



National Research University Higher School of Economics

# BasketBoBr

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## Contest (1)

genfolders.sh 6 lines

```
chmod +x bld*
for f in {A..Z}
do
    mkdir $f
    cp main.cpp bld/$f
done
```

bld 1 lines

```
g++ -std=c++20 -g -DLOCAL -fsanitize=address,bounds,undefined -o $1 $1.
    cpp
```

bldf 1 lines

```
g++ -std=c++20 -g -O2 -o $1 $1.cpp
```

hacks.sh 2 lines

```
UBSAN_OPTIONS=print_stacktrace=1 ./main
gdb rbreak regex
```

hash.sh 3 lines

```
# Hashes a file, ignoring all whitespace and comments.
# Use for verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[[:space:]]' | md5sum | cut -c-6
```

clion.cpp 2 lines

```
set(CMAKE_CXX_STANDARD 20)
set(CMAKE_CXX_FLAGS "-DLOCAL")
```

## C++ (2)

### GpHashtable.cpp

Description: Hash map with mostly the same API as unordered\_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided). 4 lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

const int RANDOM =
    chrono::high_resolution_clock::now().time_since_epoch().count();
struct hasher {
    int operator()(int x) const { return x ^ RANDOM; }
};

gp_hash_table<int, int, hasher> table;
```

### OrderedSet.cpp

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null\_type.

Time:  $\mathcal{O}(\log(n))$  37 lines

```
<bits/extc++.h>, <bits/stdc++.h> dff260, 37 lines

using namespace __gnu_pbds;
using namespace std;

template <typename T>
using ordered_set =
    tree<T, null_type, less<>, rb_tree_tag,
        tree_order_statistics_node_update>;

int main() {
    ordered_set<int> X;
    X.insert(1);
    X.insert(2);
    X.insert(4);
    X.insert(8);
    X.insert(16);

    assert(*X.find_by_order(1) == 2);
    assert(*X.find_by_order(2) == 4);
```

```
assert(*X.find_by_order(4) == 16);
assert(X.find_by_order(6) == X.end());

assert(X.order_of_key(-5) == 0);
assert(X.order_of_key(1) == 0);
assert(X.order_of_key(3) == 2);
assert(X.order_of_key(4) == 2);
assert(X.order_of_key(400) == 5);
// std::cout << *X.find_by_order(1) << std::endl; // 2
// std::cout << *X.find_by_order(2) << std::endl; // 4
// std::cout << *X.find_by_order(4) << std::endl; // 16
// std::cout << (end(X) == X.find_by_order(6)) << std::endl; // true

// std::cout << X.order_of_key(-5) << std::endl; // 0
// std::cout << X.order_of_key(1) << std::endl; // 0
// std::cout << X.order_of_key(3) << std::endl; // 2
// std::cout << X.order_of_key(4) << std::endl; // 2
// std::cout << X.order_of_key(400) << std::endl; // 5
return 0;
}
```

### bitset.cpp

Description: bitset 521d1f, 2 lines

```
bs._Find_first()
bs._Find_next(idx) - returns right after
```

### alloc.cpp

Description: fastalloc 8726b1, 11 lines

```
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX_MEM];
inline void *operator new(size_t n) {
    assert((mpos + n) <= MAX_MEM);
    return (void*)(mem + mpos - n);
}
void operator delete(void *) noexcept {
} // must have!
void operator delete(void *, size_t) noexcept {
} // must have!
```

### fastio.cpp

Description: fastio 79fd14, 52 lines

```
inline int readChar();
template <class T = int>
inline T readInt();
template <class T>
inline void writeInt(T x, char end = 0);
inline void writeChar(int x);
inline void writeWord(const char *s);
static const int buf_size = 4096;
inline int getChar() {
    static char buf[buf_size];
    static int len = 0, pos = 0;
    if (pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin);
    if (pos == len) return -1;
    return buf[pos++];
}
inline int readChar() {
    int c = getChar();
    while (c <= 32) c = getChar();
    return c;
}
template <class T>
inline T readInt() {
    int s = 1, c = readChar();
    T x = 0;
    if (c == '-') s = -1, c = getChar();
    while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
    return s == 1 ? x : -x;
}
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar(int x) {
    if (write_pos == buf_size)
        fwrite(write_buf, 1, buf_size, stdout), write_pos = 0;
    write_buf[write_pos++] = x;
```

```
}
template <class T>
inline void writeInt(T x, char end) {
    if (x < 0) writeChar('-'), x = -x;
    char s[24];
    int n = 0;
    while (x || !n) s[n++] = '0' + x % 10, x /= 10;
    while (n--) writeChar(s[n]);
    if (end) writeChar(end);
}
inline void writeWord(const char *s) {
    while (*s) writeChar(*s++);
}
struct Flusher {
    ~Flusher() {
        if (write_pos) fwrite(write_buf, 1, write_pos, stdout), write_pos = 0;
    }
} flusher;
```

## Strings (3)

### Manacher.cpp

Description: Manacher algorithm

Time:  $\mathcal{O}(n)$

a6ddfb, 27 lines

