(n \log(n))$ 5bd011, 47 lines

```
vector<int> buildSuffixArray(string &s) {
    // Remove, if you want to sort cyclic shifts
    s += (char)(1);
    int n = s.size();
    vector<int> a(n);
    iota(all(a), 0);
    stable_sort(all(a), [&](int i, int j) { return s[i] < s[j]; });
    vector<int> c(n);
    int cc = 0;
    for (int i = 0; i < n; i++) {
        if (i == 0 || s[a[i]] != s[a[i - 1]]) {
            c[a[i]] = cc++;
        } else {
            c[a[i]] = c[a[i - 1]];
        }
    }
    for (int L = 1; L < n; L *= 2) {
        vector<int> cnt(n);
        for (auto i : c) {
            cnt[i]++;
        }
        vector<int> pref(n);
        for (int i = 1; i < n; i++) {
            pref[i] = pref[i - 1] + cnt[i - 1];
        }
        vector<int> na(n);
        for (int i = 0; i < n; i++) {
            int pos = (a[i] - L + n) % n;
            na[pref[c[pos]]++] = pos;
        }
        a = na;
        vector<int> nc(n);
        cc = 0;
        for (int i = 0; i < n; i++) {
            if (i == 0 || c[a[i]] != c[a[i - 1]] ||
                c[(a[i] + L) % n] != c[(a[i - 1] + L) % n]) {
                nc[a[i]] = cc++;
            } else {
                nc[a[i]] = nc[a[i - 1]];
            }
        }
        c = nc;
    }
    a.erase(a.begin());
    s.pop_back();
    return a;
}
```

Lcp.cpp

Description: lcp array
Time: $\mathcal{O}(n)$ 1cc27c, 43 lines

```
vector<int> perm;
vector<int> buildLCP(string &s, vector<int> &a) {
    int n = s.size();
    vector<int> ra(n);
    for (int i = 0; i < n; i++) {
        ra[a[i]] = i;
    }
    vector<int> lcp(n - 1);
    int cur = 0;
    for (int i = 0; i < n; i++) {
        cur--;
        chkmax(cur, 0);
        if (ra[i] == n - 1) {
            cur = 0;
            continue;
        }
        int j = a[ra[i] + 1];
        while (s[i + cur] == s[j + cur]) cur++;
        lcp[ra[i]] = cur;
    }
    perm.resize(a.size());
    for (int i = 0; i < a.size(); ++i) perm[a[i]] = i;
    return lcp;
}
```

```
}
int cntr[MAXN];
int spt[MAXN][lgg];
void build(vector<int> &a) {
    for (int i = 0; i < a.size(); ++i) {
        spt[i][0] = a[i];
    }
    for (int i = 2; i < MAXN; ++i) cntr[i] = cntr[i / 2] + 1;
    for (int h = 1; (1 << (h - 1)) < a.size(); ++h) {
        for (int i = 0; i + (1 << (h - 1)) < a.size(); ++i) {
            spt[i][h] = min(spt[i][h - 1], spt[i + (1 << (h - 1))][h - 1]);
        }
    }
}
int getLCP(int l, int r) {
    l = perm[l], r = perm[r];
    if (l > r) swap(l, r);
    int xx = cntr[r - l];
    return min(spt[l][xx], spt[r - (1 << xx)][xx]);
}
```

Eertree.cpp

Description: Palindrome Tree
Time: $\mathcal{O}(n)$ 6e64b6, 49 lines

```
struct palindromic_tree {
    int new_node() {
        tree.push_back(node());
        return static_cast<int>(tree.size()) - 1;
    }

    int find_suffix(int v) {
        int n = str.size();
        while (tree[v].length == n - 1 || str.back() != str[n - 2 - tree[v].length]) {
            v = tree[v].suflink;
        }
        return v;
    }

    struct node {
        int length = 0, suflink = -1, to[ALPHABET];
        node() { memset(to, -1, sizeof(to)); }
    };

    int even, odd, last;
    vector<node> tree;
    vector<int> str;

    palindromic_tree() {
        odd = new_node();
        even = new_node();
        tree[even].suflink = tree[odd].suflink = odd;
        tree[odd].length = -1;
        last = even;
    }

    void add(int symbol) {
        str.push_back(symbol);
        last = find_suffix(last);
        if (tree[last].to[symbol] == -1) {
            int v = new_node();
            tree[v].length = tree[last].length + 2;
            int u = find_suffix(tree[last].suflink);
            if (tree[u].to[symbol] != -1) {
                tree[v].suflink = tree[u].to[symbol];
            }
            else {
                tree[v].suflink = even;
            }
            tree[last].to[symbol] = v;
        }
        last = tree[last].to[symbol];
    }
};
```

SuffixAutomaton.cpp

Description: Build suffix automaton.
Time: $\mathcal{O}(n)$ 340cda, 53 lines

```
const int alpha = 26;

struct state {
    int len, link;
    array<int, alpha> next;
};
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
    sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    st[0].next.fill(-1);
    ++sz;
}

int sa_cut(int v, int c) {
    assert(st[v].next[c] != -1);
    int to = st[v].next[c];
    if (st[to].len == st[v].len + 1) {
        return to;
    }
    int clone = sz++;
    st[clone].len = st[v].len + 1;
    st[clone].next = st[to].next;
    st[clone].link = st[to].link;
    for (; v != -1 && st[v].next[c] == to; v = st[v].link)
        st[v].next[c] = clone;
    st[to].link = clone;
    return clone;
}

void sa_extend(int c) {
    if (st[last].next[c] != -1) {
        int to = sa_cut(last, c);
        last = to;
        return;
    }
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].next.fill(-1);
    int v;
    for (v = last; v != -1 && st[v].next[c] == -1; v = st[v].link)
        st[v].next[c] = cur;
    if (v == -1)
        st[cur].link = 0;
    else {
        int to = st[v].next[c];
        st[cur].link = sa_cut(v, c);
    }
    last = cur;
}
```

PrefixZ.cpp

Description: Calculates Prefix,Z-functions
Time: $\mathcal{O}(n)$ 1c4e93, 25 lines

```
vector<int> pf(string s) {
    int k = 0;
    vector<int> p(s.size());
    for (int i = 1; i < s.size(); ++i) {
        while (k && s[i] != s[k])
            k = p[k - 1];
        k += (s[i] == s[k]);
        p[i] = k;
    }
    return p;
}

vector<int> zf(string s) {
    int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
    }
}
```

```

    while (i + z[i] < n && s[z[i]] == s[i + z[i]])
        ++z[i];
    if (i + z[i] - 1 > r)
        l = i, r = i + z[i] - 1;
}
z[0] = n;
return z;
}
```

MinShift.cpp

Description: Calculates min-cyclic-shift of s, Duval decomposition
Time: $\mathcal{O}(n)$ 3f0fb9, 21 lines

```

string minshift(string s) {
    int i = 0, ans = 0;
    s += s; // Remove for lyndon decomposition
    int n = s.size();
    while (i < n / 2) { // (i < n) lyndon
        ans = i;
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                ++k;
            ++j;
        }
        while (i <= k) {
            // s.substr(i,j-k) - simple string
            i += j - k;
        }
    }
    return s.substr(ans, n / 2);
}
```

SA-IS.cpp

Description: Build suffix array
Time: $\mathcal{O}(n)$ f90ffe, 87 lines

```

void induced_sort(vector<int> &vec, int LIM, vector<int> &sa, vector<bool>
    > &sl, vector<int> &fx) {
    vector<int> l(LIM), r(LIM);
    for (int c : vec) {
        if (c + 1 < LIM) {
            ++l[c + 1];
        }
        ++r[c];
    }
    partial_sum(all(l), l.begin());
    partial_sum(all(r), r.begin());
    fill(all(sa), -1);
    for (int i = fx.size() - 1; i >= 0; --i) {
        sa[--r[vec[fx[i]]]] = fx[i];
    }
    for (int i : sa) {
        if (i >= 1 && sl[i - 1]) {
            sa[l[vec[i - 1]]++] = i - 1;
        }
    }
    fill(all(r), 0);
    for (int c : vec) ++r[c];
    partial_sum(all(r), r.begin());
    for (int k = sa.size() - 1, i = sa[k]; k >= 1; --k, i = sa[k])
        if (i >= 1 && !sl[i - 1]) sa[--r[vec[i - 1]]] = i - 1;
}
vector<int> SA_IS(vector<int> &vec, int LIM) {
    const int n = vec.size();
    vector<int> sa(n), fx;
    vector<bool> sl(n);
    sl[n - 1] = false;
    for (int i = n - 2; i >= 0; --i) {
        sl[i] = (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i + 1]));
        if (sl[i] && !sl[i + 1]) {
            fx.pb(i + 1);
        }
    }
    reverse(all(fx));
}
```

MinShift SA-IS Hungarian BlossomShrinking

```

induced_sort(vec, LIM, sa, sl, fx);
vector<int> nfx(fx.size()), lmv(fx.size());
for (int i = 0, k = 0; i < n; ++i) {
    if (!sl[sa[i]] && sa[i] >= 1 && sl[sa[i] - 1]) {
        nfx[k++] = sa[i];
    }
}
int cur = 0;
sa[n - 1] = cur;
for (int k = 1; k < nfx.size(); ++k) {
    int i = nfx[k - 1], j = nfx[k];
    if (vec[i] != vec[j]) {
        sa[j] = ++cur;
        continue;
    }
    bool flag = false;
    for (int a = i + 1, b = j + 1;; ++a, ++b) {
        if (vec[a] != vec[b]) {
            flag = true;
            break;
        }
        if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
            flag = !(sl[a] && sl[a - 1]) && (!sl[b] && sl[b - 1]);
            break;
        }
    }
    sa[j] = (flag ? ++cur : cur);
}
for (int i = 0; i < fx.size(); ++i) {
    lmv[i] = sa[fx[i]];
}
if (cur + 1 < (int)fx.size()) {
    auto lms = SA_IS(lmv, cur + 1);
    for (int i = 0; i < fx.size(); ++i) {
        nfx[i] = fx[lms[i]];
    }
}
induced_sort(vec, LIM, sa, sl, nfx);
return sa;
}
template <typename T>
vector<int> suffix_array(T &s, const int LIM = 128) {
    vector<int> vec(s.size() + 1);
    copy(all(s), begin(vec));
    vec.back() = (char)(1);
    auto ret = SA_IS(vec, LIM);
    ret.erase(ret.begin());
    return ret;
}
```

Graph (4)

Hungarian.cpp

Description: Hungarian algorithm
Time: $\mathcal{O}(n^3)$ 5afee5, 41 lines

```

int n, m;
vector<vector<int>>> a;
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    int minv(m + 1, INF);
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j = 1; j <= m; ++j)
            if (!used[j]) {
                int cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j = 0; j <= m; ++j)
            if (used[j])
```

```

                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}

// matching
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j) {
    ans[p[j]] = j;
}

// cost
int cost = -v[0];
```

BlossomShrinking.cpp

Description: Maximum matching in general graph
Time: $\mathcal{O}(n^3)$ 23839d, 118 lines

```

struct Edge {
    int u, v;
};
const int N = 510;
int n, m;
vector<int> g[N];
vector<Edge> perfectMatching;
int match[N], par[N], base[N];
bool used[N], blossom[N], lcaUsed[N];
int lca(int u, int v) {
    fill(lcaUsed, lcaUsed + n, false);
    while (u != -1) {
        u = base[u];
        lcaUsed[u] = true;
        if (match[u] == -1)
            break;
        u = par[match[u]];
    }
    while (v != -1) {
        v = base[v];
        if (lcaUsed[v])
            return v;
        v = par[match[v]];
    }
    assert(false);
    return -1;
}
void markPath(int v, int myBase, int children) {
    while (base[v] != myBase) {
        blossom[v] = blossom[match[v]] = true;
        par[v] = children;
        children = match[v];
        v = par[match[v]];
    }
}
int findPath(int root) {
    iota(base, base + n, 0);
    fill(par, par + n, -1);
    fill(used, used + n, false);
    queue<int> q;
    q.push(root);
    used[root] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto to : g[v]) {
            if (match[v] == to)
                continue;
            if (base[v] == base[to])
                continue;
            if (to == root || (match[to] != -1 && par[match[to]] != -1))
                fill(blossom, blossom + n, false);
        }
    }
}
```

```
int myBase = lca(to, v);
markPath(v, myBase, to);
markPath(to, myBase, v);
for (int u = 0; u < n; ++u) {
    if (!blossom[base[u]])
        continue;
    base[u] = myBase;
    if (used[u])
        continue;
    used[u] = true;
    q.push(u);
}
} else if (par[to] == -1) {
    par[to] = v;
    if (match[to] == -1) {
        return to;
    }
    used[match[to]] = true;
    q.push(match[to]);
}
}
}
return -1;
}
void blossomShrinking() {
    fill(match, match + n, -1);
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1)
            continue;
        int nxt = findPath(v);
        while (nxt != -1) {
            int parV = par[nxt];
            int parParV = match[parV];
            match[nxt] = parV;
            match[parV] = nxt;
            nxt = parParV;
        }
    }
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1 && v < match[v]) {
            perfectMatching.push_back({v, match[v]});
        }
    }
}
signed main() {
    cin >> n;
    int u, v;
    set<pair<int, int>> edges;
    while (cin >> u >> v) {
        --u;
        --v;
        if (u > v)
            swap(u, v);
        if (edges.count({u, v}))
            continue;
        edges.insert({u, v});
        g[u].push_back(v);
        g[v].push_back(u);
    }
    blossomShrinking();
    cout << perfectMatching.size() * 2 << '\n';
    for (auto i : perfectMatching) {
        cout << i.u + 1 << " " << i.v + 1 << "\n";
    }
    return 0;
}
```

Lct.cpp
Description: link-cut tree
Time: $\mathcal{O}(n \log(n))$

cfc529, 143 lines

```
struct link_cut {
    struct node {
        int par;
        array<int, 2> sons;
        bool inv;
        int size;

        node() : par(-1), sons({ -1, -1 }), inv(false), size(1) {}
    };
};
```

```
};

vector<node> t;

int size() const {
    return t.size();
}

void push(int v) {
    if (t[v].inv) {
        t[v].inv = false;
        swap(t[v].sons[0], t[v].sons[1]);
        for (const auto u : t[v].sons)
            if (u != -1)
                t[u].inv ^= 1;
    }
}

void relax(int v) {
    push(v);
    t[v].size = 1;
    for (const auto x : t[v].sons)
        if (x != -1)
            t[v].size += t[x].size;
}

void rotate(int v) {
    int u = t[v].par, w = t[u].par;
    push(u), push(v);
    t[v].par = w;
    if (w != -1)
        for (auto& x : t[w].sons)
            if (x == u)
                x = v;
    int i = t[u].sons[1] == v;
    t[u].sons[i] = t[v].sons[i ^ 1];
    if (t[v].sons[i ^ 1] != -1)
        t[t[v].sons[i ^ 1]].par = u;
    t[v].sons[i ^ 1] = u;
    t[u].par = v;
    relax(u), relax(v);
}

bool is_root(int v) const {
    return t[v].par == -1 || (t[t[v].par].sons[0] != v && t[t[v].par].sons[1] != v);
}

void splay(int v) {
    while (!is_root(v)) {
        int u = t[v].par;
        if (!is_root(u))
            rotate((t[t[u].par].sons[0] == u) == (t[u].sons[0] == v)
                ? u : v);
        rotate(v);
    }
    push(v);
}

int expose(int v) {
    int prev = -1;
    for (int u = v; u != -1; prev = u, u = t[u].par) {
        splay(u);
        t[u].sons[1] = prev;
        relax(u);
    }
    splay(v);
    assert(t[v].sons[1] == -1 && t[v].par == -1);
    return prev;
}

link_cut(int n = 0) : t(n) {}

int add() {
    t.push_back(node());
    return int(t.size()) - 1;
}

void set_root(int root) {
};
```

```
expose(root);
t[root].inv ^= 1;
push(root);
}

bool connected(int v, int u) {
    if (v == u)
        return true;
    expose(v), expose(u);
    return t[v].par != -1;
}

bool link(int v, int u) {
    if (connected(v, u))
        return false;
    t[u].inv ^= 1;
    t[u].par = v;
    expose(u);
    return true;
}

bool cut(int v, int u) {
    if (v == u)
        return false;
    set_root(v), expose(u);
    if (t[u].sons[0] != v)
        return false;
    t[u].sons[0] = -1;
    relax(u);
    t[v].par = -1;
    return true;
}

int par(int v, int root) {
    if (!connected(v, root))
        return -1;
    set_root(root), expose(v);
    if (t[v].sons[0] == -1)
        return -1;
    v = t[v].sons[0];
    while (push(v), t[v].sons[1] != -1)
        v = t[v].sons[1];
    splay(v);
    return v;
}

int distance(int v, int u) {
    if (!connected(v, u))
        return -1;
    set_root(v), expose(u);
    return t[u].sons[0] == -1 ? 0 : t[t[u].sons[0]].size;
}

int lca(int v, int u, int root) {
    set_root(root), expose(v);
    return expose(u);
}
};
```