```
vector<int> manacherOdd(string s) {
    int n = s.size();
    vector<int> d1(n);
    int l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
        while (i + k < n && i - k >= 0 && s[i + k] == s[i - k])
            ++k;
        d1[i] = k;
        if (i + k - 1 > r)
            l = i - k + 1, r = i + k - 1;
    }
}
```

```
vector<int> manacherEven(string s) {
    int n = s.size();
    vector<int> d2(n);
    l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1])
            ++k;
        d2[i] = k;
        if (i + k - 1 > r)
            l = i - k, r = i + k - 1;
    }
}
```

### AhoCorasick.cpp

Description: Build aho-corasick automaton.

Time:  $\mathcal{O}(n)$

```
int go(int v, char c);

int get_link(int v) {
    if (t[v].link == -1)
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    return t[v].link;
}

int go(int v, char c) {
    if (t[v].go[c] == -1)
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), c);
    return t[v].go[c];
}
```

**SuffixArray.cpp**

Description: Build suffix array  
Time:  $\mathcal{O}(n \log(n))$

```
vector<int> buildSuffixArray(string &s) {
    // Remove, if you want to sort cyclic shifts
    s += (char)(1);
    int n = s.size();
    vector<int> a(n);
    iota(all(a), 0);
    stable_sort(all(a), [&](int i, int j) { return s[i] < s[j]; });
    vector<int> c(n);
    int cc = 0;
    for (int i = 0; i < n; i++) {
        if (i == 0 || s[a[i]] != s[a[i - 1]]) {
            c[a[i]] = cc++;
        } else {
            c[a[i]] = c[a[i - 1]];
        }
    }
    for (int L = 1; L < n; L *= 2) {
        vector<int> cnt(n);
        for (auto i : c) {
            cnt[i]++;
        }
        vector<int> pref(n);
        for (int i = 1; i < n; i++) {
            pref[i] = pref[i - 1] + cnt[i - 1];
        }
        vector<int> na(n);
        for (int i = 0; i < n; i++) {
            int pos = (a[i] - L + n) % n;
            na[pref[c[pos]]++].push_back(i);
        }
        a = na;
        vector<int> nc(n);
        cc = 0;
        for (int i = 0; i < n; i++) {
            if (i == 0 || c[a[i]] != c[a[i - 1]] ||
                c[(a[i] + L) % n] != c[(a[i - 1] + L) % n]) {
                nc[a[i]] = cc++;
            } else {
                nc[a[i]] = nc[a[i - 1]];
            }
        }
        c = nc;
    }
    a.erase(a.begin());
    s.pop_back();
    return a;
}
```

**Lcp.cpp**

Description: lcp array  
Time:  $\mathcal{O}(n)$

```
vector<int> perm;
vector<int> buildLCP(string &s, vector<int> &a) {
    int n = s.size();
    vector<int> ra(n);
    for (int i = 0; i < n; i++) {
        ra[a[i]] = i;
    }
    vector<int> lcp(n - 1);
    int cur = 0;
    for (int i = 0; i < n; i++) {
        cur--;
        chkmax(cur, 0);
        if (ra[i] == n - 1) {
            cur = 0;
            continue;
        }
        int j = a[ra[i] + 1];
        while (s[i + cur] == s[j + cur]) cur++;
        lcp[ra[i]] = cur;
    }
    perm.resize(a.size());
    for (int i = 0; i < a.size(); ++i) perm[a[i]] = i;
    return lcp;
}
```

**SuffixArray Lcp Eertree SuffixAutomaton PrefixZ**

```
}
int cntr[MAXN];
int spt[MAXN][MAXN];
void build(vector<int> &a) {
    for (int i = 0; i < a.size(); ++i) {
        spt[i][0] = a[i];
    }
    for (int i = 2; i < MAXN; ++i) cntr[i] = cntr[i / 2] + 1;
    for (int h = 1; (1 << (h - 1)) < a.size(); ++h) {
        for (int i = 0; i + (1 << (h - 1)) < a.size(); ++i) {
            spt[i][h] = min(spt[i][h - 1], spt[i + (1 << (h - 1))][h - 1]);
        }
    }
}
int getLCP(int l, int r) {
    l = perm[l], r = perm[r];
    if (l > r) swap(l, r);
    int xx = cntr[r - 1];
    return min(spt[l][xx], spt[r - (1 << xx)][xx]);
}
```

**Eertree.cpp**

Description: Palindrome Tree

Time:  $\mathcal{O}(n)$

6e64b6, 49 lines

```
struct palindromic_tree {
    int new_node() {
        tree.push_back(node());
        return static_cast<int>(tree.size()) - 1;
    }

    int find_suffix(int v) {
        int n = str.size();
        while (tree[v].length == n - 1 || str.back() != str[n - 2 - tree[v].length]) {
            v = tree[v].suflink;
        }
        return v;
    }

    struct node {
        int length = 0, suflink = -1, to[ALPHABET];
        node() { memset(to, -1, sizeof(to)); }
    };

    int even, odd, last;
    vector<node> tree;
    vector<int> str;

    palindromic_tree() {
        odd = new_node();
        even = new_node();
        tree[even].suflink = tree[odd].suflink = odd;
        tree[odd].length = -1;
        last = even;
    }

    void add(int symbol) {
        str.push_back(symbol);
        last = find_suffix(last);
        if (tree[last].to[symbol] == -1) {
            int v = new_node();
            tree[v].length = tree[last].length + 2;
            int u = find_suffix(tree[last].suflink);
            if (tree[u].to[symbol] == -1) {
                tree[v].suflink = tree[u].to[symbol];
            } else {
                tree[v].suflink = even;
            }
            tree[last].to[symbol] = v;
        }
        last = tree[last].to[symbol];
    }
};
```

**SuffixAutomaton.cpp**

Description: Build suffix automaton.  
Time:  $\mathcal{O}(n)$

```
const int alpha = 26;

struct state {
    int len, link;
    array<int, alpha> next;
};
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
    sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    st[0].next.fill(-1);
    ++sz;
}

int sa_cut(int v, int c) {
    assert(st[v].next[c] != -1);
    int to = st[v].next[c];
    if (st[to].len == st[v].len + 1) {
        return to;
    }
    int clone = sz++;
    st[clone].len = st[v].len + 1;
    st[clone].next = st[to].next;
    st[clone].link = st[to].link;
    for (; v != -1 && st[v].next[c] == to; v = st[v].link)
        st[v].next[c] = clone;
    st[to].link = clone;
    return clone;
}

void sa_extend(int c) {
    if (st[last].next[c] != -1) {
        int to = sa_cut(last, c);
        last = to;
        return;
    }
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].next.fill(-1);
    int v;
    for (v = last; v != -1 && st[v].next[c] == -1; v = st[v].link)
        st[v].next[c] = cur;
    if (v == -1)
        st[cur].link = 0;
    else {
        int to = st[v].next[c];
        st[cur].link = sa_cut(v, c);
    }
    last = cur;
}

vector<int> pf(string s) {
    int k = 0;
    vector<int> p(s.size());
    for (int i = 1; i < s.size(); ++i) {
        while (k && s[i] != s[k])
            k = p[k - 1];
        k += (s[i] == s[k]);
        p[i] = k;
    }
    return p;
}

vector<int> zf(string s) {
    int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - 1]);
    }
}
```

**PrefixZ.cpp**

Description: Calculates Prefix,Z-functions  
Time:  $\mathcal{O}(n)$

1c4e93, 25 lines

```

    while (i + z[i] < n && s[z[i]] == s[i + z[i]])
        ++z[i];
    if (i + z[i] - 1 > r)
        l = i, r = i + z[i] - 1;
}
z[0] = n;
return z;
}

```

MinShift.cpp

**Description:** Calculates min-cyclic-shift of  $s$ , Duval decomposition  
**Time:**  $\mathcal{O}(n)$

Time:  $\mathcal{O}(n)$

---

```

string minshift(string s) {
    int i = 0, ans = 0;
    s += s; // Remove for lyndon decomposition
    int n = s.size();
    while (i < n / 2) { // (i < n) lyndon
        ans = i;
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                ++k;
            ++j;
        }
        while (i <= k) {
            // s.substr(i,j-k) - simple string
            i += j - k;
        }
    }
    return s.substr(ans, n / 2);
}

```

SA-IS.cpp

Description: Build suffix array  
Time:  $\mathcal{O}(n)$

**Time:**  $\mathcal{O}(n)$

```

void induced_sort(vector<int> &vec, int LIM, vector<int> &sa, vector<bool>
    >&sl,
                    vector<int> &fx) {
vector<int> l(LIM), r(LIM);
for (int c : vec) {
    if (c + 1 < LIM) {
        ++l[c + 1];
    }
    ++r[c];
}
partial_sum(all(l), l.begin());
partial_sum(all(r), r.begin());
fill(all(sa), -1);
for (int i = fx.size() - 1; i >= 0; --i) {
    sa[--r[vec[fx[i]]]] = fx[i];
}
for (int i : sa) {
    if (i >= 1 && sl[i - 1]) {
        sa[l[vec[i - 1]]++] = i - 1;
    }
}
fill(all(r), 0);
for (int c : vec) ++r[c];
partial_sum(all(r), r.begin());
for (int k = sa.size() - 1, i = sa[k]; k >= 1; --k, i = sa[k])
    if (i >= 1 && !sl[i - 1]) sa[--r[vec[i - 1]]] = i - 1;
}

```