MaxFlow.cpp

Description: Dinic

Time: $\mathcal{O}(n^2m)$

1c1bc8, 72 lines

struct MaxFlow {
 const int inf = 1e9 + 20;
 struct edge {
 int a, b, cap;
 };
 int n;
 vector<edge> e;
 vector<vector<int>> g;
 MaxFlow() {}
 int s, t;
 vector<int> d, ptr;
 void init(int n_, int s_, int t_) {
 s = s_, t = t_, n = n_;
 g.resize(n);
 ptr.resize(n);
 };
};

```

}
void addedge(int a, int b, int cap) {
    g[a].pbc(e.size());
    e.pbc({a, b, cap});
    g[b].pbc(e.size());
    e.pbc({b, a, 0});
}
bool bfs() {
    d.assign(n, inf);
    d[s] = 0;
    queue<int> q;
    q.push(s);
    while (q.size()) {
        int v = q.front();
        q.pop();
        for (int i : g[v]) {
            if (e[i].cap > 0) {
                int b = e[i].b;
                if (d[b] > d[v] + 1) {
                    d[b] = d[v] + 1;
                    q.push(b);
                }
            }
        }
    }
    return d[t] != inf;
}
int dfs(int v, int flow) {
    if (v == t) return flow;
    if (!flow) return 0;
    int sum = 0;
    for (; ptr[v] < g[v].size(); ++ptr[v]) {
        int b = e[g[v][ptr[v]]].b;
        int cap = e[g[v][ptr[v]]].cap;
        if (cap <= 0) continue;
        if (d[b] != d[v] + 1) continue;
        int x = dfs(b, min(flow, cap));
        int id = g[v][ptr[v]];
        e[id].cap -= x;
        e[id ^ 1].cap += x;
        flow -= x;
        sum += x;
    }
    return sum;
}
int dinic() {
    int ans = 0;
    while (1) {
        if (!bfs()) break;
        ptr.assign(n, 0);
        int x = dfs(s, inf);
        if (!x) break;
        ans += x;
    }
    return ans;
}
};
```

MCMF.cpp
Description: Min cost
Time: $\mathcal{O}(?)$

32340a, 61 lines

```

struct MCMF {
    struct edge {
        int a, b, cap, cost;
    };
    vector<edge> e;
    vector<vector<int>>> g;
    int s, t;
    int n;
    void init(int N, int S, int T) {
        s = S, t = T, n = N;
        g.resize(N);
        e.clear();
    }
    void addedge(int a, int b, int cap, int cost) {
        g[a].pbc(e.size());
        e.pbc({a, b, cap, cost});
        g[b].pbc(e.size());
    }
};
```

```

    e.pbc({b, a, 0, -cost});
}
int getcost(int k) {
    int flow = 0;
    int cost = 0;
    while (flow < k) {
        vector<int> d(n, INF);
        vector<int> pr(n);
        d[s] = 0;
        queue<int> q;
        q.push(s);
        while (q.size()) {
            int v = q.front();
            q.pop();
            for (int i : g[v]) {
                int u = e[i].b;
                if (e[i].cap && d[u] > d[v] + e[i].cost) {
                    d[u] = d[v] + e[i].cost;
                    q.push(u);
                    pr[u] = i;
                }
            }
        }
        if (d[t] == INF) return INF;
        int gf = k - flow;
        int v = t;
        while (v != s) {
            int id = pr[v];
            chkmin(gf, e[id].cap);
            v = e[id].a;
        }
        v = t;
        while (v != s) {
            int id = pr[v];
            e[id].cap -= gf;
            e[id ^ 1].cap += gf;
            cost += e[id].cost * gf;
            v = e[id].a;
        }
        flow += gf;
    }
    return cost;
}
};
```

MCMFfast.cpp
Description: Min cost with potentials
Time: $\mathcal{O}(?)$

363228, 86 lines

```

struct MCMF {
    struct edge {
        int a, b, cap, cost;
    };
    vector<edge> e;
    vector<vector<int>>> g;
    vector<ll> po;
    int s, t;
    int n;
    void init(int N, int S, int T) {
        s = S, t = T, n = N;
        g.resize(N);
        e.clear();
    }
    void addedge(int a, int b, int cap, int cost) {
        g[a].pbc(e.size());
        e.pbc({a, b, cap, cost});
        g[b].pbc(e.size());
        e.pbc({b, a, 0, -cost});
    }
    void calc_p() {
        po.assign(n, INF);
        vector<int> inq(n);
        queue<int> q;
        q.push(s);
        po[s] = 0;
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            inq[v] = 0;
        }
    }
};
```

```

    for (auto i : g[v]) {
        if (po[e[i].b] > po[v] + e[i].cost && e[i].cap) {
            po[e[i].b] = po[v] + e[i].cost;
            if (!inq[e[i].b]) q.push(e[i].b);
            inq[e[i].b] = 1;
        }
    }
}
}
ll getcost(int k) {
    calc_p();
    int flow = 0;
    ll cost = 0;
    while (flow < k) {
        vector<ll> d(n, INF);
        vector<int> pr(n);
        d[s] = 0;
        set<pair<ll, int>> q;
        q.insert(mp(0ll, s));
        while (q.size()) {
            int v = q.begin()->second;
            q.erase(q.begin());
            for (int i : g[v]) {
                int u = e[i].b;
                if (e[i].cap && d[u] > d[v] + e[i].cost + po[v] - po[e[i].b]) {
                    q.erase(mp(d[u], u));
                    d[u] = d[v] + e[i].cost + po[v] - po[e[i].b];
                    q.insert(mp(d[u], u));
                    pr[u] = i;
                }
            }
        }
        if (d[t] == INF) return INF;
        for (int i = 0; i < n; ++i) {
            if (d[i] != INF) po[i] += d[i];
        }
        int gf = k - flow;
        int v = t;
        while (v != s) {
            int id = pr[v];
            chkmin(gf, e[id].cap);
            v = e[id].a;
        }
        v = t;
        while (v != s) {
            int id = pr[v];
            e[id].cap -= gf;
            e[id ^ 1].cap += gf;
            cost += 1ll * e[id].cost * gf;
            v = e[id].a;
        }
        flow += gf;
    }
    return cost;
}
};
```

GlobalMincut.cpp
Description: Global min cut
Time: $\mathcal{O}(n^3)$

7b8a6b, 35 lines

```

const int MAXN = 500;
int n, g[MAXN][MAXN];
int best_cost = 1000000000;
vector<int> best_cut;
void mincut() {
    vector<int> v[MAXN];
    for (int i = 0; i < n; ++i)
        v[i].assign(1, i);
    int w[MAXN];
    bool exist[MAXN], in_a[MAXN];
    memset(exist, true, sizeof exist);
    for (int ph = 0; ph < n - 1; ++ph) {
        memset(in_a, false, sizeof in_a);
        memset(w, 0, sizeof w);
        for (int it = 0, prev; it < n - ph; ++it) {
            int sel = -1;
            for (int i = 0; i < n; ++i)
```

```
struct DominatorTree {
    vector<basic_string<int>> g, rg, bucket;
    basic_string<int> arr, par, rev, sdom, dom, dsu, label;
    int n, t;
```

```
DominatorTree(int n) : g(n), rg(n), bucket(n), arr(n, -1), par(n, -1)
, rev(n, -1),
sdom(n, -1), dom(n, -1), dsu(n, 0), label(n, 0), n(n), t(0) {}
void add_edge(int u, int v) {
    g[u] += v;
}
void dfs(int u) {
    arr[u] = t;
    rev[t] = u;
    label[t] = sdом[t] = dsu[t] = t;
    t++;
    for (int w : g[u]) {
        if (arr[w] == -1) {
            dfs(w);
            par[arr[w]] = arr[u];
        }
        rg[arr[w]] += arr[u];
    }
}
int find(int u, int x=0) {
    if (u == dsu[u]) return x ? -1 : u;
    int v = find(dsu[u], x + 1);
    if (v < 0) return u;
    if (sdом[label[dsu[u]]] < sdом[label[u]])
        label[u] = label[dsu[u]];
    dsu[u] = v;
    return x ? v : label[u];
}
vector<int> run(int root) {
    dfs(root);
    iota(dom.begin(), dom.end(), 0);
    for (int i = t - 1; i >= 0; --i) {
        for (int w : rg[i]) sdом[i] = min(sdом[i], sdом[find(w)]);
        if (i) bucket[sdом[i]] += i;
        for (int w : bucket[i]) {
            int v = find(w);
            if (sdом[v] == sdом[w]) dom[w] = sdом[w];
            else dom[w] = v;
        }
        if (i > 1) dsu[i] = par[i];
    }
    for (int i = 1; i < t; i++) if (dom[i] != sdом[i]) dom[i] = dom[
        dom[i]];
    vector<int> outside_dom(n, -1);
    for (int i = 1; i < t; i++) outside_dom[rev[i]] = rev[dom[i]];
    //−1 if vertex is not reachable
    return outside_dom;
}
};
```

OrientedSpanningTree.cpp

Description: Oriented Spanning Tree

Time: $\mathcal{O}(n\log n?)$ 3d7a73, 96 lines

```
struct RollbackUF {
    vector<int> p, sz;
    vector<int> changes;
    RollbackUF(int n) {
        p.resize(n);
        changes.reserve(n);
        sz.resize(n, 1);
        for (int i = 0; i < n; ++i) p[i] = i;
    }
    int time() {
        return changes.size();
    }
    int find(int v) {
        if (v == p[v]) return v;
        return find(p[v]);
    }
    bool join(int a, int b) {
        a = find(a);
        b = find(b);
        if (a == b) return false;
        if (sz[a] > sz[b]) swap(a, b);
        changes.push_back(a);
        sz[b] += sz[a];
        p[a] = b;
        return true;
    }
};
```

```
void rollback(int t) {
    while (changes.size() > t) {
        int v = changes.back();
        sz[p[v]] -= sz[v];
        p[v] = v;
        changes.pop_back();
    }
};
struct Edge { int a, b; ll w; };
struct Node {
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ? b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}
void pop(Node& a) { a->prop(); a = merge(a->l, a->r); }
pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node(e));
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1, -1}), comp;
    deque<tuple<int, int, vector<Edge>>> cyps;
    for (int s = 0; s < n; ++s) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}};
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) {
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do {
                    cyc = merge(cyc, heap[w = path[--qi]]);
                } while (uf.join(u, w));
                u = uf.find(u), heap[u] = cyc, seen[u] = -1;
                cyps.push_front({u, time, {Q[qi], Q[end]}});
            }
        }
        for (int i = 0; i < qi; ++i) {
            in[uf.find(Q[i].b)] = Q[i];
        }
    }
    for (auto& [u, t, comp] : cyps) { // restore so l ( optional )
        uf.rollback(t);
        Edge inEdge = in[u];
        for (auto& e : comp) in[uf.find(e.b)] = e;
        in[uf.find(inEdge.b)] = inEdge;
    }
    for (int i = 0; i < n; ++i) par[i] = in[i].a;
    return {res, par};
}
```

MatroidIntersection.cpp

Description: matroid interestion

Time: ? d2387f, 71 lines

```
template<typename T, typename A, typename B>
vector<T> matroid_intersection(const std::vector<T> &ground_set, const A
&matroid1, const B &matroid2) {
```

```
//weighted − minimize (weight, cnt edges) with dijkstra
int n = ground_set.size();
vector<char> in_set(n), inm1(n), inm2(n);
vector<bool> used(n);
vi par(n), left, right;
while (true) {
    A m1 = matroid1;
    B m2 = matroid2;
    left.clear(); right.clear();
    for (int i = 0; i < n; i++)
        if (in_set[i]) {
            m1.add(ground_set[i]);
            m2.add(ground_set[i]);
            left.push_back(i);
        } else {
            right.push_back(i);
        }
    fill(all(inm1), 0); fill(all(inm2), 0);
    bool found = false;
    for (int i : right) {
        inm1[i] = m1.independed_with(ground_set[i]);
        inm2[i] = m2.independed_with(ground_set[i]);
        if (inm1[i] && inm2[i]) {
            in_set[i] = 1;
            found = true;
            break;
        }
    }
    if (found) continue;
    fill(all(used), false); fill(all(par), -1);
    queue<int> que;
    for (int i : right) if (inm1[i]) {
        used[i] = true;
        que.push(i);
    }
    while (!que.empty() && !found) {
        int v = que.front();
        que.pop();
        if (in_set[v]) {
            A m = matroid1;
            for (int i : left) if (i != v) m.add(ground_set[i]);
            for (int u : right)
                if (!used[u] && m.independed_with(ground_set[u])) {
                    par[u] = v;
                    used[u] = true;
                    que.push(u);
                    if (inm2[u]) {
                        found = true;
                        for (; u != -1; u = par[u]) in_set[u] ^= 1;
                        break;
                    }
                }
        } else {
            B m = m2;
            m.add_extra(ground_set[v]);
            for (auto u : left)
                if (!used[u] && m.independed_without(ground_set[u])) {
                    par[u] = v;
                    used[u] = true;
                    que.push(u);
                }
        }
    }
    if (!found) break;
}
vector<T> res;
for (int i = 0; i < n; i++) if (in_set[i]) res.push_back(ground_set[i
]);
return res;
}
```

MinMeanCycle.cpp

Description:	
<div><div>MINIMUM MEAN CYCLE ALGORITHM</div><div><i>Input:</i> A digraph G, weights $c : E(G) \rightarrow \mathbb{R}$.</div><div><i>Output:</i> A circuit C with minimum mean weight or the information that G is acyclic.</div></div>	
①	Add a vertex s and edges (s, x) with $c((s, x)) := 0$ for all $x \in V(G)$ to G .
②	Set $n := V(G) $, $F_0(s) := 0$, and $F_0(x) := \infty$ for all $x \in V(G) \setminus \{s\}$.
③	For $k := 1$ to n do : For all $x \in V(G)$ do : Set $F_k(x) := \infty$. For all $(w, x) \in \delta^-(x)$ do : If $F_{k-1}(w) + c((w, x)) < F_k(x)$ then : Set $F_k(x) := F_{k-1}(w) + c((w, x))$ and $p_k(x) := w$.
④	If $F_n(x) = \infty$ for all $x \in V(G)$ then stop (G is acyclic).
⑤	Let x be a vertex for which $\max_{\substack{0 \leq k \leq n-1 \\ F_k(x) < \infty}} \frac{F_n(x) - F_k(x)}{n - k}$ is minimum.
⑥	Let C be any circuit in the edge progression given by $p_n(x), p_{n-1}(p_n(x)), p_{n-2}(p_{n-1}(p_n(x))), \dots$
d41d8c, 1 lines	
// ?	