```

vector<int> SA_IS(vector<int> &vec, int LIM) {
    const int n = vec.size();
    vector<int> sa(n), fx;
    vector<bool> sl(n);
    sl[n - 1] = false;
    for (int i = n - 2; i >= 0; --i) {
        sl[i] = (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i + 1]));
        if (sl[i] && !sl[i + 1]) {
            fx.pbc(i + 1);
        }
    }
    reverse(all(fx));
}

```

## MinShift SA-IS Hungarian BlossomShrinking

```

induced_sort(vec, LIM, sa, sl, fx);
vector<int> nfx(fx.size()), lmv(fx.size());
for (int i = 0, k = 0; i < n; ++i) {
    if (!sl[sa[i]] && sa[i] >= 1 && sl[sa[i] - 1]) {
        nfx[k++] = sa[i];
    }
}
int cur = 0;
sa[n - 1] = cur;
for (int k = 1; k < nfx.size(); ++k) {
    int i = nfx[k - 1], j = nfx[k];
    if (vec[i] != vec[j]) {
        sa[j] = ++cur;
        continue;
    }
    bool flag = false;
    for (int a = i + 1, b = j + 1; ++a, ++b) {
        if (vec[a] != vec[b]) {
            flag = true;
            break;
        }
        if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
            flag = !((!sl[a] && sl[a - 1]) && (!sl[b] && sl[b - 1]));
            break;
        }
    }
    sa[j] = (flag ? ++cur : cur);
}
for (int i = 0; i < fx.size(); ++i) {
    lmv[i] = sa[fx[i]];
}
if (cur + 1 < (int)fx.size()) {
    auto lms = SA_IS(lmv, cur + 1);
    for (int i = 0; i < fx.size(); ++i) {
        nfx[i] = fx[lms[i]];
    }
}
induced_sort(vec, LIM, sa, sl, nfx);
return sa;

```

---

```

plate <typename T>
vector<int> suffix_array(T &s, const int LIM = 128) {
    vector<int> vec(s.size() + 1);
    copy(all(s), begin(vec));
    vec.back() = (char)(1);
    auto ret = SA_IS(vec, LIM);
    ret.erase(ret.begin());
    return ret;
}

```

### Graph (4)

Hungarian.cpp

**Description:** Hungarian algorithm

**Time:**  $\mathcal{O}(n^3)$

5 - f. 5 - 5 - 41

```

int n, m;
vector<vector<int>> a;
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j = 1; j <= m; ++j)
            if (!used[j]) {
                int cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j = 0; j <= m; ++j)
            if (used[j])

```

```

        u[p[j]] += delta, v[j] -= delta;
    else
        minv[j] -= delta;
    j0 = j1;
} while (p[j0] != 0);
do {
    int j1 = way[j0];
    p[j0] = p[j1];
    j0 = j1;
} while (j0);
}

// matching
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j) {
    ans[p[j]] = j;
}

// cost
int cost = -v[0];

```

**BlossomShrinking.cpp**  
**Description:** Maximum matching in general graph  
**Time:**  $\mathcal{O}(n^3)$  23839d, 118 lines

---

```

struct Edge {
    int u, v;
};

const int N = 510;
int n, m;
vector<int> g[N];
vector<Edge> perfectMatching;
int match[N], par[N], base[N];
bool used[N], blossom[N], lcaUsed[N];
int lca(int u, int v) {
    fill(lcaUsed, lcaUsed + n, false);
    while (u != -1) {
        u = base[u];
        lcaUsed[u] = true;
        if (match[u] == -1)
            break;
        u = par[match[u]];
    }
    while (v != -1) {
        v = base[v];
        if (lcaUsed[v])
            return v;
        v = par[match[v]];
    }
    assert(false);
    return -1;
}
void markPath(int v, int myBase, int children) {
    while (base[v] != myBase) {
        blossom[v] = blossom[match[v]] = true;
        par[v] = children;
        children = match[v];
        v = par[match[v]];
    }
}
int findPath(int root) {
    iota(base, base + n, 0);
    fill(par, par + n, -1);
    fill(used, used + n, false);
    queue<int> q;
    q.push(root);
    used[root] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto to : g[v]) {
            if (match[v] == to)
                continue;
            if (base[v] == base[to])
                continue;
            if (to == root || (match[to] != -1 && par[match[to]] != -1))
                {
                    fill(blossom, blossom + n, false);
                    blossom[v] = blossom[to] = true;
                    par[to] = v;
                    used[to] = true;
                    q.push(to);
                }
        }
    }
}

```

```

int myBase = lca(to, v);
markPath(v, myBase, to);
markPath(to, myBase, v);
for (int u = 0; u < n; ++u) {
    if (!blossom[base[u]])
        continue;
    base[u] = myBase;
    if (used[u])
        continue;
    used[u] = true;
    q.push(u);
}
} else if (par[to] == -1) {
    parto = v;
    if (match[to] == -1) {
        return to;
    }
    used[match[to]] = true;
    q.push(match[to]);
}
}
return -1;
}

void blossomShrinking() {
    fill(match, match + n, -1);
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1)
            continue;
        int nxt = findPath(v);
        while (nxt != -1) {
            int parV = par[nxt];
            int parParV = match[parV];
            match[nxt] = parV;
            match[parV] = nxt;
            nxt = parParV;
        }
    }
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1 && v < match[v]) {
            perfectMatching.push_back({v, match[v]});
        }
    }
}

signed main() {
    cin >> n;
    int u, v;
    set<pair<int, int>> edges;
    while (cin >> u >> v) {
        --u;
        --v;
        if (u > v)
            swap(u, v);
        if (edges.count({u, v}))
            continue;
        edges.insert({u, v});
        g[u].push_back(v);
        g[v].push_back(u);
    }
    blossomShrinking();
    cout << perfectMatching.size() * 2 << '\n';
    for (auto i : perfectMatching) {
        cout << i.u + 1 << " " << i.v + 1 << "\n";
    }
    return 0;
}

```

**Lct.cpp**

Description: link-cut tree  
Time:  $\mathcal{O}(n \log(n))$

cfc529, 143 lines

```

struct link_cut {
    struct node {
        int par;
        array<int, 2> sons;
        bool inv;
        int size;
    };
    node() : par(-1), sons({-1, -1}), inv(false), size(1) {}
}

```

```

};

vector<node> t;

int size() const {
    return t.size();
}

void push(int v) {
    if (t[v].inv) {
        t[v].inv = false;
        swap(t[v].sons[0], t[v].sons[1]);
        for (const auto u : t[v].sons)
            if (u != -1)
                t[u].inv ^= 1;
    }
}

void relax(int v) {
    push(v);
    t[v].size = 1;
    for (const auto x : t[v].sons)
        if (x != -1)
            t[v].size += t[x].size;
}

void rotate(int v) {
    int u = t[v].par, w = t[u].par;
    push(u), push(v);
    t[v].par = w;
    if (w != -1)
        for (auto& x : t[w].sons)
            if (x == u)
                x = v;
    int i = t[u].sons[1] == v;
    t[u].sons[i] = t[v].sons[i ^ 1];
    if (t[v].sons[i ^ 1] != -1)
        t[t[v].sons[i ^ 1]].par = u;
    t[v].sons[i ^ 1] = u;
    t[u].par = v;
    relax(u), relax(v);
}

bool is_root(int v) const {
    return t[v].par == -1 || (t[t[v].par].sons[0] != v && t[t[v].par].sons[1] != v);
}

void splay(int v) {
    while (!is_root(v)) {
        int u = t[v].par;
        if (!is_root(u))
            rotate((t[t[u].par].sons[0] == u) == (t[u].sons[0] == v) ? u : v);
        rotate(v);
        push(v);
    }
}

int expose(int v) {
    int prev = -1;
    for (int u = v; u != -1; prev = u, u = t[u].par) {
        splay(u);
        t[u].sons[1] = prev;
        relax(u);
    }
    splay(v);
    assert(t[v].sons[1] == -1 && t[v].par == -1);
    return prev;
}

link_cut(int n = 0) : t(n) {}

int add() {
    t.push_back(node());
    return int(t.size()) - 1;
}

void set_root(int root) {

```

```

    expose(root);
    t[root].inv ^= 1;
    push(root);
}

bool connected(int v, int u) {
    if (v == u)
        return true;
    expose(v), expose(u);
    return t[v].par != -1;
}

bool link(int v, int u) {
    if (connected(v, u))
        return false;
    t[u].inv ^= 1;
    t[u].par = v;
    expose(u);
    return true;
}

bool cut(int v, int u) {
    if (v == u)
        return false;
    set_root(v), expose(u);
    if (t[u].sons[0] != v)
        return false;
    t[u].sons[0] = -1;
    relax(u);
    t[v].par = -1;
    return true;
}

int par(int v, int root) {
    if (!connected(v, root))
        return -1;
    set_root(root), expose(v);
    if (t[v].sons[0] == -1)
        return -1;
    v = t[v].sons[0];
    while (push(v), t[v].sons[1] != -1)
        v = t[v].sons[1];
    splay(v);
    return v;
}

int distance(int v, int u) {
    if (!connected(v, u))
        return -1;
    set_root(v), expose(u);
    return t[u].sons[0] == -1 ? 0 : t[t[u].sons[0]].size;
}

int lca(int v, int u, int root) {
    set_root(root), expose(v);
    return expose(u);
}
};


```

**MaxFlow.cpp**

Description: Dinic  
Time:  $\mathcal{O}(n^2m)$

1c1bc8, 72 lines

```

struct MaxFlow {
    const int inf = 1e9 + 20;
    struct edge {
        int a, b, cap;
    };
    int n;
    vector<edge> e;
    vector<vector<int>> g;
    MaxFlow() {}
    int s, t;
    vector<int> d, ptr;
    void init(int n_, int s_, int t_) {
        s = s_, t = t_, n = n_;
        g.resize(n);
        ptr.resize(n);
    }
}

```