Geometry (5)

Point.cpp	
Description: struct Point	80dfd5, 80 lines
<pre>const ld EPS = 1e-7;</pre>	
<pre>ld sq(ld x) { return x * x; }</pre>	
<pre>int sign(ld x) { if (x < -EPS) { return -1; } if (x > EPS) { return 1; } return 0; }</pre>	
<pre>#define vec point struct point { // % - cross, * - dot ld x, y; auto operator<=(const point&) const = default; }; ld operator*(const point &a, const point &b) { return a.x * b.x + a.y * b.y; } ld operator%(const point &a, const point &b) { return a.x * b.y - a.y * b.x; } point operator-(const point &a, const point &b) { return {a.x - b.x, a.y - b.y}; } point operator+(const point &a, const point &b) { return {a.x + b.x, a.y + b.y}; } point operator*(const point &a, ld b) { return {a.x * b, a.y * b}; } point operator/(const point &a, ld b) { return {a.x / b, a.y / b}; } bool operator<(const point &a, const point &b) { if (sign(a.y - b.y) != 0) { return a.y < b.y; } else if (sign(a.x - b.x) != 0) {</pre>	

<pre> return a.x < b.x; } return 0; } ld len2(const point &a) { return sq(a.x) + sq(a.y); } ld len(const point &a) { return sqrt(len2(a)); } point norm(point a) { return a / len(a); } int half(point a) { return (sign(a.y) == -1 (sign(a.y) == 0 && a.x < 0)); } point ort(point a) { return {-a.y, a.x}; } point turn(point a, ld ang) { return {a.x * cos(ang) - a.y * sin(ang), a.x * sin(ang) + a.y * cos(ang)}; } ld getAngle(point &a, point &b) { return atan2(a % b, a * b); } } bool cmpHalf(const point &a, const point &b) { if (half(a) != half(b)) { return half(b); } else { int sgn = sign(a % b); if (!sgn) { return len2(a) < len2(b); } else { return sgn == 1; } } }</pre>	
Line.cpp	
Description: struct Line	887306, 26 lines
<pre>struct line { ld a, b, c; void norm() { // for half planes ld d = len({a, b}); assert(sign(d) > 0); a /= d; b /= d; c /= d; } ld eval(point p) const { return a * p.x + b * p.y + c; } bool isIn(point p) const { return sign(eval(p)) >= 0; } bool operator==(const line &other) const { return sign(a * other.b - b * other.a) == 0 && sign(a * other.c - c * other.a) == 0 && sign(b * other.c - c * other.b) == 0; } }; line getLn(point a, point b) { line res; res.a = a.y - b.y; res.b = b.x - a.x; res.c = -(res.a * a.x + res.b * a.y); res.norm(); return res; }</pre>	

Intersections.cpp	
Description: Geometry intersections	45d7d9, 75 lines
<pre>bool isCrossed(ld lx, ld rx, ld ly, ld ry) { if (lx > rx) swap(lx, rx); if (ly > ry) swap(ly, ry); return sign(min(rx, ry) - max(lx, ly)) >= 0; }</pre>	

<pre>// if two segments [a, b] and [c, d] has AT LEAST one common point -> true bool intersects(const point &a, const point &b, const point &c, const point &d) { if (!isCrossed(a.x, b.x, c.x, d.x)) return false; if (!isCrossed(a.y, b.y, c.y, d.y)) return false; if (sign((b - a) % (c - a)) * sign((b - a) % (d - a)) == 1) return 0; if (sign((d - c) % (a - c)) * sign((d - c) % (b - c)) == 1) return 0; return 1; } //intersecting lines bool intersect(line l, line m, point &I) { ld d = l.b * m.a - m.b * l.a; if (sign(d) == 0) { return false; } ld dx = m.b * l.c - m.c * l.b; ld dy = m.c * l.a - l.c * m.a; I = {dx / d, dy / d}; return true; } //intersecting circles int intersect(point o1, ld r1, point o2, ld r2, point &i1, point &i2) { if (r1 < r2) { swap(o1, o2); swap(r1, r2); } if (sign(r1 - r2) == 0 && len2(o2 - o1) < EPS) { return 3; } ld ln = len(o1 - o2); if (sign(ln - r1 - r2) == 1 sign(r1 - ln - r2) == 1) { return 0; } ld d = (sq(r1) - sq(r2) + sq(ln)) / 2 / ln; vec v = norm(o2 - o1); point a = o1 + v * d; if (sign(ln - r1 - r2) == 0 sign(ln + r2 - r1) == 0) { i1 = a; return 1; } v = ort(v) * sqrt(sq(r1) - sq(d)); i1 = a + v; i2 = a - v; return 2; } } //intersecting line and circle, line should be normed int intersect(point o, ld r, line l, point &i1, point &i2) { ld len = abs(l.eval(o)); int sgn = sign(len - r); if (sgn == 1) { return 0; } vec v = norm(vec{l.a, l.b}) * len; if (sign(l.eval(o + v)) != 0) { v = vec{0, 0} - v; } point a = o + v; if (sgn == 0) { i1 = a; return 1; } v = norm({-l.b, l.a}) * sqrt(sq(r) - sq(len)); i1 = a + v; i2 = a - v; return 2; } }</pre>	
Tangents.cpp	
Description: Tangents to circles.	c73373, 43 lines
<pre>// tangents from point to circle int tangents(point &o, ld r, point &p, point &i1, point &i2) { ld ln = len(o - p); int sgn = sign(ln - r); if (sgn == -1) {</pre>	

```

        return 0;
    } else if (sgn == 0) {
        il = p;
        return 1;
    } else {
        ld x = sq(r) / ln;
        vec v = norm(p - o) * x;
        point a = o + v;
        v = ort(norm(p - o)) * sqrt(sq(r) - sq(x));
        il = a + v;
        i2 = a - v;
        return 2;
    }
}

void _tangents(point c, ld r1, ld r2, vector<line> &ans) {
    ld r = r2 - r1;
    ld z = sq(c.x) + sq(c.y);
    ld d = z - sq(r);
    if (sign(d) == -1)
        return;
    d = sqrt(abs(d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}

// tangents between two circles
vector<line> tangents(point o1, ld r1, point o2, ld r2) {
    vector<line> ans;
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            _tangents(o2 - o1, r1 * i, r2 * j, ans);
    for (int i = 0; i < (int)ans.size(); ++i)
        ans[i].c -= ans[i].a * o1.x + ans[i].b * o1.y;
    return ans;
}

```

Hull.cpp

Description: Polygon functions
 fc1928, 16 lines

```

vector<point> hull(vector<point> p, bool need_all=false) {
    sort(all(p));
    p.erase(unique(all(p)), end(p));
    int n = p.size(), k = 0;
    if (n <= 2) return p;
    vector<point> ch(2 * n);
    ld th = need_all ? -EPS : +EPS; // 0 : 1 if int
    for (int i = 0; i < n; ch[k++] = p[i++]) {
        while (k >= 2 && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
            --k;

        for (int i = n - 2, t = k + 1; i >= 0; ch[k++] = p[i--]) {
            while (k >= t && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
                --k;
        }
        ch.resize(k - 1);
        return ch;
    }
}

```

IsInPolygon.cpp

Description: Is in polygon functions
 f17b31, 65 lines

```

bool isOnSegment(point &a, point &b, point &x) {
    if (sign(len2(a - b)) == 0) {
        return sign(len(a - x)) == 0;
    }
    return sign((b - a) % (x - a)) == 0 && sign((b - x) * (a - x)) <= 0;
    // optional (slower, but works better if there are some precision
    // problems) return sign((b - a).len() - (x - a).len() - (x - b).len
    // ())
    // == 0;
}

int isIn(vector<point> &p, point &a) {
    int n = p.size();
    // depends on limitations(2*MAXC + 228)
    point b = a + point{2e9 + 228, 1};

```

```

    int cnt = 0;
    for (int i = 0; i < n; ++i) {
        point x = p[i];
        point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return 1;
        }
        cnt += intersects(x, y, a, b);
    }
    return 2 * (cnt % 2 == 1);
    /*optional (atan2 is VERY SLOW)!
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
        Point x = p[i];
        Point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return true;
        }
        x = x - a;
        y = y - a;
        ans += atan2(x ^ y, x * y);
    }
    return abs(ans) > 1;*/
}

bool isInTriangle(point &a, point &b, point &c, point &x) {
    return sign((b - a) % (x - a)) >= 0 && sign((c - b) % (x - b)) >= 0
        &&
        sign((a - c) % (x - c)) >= 0;
}

// points should be in the counterclockwise order
bool isInConvex(vector<point> &p, point &a) {
    int n = p.size();
    assert(n >= 3);
    // assert(isConvex(p));
    // assert(isCounterclockwise(p));
    if (sign((p[1] - p[0]) % (a - p[0])) < 0)
        return 0;
    if (sign((p[n - 1] - p[0]) % (a - p[0])) > 0)
        return 0;
    int pos = lower_bound(p.begin() + 2, p.end(), a,
        [&](point a, point b) -> bool {
            return sign((a - p[0]) % (b - p[0])) > 0;
        }) -
        p.begin();
    assert(pos > 1 && pos < n);
    return isInTriangle(p[0], p[pos - 1], p[pos], a);
}

```

```

}

// points should be in the counterclockwise order
bool isInConvex(vector<point> &p, point &a) {
    int n = p.size();
    assert(n >= 3);
    // assert(isConvex(p));
    // assert(isCounterclockwise(p));
    if (sign((p[1] - p[0]) % (a - p[0])) < 0)
        return 0;
    if (sign((p[n - 1] - p[0]) % (a - p[0])) > 0)
        return 0;
    int pos = lower_bound(p.begin() + 2, p.end(), a,
        [&](point a, point b) -> bool {
            return sign((a - p[0]) % (b - p[0])) > 0;
        }) -
        p.begin();
    assert(pos > 1 && pos < n);
    return isInTriangle(p[0], p[pos - 1], p[pos], a);
}

```

Diameter.cpp

Description: Rotating calipers.
 Time: $O(n)$
 0f341c, 21 lines

```

ld diameter(vector<point> p) {
    p = hull(p);
    int n = p.size();
    if (n <= 1) {
        return 0;
    }
    if (n == 2) {
        return len(p[0] - p[1]);
    }
    ld ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (sign((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] - p[j])) >=
            0) {
            chkmax(ans, len(p[i] - p[j]));
            j = (j + 1) % n;
        }
        chkmax(ans, len(p[i] - p[j]));
        ++i;
    }
    return ans;
}

```

TangentsAlex.cpp

Description: Find both tangets to the convex polygon.
 (Zakaldovany algos mozhet sgonyat za pivom tak zhe).
 Time: $O(\log(n))$
 2eeea8, 17 lines

```

pair<int, int> tangents_alex(vector<point> &p, point &a) {
    int n = p.size();
    int l = __lg(n);
    auto findWithSign = [&](int val) {
        int i = 0;
        for (int k = 1; k >= 0; --k) {
            int i1 = (i - (1 << k) + n) % n;
            int i2 = (i + (1 << k)) % n;
            if (sign((p[i1] - a) % (p[i] - a)) == val)
                i = i1;
            if (sign((p[i2] - a) % (p[i] - a)) == val)
                i = i2;
        }
        return i;
    };
    return {findWithSign(1), findWithSign(-1)};
}

```

IsHpiEmpty.cpp

Description: Determines is half plane intersectinos.
 Time: $O(n)$ (expected)
 3b5e69, 42 lines

```

// all lines must be normed!!!!, sign > 0
bool isHpiEmpty(vector<line> lines) {
    // return hpi(lines).empty();
    // overflow/precision problems?
    shuffle(all(lines), rnd);
    const ld C = 1e9;
    point ans(C, C);
    vector<point> box = {{-C, -C}, {C, -C}, {C, C}, {-C, C}};
    for (int i = 0; i < 4; ++i)
        lines.push_back(getln(box[i], box[(i + 1) % 4]));
    int n = lines.size();
    for (int i = n - 4; i >= 0; --i) {
        if (lines[i].isIn(ans))
            continue;
        point up(0, C + 1), down(0, -C - 1), pi = {lines[i].b, -lines[i].
            a};
        for (int j = i + 1; j < n; ++j) {
            if (lines[i] == lines[j])
                continue;
            point p, pj = {lines[j].b, -lines[j].a};
            if (!intersect(lines[i], lines[j], p)) {
                if (sign(pi * pj) != -1)
                    continue;
                if (sign(lines[i].c + lines[j].c) *
                    (!sign(pi.y) ? sign(pi.x) : -1) ==
                    1)
                    return true;
            }
        }
        if ((ans = up) < down)
            return true;
    }
    // for (int i = 0; i < n; ++i) {
    //     assert(lines[i].eval(ans) < EPS);
    // }
    return false;
}

```

HalfPlaneIntersection.cpp

Description: Find the intersection of the half planes.
 Time: $O(n \log(n))$
 fdf28f, 62 lines

```

vec getPoint(line l) { return {-l.b, l.a}; }

bool bad(line a, line b, line c) {

```

```
point x;
assert(intersect(b, c, x) == 1);
return a.eval(x) < 0;
}

// Do not forget about the bounding box
vector<point> hpi(vector<line> lines) {
    sort(all(lines), [](line al, line bl) -> bool {
        point a = getPoint(al);
        point b = getPoint(bl);
        if (half(a) != half(b)) {
            return half(a) < half(b);
        }
        return a % b > 0;
    });

    vector<pair<line, int>> st;
    for (int it = 0; it < 2; it++) {
        for (int i = 0; i < (int)lines.size(); i++) {
            bool flag = false;
            while (!st.empty()) {
                if (len(getPoint(st.back().first) - getPoint(lines[i])) < EPS) {
                    if (lines[i].c >= st.back().first.c) {
                        flag = true;
                        break;
                    } else {
                        st.pop_back();
                    }
                } else if (getPoint(st.back().first) % getPoint(lines[i]) < EPS / 2) {
                    return {};
                } else if (st.size() >= 2 && bad(st[st.size() - 2].first, st[st.size() - 1].first, lines[i])) {
                    st.pop_back();
                } else {
                    break;
                }
            }
            if (!flag)
                st.push_back({lines[i], i});
        }
    }

    vector<int> en(lines.size(), -1);
    vector<point> ans;
    for (int i = 0; i < (int)st.size(); i++) {
        if (en[st[i].second] == -1) {
            en[st[i].second] = i;
            continue;
        }
        for (int j = en[st[i].second]; j < i; j++) {
            point I;
            assert(intersect(st[j].first, st[j + 1].first, I) == 1);
            ans.push_back(I);
        }
        break;
    }
    return ans;
}
```

CHT.cpp
Description: CHT for minimum, k is decreasing, works for equal slopes

```
struct line {
    int k, b;
    int eval(int x) {
        return k * x + b;
    }
};

struct part {
    line a;
    ld x;
};

ld intersection(line a, line b) {
    return (ld) (a.b - b.b) / (b.k - a.k);
}
```

```
struct ConvexHullMin {
    vector<part> st;
    void add(line a) {
        if (!st.empty() && st.back().a.k == a.k) {
            if (st.back().a.b > a.b) st.pop_back();
            else return;
        }
        while (st.size() > 1 && intersection(st[st.size() - 2].a, a) <= st[st.size() - 2].x) st.pop_back();
        if (!st.empty()) st.back().x = intersection(st.back().a, a);
        st.push_back({a, INF});
    }

    int get_val(int x) {
        int l = -1, r = (int)st.size() - 1;
        while (r - l > 1) {
            int m = (l + r) / 2;
            if (st[m].x < x) l = m;
            else r = m;
        }
        return st[r].a.eval(x);
    }
};
```

DynamicCHT.cpp
Description: Dynamic CHT for maximum

```
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {
        return Q ? p < o.p : k < o.k;
    }
};

struct LineContainer : multiset<Line> {
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }

    void add(ll k, ll m) {
        auto z = insert({k, m, 0}); y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }

    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0, 0, x}); Q = 0;
        return l.k * x + l.m;
    }
};
```

MinPlusConv.cpp
Description: Min-Plusconv, A is convex down
Time: $\mathcal{O}(n \log n \text{ fast})$

```
// Assumptions: 'a' is convex, 'opt' has size 'n+m-1'
// 'opt[k]' will be equal to 'arg min(a[k-i] + b[i])'
template<typename T>
void convex_arbitrary_min_plus_conv(T *a, int n, T *b, int m, int *opt) {
    auto rec = [&](auto &&self, int lx, int rx, int ly, int ry) -> void {
        if (lx > rx) return;
        int mx = (lx + rx) >> 1;
        opt[mx] = ly;
        for (int i = ly; i <= ry; ++i)
            if (mx >= i && (mx - opt[mx] >= n || a[mx - opt[mx]] + b[opt[mx]] > a[mx - i] + b[i]))
                opt[mx] = i;
        self(self, lx, mx - 1, ly, opt[mx]);
        self(self, mx + 1, rx, opt[mx], ry);
    };
    rec(rec, 0, n + m - 2, 0, m - 1);
}
```

```
// Assumptions: 'a' is convex down
template<typename T>
std::vector<T> convex_arbitrary_min_plus_conv(const std::vector<T> &a,
const std::vector<T> &b) {
    int n = a.size(), m = b.size();
    int *opt = (int*) malloc(sizeof(int) * (n + m - 1));
    convex_arbitrary_min_plus_conv(a.data(), n, b.data(), m, opt);
    std::vector<T> ans(n + m - 1);
    for (int i = 0; i < n + m - 1; ++i) ans[i] = a[i - opt[i]] + b[opt[i]];
    free(opt);
    return ans;
}
```

Kinetic.cpp
Description: kinetic segment tree
Time: $\mathcal{O}(hz)$

```
//vnutrennii functions — poluintervali, vneshnie — otrezki. ishet min priamuy
struct line {
    ll k, b, temp;
    ll eval() const {
        return k * temp + b;
    }
    ll melting_point(const line& other) const {
        ll val1 = eval();
        ll val2 = other.eval();
        assert(val1 <= val2);
        if (other.k >= k) {
            return INF;
        }
        ll delta_val = val2 - val1;
        ll delta_k = k - other.k;
        assert(delta_val >= 0 && delta_k > 0);
        return (delta_val + delta_k - 1) / delta_k;
    }
};

struct kinetic_segtree {
    struct node {
        ll lazy_b = 0, lazy_temp = 0, melt = INF;
        line best;
        node(line best = line()) : best(best) {}
    };

    int n;
    vector<node> tree;
    void update(int v) {
        if (make_pair(tree[v << 1].best.eval(), tree[v << 1].best.k) < make_pair(tree[v << 1 | 1].best.eval(), tree[v << 1 | 1].best.k)) {
            tree[v].best = tree[v << 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1 | 1].best);
        } else {
            tree[v].best = tree[v << 1 | 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1].best);
        }

        tree[v].melt = min({tree[v].melt, tree[v << 1].melt, tree[v << 1 | 1].melt});
        assert(tree[v].melt > 0);
    }

    void apply(int v, int vl, int vr, ll delta_b, ll delta_temp) {
        tree[v].lazy_b += delta_b;
        tree[v].lazy_temp += delta_temp;

        tree[v].best.b += delta_b;
        tree[v].best.temp += delta_temp;

        tree[v].melt -= delta_temp;
        if (tree[v].melt <= 0) {
            push(v, vl, vr);
            update(v);
        }
    }

    void push(int v, int vl, int vr) {
        int vm = (vl + vr) / 2;
        apply(v << 1, vl, vm, tree[v].lazy_b, tree[v].lazy_temp);
        apply(v << 1 | 1, vm, vr, tree[v].lazy_b, tree[v].lazy_temp);
    }
};
```

```
        tree[v].lazy_b = 0;
        tree[v].lazy_temp = 0;
    }
    void build(int v, int vl, int vr, const vector<line> &lines) {
        if (vr - vl == 1) {
            tree[v] = node(lines[vl]);
            return;
        }
        int vm = (vl + vr) / 2;
        build(v << 1, vl, vm, lines);
        build(v << 1 | 1, vm, vr, lines);
        update(v);
    }
    void add(int v, int vl, int vr, int l, int r, ll delta_b, ll delta_temp) {
        delta_temp) {
            if (r <= vl || vr <= l) {
                return;
            }
            if (l <= vl && vr <= r) {
                apply(v, vl, vr, delta_b, delta_temp);
                return;
            }
            push(v, vl, vr);
            int vm = (vl + vr) / 2;
            add(v << 1, vl, vm, l, r, delta_b, delta_temp);
            add(v << 1 | 1, vm, vr, l, r, delta_b, delta_temp);
            update(v);
        }
        void change_line(int v, int vl, int vr, int pos, const line &new_line)
        ) {
            if (vr - vl == 1) {
                tree[v].best = new_line;
                return;
            }
            push(v, vl, vr);
            int vm = (vl + vr) / 2;
            if (pos < vm) {
                change_line(v << 1, vl, vm, pos, new_line);
            } else {
                change_line(v << 1 | 1, vm, vr, pos, new_line);
            }
            update(v);
        }
        ll query(int v, int vl, int vr, int l, int r) {
            if (r <= vl || vr <= l) {
                return INF;
            }
            if (l <= vl && vr <= r) {
                return tree[v].best.eval();
            }
            push(v, vl, vr);
            int vm = (vl + vr) / 2;
            return min(query(v << 1, vl, vm, l, r), query(v << 1 | 1, vm, vr, l, r));
        }
        kinetic_segtree(const vector<line> &lines) : n(lines.size()), tree(4
            * n) {
            build(1, 0, n, lines);
        }
        kinetic_segtree(int n) : n(n), tree(4 * n) {
            vector <line> lines(n, {0, INF, 0});
            build(1, 0, n, lines);
        }
        void add(int l, int r, ll delta_b, ll delta_temp) {
            assert(delta_temp >= 0);
            add(1, 0, n, l, r + 1, delta_b, delta_temp);
        }
        void change_line(int pos, const line &new_line) {
            assert(0 <= pos && pos < n);
            change_line(1, 0, n, pos, new_line);
        }
        ll query(int l, int r) {
            return query(1, 0, n, l, r + 1);
        }
    }
```

GoldenSearch.cpp

Description: Golden Search	31d45b, 14 lines
----------------------------	------------------

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5) - 1) / 2, eps = 1e-7;
    double x1 = b - r * (b - a), x2 = a + r * (b - a);
    double f1 = f(x1), f2 = f(x2);
    while (b - a > eps)
        if (f1 < f2) { // change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r * (b - a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r * (b - a); f2 = f(x2);
        }
    return a;
}
```

3dBasic.cpp

Description: Basic 3d geom things	467773, 79 lines
-----------------------------------	------------------

```
const int inf = int(1e9) + int(1e5);
const ll inf1 = ll(2e18) + ll(1e10);
const ld eps = 1e-9;
bool ze(ld x) {
    return fabs1(x) < eps;
}
struct pt {
    ld x, y, z;
    pt operator+(const pt &p) const {
        return pt{x + p.x, y + p.y, z + p.z};
    }
    pt operator-(const pt &p) const {
        return pt{x - p.x, y - p.y, z - p.z};
    }
    ld operator*(const pt &p) const {
        return x * p.x + y * p.y + z * p.z;
    }
    pt operator*(ld a) const {
        return pt{x * a, y * a, z * a};
    }
    pt operator%(const pt &p) const {
        return pt{y * p.z - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x};
    }
    ld abs() const {
        return sqrt1(*this * *this);
    }
    ld abs2() const {
        return *this * *this;
    }
    pt norm() const {
        ld d = abs();
        return pt{x / d, y / d, z / d};
    }
};
// BEGIN.CODE
struct Plane {
    pt v;
    ld c;
    Plane(pt a, pt b, pt c) {
        v = ((b - a) % (c - a)).norm();
        this->c = a * v;
    }
    ld dist(pt p) {
        return p * v - c;
    }
};
pt projection(pt p, pt a, pt b) {
    pt v = b - a;
    if (ze(v.abs2())) {
        // stub : bad line
        return a;
    }
    return a + v * (((p - a) * v) / (v * v));
}
pair<pt, pt> planesIntersection(Plane a, Plane b) {
    pt dir = a.v % b.v;
    if (ze(dir.abs2())) {
        // stub : parallel planes
    }
```

```
        return {pt{1e18, 1e18, 1e18}, pt{1e18, 1e18, 1e18}};
    }
    ld s = a.v * b.v;
    pt v3 = b.v - a.v * s;
    pt h = a.v * a.c + v3 * ((b.c - a.c * s) / (v3 * v3));
    return {h, h + dir};
}
pair<pt, pt> commonPerpendicular(pt a, pt b, pt c, pt d) {
    pt v = (b - a) % (d - c);
    ld S = v.abs();
    if (ze(S)) {
        // stub : parallel lines
        return {pt{1e18, 1e18, 1e18}, pt{1e18, 1e18, 1e18}};
    }
    v = v.norm();
    pt sh = v * (v * c - v * a);
    pt a2 = a + sh;
    ld s1 = ((c - a2) % (d - a2)) * v;
    pt p = a + (b - a) * (s1 / S);
    return {p, p + sh};
}
```

NDHull.cpp

Description: Hull in arbitrary number of dimensions Time: $O(N * Dim * Hull)$	cf8067, 77 lines
--	------------------

```
const int DIM = 4;
typedef array<ll, DIM> pt;
pt operator-(const pt &a, const pt &b) {
    pt res;
    forn(i, DIM) res[i] = a[i] - b[i];
    return res;
}
typedef array<pt, DIM - 1> Edge;
typedef array<pt, DIM> Face;
vector<Face> faces;
ll det(pt *a) {
    int p[DIM];
    iota(p, p + DIM, 0);
    ll res = 0;
    do {
        ll x = 1;
        forn(i, DIM) {
            forn(j, i) if (p[j] > p[i]) x *= -1;
            x *= a[i][p[i]];
        }
        res += x;
    } while (next_permutation(p, p + DIM));
    return res;
}
ll V(Face f, pt pivot) {
    pt p[DIM];
    forn(i, DIM) p[i] = f[i] - pivot;
    return det(p);
}
void init(vector<pt> p) {
    forn(i, DIM + 1) {
        Face a;
        int q = 0;
        forn(j, DIM + 1) if (j != i) a[q++] = p[j];
        ll v = V(a, p[i]);
        assert(v != 0);
        if (v < 0) swap(a[0], a[1]);
        faces.push_back(a);
    }
}
void add(pt p) {
    vector<Face> newf, bad;
    for (auto f : faces) {
        if (V(f, p) < 0)
            bad.push_back(f);
        else
            newf.push_back(f);
    }
    if (bad.empty()) {
        cout << " Ignore \n";
        return;
    }
    cout << " Rebuild \n";
}
```

```
faces = newf;
vector<pair<Edge, pt>> edges;
for (auto f : bad) {
    sort(all(f));
    forn(i, DIM) {
        Edge e;
        int q = 0;
        forn(j, DIM) if (i != j) e[q++] = f[j];
        edges.emplace_back(e, f[i]);
    }
}
sort(all(edges));
forn(i, sz(edges)) {
    if (i + 1 < sz(edges) && edges[i + 1].first == edges[i].first) {
        ++i;
        continue;
    }
    Face f;
    forn(j, DIM - 1) f[j] = edges[i].first[j];
    f[DIM - 1] = p;
    if (V(f, edges[i].second) < 0) swap(f[0], f[1]);
    faces.push_back(f);
}
}
```

GenerateNonConvex.cpp

Description: Non convex polygon generation 2a7d37, 74 lines

```
vector<vec> pointsInGeneralPosition(int n, int maxC) {
    vector<vec> arr(n);
    for (int i = 0; i < n; ++i) {
        arr[i].x = randint(0, maxC);
        arr[i].y = randint(0, maxC);
    }
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < i; ++j) {
            for (int k = 0; k < j; ++k) {
                if (sign((arr[i] - arr[j]) % (arr[j] - arr[k])) == 0) {
                    return pointsInGeneralPosition(n, maxC);
                }
            }
        }
    }
    return arr;
}

vector<vec> pointsDifferent(int n, int maxC) {
    vector<vec> arr;
    while (arr.size() < n) {
        vec v;
        v.x = randint(0, maxC);
        v.y = randint(0, maxC);
        if (binary_search(all(arr), v)) {
            continue;
        }
        arr.pbc(v);
        sort(all(arr));
    }
    shuffle(all(arr), rnd);
    return arr;
}

vector<vec> generateNonconvex(int n, int maxC) {
    vector<vec> arr = pointsDifferent(n, maxC);
    bool was = 1;
    while (was) {
        was = 0;
        for (int i = 0; i < n; ++i) {
            for (int j = i + 2; j < n; ++j) {
                if ((j + 1) % n == i) continue;
                if (intersects(arr[i], arr[(i + 1) % n], arr[j], arr[(j + 1) % n])) {
                    reverse(arr.begin() + i + 1, arr.begin() + j + 1);
                    was = 1;
                }
            }
        }
    }
    if (area(arr) < 0) {
```

GenerateNonConvex MinDisk ClosestPair Faces

```
reverse(all(arr));
}
if (sign(area(arr)) == 0) {
    return generateNonconvex(n, maxC);
}
return arr;
}

template<typename T>
vector<vec<T>> polyRemoveOnOneLine(vector<vec<T>> arr) {
    int n = arr.size();
    for (int it = 0; it < 3; ++it) {
        vector<vec<T>> res;
        for (auto el : arr) {
            if (res.size() >= 2 && sign((res[res.size() - 2] - el) % (res.back() - el)) == 0) {
                res.pop_back();
            }
            res.pbc(el);
        }
        arr = res;
        rotate(arr.begin(), 1 + all(arr));
    }
    return arr;
}
}
```

MinDisk.cpp

Description: Computes the minimum circle that encloses a set of points. Time: expected $\mathcal{O}(n)$ 3b8fcd, 31 lines

```
ld ccRadius(const vec& A, const vec& B, const vec& C) {
    return len(B-A)*len(C-B)*len(A-C)/abs((B-A)%(C-A))/2;
}
vec circumcenter(const vec& A, const vec& B, const vec& C) {
    vec b = C-A, c = B-A;
    return A + ort(b*(c*c)-c*(b*b))/(b%c)/2;
}

pair<vec, ld> mindisk(vector<vec> ps) {
    shuffle(all(ps), rnd);
    vec o = ps[0];
    ld r = 0, EPS = 1 + 1e-8;
    for (int i = 0; i < ps.size(); ++i) {
        if (len(o - ps[i]) > r * EPS) {
            o = ps[i], r = 0;
            for (int j = 0; j < i; ++j) {
                if (len(o - ps[j]) > r * EPS) {
                    o = (ps[i] + ps[j]) / 2;
                    r = len(o - ps[i]);
                    for (int k = 0; k < j; ++k) {
                        if (len(o - ps[k]) > r * EPS) {
                            o = circumcenter(ps[i], ps[j], ps[k]);
                            r = len(o - ps[i]);
                        }
                    }
                }
            }
        }
    }
    return {o, r};
}
}
```

ClosestPair.cpp

Description: Finds the closest pair of points. Time: $\mathcal{O}(n \log n)$ 8c39c9, 17 lines

```
// assumes points are long long, long double probably should work, but is slow
pair<vec, vec> closest(vector<vec> v) {
    assert(v.size() > 1);
    set<vec> S;
    sort(all(v), [](vec a, vec b) { return a.y < b.y; });
    pair<ll, pair<vec, vec>> ret{LLONG_MAX, {{0,0}, {0,0}}};
    int j = 0;
    for (vec p : v) {
        vec d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
```

```
ret = min(ret, {len2(*lo - p), {*lo, p}});
S.insert(p);
}
return ret.second;
}

Faces.cpp
Description: dealing with planar graphs b0fad2, 170 lines

// returns faces.size(), if v in the outter face
int find_face(const vector<vec> &pts, const vector<vector<int>> &faces,
              vec v) {
    int res = faces.size();
    ld resarea = 0;
    vector<vec> face;
    for (int i = 0; i < (int)faces.size(); ++i) {
        face.clear();
        for (int j : faces[i]) {
            face.push_back(pts[j]);
        }
        ld area = get_area(face);
        if (sign(area) > 0) {
            if (isIn(face, v)) {
                // return faces.size(); // if faces are connected
                if (res == (int)faces.size() || area < resarea) {
                    res = i;
                    resarea = area;
                }
            }
        }
    }
    return res;
}
}
```

```
// g.size() == pts.size() + 1, so that there is one new outter face
// all previously outter faces will have g[v].size() == 0
vector<vector<int>> build_faces_graph(const vector<vec> &pts, const
vector<vector<int>> &faces) {
    vector<int> realface(faces.size());
    iota(all(realface), 0);
    vector<vec> qq;
    vector<int> ind;
    for (int i = 0; i < (int) faces.size(); ++i) {
        vector<vec> face;
        for (int j : faces[i]) {
            face.pbc(pts[j]);
        }
        ld a = get_area(face);
        if (a < 0) {
            // if only one outter face, then realface[i] = faces.size();
            // otherwise following code
            vec v = *min_element(all(face));
            v.x -= 10 * EPS;
            qq.pbc(v);
            ind.pbc(i);
            // realface[i] = find_face(pts, faces, v);
            // assert(realface[i] != i);
        }
    }
    if (1) { // slow, but easy to write
        for (int i = 0; i < (int)qq.size(); ++i) {
            int j = find_face(pts, faces, qq[i]);
            assert(j != ind[i]);
            realface[ind[i]] = j;
        }
    } else {
        vector<int> res = point_location(pts, faces, qq);
        for (int i = 0; i < (int)qq.size(); ++i) {
            int j = res[i];
            assert(j != ind[i]);
            realface[ind[i]] = j;
        }
    }
    map<pair<int, int>, int> edge2face;
    for (int i = 0; i < (int) faces.size(); ++i) {
        for (int j = 0; j < (int) faces[i].size(); ++j) {
            int a = faces[i][j];
            int b = faces[i][(j + 1) % faces[i].size()];
            edge2face[{a, b}] = realface[i];
        }
    }
```

```
    }
}
vector<vector<int>> g(faces.size() + 1);
for (auto [pp, c] : edge2face) {
    g[c].pbc(edge2face[{pp.second, pp.first}]);
}
for (auto &el : g) {
    sort(all(el));
    el.erase(unique(all(el)), el.end());
}
return g;
}

vector<vector<int>> get_faces(const vector<vec> &pts, const vector<vector<int>> &g) {
    int n = g.size();
    vector<vector<pair<int, int>>> g2(n);
    int cur_edge = 0;
    for (int i = 0; i < n; ++i) {
        for (int j : g[i]) {
            if (i < j) {
                g2[j].pbc({i, cur_edge});
                g2[i].pbc({j, cur_edge ^ 1});
                cur_edge += 2;
            }
        }
    }
    vector<int> ind(cur_edge), used(cur_edge);
    for (int i = 0; i < n; ++i) {
        sort(all(g2[i]), [&](auto a, auto b) {
            auto va = pts[a.first] - pts[i];
            auto vb = pts[b.first] - pts[i];
            return mp(half(va), (ld)0) < mp(half(vb), va % vb);
        });
        for (int j = 0; j < (int) g2[i].size(); ++j) {
            ind[g2[i][j].second] = j;
        }
    }
    vector<vector<int>> faces;
    for (int i = 0; i < n; ++i) {
        for (int ei = 0; ei < (int)g[i].size(); ++ei) {
            if (used[g2[i][ei].second]) continue;
            vector<int> face;
            int v = i;
            int e = g2[v][ei].second;
            while (!used[e]) {
                used[e] = 1;
                face.pbc(v);
                int u = g2[v][ind[e]].first;
                int newe = g2[u][ind[e ^ 1] - 1 + g2[u].size()] % g2[u].size();
                v = u;
                e = newe;
            }
            faces.push_back(face);
        }
    }
    return faces;
}

pair<vector<vec>, vector<vector<int>>> build_graph(vector<pair<vec, vec>> segs) {
    vector<vec> p;
    vector<vector<int>> g;
    map<pair<ll, ll>, int> id;
    auto getid = [&](vec v) {
        auto r = mp(ll(round(v.x * 1'000'000'000 + EPS * sign(v.x))), ll(round(v.y * 1'000'000'000 + EPS * sign(v.y))));
        if (!id.count(r)) {
            g.pbc({});
            int i = id.size();
            id[r] = i;
            p.pbc(v);
            return i;
        }
    }
    return id[r];
}

pair<vector<vec>, vector<vector<int>>> build_graph(vector<pair<vec, vec>> segs) {
    vector<vec> p;
    vector<vector<int>> g;
    map<pair<ll, ll>, int> id;
    auto getid = [&](vec v) {
        auto r = mp(ll(round(v.x * 1'000'000'000 + EPS * sign(v.x))), ll(round(v.y * 1'000'000'000 + EPS * sign(v.y))));
        if (!id.count(r)) {
            g.pbc({});
            int i = id.size();
            id[r] = i;
            p.pbc(v);
            return i;
        }
    }
    return id[r];
}

for (int i = 0; i < (int)segs.size(); ++i) {
    vector<int> cur = {getid(segs[i].first), getid(segs[i].second)};
}
```