```

}
void addedge(int a, int b, int cap) {
    g[a].pb(e.size());
    e.pbc({a, b, cap});
    g[b].pb(e.size());
    e.pbc({b, a, 0});
}
bool bfs() {
    d.assign(n, inf);
    d[s] = 0;
    queue<int> q;
    q.push(s);
    while (q.size()) {
        int v = q.front();
        q.pop();
        for (int i : g[v]) {
            if (e[i].cap > 0) {
                int b = e[i].b;
                if (d[b] > d[v] + 1) {
                    d[b] = d[v] + 1;
                    q.push(b);
                }
            }
        }
    }
    return d[t] != inf;
}
int dfs(int v, int flow) {
    if (v == t) return flow;
    if (!flow) return 0;
    int sum = 0;
    for (; ptr[v] < g[v].size(); ++ptr[v]) {
        int b = e[g[v][ptr[v]].b];
        int cap = e[g[v][ptr[v]].cap];
        if (cap <= 0) continue;
        if (d[b] != d[v] + 1) continue;
        int x = dfs(b, min(flow, cap));
        int id = g[v][ptr[v]];
        e[id].cap -= x;
        e[id ^ 1].cap += x;
        flow -= x;
        sum += x;
    }
    return sum;
}
int dinic() {
    int ans = 0;
    while (1) {
        if (!bfs()) break;
        ptr.assign(n, 0);
        int x = dfs(s, inf);
        if (!x) break;
        ans += x;
    }
    return ans;
}

```

**MCMF.cpp**

Description: Min cost  
Time:  $\mathcal{O} (?)$

32340a, 61 lines

```

struct MCMF {
    struct edge {
        int a, b, cap, cost;
    };
    vector<edge> e;
    vector<vector<int>> g;
    int s, t;
    int n;
    void init(int N, int S, int T) {
        s = S, t = T, n = N;
        g.resize(N);
        e.clear();
    }
    void addedge(int a, int b, int cap, int cost) {
        g[a].pb(e.size());
        e.pbc({a, b, cap, cost});
        g[b].pb(e.size());
    }

```

```

        e.pbc({b, a, 0, -cost});
    }
    int getcost(int k) {
        int flow = 0;
        int cost = 0;
        while (flow < k) {
            vector<int> d(n, INF);
            vector<int> pr(n);
            d[s] = 0;
            queue<int> q;
            q.push(s);
            while (q.size()) {
                int v = q.front();
                q.pop();
                for (int i : g[v]) {
                    int u = e[i].b;
                    if (e[i].cap && d[u] > d[v] + e[i].cost) {
                        d[u] = d[v] + e[i].cost;
                        q.push(u);
                        pr[u] = i;
                    }
                }
            }
            if (d[t] == INF) return INF;
            int gf = k - flow;
            int v = t;
            while (v != s) {
                int id = pr[v];
                chkmin(gf, e[id].cap);
                v = e[id].a;
            }
            v = t;
            while (v != s) {
                int id = pr[v];
                e[id].cap -= gf;
                e[id ^ 1].cap += gf;
                cost += e[id].cost * gf;
                v = e[id].a;
            }
            flow += gf;
        }
        return cost;
    }
};

```

**MCMFFast.cpp**

Description: Min cost with potentials  
Time:  $\mathcal{O} (?)$

363228, 86 lines

```

struct MCMF {
    struct edge {
        int a, b, cap, cost;
    };
    vector<edge> e;
    vector<vector<int>> g;
    vector<ll> po;
    int s, t;
    int n;
    void init(int N, int S, int T) {
        s = S, t = T, n = N;
        g.resize(N);
        e.clear();
    }
    void addedge(int a, int b, int cap, int cost) {
        g[a].pb(e.size());
        e.pbc({a, b, cap, cost});
        g[b].pb(e.size());
        e.pbc({b, a, 0, -cost});
    }
    void calc_p() {
        po.assign(n, INF);
        vector<int> inq(n);
        queue<int> q;
        q.push(s);
        po[s] = 0;
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            inq[v] = 0;

```