```
for (int j = 0; j < (int)segs.size(); ++j) {
    if (i != j) {
        if (intersects(segs[i].first, segs[i].second, segs[j].first, segs[j].second)) {
            vec res;
            if (intersect(getln(segs[i].first, segs[i].second), getln(segs[j].first, segs[j].second), res)) {
                cur.pbc(getid(res));
            } else {
                if (isOnSegment(segs[i].first, segs[i].second, segs[j].first))
                    cur.pbc(getid(segs[j].first));
                if (isOnSegment(segs[i].first, segs[i].second, segs[j].second))
                    cur.pbc(getid(segs[j].second));
            }
        }
    }
}
sort(all(cur), [&](int i, int j) { return p[i] < p[j]; });
cur.erase(unique(all(cur)), cur.end());
for (int j = 1; j < (int)cur.size(); ++j) {
    g[cur[j]].pbc(cur[j - 1]);
    g[cur[j - 1]].pbc(cur[j]);
}
}
}
for (auto &el : g) {
    sort(all(el));
    el.erase(unique(all(el)), el.end());
}
return {p, g};
}
```

PointLocation.cpp

Description: Point location xd573c9d, 276 lines

const vec arb = {(int)1e9 + 228, (int)1e9 + 228}; // ne soupadaet s drygimi tochkami

bool ge(const ll& a, const ll& b) { return a >= b; }
bool le(const ll& a, const ll& b) { return a <= b; }
bool eq(const ll& a, const ll& b) { return a == b; }
bool gt(const ll& a, const ll& b) { return a > b; }
bool lt(const ll& a, const ll& b) { return a < b; }

ll vec::dot(const vec &a) const {
 return *this * a;
}
ll vec::cross(const vec &a) const {
 return *this % a;
}
ll vec::dot(const vec &a, const vec &b) const {
 return (a - *this) * (b - *this);
}
ll vec::cross(const vec &a, const vec &b) const {
 return (a - *this) % (b - *this);
}
}

struct Edge {
 vec l, r;
 auto operator<=>(const Edge &) const = default;
};

bool edge_cmp(const Edge& edge1, const Edge& edge2) {
 {
 const vec a = edge1.l, b = edge1.r;
 const vec c = edge2.l, d = edge2.r;
 int val = sign(a.cross(b, c)) + sign(a.cross(b, d));
 if (val != 0)
 return val > 0;
 val = sign(c.cross(d, a)) + sign(c.cross(d, b));
 return val < 0;
 }
}

enum EventType { DEL = 2, ADD = 3, GET = 1, VERT = 0 };

struct Event {
 EventType type;
 int pos;
}

```
bool operator<(const Event& event) const { return type < event.type;
}

};

vector<Edge> sweepline(vector<Edge> planar, vector<vec> queries) {
    {
        using vec_type = decltype(vec::x);

        // collect all x-coordinates
        auto s = set<vec_type, std::function<bool(const vec_type&, const vec_type&)>>(lt);
        for (vec p : queries)
            s.insert(p.x);
        for (auto e : planar) {
            s.insert(e.l.x);
            s.insert(e.r.x);
        }

        // map all x-coordinates to ids
        int cid = 0;
        auto id = map<vec_type, int, std::function<bool(const vec_type&, const vec_type&)>>(lt);
        for (auto x : s)
            id[x] = cid++;

        // create events
        auto t = set<Edge, decltype(*edge_cmp)>(edge_cmp);
        auto vert_cmp = [] (const pair<vec_type, int>& l, const pair<vec_type, int>& r) {
            if (!eq(l.first, r.first))
                return lt(l.first, r.first);
            return l.second < r.second;
        };
        auto vert = set<pair<vec_type, int>, decltype(vert_cmp)>(vert_cmp);
        vector<vector<Event>> events(cid);
        for (int i = 0; i < (int)queries.size(); i++) {
            int x = id[queries[i].x];
            events[x].push_back(Event{GET, i});
        }
        for (int i = 0; i < (int)planar.size(); i++) {
            int lx = id[planar[i].l.x], rx = id[planar[i].r.x];
            if (lx > rx) {
                swap(lx, rx);
                swap(planar[i].l, planar[i].r);
            }
            if (lx == rx) {
                events[lx].push_back(Event{VERT, i});
            } else {
                events[lx].push_back(Event{ADD, i});
                events[rx].push_back(Event{DEL, i});
            }
        }

        // perform sweep line algorithm
        vector<Edge> ans(queries.size(), {arb, arb});
        for (int x = 0; x < cid; x++) {
            sort(events[x].begin(), events[x].end());
            vert.clear();
            for (Event event : events[x]) {
                if (event.type == DEL) {
                    t.erase(planar[event.pos]);
                }
                if (event.type == VERT) {
                    vert.insert(make_pair(
                        min(planar[event.pos].l.y, planar[event.pos].r.y),
                        event.pos));
                }
                if (event.type == ADD) {
                    t.insert(planar[event.pos]);
                }
                if (event.type == GET) {
                    auto jt = vert.upper_bound(
                        make_pair(queries[event.pos].y, planar.size()));
                    if (jt != vert.begin()) {
                        --jt;
                        int i = jt->second;
                    }
                }
            }
        }
    }
}
```

```
        if (ge(max(planar[i].l.y, planar[i].r.y),
            queries[event.pos].y)) {
            ans[event.pos] = planar[i];
            continue;
        }
    }
    Edge e;
    e.l = e.r = queries[event.pos];
    auto it = t.upper_bound(e);
    if (it != t.begin()) {
        ans[event.pos] = *(--it);
    }
}

for (Event event : events[x]) {
    if (event.type != GET)
        continue;
    if (ans[event.pos].l != arb &&
        eq(ans[event.pos].l.x, ans[event.pos].r.x))
        continue;

    Edge e;
    e.l = e.r = queries[event.pos];
    auto it = t.upper_bound(e);
    if (it == t.begin())
        e = {arb, arb};
    else
        e = *(--it);
    if (ans[event.pos].l == arb) {
        ans[event.pos] = e;
        continue;
    }
    if (e.l == arb)
        continue;
    if (e == ans[event.pos])
        continue;
    if (id[ans[event.pos].r.x] == x) {
        if (id[e.l.x] == x) {
            if (gt(e.l.y, ans[event.pos].r.y))
                ans[event.pos] = e;
        }
        else {
            ans[event.pos] = e;
        }
    }
}
return ans;
}

struct DCEL {
    struct Edge {
        vec origin;
        int nxt;
        int twin;
        int face;
    };
    vector<Edge> body;
};

// outer face is -1, returns (1,i) is point is strictly inside face i,
// and (0,1) if point lies on the edge i
vector<pair<int, int>> point_location(DCEL planar, vector<vec> queries)
{
    vector<pair<int, int>> ans(queries.size());
    vector<Edge> planar2;
    map<Edge, int> pos;
    map<Edge, int> added_on;
    int n = planar.body.size();
    for (int i = 0; i < n; i++) {
        if (planar.body[i].face > planar.body[planar.body[i].twin].face)
            continue;
        Edge e;
        e.l = planar.body[i].origin;
        e.r = planar.body[planar.body[i].twin].origin;
        if (e.r.x < e.l.x) swap(e.l, e.r);
        added_on[e] = i;
        pos[e] =
            lt(planar.body[i].origin.x, planar.body[planar.body[i].twin].
                origin.x)
```

```
        ? planar.body[i].face
        : planar.body[planar.body[i].twin].face;
    planar2.push_back(e);
}
auto res = sweepline(planar2, queries);
for (int i = 0; i < (int)queries.size(); i++) {
    if (res[i].l == arb) {
        ans[i] = make_pair(1, -1);
        continue;
    }
    vec p = queries[i];
    vec l = res[i].l, r = res[i].r;
    if (eq(p.cross(l, r), 0) && le(p.dot(l, r), 0)) {
        ans[i] = make_pair(0, added_on[res[i]]);
        continue;
    }
    ans[i] = make_pair(1, pos[res[i]]);
}
return ans;
}

DCEL buildDCEL(const vector<vec> &pts, const vector<vector<int>> &g) {
    int n = g.size();
    vector<vector<pair<int, int>>> g2(n);
    int cur_edge = 0;
    for (int i = 0; i < n; ++i) {
        for (int j : g[i]) {
            if (i < j) {
                g2[j].pbc({i, cur_edge});
                g2[i].pbc({j, cur_edge ^ 1});
                cur_edge += 2;
            }
        }
    }
    vector<int> ind(cur_edge), used(cur_edge);
    for (int i = 0; i < n; ++i) {
        sort(all(g2[i]), [&](auto a, auto b) {
            auto va = pts[a.first] - pts[i];
            auto vb = pts[b.first] - pts[i];
            return mp(half(va), 0LL) < mp(half(vb), va % vb);
        });
        for (int j = 0; j < (int) g2[i].size(); ++j) {
            ind[g2[i][j].second] = j;
        }
    }
    using Edge = DCEL::Edge;
    vector<Edge> edges(cur_edge);
    for (int i = 0; i < cur_edge; ++i) {
        edges[i].twin = i ^ 1;
    }
    int cur_face = 0;
    for (int i = 0; i < n; ++i) {
        for (int ei = 0; ei < (int)g[i].size(); ++ei) {
            if (used[g2[i][ei].second]) continue;
            vector<vec> face;
            vector<int> inds;
            int v = i;
            int e = g2[v][ei].second;
            while (!used[e]) {
                edges[e].origin = pts[v];
                edges[e].face = cur_face;
                inds.pbc(e);
                used[e] = 1;
                face.pbc(pts[v]);
                int u = g2[v][ind[e]].first;
                int newe = g2[u][ind[e ^ 1] - 1 + g2[u].size()] % g2[u].
                    size()].second;
                edges[e].nxt = newe;
                v = u;
                e = newe;
            }
            if (sign(get_area(face)) <= 0) {
                for (int i : inds) {
                    edges[i].face = -1;
                }
            }
            else {
                ++cur_face;
            }
        }
    }
}
```

```
    return {edges};
}

Svg.cpp
Description: geometry visualizer
e9032a, 36 lines

struct SVG {
    FILE *out;
    ld sc = 50;

    void open() {
        out = fopen("image.svg", "w");
        fprintf(out, "<svg xmlns='http://www.w3.org/2000/svg' viewBox
            ='-1000 -1000 2000 2000'>\n");
    }

    void line(vec a, vec b) {
        a = a * sc, b = b * sc;
        fprintf(out, "<line x1='%Lf' y1='%Lf' x2='%Lf' y2='%Lf' stroke='
            black'/>\n", a.x, -a.y, b.x, -b.y);
    }

    void circle(vec a, ld r = -1, string col = "red") {
        r = (r == -1 ? 10 : sc * r);
        a = a * sc;
        fprintf(out, "<circle cx='%Lf' cy='%Lf' r='%Lf' fill='%s'/>\n", a
            .x, -a.y, r, col.c_str());
    }

    void text(vec a, string s) {
        a = a * sc;
        fprintf(out, "<text x='%Lf' y='%Lf' font-size='10px'>%s</text>\n"
            , a.x, -a.y, s.c_str());
    }

    void close() {
        fprintf(out, "</svg>\n");
        fclose(out);
        out = 0;
    }

    ~SVG() {
        if (out)
            close();
    }
} svg;
```

```
Delauney.cpp
Description: Fast Delaunay triangulation. Each circumcircle contains none of the
input points. There must be no duplicate points. If all points are on a line, no
triangles will be returned. Should work for doubles as well, though there may be
precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0],
...}, all counter-clockwise.
Time: O(n log n)
9e818a, 97 lines

typedef vec P;
typedef struct Quad* Q;
// using lll = _int128_t; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point
#define rep(i,a,b) for (int i=a;i<b;++i)

lll vec::cross(const vec &b) const {
    return *this % b;
}

lll vec::cross(const vec &b, const vec &c) const {
    return (b - *this) % (c - *this);
}

struct Quad {
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = len2(p), A = len2(a)-p2,
```

```
B = len2(b)-p2, C = len2(c)-p2;
return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
Q r = H ? H : new Quad{new Quad{new Quad{0}}};
H = r->o; r->r()->r() = r;
rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
r->p = orig; r->F() = dest;
return r;
}
void splice(Q a, Q b) {
swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
Q q = makeEdge(a->F(), b->p);
splice(q, a->next());
splice(q->r(), b);
return q;
}
pair<Q,Q> rec(const vector<P>& s) {
if (s.size() <= 3) {
Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
if (s.size() == 2) return { a, a->r() };
splice(a->r(), b);
auto side = s[0].cross(s[1], s[2]);
Q c = side ? connect(b, a) : 0;
return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
}
}
#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
Q A, B, ra, rb;
int half = s.size() / 2;
tie(ra, A) = rec({all(s) - half});
tie(B, rb) = rec({s.size() - half + all(s)});
while ((B->p.cross(H(A)) < 0 && (A = A->next()) ||
(A->p.cross(H(B)) > 0 && (B = B->r()->o)));
Q base = connect(B->r(), A);
if (A->p == ra->p) ra = base->r();
if (B->p == rb->p) rb = base;
#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
while (circ(e->dir->F(), H(base), e->F())) { \
Q t = e->dir; \
splice(e, e->prev()); \
splice(e->r(), e->r()->prev()); \
e->o = H; H = e; e = t; \
}
for (;) {
DEL(LC, base->r(), o); DEL(RC, base, prev());
if (!valid(LC) && !valid(RC)) break;
if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
base = connect(RC, base->r());
else
base = connect(base->r(), LC->r());
}
return { ra, rb };
}
vector<P> triangulate(vector<P> pts) {
sort(all(pts)); assert(unique(all(pts)) == pts.end());
if (pts.size() < 2) return {};
Q e = rec(pts).first;
vector<Q> q = {e};
int qi = 0;
while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
q.push_back(c->r()); c = c->next(); } while (c != e); }
ADD; pts.clear();
while (qi < q.size()) if (!(e = q[qi++])->mark) ADD;
return pts;
}
SegmentInPolygon.cpp
Description: length of longest segment inside polygon
<bits/stdc++.h>
509dfa, 40 lines
#define ll long long
```

```
using namespace std;
const int N=210;
int n,w;
double s,res,ans;
int sgn(ll x){return !x?0:(x>0?1:-1);}
struct point{
int x,y;
point operator-(point a){return {x-a.x,y-a.y};}
ll operator|(point a){return 1ll*x*a.y-1ll*y*a.x;}
double len(){return sqrt(1ll*x*x+1ll*y*y);}
}p[N];
vector<pair<double,int> >v;
double isp(point x1,point y1,point x2,point y2){
return 1.0*((x2-x1)|(y2-x1))/((y2-x2)|(y1-x1));
}
double calc(point a,point b){
v.clear(),w=s=res=0;
for(int i=1;i<=n;i++){
int x=sgn((b-a)|(p[i-1]-a)),y=sgn((b-a)|(p[i]-a));
if(x==y) continue;
v.push_back({isp(a,b,p[i-1],p[i]),(x<y?-1):(x&&?2:1)});
}
sort(v.begin(),v.end());
for(int i=0;i<(int)v.size();i++){
if(w) s+=v[i].first-v[i-1].first;
else res=max(res,s),s=0;
w+=v[i].second;
}
return max(res,s)*((b-a).len());
}
signed main(){
scanf("%d",&n);
for(int i=1;i<=n;i++) scanf("%d%d",&p[i].x,&p[i].y);
p[0]=p[n];
for(int i=1;i<=n;i++)
for(int j=i+1;j<=n;j++) ans=max(ans,calc(p[i],p[j]));
printf("%.9lf\n",ans);
return 0;
}
Math (6)
BerlekampMassey.cpp
Description: Find the shortest linear-feedback shift register
Time: O(n^2)
08eddc, 86 lines
vector<int> berlekamp(vector<int> x) {
vector<int> ls, cur;
int lf = 0, d = 0;
for (int i = 0; i < x.size(); ++i) {
ll t = 0;
for (int j = 0; j < cur.size(); ++j) {
t = (t + (ll) x[i - j - 1] * cur[j]) % MOD;
}
if ((t - x[i]) % MOD == 0)
continue;
if (cur.empty()) {
cur.resize(i + 1);
lf = i;
d = (t - x[i]) % MOD;
continue;
}
ll k = -(x[i] - t) * powmod(d, MOD - 2) % MOD;
vector<int> c(i - lf - 1);
c.push_back(k);
for (auto &j : ls)
c.push_back(-j * k % MOD);
if (c.size() < cur.size())
c.resize(cur.size());
for (int j = 0; j < cur.size(); ++j) {
c[j] = (c[j] + cur[j]) % MOD;
}
if (i - lf + (int)ls.size() >= (int)cur.size()) {
tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % MOD);
}
cur = c;
}
}
```