```

            for (auto i : g[v]) {
                if (po[e[i].b] > po[v] + e[i].cost && e[i].cap) {
                    po[e[i].b] = po[v] + e[i].cost;
                    if (!inq[e[i].b]) q.push(e[i].b);
                    inq[e[i].b] = 1;
                }
            }
        }
        ll getcost(int k) {
            calc_p();
            int flow = 0;
            ll cost = 0;
            while (flow < k) {
                vector<ll> d(n, INF);
                vector<int> pr(n);
                d[s] = 0;
                set<pair<ll, int>> q;
                q.insert(mp(0ll, s));
                while (q.size()) {
                    int v = q.begin()->second;
                    q.erase(q.begin());
                    for (int i : g[v]) {
                        int u = e[i].b;
                        if (e[i].cap && d[u] > d[v] + e[i].cost + po[v] - po[e[i].b]) {
                            q.erase(mp(d[u], u));
                            d[u] = d[v] + e[i].cost + po[v] - po[e[i].b];
                            q.insert(mp(d[u], u));
                            pr[u] = i;
                        }
                    }
                }
                if (d[t] == INF) return INF;
                for (int i = 0; i < n; ++i) {
                    if (d[i] != INF) po[i] += d[i];
                }
                int gf = k - flow;
                int v = t;
                while (v != s) {
                    int id = pr[v];
                    chkmin(gf, e[id].cap);
                    v = e[id].a;
                }
                v = t;
                while (v != s) {
                    int id = pr[v];
                    e[id].cap -= gf;
                    e[id ^ 1].cap += gf;
                    cost += 111 * e[id].cost * gf;
                    v = e[id].a;
                }
                flow += gf;
            }
            return cost;
        }
    };

```

**GlobalMincut.cpp**

Description: Global min cut  
Time:  $\mathcal{O} (n^3)$

7b8a6b, 35 lines

```

const int MAXN = 500;
int n, g[MAXN][MAXN];
int best_cost = 1000000000;
vector<int> best_cut;
void mincut() {
    vector<int> v[MAXN];
    for (int i = 0; i < n; ++i)
        v[i].assign(1, i);
    int w[MAXN];
    bool exist[MAXN], in_a[MAXN];
    memset(exist, true, sizeof(exist));
    for (int ph = 0; ph < n - 1; ++ph) {
        memset(in_a, false, sizeof in_a);
        memset(w, 0, sizeof w);
        for (int it = 0, prev; it < n - ph; ++it) {
            int sel = -1;
            for (int i = 0; i < n; ++i)

```

## WeightedMatching DominatorTree

```

        if (exist[i] && !in_a[i] && (sel == -1 || w[i] > w[sel]))
            sel = i;
    if (it == n - ph - 1) {
        if (w[sel] < best_cost)
            best_cost = w[sel], best_cut = v[sel];
        v[prev].insert(v[prev].end(), v[sel].begin(), v[sel].end()
                      ());
        for (int i = 0; i < n; ++i)
            g[prev][i] = g[i][prev] += g[sel][i];
        exist[sel] = false;
    } else {
        in_a[sel] = true;
        for (int i = 0; i < n; ++i)
            w[i] += g[sel][i];
        prev = sel;
    }
}
}

```

## WeightedMatching.cpp

**Description:** Max weighted matching

**Time:**  $\mathcal{O}(N^3)$  or so

c3f149, 193 line

```

#define Dist(e) (lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2)
const int N = 1023, INF = 1e9;
struct Edge {
    int u, v, w;
} g[N][N];
int n, m, n_x, lab[N], match[N], slack[N], st[N], pa[N], flower_from[N][N],
    S[N], vis[N];
vector<int> flower[N];
deque<int> q;
void update_slack(int u, int x) {
    if (!slack[x] || Dist(g[u][x]) < Dist(g[slack[x]][x])) slack[x] = u;
}
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0) update_slack(u, x);
}
void q_push(int x) {
    if (x <= n) return q.push_back(x);
    for (int i = 0; i < flower[x].size(); ++i) q_push(flower[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x <= n) return;
    for (int i = 0; i < flower[x].size(); ++i) set_st(flower[x][i], b);
}
int get_pr(int b, int xr) {
    int pr = find(flower[b].begin(), flower[b].end(), xr) - flower[b].begin();
    if (pr % 2 == 1) {
        reverse(flower[b].begin() + 1, flower[b].end());
        return (int)flower[b].size() - pr;
    } else return pr;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    Edge e = g[u][v];
    int xr = flower_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flower[u][i], flower[u][i ^ 1]);
    set_match(xr, v);
    rotate(flower[u].begin(), flower[u].begin() + pr, flower[u].end());
}
void augment(int u, int v) {
    int xnv = st[match[u]];
    set_match(u, v);
    if (!xnv) return;
    set_match(xnv, st[pa[xnv]]);
    augment(st[pa[xnv]], xnv);
}
int get_lca(int u, int v) {
    static int t = 0;
    if (st[u] == st[v])
        return t;
    if (st[u] < st[v])
        swap(u, v);
    for (int i = 0; i < st[u]; ++i)
        if (st[u] < st[match[i]]) return i;
    for (int i = 0; i < st[v]; ++i)
        if (st[v] < st[match[i]]) return i;
    return t;
}
bool on_found_Edge(const Edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), 1;
        else add_blossom(u, lca, v);
    }
    return 0;
}
bool matching() {
    fill(S, S + n_x + 1, -1), fill(slack, slack + n_x + 1, 0);
    q.clear();
    for (int x = 1; x <= n_x; ++x) if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q.push(x);
    if (q.empty()) return 0;
    while(1) {
        while (q.size()) {
            int u = q.front();
            q.pop_front();

```