```
for (auto &i : cur)
i = (i % MOD + MOD) % MOD;
return cur;
}
// for a_i = 2 * a_{i-1} + a_{i-2} returns {2, 1}
// kth element of p/q as fps
int getkfps(vector<int> p, vector<int> q, ll k) {
assert(q[0] != 0);
while (k) {
auto f = q;
for (int i = 1; i < (int) f.size(); i += 2) {
f[i] = sub(0, f[i]);
}
auto p2 = conv(p, f);
auto q2 = conv(q, f);
p.clear(), q.clear();
for (int i = k % 2; i < (int) p2.size(); i += 2) {
p.pbc(p2[i]);
}
for (int i = 0; i < (int)q2.size(); i += 2) {
q.pbc(q2[i]);
}
k >>= 1;
}
return mul(p[0], inv(q[0]));
}
// vals - initials values of recurrence, c - result of belekamp on vals
int getk(const vector<int> &vals, vector<int> c, ll k) {
int d = c.size();
c.insert(c.begin(), MOD-1);
while (c.back() == 0) {
c.pop_back();
}
for (auto &el : c) {
el = sub(0, el);
}
vector<int> p(d);
copy(vals.begin(), vals.begin() + d, p.begin());
p = conv(p, c);
p.resize(d);
return getkfps(p, c, k);
}
vector<int> getmod(vector<int> a, vector<int> md) {
for (int i = a.size() - 1; i + 1 >= md.size(); --i) {
int v = mul(a[i], inv(md.back()));
for (int j = 0; j < md.size(); ++j) {
a[i - md.size() + 1 + j] = sub(a[i - md.size() + 1 + j], mul(
md[j], v));
}
a.pop_back();
}
return a;
}
GoncharFedor.cpp
Description: Calculating number of points x,y ≥ 0, Ax+By ≤ C
Time: O(log(C))
0ef10e, 11 lines
ll solve_triangle(ll A, ll B, ll C) { // x,y ≥ 0, Ax+By ≤ C
if (C < 0)
return 0;
if (A > B)
swap(A, B);
ll p = C / B;
ll k = B / A;
ll d = (C - p * B) / A;
return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
(p + 1) * (d + 1) + k * p * (p + 1) / 2;
}
CRT.cpp
Description: CRT for arbitrary modulus
28309e, 25 lines
int extgcd(int a, int b, int &x, int &y) { // define int ll
if (a == 0) {
x = 0, y = 1;
}
```



```
        return b;
    }
    int x1, y1;
    int g = extgcd(b % a, a, x1, y1);
    x = y1 - x1 * (b / a);
    y = x1;
    return g;
}

int lcm(int a, int b) { return a / __gcd(a, b) * b; }
int crt(int mod1, int mod2, int rem1, int rem2) {
    int r = (rem2 - (rem1 % mod2) + mod2) % mod2;
    int x, y;
    int g = extgcd(mod1, mod2, x, y);
    if (r % g) return -1;
    x %= mod2;
    if (x < 0) x += mod2;
    int ans = (x * (r / g)) % mod2;
    ans = ans * mod1 + rem1;
    assert(ans % mod1 == rem1);
    assert(ans % mod2 == rem2);
    return ans % lcm(mod1, mod2);
}
```

Fastmod.cpp

Description: Fast multiplication by modulo(in [0;2b)) 38ea39, 7 lines

```
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m((-1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)(__uint128_t(m) * a) >> 64) * b;
    }
};
```

ModularSqrt.cpp

Description: Calculating sqrt modulo smth Time: $\mathcal{O}(\log^2)$ 19a793, 23 lines

```
ll sqrt(ll a, ll p) {
    a %= p;
    if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p - 1) / 2, p) == 1); // e lse no so lution
    if (p % 4 == 3) return modpow(a, (p + 1) / 4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n=1)/4 works i f p % 8 == 5
    ll s = p - 1, n = 2;
    int r = 0, m;
    while (s % 2 == 0) ++r, s /= 2;
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p);
    ll b = modpow(a, s, p), g = modpow(n, s, p);
    for (;;) r = m) {
        ll t = b;
        for (m = 0; m < r && t != 1; ++m) t = t * t % p;
        if (m == 0) return x;
        ll gs = modpow(g, 1LL << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p;
    }
}
```

DiscreteLog.cpp

Description: Discrete log Time: $\mathcal{O}(\sqrt{n})$ 1cc247, 9 lines

```
ll modLog(ll a, ll b, ll m) {
    ll n = (ll)sqrt(m) + 1, e = 1, f = 1, j = 1;
    unordered_map<ll, ll> A;
    while (j <= n && (e = f * e * a % m) != b % m) A[e * b % m] = j++;
    if (e == b % m) return j;
    if (__gcd(m, e) == __gcd(m, b))
        for (int i = 2; i < n + 2; ++i)
            if (A.count(e = e * f % m)) return n * i - A[e];
    return -1;
}
```

PrimalityTest.cpp

Description: Checking primality of p Time: $\mathcal{O}(\log(C))$ acd8f1, 44 lines

```
ll binpow(ll x, ll n, ll mod) {
    ll res = 1;
    for (ll i = 1; i <= n; i *= 2, x = (__int128_t)x * x % mod) {
        if (n & i) {
            res = (__int128_t)res * x % mod;
        }
    }
    return res;
}

bool isprime(ll p) {
    if (p == 1 || p == 4) return 0;
    if (p == 2 || p == 3) return 1;
    // for(ll a: {2, 7, 61}){
    // for(ll a: {2, 325, 9375, 28178, 450775, 9780504, 1795265022}){
    for (ll a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        if (a % p == 0) continue;
        ll nw = p - 1;
        while (nw % 2 == 0) nw /= 2;
        ll x = binpow(a, nw, p); // int128
        if (x == 1) continue;
        ll last = x;
        nw *= 2;
        while (nw <= p - 1) {
            x = (__int128_t)x * x % p;
            if (x == 1) {
                if (last != p - 1) {
                    return 0;
                }
                break;
            }
            last = x;
            nw *= 2;
        }
        if (x != 1) return 0;
    }
    return 1;
}
```

XorConvolution.cpp

Description: Calculating xor-convolution of 2 vectors modulo smth Time: $\mathcal{O}(n \log(n))$ 454afd, 23 lines

```
void fwht(vector<int> &a) {
    int n = a.size();
    for (int l = 1; l < n; l <= 1) {
        for (int i = 0; i < n; i += 2 * l) {
            for (int j = 0; j < l; ++j) {
                int u = a[i + j], v = a[i + j + l];
                a[i + j] = add(u, v), a[i + j + l] = sub(u, v);
            }
        }
    }
}

// https://judge.yosupo.jp/problem/bitwise_xor_convolution
vector<int> xorconvo(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    fwht(a), fwht(b);
    int in = inv(n);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], mul(b[i], in));
    fwht(a);
    return a;
}
```

Factorization.cpp

Description: Factorizing a number real quick Time: $\mathcal{O}(n^{\frac{1}{4}})$ f0d7c6, 51 lines

```
ll gcd(ll a, ll b) {
    while (b)
        a %= b, swap(a, b);
    return a;
}

ll f(ll a, ll n) { return ((__int128_t)a * a % n + 1) % n; }

vector<ll> factorize(ll n) {
    if (n <= 1e6) { // can add primality check for speed?
        vector<ll> res;
        for (ll i = 2; i * i <= n; ++i) {
            while (n % i == 0) {
                res.pb(i);
                n /= i;
            }
        }
        if (n != 1)
            res.pb(n);
        return res;
    }
    ll x = rnd() % (n - 1) + 1;
    ll y = x;
    ll tries = 10 * sqrt(sqrt(n));
    const int C = 60;
    for (ll i = 0; i < tries; i += C) {
        ll xs = x;
        ll ys = y;
        ll m = 1;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            m = (__int128_t)m * abs(x - y) % n;
        }
        if (gcd(n, m) == 1) continue;
        x = xs, y = ys;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            ll res = gcd(n, abs(x - y));
            if (res != 1 && res != n) {
                vector<ll> v1 = factorize(res), v2 = factorize(n / res);
                for (auto j : v2)
                    v1.pb(j);
                return v1;
            }
        }
    }
    return {n};
}
```

PrimeCount.cpp

Description: counting number of primes below N Time: $\mathcal{O}(N^{\frac{2}{3}})$ a8507c, 53 lines

```
ll prime_pi(const ll N) {
    if (N <= 1) return 0;
    if (N == 2) return 1;
    const int v = sqrt(N);
    int s = (v + 1) / 2;
    vector<int> smalls(s);
    for (int i = 1; i < s; i++) smalls[i] = i;
    vector<int> roughs(s);
    for (int i = 0; i < s; i++) roughs[i] = 2 * i + 1;
    vector<ll> larges(s);
    for (int i = 0; i < s; i++) larges[i] = (N / (2 * i + 1) - 1) / 2;
    vector<bool> skip(v + 1);
    const auto divide = [](ll n, ll d) -> int { return n / d; };
    const auto half = [](int n) -> int { return (n - 1) >> 1; };
    int pc = 0;
    for (int p = 3; p <= v; p += 2)
        if (!skip[p]) {
            int q = p * p;
        }
}
```

```
    if ((ll)q * q > N) break;
    skip[p] = true;
    for (int i = q; i <= v; i += 2 * p) skip[i] = true;
    int ns = 0;
    for (int k = 0; k < s; k++) {
        int i = roughs[k];
        if (skip[i]) continue;
        ll d = (ll)i * p;
        larges[ns] = larges[k] -
            (d <= v ? larges[smalls[d >> 1] - pc]
              : smalls[half(divide(N, d))]) +
            pc;
        roughs[ns++] = i;
    }
    s = ns;
    for (int i = half(v), j = ((v / p) - 1) | 1; j >= p; j -= 2) {
        int c = smalls[j >> 1] - pc;
        for (int e = (j * p) >> 1; i >= e; i--) smalls[i] -= c;
    }
    pc++;
}
larges[0] += (ll)(s + 2 * (pc - 1)) * (s - 1) / 2;
for (int k = 1; k < s; k++) larges[0] -= larges[k];
for (int l = 1; l < s; l++) {
    ll q = roughs[l];
    ll M = N / q;
    int e = smalls[half(M / q)] - pc;
    if (e < 1 + 1) break;
    ll t = 0;
    for (int k = 1 + 1; k <= e; k++)
        t += smalls[half(divide(M, roughs[k]))];
    larges[0] += t - (ll)(e - 1) * (pc + 1 - 1);
}
return larges[0] + 1;
}
```

NTT.cpp

Description: Fast FFT!

Time: $\mathcal{O}(n \log(n))$ e7ea21, 272 lines

// Don't use Ofast, potential slow down by 2x!
// Write mint first!

```
int maxn, maxk;
vector<int> rvi;
vector<mint> wpws;

void build_fft(int _maxk) {
    maxk = _maxk;
    maxn = (1 << maxk);
    rvi.resize(maxn);
    rvi[0] = 0;
    for (int i = 1; i < maxn; i += 1) {
        rvi[i] = (rvi[i >> 1] >> 1);
        if (i & 1) {
            rvi[i] |= (1 << (maxk - 1));
        }
    }
    mint w = mint(3).pow((mod - 1) / maxn);
    mint pw = 1;
    wpws.resize(maxn);
    rep(i, maxn) {
        wpws[rvi[i]] = pw;
        pw *= w;
    }
}

void fft(vector<mint>& a, int k) {
    int n = (1 << k);
    for (int ln = n / 2; ln >= 1; ln /= 2) {
        int ln2 = ln * 2;
        for (int i = 0; i < n; i += ln2) {
            auto w = wpws[i / ln];
            for (int j = i; j < i + ln; j += 1) {
                auto u = a[j];
                auto v = a[j + ln] * w;
                a[j] = u + v;
                a[j + ln] = u - v;
            }
        }
    }
}
```

```
    }
}
rep(i, n) {
    int mrv = (rvi[i] >> (maxk - k));
    if (mrv < i) {
        swap(a[i], a[mrv]);
    }
}
}

void inv_fft(vector<mint>& a, int k) {
    fft(a, k);
    int n = (1 << k);
    mint invn = mint(n).inv();
    rep(i, n) {
        a[i] *= invn;
    }
    reverse(a.begin() + 1, a.end());
}

vector<mint> mul(vector<mint> a, vector<mint> b) {
    if (a.empty() or b.empty()) {
        return {};
    }
    auto ca = a;
    auto cb = b;
    int lna = len(a);
    int lnb = len(b);
    int k = __lg(lna + lnb - 1);
    if (lna + lnb - 1 == (1 << k) + 1) {
        auto c = mul(vector<mint>(a.begin(), a.end() - 1), b);
        c.resize(lna + lnb - 1);
        rep(j, lnb) {
            c[lna - 1 + j] += a[lna - 1] * b[j];
        }
        return c;
    }
    if (lna + lnb - 1 > (1 << k)) {
        k += 1;
    }
    int n = (1 << k);
    a.resize(n);
    b.resize(n);
    fft(a, k);
    fft(b, k);
    rep(i, n) {
        a[i] *= b[i];
    }
    inv_fft(a, k);
    a.resize(lna + lnb - 1);
    return a;
}

vector<mint> operator+(vector<mint> a, vector<mint> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int)b.size(); ++i) {
        a[i] += b[i];
    }
    return a;
}

vector<mint> operator-(vector<mint> a, vector<mint> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int)b.size(); ++i) {
        a[i] -= b[i];
    }
    return a;
}

vector<mint> inv(const vector<mint>& a, int need) {
    vector<mint> b = { a[0].inv() };
    while ((int)b.size() < need) {
        vector<mint> al = a;
        int m = b.size();
        al.resize(min((int)al.size(), 2 * m));
        b = mul(b, vector<mint>(2) - mul(al, b));
        b.resize(2 * m);
    }
    b.resize(need);
}
```

```
    return b;
}

vector<mint> mul2(vector<mint> a, vector<mint> b) {
    int lna = len(a);
    int lnb = len(b);
    int k = 0;
    while ((1 << k) < lna) {
        ++k;
    }
    int n = (1 << k);
    a.resize(n);
    reverse(all(b));
    b.resize(n);
    fft(a, k);
    fft(b, k);
    rep(i, n) a[i] *= b[i];
    inv_fft(a, k);
    vector<mint> c(lna - lnb + 1);
    rep(i, len(c)) {
        c[i] = a[lnb - 1 + i];
    }
    return c;
}

vector<mint> multipoint(vector<mint> a, vector<mint> x) {
    int n = x.size();
    int m = len(a);
    vector<vector<mint>> tree(2 * n);
    for (int i = 0; i < n; ++i) {
        tree[i + n] = { 1, 0 - x[i] };
    }
    for (int i = n - 1; i; --i) {
        tree[i] = mul(tree[2 * i], tree[2 * i + 1]);
    }
    auto tinv = inv(tree[1], m);
    a.resize(n + m - 1);
    auto c = mul2(a, tinv);
    tree[1] = c;
    for (int i = 1; i < n; i += 1) {
        auto x = tree[i + 1];
        auto y = tree[i + i + 1];
        tree[i + i] = mul2(tree[i], y);
        tree[i + i + 1] = mul2(tree[i], x);
    }
    vector<mint> res(n);
    for (int i = 0; i < n; ++i) {
        res[i] = tree[i + n][0];
    }
    return res;
}

vector<mint> div(vector<mint> a, vector<mint> b) {
    int n = a.size() - 1;
    int m = b.size() - 1;
    if (n < m) return { 0 };
    reverse(all(a));
    reverse(all(b));
    a.resize(n - m + 1);
    b.resize(n - m + 1);
    vector<mint> c = inv(b, b.size());
    vector<mint> q = mul(a, c);
    q.resize(n - m + 1);
    reverse(all(q));
    return q;
}

vector<mint> mod_poly(vector<mint> a, vector<mint> b) {
    auto res = a - mul(b, div(a, b));
    res.resize(len(b) - 1);
    return res;
}

vector<mint> deriv(vector<mint> a) {
    for (int i = 1; i < (int)a.size(); ++i) {
        a[i - 1] = a[i] * i;
    }
    a.back() = 0;
    if (a.size() > 1) {

```

```
        a.pop_back();
    }
    return a;
}

vector<mint> integ(vector<mint> a) {
    a.push_back(0);
    for (int i = (int)a.size() - 1; i; --i) {
        a[i] = a[i - 1] * mint(i).inv();
    }
    a[0] = 0;
    return a;
}

vector<mint> log(vector<mint> a, int n) {
    auto res = integ(mul(deriv(a), inv(a, n)));
    res.resize(n);
    return res;
}

vector<mint> exp(vector<mint> a, int need) {
    vector<mint> b = { 1 };
    while ((int)b.size() < need) {
        vector<mint> a1 = a;
        int m = b.size();
        a1.resize(min((int)a1.size(), 2 * m));
        a1[0] += 1;
        b = mul(b, a1 - log(b, 2 * m));
        b.resize(2 * m);
    }
    b.resize(need);
    return b;
}

vector<mint> gf_projection(vector<mint> f) { // ensure that f[0]=0
    int lnf = len(f);
    int n = 1;
    while (n < len(f)) n *= 2;
    vector<mint> g(n);
    g[n - lnf] = 1;
    f.resize(n);
    rep(i, n) f[i] = 0 - f[i];
    int m = 1;
    while (n > 1) {
        f.resize(4 * n * m);
        f[2 * n * m] = 1;
        g.resize(4 * n * m);
        fft(f);
        fft(g);
        auto q = f;
        rotate(q.begin(), q.begin() + 2 * n * m, q.end());
        vector<mint> gf(4 * n * m), ff(4 * n * m);
        rep(i, 4 * n * m) {
            gf[i] = g[i] * q[i];
            ff[i] = f[i] * q[i];
        }
        inv_fft(gf);
        inv_fft(ff);
        ff[0] -= 1;
        f.assign(2 * n * m, 0);
        g.assign(2 * n * m, 0);
        rep(i, n / 2) rep(j, 2 * m) {
            f[j * n + i] = ff[j * (2 * n) + 2 * i];
            g[j * n + i] = gf[j * (2 * n) + 2 * i + 1];
        }
        n /= 2; m *= 2;
    }
    vector<mint> res(m);
    rep(i, m) {
        res[i] = g[2 * i];
    }
    reverse(all(res));
    res.resize(lnf);
    return res;
}
```

AndConvolution.cpp

Description: Calculating and-convolution modulo smth

Time: $\mathcal{O}(n \log(n))$ 5dedf4, 24 lines

```
void conv(vector<int> &a, bool x) {
    int n = a.size();
    for (int j = 0; (1 << j) < n; ++j) {
        for (int i = 0; i < n; ++i) {
            if (!(i & (1 << j))) {
                if (x)
                    a[i] = add(a[i], a[i | (1 << j)]);
                else
                    a[i] = sub(a[i], a[i | (1 << j)]);
            }
        }
    }
}

vector<int> andcon(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    conv(a, 1), conv(b, 1);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], b[i]);
    conv(a, 0);
    return a;
}
```

SubsetConvolution.cpp

Description: subset convolution

Time: $\mathcal{O}(2^n * n^2)$ (500 ms n = 20 with pragms) a47122, 39 lines

```
void transform(int n, int N, vector<int>& b, const vector<int>& a,
               const vector<int>& pc, bool rev) {
    if (!rev) {
        b.assign(N << n, 0);
        for(int i = 0; i < (int)a.size(); ++i) b[pc[i] + i*N] = a[i];
    }
    for(int w = 1; w <= (1<<n); ++w) {
        for(int d = 0; !(w&(1<<d)); ++d) {
            int W = N * (w - (1<<d)), dd = N<<d;
            for(int i = N * (w - (2<<d)); i < W; ++i) {
                if (!rev) b[i + dd] = add(b[i + dd], b[i]);
                else b[i + dd] = sub(b[i + dd], b[i]);
            }
        }
    }
}

vector<int> SubsetConvolution(const vector<int>& a, const vector<int>& b)
{
    int n = 0;
    while((1 << n) < max(a.size(),b.size())) n++;
    int N = n+1;
    vector<int> pc(1<<n,0);
    for(int i = 1; i < (1<<n); ++i) pc[i] = pc[i - (i&-i)] + 1;
    vector<int> bufA, bufB;
    transform(n, N, bufA, a, pc, false);
    transform(n, N, bufB, b, pc, false);
    for(int i = 0; i < (1<<n); i++) {
        int I = i * N;
        vector<int> Q(N);
        for(int ja = 0; ja <= pc[i]; ++ja) {
            for(int jb = pc[i] - ja, x = min(n - ja, pc[i]); jb <= x; ++jb){
                Q[ja + jb] = add(Q[ja + jb], mul(bufA[ja + I], bufB[jb + I]));
            }
        }
        copy(Q.begin(), Q.end(), bufA.begin() + I);
    }
    transform(n, N, bufA, a, pc, true);
    vector<int> res(1<<n);
    for(int i = 0; i<(1<<n); ++i) res[i] = bufA[pc[i] + i*N];
    return res;
}
```

Simplex.cpp

Description: Simplex

Time: exponential XD(ok for 200-300 variables/bounds) 4dda3c, 99 lines

```
/* solver for linear programs of the form
maximize c^T x, subject to A x <= b, x >= 0
outputs target function for optimal solution and
the solution by reference
if unbounded above : returns inf, if infeasible : returns -inf
create Simplex_Steep<ld> LP(A, b, c), then call LP. Solve (x)
*/
template <typename DOUBLE>
struct Simplex_Steep {
    using VD = vector<DOUBLE>;
    using VVD = vector<VD>;
    using VI = vector<int>;
    DOUBLE EPS = 1e-12;
    int m, n;
    VI B, N;
    VVD D;
    Simplex_Steep(const VVD &A, const VD &b, const VD &c)
        : m(b.size()), n(c.size()), B(m), N(n + 1), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++)
            for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) {
            B[i] = n + i;
            D[i][n] = -1;
            D[i][n + 1] = b[i];
        }
        for (int j = 0; j < n; j++) {
            N[j] = j;
            D[m][j] = -c[j];
        }
        N[n] = -1;
        D[m + 1][n] = 1;
    }
    void Pivot(int r, int s) {
        for (int i = 0; i < m + 2; i++)
            if (i != r)
                for (int j = 0; j < n + 2; j++)
                    if (j != s) D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n + 2; j++)
            if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m + 2; i++)
            if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }
    bool Simplex(int phase) {
        int x = m + (int)(phase == 1);
        while (true) {
            int s = -1;
            DOUBLE c_val = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                DOUBLE norm_sq = 0;
                for (int k = 0; k <= m; k++) norm_sq += D[k][j] * D[k][j];
                norm_sq = max(norm_sq, EPS);
                DOUBLE c_val_j = D[x][j] / sqrtl(norm_sq);
                if (s == -1 || c_val_j < c_val ||
                    (c_val == c_val_j && N[j] < N[s])) {
                    s = j;
                    c_val = c_val_j;
                }
            }
            if (D[x][s] >= -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] <= EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                    D[i][n + 1] / D[i][s] == D[r][n + 1] / D[r][s] &&
                    B[i] < B[r])
                    r = i;
            }
            if (r == -1) return false;
            Pivot(r, s);
        }
    }
};
```

```

}
DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++)
        if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] <= -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
            return -numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++)
            if (B[i] == -1) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                    if (s == -1 || D[i][j] < D[i][s] ||
                        (D[i][j] == D[i][s] && N[j] < N[s]))
                        s = j;
                Pivot(i, s);
            }
    }

    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++)
        if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
}
};
```

DeterminantLd.cpp

Description: Determinant in ld1a6123, 18 lines

```
double det(vector<vector<double>>& a) {
    int n = sz(a);
    double res = 1;
    for (int i = 0; i < n; ++i) {
        int b = i;
        for (int j = i + 1; j < n; ++j)
            if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        for (int j = i + 1; j < n; ++j) {
            double v = a[j][i] / a[i][i];
            if (v != 0)
                for (int k = i + 1; k < n; ++k) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}
```

DeterminantInt.cpp

Description: Determinant in ints c2ab5a, 19 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a);
    ll ans = 1;
    for (int i = 0; i < n; ++i) {
        for (int j = i + 1; j < n; ++j) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i];
                if (t)
                    for (int k = i; k < n; ++k)
                        a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            }
        }
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    }
    return (ans + mod) % mod;
}
```

TridiagSLE.cpp

Description: Tridiagonal SLE solver(didnt test yet) Time: O(N)532e1d, 16 lines

```
vector<ld> trisle(vector<ld> a, vector<ld> b, vector<ld> c) {
    // a[i] * x[i - 1] + c[i] * x[i] + b[i] * x[i + 1] == f[i]
    int n = a.size(); // a[0] == 0, b[n - 1] == 0
```

```
    alpha[1] = -(ld)b[0] / c[0];
    beta[1] = (ld)f[0] / c[0];
    for (int i = 1; i < n - 1; i++) {
        ld zn = (ld)a[i] * alpha[i] + c[i];
        alpha[i + 1] = -(ld)b[i] / zn;
        beta[i + 1] = (f[i] - (ld)a[i] * beta[i]) / zn;
    }
    x[n - 1] = (f[n - 1] - a[n - 1] * beta[n - 1]) /
        (a[n - 1] * alpha[n - 1] + c[n - 1]);
    for (int i = n - 2; i >= 0; i--)
        x[i] = alpha[i + 1] * x[i + 1] + beta[i + 1];
    return x;
}
```

Gauss.cpp

Description: Solving linear systems Time: O(n^3)a45131, 35 lines

```
typedef vector<double> vd;
const double eps = 1e-12; // rep(i,a,b) = for(int i=a;i<b;++i)
int gauss(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m);
    iota(all(col), 0);
    rep(i, 0, n) {
        double v, bv = 0;
        rep(r, i, n) rep(c, i, m) if ((v = fabs(A[r][c])) > bv) br = r, bc = c, bv = v;

        if (bv <= eps) {
            rep(j, i, n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j, 0, n) swap(A[j][i], A[j][bc]);
        bv = 1 / A[i][i];
        rep(j, i + 1, n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k, i + 1, m) A[j][k] -= fac * A[i][k];
        }
        rank++;
    }
    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i];
        rep(j, 0, i) b[j] -= A[j][i] * b[i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

GaussBinary.cpp

Description: Solving linear systems modulo 2 (returns solution and rank) Time: O(n^3/w)8f1f50, 37 lines

```
using bt = bitset<maxn>;
pair<bt, int> gauss_binary(vector<bt> a, int n, int m) {
    int row = 0;
    for (int col = 0; col < m and row < n; col += 1) {
        for (int i = row; i < n; i += 1) {
            if (a[i][col] != 0) {
                swap(a[row], a[i]);
                break;
            }
        }
        if (a[row][col] == 0)
            continue;
        for (int i = row + 1; i < n; i += 1) {
            if (a[i][col] == 0)
                continue;
        }
        a[i] ^= a[row];
    }
    ++row;
}
```

```

}
for (int i = row; i < n; i += 1) {
    if (a[i][m] != 0) {
        return { bt(i), -1 };
    }
}
bt cur;
cur.reset();
for (int i = row - 1; i >= 0; i -= 1) {
    int value = (cur & a[i]).count() % 2;
    if (value != a[i][m]) {
        int pos = a[i].Find_first();
        assert(pos < m);
        cur.set(pos);
    }
}
return { cur, row };
}
```

PolyInter.cpp

Description: Interpolating polynomials Time: O(n^2)4edad5, 14 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    for (int k = 0; k < n - 1; ++k)
        for (int i = k + 1; i < n; ++i) y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0;
    temp[0] = 1;
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i) {
            res[i] += y[k] * temp[i];
            swap(last, temp[i]);
            temp[i] -= last * x[k];
        }
    return res;
}
```

CharPoly.cpp

Description: det(a - xI)666c0e, 37 lines

```
vector<int> CharacteristicPolynomial(vector<vector<int>> a) {
    int n = a.size();
    for(int j = 0; j < n - 2; j++) {
        for(int i = j + 1; i < n; i++) {
            if(a[i][j] != 0) {
                swap(a[j + 1], a[i]);
                for(int k = 0; k < n; k++) swap(a[k][j + 1], a[k][i]);
                break;
            }
        }
        if(a[j + 1][j] != 0) {
            int flex = inv(a[j + 1][j]);
            for(int i = j + 2; i < n; i++) {
                if(a[i][j] == 0) continue;
                int coe = mul(flex, a[i][j]);
                for(int l = j; l < n; l++) a[i][l] = sub(a[i][l], mul(coe, a[j + 1][l]));
            }
            for(int k = 0; k < n; k++) a[k][j + 1] = add(a[k][j + 1], mul(coe, a[k][i]));
        }
    }
    vector<vector<int>> p(n + 1);
    p[0] = {1};
    for(int i = 1; i <= n; i++) {
        p[i].resize(i + 1);
        for(int j = 0; j < i; j++) {
            p[i][j + 1] = sub(p[i][j + 1], p[i - 1][j]);
            p[i][j] = add(p[i][j], mul(p[i - 1][j], a[i - 1][i - 1]));
        }
        int x = 1;
        for(int m = 1; m < i; m++) {
            x = mul(x, sub(0, a[i - m][i - m - 1]));
            int coe = mul(x, a[i - m - 1][i - 1]);
            for(int j = 0; j < i - m; j++) p[i][j] = add(p[i][j], mul(coe, p[i - m - 1][j]));
        }
    }
}
```

```
    return p[n];
}

FloorSum.cpp
Description: finds  $\sum_{x=0}^{n-1} \lfloor (kx+b)/m \rfloor$ . Require  $k \geq 0, b \geq 0, m \geq 0, 0 \leq n < m$ . 326c, 041 lines

template<typename T>
T floor_sum(T k, T b, T m, T n) {
    if (k == 0) {
        return (b / m) * n;
    }
    if (k >= m || b >= m) {
        return n * (n - 1) / 2 * (k / m) + n * (b / m) + floor_sum(k % m,
            b % m, m, n);
    }
    T ymax = (k * (n - 1) + b) / m;
    return n * ymax - floor_sum(m, m + k - b - 1, k, ymax);
}
```

```
WaysCount.cpp
Description: Find number of right-up paths from (0, 0) to (x, y), not touching
lines y=x+l and y=x+r Time: O((x+y)/(r-l)) 57f1b0, 12 lines

mint flex(ll x, ll y, ll l, ll r) {
    if (l >= 0 or r <= 0) {
        return 0;
    }
    ll n = x + y;
    mint res = 0;
    for (ll k = -(n / (r - l)); k <= n / (r - l); k += 1) {
        res += cnk(n, x + k * (r - l));
        res -= cnk(n, y - r + k * (r - l));
    }
    return res;
}
```

6.1 Fun things

$$\begin{aligned} ClassesCount &= \frac{1}{|G|} \sum_{\pi \in G} I(\pi) \\ ClassesCount &= \frac{1}{|G|} \sum_{\pi \in G} k^{C(\pi)} \\ \text{Stirling 2kind - count of partitions of } n \text{ objects into } k \text{ nonempty sets:} \\ S(n, k) &= S(n-1, k-1) + kS(n-1, k) \\ S(n, k) &= \sum_{j=0}^{n-1} \binom{n-1}{j} S(j, k-1) \\ S(n, k) &= \frac{1}{k!} \sum_{j=0}^k (-1)^{k+j} \binom{k}{j} j^n \\ n! &\approx \sqrt{2n\pi} \left(\frac{n}{e}\right)^n \\ \binom{n}{k} &\equiv \prod_i \binom{n_i}{k_i}, n_i, k_i - \text{digits of } n, k \text{ in } p\text{-adic system} \\ \int_a^b f(x) dx &\approx \frac{b-a}{6} (f(a) + 4f(\frac{a+b}{2}) + f(b)) \\ x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)}, O(loglog) \\ G(n) &= n \oplus (n \gg 1) \\ g(n) &= \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} g(d) \mu(\frac{n}{d}) \\ \sum_{d|n} \mu(d) &= [n=1], \mu(1) = 1, \mu(p) = -1, \mu(p^k) = 0 \\ \sin(a \pm b) &= \sin a \cos b \pm \sin b \cos a \\ \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\ \operatorname{tg}(a \pm b) &= \frac{\operatorname{tg} a \pm \operatorname{tg} b}{1 \mp \operatorname{tg} a \operatorname{tg} b} \\ \operatorname{ctg}(a \pm b) &= \frac{\operatorname{ctg} a \operatorname{ctg} b \mp 1}{\operatorname{ctg} b \pm \operatorname{ctg} a} \\ \sin \frac{a}{2} &= \pm \sqrt{\frac{1 - \cos a}{2}} \\ \cos \frac{a}{2} &= \pm \sqrt{\frac{1 + \cos a}{2}} \\ \operatorname{tg} \frac{a}{2} &= \frac{\sin a}{1 - \cos a} = \frac{1 - \cos a}{\sin a} \\ \sin \alpha &= \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}} \\ \cos \alpha &= \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}} \\ \operatorname{tg} \alpha &= \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 - \operatorname{tg}^2 \frac{\alpha}{2}} \\ \sin^2 \alpha &= \frac{1 - \cos 2\alpha}{2} \\ \sin^3 \alpha &= \frac{3 \sin \alpha - \sin 3\alpha}{4} \\ \cos^2 \alpha &= \frac{1 + \cos 2\alpha}{2} \\ \cos^3 \alpha &= \frac{3 \cos \alpha + \cos 3\alpha}{4} \\ \sin a \sin b &= \frac{\cos(a-b) - \cos(a+b)}{2} \\ \sin a \cos b &= \frac{\sin(a-b) + \sin(a+b)}{2} \\ \cos a \cos b &= \frac{\cos(a-b) + \cos(a+b)}{2} \\ 1 \text{ jan } 2000 - \text{ saturday, } 1 \text{ jan } 1900 - \text{ monday, } 14 \text{ apr } 1961 - \text{ friday} \end{aligned}$$

Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751, 22:4506715738447323, 23:44152005855084346
Fibonacci: 45:1134903170. 46:1836311903(max int), 91: 4660046610375530309
Highly composite numbers:
 $\leq 1000 : d(840) = 32, \leq 10^4 : d(9240) = 64, \leq 10^5 : d(83160) = 128, \leq 10^6 : d(720720) = 240, \leq 10^7 : d(8648640) = 448, \leq 10^8 : d(91891800) = 768, \leq 10^9 : d(931170240) = 1344, \leq 10^{11} : d(97772875200) = 4032, \leq 10^{15} : d(866421317361600) = 26880, \leq 10^{18} : d(897612484786617600) = 103680$
BEST Theorem:
 $ec(G) = \#SpanningTrees(G) \cdot \prod_{v \in V} (deg(v) - 1)!$
Erdos: Graph exists
 $\Leftrightarrow d_1 \geq .. \geq d_n, \forall k \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n min(d_i, k)$
Pick: $Area = Interior + \frac{Bounds}{2} - 1$
Euler: $V - E + F = 1 + C$
Kirchhoff: put degree on diagonal, -1 for each edge, cut out first row + column, calc det - result is $\#SpanningTrees$
Tree Hash: for vertex v calculate $\prod_i (c_i + d_{h_i})$, where c_i - hash of ith child, d_{h_i} - random number associated to depth of current child
Get position of Gray Code g: int n = 0; for (; g; g>>= 1) n xor= g; return n;

Table of Basic Integrals (7)

Basic Forms

∫ x^n dx = 1/(n+1) x^{n+1}, n ≠ -1 (7.1)

∫ 1/x dx = ln |x| (7.2)

∫ u dv = uv - ∫ v du (7.3)

∫ 1/(ax+b) dx = 1/a ln |ax+b| (7.4)

Integrals of Rational Functions

∫ 1/(x+a)^2 dx = -1/(x+a) (7.5)

∫ (x+a)^n dx = (x+a)^{n+1}/(n+1), n ≠ -1 (7.6)

∫ x(x+a)^n dx = (x+a)^{n+1}((n+1)x-a)/((n+1)(n+2)) (7.7)

∫ 1/(1+x^2) dx = tan^-1 x (7.8)

∫ 1/(a^2+x^2) dx = 1/a tan^-1 x/a (7.9)

∫ x/(a^2+x^2) dx = 1/2 ln |a^2+x^2| (7.10)

∫ x^2/(a^2+x^2) dx = x - a tan^-1 x/a (7.11)

∫ x^3/(a^2+x^2) dx = 1/2 x^2 - 1/2 a^2 ln |a^2+x^2| (7.12)

∫ 1/(ax^2+bx+c) dx = 2/(sqrt(4ac-b^2)) tan^-1 (2ax+b)/sqrt(4ac-b^2) (7.13)

∫ 1/((x+a)(x+b)) dx = 1/(b-a) ln (a+x)/(b+x), a ≠ b (7.14)

∫ x/(x+a)^2 dx = a/(a+x) + ln |a+x| (7.15)

∫ x/(ax^2+bx+c) dx = 1/2a ln |ax^2+bx+c| - b/(a sqrt(4ac-b^2)) tan^-1 (2ax+b)/sqrt(4ac-b^2) (7.16)

Integrals with Roots

∫ sqrt(x-a) dx = 2/3 (x-a)^{3/2} (7.17)

∫ 1/sqrt(x±a) dx = 2 sqrt(x±a) (7.18)

∫ 1/sqrt(a-x) dx = -2 sqrt(a-x) (7.19)

∫ x sqrt(x-a) dx = { 2a/3 (x-a)^{3/2} + 2/5 (x-a)^{5/2}, or 2/3 x(x-a)^{3/2} - 4/15 (x-a)^{5/2}, or 2/15 (2a+3x)(x-a)^{3/2} } (7.20)

∫ sqrt(ax+b) dx = (2b/3a + 2x/3) sqrt(ax+b) (7.21)

∫ (ax+b)^{3/2} dx = 2/5a (ax+b)^{5/2} (7.22)

∫ x/sqrt(x±a) dx = 2/3 (x ∓ 2a) sqrt(x±a) (7.23)

∫ sqrt(x/(a-x)) dx = -sqrt(x(a-x)) - a tan^-1 sqrt(x(a-x))/(x-a) (7.24)

∫ sqrt(x/(a+x)) dx = sqrt(x(a+x)) - a ln [sqrt(x)+sqrt(x+a)] (7.25)

∫ x sqrt(ax+b) dx = 2/15a^2 (-2b^2+abx+3a^2x^2) sqrt(ax+b) (7.26)

∫ sqrt(x(ax+b)) dx = 1/(4a^{3/2}) [(2ax+b) sqrt(ax(ax+b)) - b^2 ln |a sqrt(x)+sqrt(a(ax+b))|] (7.27)

∫ sqrt(x^3(ax+b)) dx = [b/12a - b^2/(8a^2x) + x/3] sqrt(x^3(ax+b)) + b^3/(8a^{5/2}) ln |a sqrt(x)+sqrt(a(ax+b))| (7.28)

∫ sqrt(x^2±a^2) dx = 1/2 x sqrt(x^2±a^2) ± 1/2 a^2 ln |x+sqrt(x^2±a^2)| (7.29)

∫ sqrt(a^2-x^2) dx = 1/2 x sqrt(a^2-x^2) + 1/2 a^2 tan^-1 x/sqrt(a^2-x^2) (7.30)

∫ x sqrt(x^2±a^2) dx = 1/3 (x^2±a^2)^{3/2} (7.31)

∫ 1/sqrt(x^2±a^2) dx = ln |x+sqrt(x^2±a^2)| (7.32)

∫ 1/sqrt(a^2-x^2) dx = sin^-1 x/a (7.33)

∫ x/sqrt(x^2±a^2) dx = sqrt(x^2±a^2) (7.34)

∫ x/sqrt(a^2-x^2) dx = -sqrt(a^2-x^2) (7.35)

∫ x^2/sqrt(x^2±a^2) dx = 1/2 x sqrt(x^2±a^2) ± 1/2 a^2 ln |x+sqrt(x^2±a^2)| (7.36)

∫ sqrt(ax^2+bx+c) dx = (b+2ax)/(4a) sqrt(ax^2+bx+c) + 4ac-b^2/(8a^{3/2}) ln |2ax+b+2 sqrt(a(ax^2+bx+c))| (7.37)

∫ x sqrt(ax^2+bx+c) dx = 1/(48a^{5/2}) (2 sqrt(a) sqrt(ax^2+bx+c) (-3b^2+2abx+8a(c+ax^2)) + 3(b^3-4abc) ln |b+2ax+2 sqrt(a) sqrt(ax^2+bx+c)|) (7.38)

∫ 1/sqrt(ax^2+bx+c) dx = 1/sqrt(a) ln |2ax+b+2 sqrt(a(ax^2+bx+c))| (7.39)

∫ x/sqrt(ax^2+bx+c) dx = 1/a sqrt(ax^2+bx+c) - b/(2a^{3/2}) ln |2ax+b+2 sqrt(a(ax^2+bx+c))| (7.40)

∫ dx/((a^2+x^2)^{3/2}) = x/(a^2 sqrt(a^2+x^2)) (7.41)

Integrals with Logarithms

∫ ln ax dx = x ln ax - x (7.42)

∫ x ln x dx = 1/2 x^2 ln x - x^2/4 (7.43)

∫ x^2 ln x dx = 1/3 x^3 ln x - x^3/9 (7.44)

∫ x^n ln x dx = x^{n+1} (ln x/(n+1) - 1/((n+1)^2)), n ≠ -1 (7.45)

∫ ln x/x dx = 1/2 (ln ax)^2 (7.46)

∫ ln x/x^2 dx = -1/x - ln x/x (7.47)

∫ ln(ax+b) dx = (x+b/a) ln(ax+b) - x, a ≠ 0 (7.48)

∫ ln(x^2+a^2) dx = x ln(x^2+a^2) + 2a tan^-1 x/a - 2x (7.49)

∫ ln(x^2-a^2) dx = x ln(x^2-a^2) + a ln (x+a)/(x-a) - 2x (7.50)

∫ ln (ax^2+bx+c) dx = 1/a sqrt(4ac-b^2) tan^-1 (2ax+b)/sqrt(4ac-b^2) - 2x + (b/2a+x) ln (ax^2+bx+c) (7.51)

Integrals with Exponentials

Integrals with Trigonometric Functions

∫ x ln(ax + b) dx = (bx)/(2a) - 1/4 x^2 + 1/2 (x^2 - (b^2)/(a^2)) ln(ax + b) (7.52)

∫ x ln(a^2 - b^2 x^2) dx = -1/2 x^2 + 1/2 (x^2 - (a^2)/(b^2)) ln(a^2 - b^2 x^2) (7.53)

∫ (ln x)^2 dx = 2x - 2x ln x + x(ln x)^2 (7.54)

∫ (ln x)^3 dx = -6x + x(ln x)^3 - 3x(ln x)^2 + 6x ln x (7.55)

∫ x(ln x)^2 dx = (x^2)/4 + 1/2 x^2 (ln x)^2 - 1/2 x^2 ln x (7.56)

∫ x^2 (ln x)^2 dx = (2x^3)/(27) + 1/3 x^3 (ln x)^2 - 2/9 x^3 ln x (7.57)

∫ e^{ax} dx = 1/a e^{ax} (7.58)

∫ √x e^{ax} dx = 1/a √x e^{ax} + (i√π)/(2a^{3/2}) erf(i√ax), where erf(x) = 2/√π ∫_0^x e^{-t^2} dt (7.59)

∫ x e^x dx = (x - 1)e^x (7.60)

∫ x e^{ax} dx = (x/a - 1/a^2) e^{ax} (7.61)

∫ x^2 e^x dx = (x^2 - 2x + 2) e^x (7.62)

∫ x^2 e^{ax} dx = (x^2/a - 2x/a^2 + 2/a^3) e^{ax} (7.63)

∫ x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x (7.64)

∫ x^n e^{ax} dx = (x^n e^{ax})/a - n/a ∫ x^{n-1} e^{ax} dx (7.65)

∫ x^n e^{ax} dx = ((-1)^n)/(a^{n+1}) Γ[1 + n, -ax], where Γ(a, x) = ∫_x^∞ t^{a-1} e^{-t} dt (7.66)

∫ e^{ax^2} dx = -(i√π)/(2√a) erf(ix√a) (7.67)

∫ e^{-ax^2} dx = (√π)/(2√a) erf(x√a) (7.68)

∫ x e^{-ax^2} dx = -1/(2a) e^{-ax^2} (7.69)

∫ x^2 e^{-ax^2} dx = 1/4 √(π/a^3) erf(x√a) - x/(2a) e^{-ax^2} (7.70)

∫ sin ax dx = -1/a cos ax (7.71)

∫ sin^2 ax dx = x/2 - (sin 2ax)/(4a) (7.72)

∫ sin^3 ax dx = -(3 cos ax)/(4a) + (cos 3ax)/(12a) (7.73)

∫ sin^n ax dx = -1/a cos ax {}_2F_1[1/2, (1-n)/2, 3/2, cos^2 ax] (7.74)

∫ cos ax dx = 1/a sin ax (7.75)

∫ cos^2 ax dx = x/2 + (sin 2ax)/(4a) (7.76)

∫ cos^3 ax dx = (3 sin ax)/(4a) + (sin 3ax)/(12a) (7.77)

∫ cos^p ax dx = -1/(a(1+p)) cos^{1+p} ax × {}_2F_1[1/2+p, 1/2, 3/2+p, cos^2 ax] (7.78)

∫ cos x sin x dx = 1/2 sin^2 x + c_1 = -1/2 cos^2 x + c_2 = -1/4 cos 2x + c_3 (7.79)

∫ cos ax sin bx dx = (cos[(a-b)x])/2(a-b) - (cos[(a+b)x])/2(a+b), a ≠ b (7.80)

∫ sin^2 ax cos bx dx = -(sin[(2a-b)x])/4(2a-b) + (sin bx)/(2b) - (sin[(2a+b)x])/4(2a+b) (7.81)

∫ sin^2 x cos x dx = 1/3 sin^3 x (7.82)

∫ cos^2 ax sin bx dx = (cos[(2a-b)x])/4(2a-b) - (cos bx)/(2b) - (cos[(2a+b)x])/4(2a+b) (7.83)

∫ cos^2 ax sin ax dx = -1/3a cos^3 ax (7.84)

∫ sin^2 ax cos^2 bxdx = x/4 - (sin 2ax)/(8a) - (sin[2(a-b)x])/16(a-b) + (sin 2bx)/(8b) - (sin[2(a+b)x])/16(a+b) (7.85)

∫ sin^2 ax cos^2 ax dx = x/8 - (sin 4ax)/(32a) (7.86)

∫ tan ax dx = -1/a ln cos ax (7.87)

∫ tan^2 ax dx = -x + 1/a tan ax (7.88)

∫ tan^n ax dx = (tan^{n+1} ax)/(a(1+n)) × {}_2F_1((n+1)/2, 1, (n+3)/2, -tan^2 ax) (7.89)

∫ tan^3 ax dx = 1/a ln cos ax + 1/(2a) sec^2 ax (7.90)

∫ sec x dx = ln |sec x + tan x| = 2 tanh^{-1} (tan x/2) (7.91)

∫ sec^2 ax dx = 1/a tan ax (7.92)

∫ sec^3 x dx = 1/2 sec x tan x + 1/2 ln |sec x + tan x| (7.93)

∫ sec x tan x dx = sec x (7.94)

∫ sec^2 x tan x dx = 1/2 sec^2 x (7.95)

∫ sec^n x tan x dx = 1/n sec^n x, n ≠ 0 (7.96)

∫ csc x dx = ln |tan x/2| = ln |csc x - cot x| + C (7.97)

∫ csc^2 ax dx = -1/a cot ax (7.98)

∫ csc^3 x dx = -1/2 cot x csc x + 1/2 ln |csc x - cot x| (7.99)

∫ csc^n x cot x dx = -1/n csc^n x, n ≠ 0 (7.100)

∫ sec x csc x dx = ln |tan x| (7.101)

Products of Trigonometric Functions and Monomials

$$\int x \cos x \, dx = \cos x + x \sin x \quad (7.102)$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \quad (7.103)$$

$$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2) \sin x \quad (7.104)$$

$$\int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \quad (7.105)$$

$$\int x^n \cos x \, dx = -\frac{1}{2}(i)^{n+1} [\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)] \quad (7.106)$$

$$\int x^n \cos ax \, dx = \frac{1}{2}(ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)] \quad (7.107)$$

$$\int x \sin x \, dx = -x \cos x + \sin x \quad (7.108)$$

$$\int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (7.109)$$

$$\int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x \quad (7.110)$$

$$\int x^2 \sin ax \, dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (7.111)$$

$$\int x^n \sin x \, dx = -\frac{1}{2}(i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)] \quad (7.112)$$

$$\int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x \quad (7.113)$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x \quad (7.114)$$

$$\int x \tan^2 x \, dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \quad (7.115)$$

$$\int x \sec^2 x \, dx = \ln \cos x + x \tan x \quad (7.116)$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (7.117)$$

$$\int e^{bx} \sin ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \quad (7.118)$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (7.119)$$

$$\int e^{bx} \cos ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \quad (7.120)$$

$$\int x e^x \sin x \, dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (7.121)$$

$$\int x e^x \cos x \, dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (7.122)$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \, dx = \frac{1}{a} \sinh ax \quad (7.123)$$

$$\int e^{ax} \cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \quad (7.124)$$

$$\int \sinh ax \, dx = \frac{1}{a} \cosh ax \quad (7.125)$$

$$\int e^{ax} \sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \quad (7.126)$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \quad (7.127)$$

$$\int e^{ax} \tanh bx \, dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1 \left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\ \quad - \frac{1}{a} e^{ax} {}_2F_1 \left[1, \frac{a}{2b}, 1 + \frac{a}{2b}, -e^{2bx} \right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \quad (7.128)$$

$$\int \cos ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] \quad (7.129)$$

$$\int \cos ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] \quad (7.130)$$

$$\int \sin ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] \quad (7.131)$$

$$\int \sin ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \quad (7.132)$$

$$\int \sinh ax \cosh ax \, dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (7.133)$$

$$\int \sinh ax \cosh bx \, dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \quad (7.134)$$

Problem	Status	Comment	pandapythoner	mangooste	allvik
A - 1					
B - 2					
C - 3					
D - 4					
E - 5					
F - 6					
G - 7					
H - 8					
I - 9					
J - 10					
K - 11					
L - 12					
M - 13					
N - 14					
O - 15					