```

if (S[st[u]] == 1) continue;
for (int v = 1; v <= n; ++v) {
    if (g[u][v].w > 0 && st[u] != st[v]) {
        if (Dist(g[u][v]) == 0) {
            if (on_found_Edge(g[u][v])) return 1;
        } else
            update_slack(u, st[v]);
    }
}

nt d = INF;
or (int b = n + 1; b <= n_x; ++b) if (st[b] == b && S[b] == 1)
    chkmn(d, lab[b] / 2);
or (int x = 1; x <= n_x; ++x) {
    if (st[x] == x && slack[x]) {
        if (S[x] == -1)
            d = min(d, Dist(g[slack[x]][x]));
        else if (S[x] == 0)
            d = min(d, Dist(g[slack[x]][x]) / 2);
    }
}

or (int u = 1; u <= n; ++u) {
    if (S[st[u]] == 0) {
        if (lab[u] <= d) return 0;
        lab[u] -= d;
    } else if (S[st[u]] == 1)
        lab[u] += d;
}

or (int b = n + 1; b <= n_x; ++b) {
    if (st[b] == b) {
        if (S[st[b]] == 0)
            lab[b] += d * 2;
        else if (S[st[b]] == 1)
            lab[b] -= d * 2;
    }
}

.clear();
or (int x = 1; x <= n_x; ++x) {
    if (st[x] == x && slack[x] && st[slack[x]] != x &&
        Dist(g[slack[x]][x]) == 0)
        if (on_found_Edge(g[slack[x]][x])) return 1;

or (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0) expand_blossom(b);
}

n 0;

int> weight_blossom() {
match, match + n + 1, 0);
n;
_nmatches = 0;
_t_weight = 0;
int u = 0; u <= n; ++u) st[u] = u, flower[u].clear();
_max = 0;
int u = 1; u <= n; ++u) {
or (int v = 1; v <= n; ++v) {
    flower_from[u][v] = (u == v ? u : 0);
    w_max = max(w_max, g[u][v].w);
}

swer = 0;
int u = 1; u <= n; ++u) lab[u] = w_max;
(matching()) +n_matches;
nt u=1; u<=n; ++u)
    f(match[u]&match[u]u)
        tot_weight+g[u][match[u]].w;
n make_pair(tot_weight,n_matches);

```

## DominatorTree.cpp

**Description:** Dominator tree

Time: ?

e82004 52 lines

```

DominatorTree(int n) : g(n), rg(n), bucket(n), arr(n, -1), par(n, -1)
    , rev(n, -1),
    sdom(n, -1), dom(n, -1), dsu(n, 0), label(n, 0), n(n), t(0) {}
void add_edge(int u, int v) {
    g[u] += v;
}
void dfs(int u) {
    arr[u] = t;
    rev[t] = u;
    label[t] = sdom[t] = dsu[t] = t;
    t++;
    for (int w : g[u]) {
        if (arr[w] == -1) {
            dfs(w);
            par[arr[w]] = arr[u];
        }
        rg[arr[w]] += arr[u];
    }
}
int find(int u, int x=0) {
    if (x == dsu[u]) return x ? -1 : u;
    int v = find(dsu[u], x + 1);
    if (v < 0) return u;
    if (sdom[label[dsu[u]]] < sdom[label[u]])
        label[u] = label[dsu[u]];
    dsu[u] = v;
    return x ? v : label[u];
}
vector<int> run(int root) {
    dfs(root);
    iota(dom.begin(), dom.end(), 0);
    for (int i = t - 1; i >= 0; --i) {
        for (int w : rg[i]) sdom[i] = min(sdom[i], sdom[find(w)]);
        if (i < sdom[sdom[i]]) += i;
        for (int w : bucket[i]) {
            int v = find(w);
            if (sdom[v] == sdom[w]) dom[w] = sdom[w];
            else dom[w] = v;
        }
        if (i > 1) dsu[i] = par[i];
    }
    for (int i = 1; i < t; i++) if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
    vector<int> outside_dom(n, -1);
    for (int i = 1; i < t; i++) outside_dom[rev[i]] = rev[dom[i]];
    //--1 if vertex is not reachable
    return outside_dom;
}

```

**OrientedSpanningTree.cpp**

Description: Oriented Spanning Tree  
Time:  $\mathcal{O}(n \log n)$

3d7a73, 96 lines

```

struct RollbackUF {
    vector<int> p, sz;
    vector<int> changes;
    RollbackUF(int n) {
        p.resize(n);
        changes.reserve(n);
        sz.resize(n, 1);
        for (int i = 0; i < n; ++i) p[i] = i;
    }
    int time() {
        return changes.size();
    }
    int find(int v) {
        if (v == p[v]) return v;
        return find(p[v]);
    }
    bool join(int a, int b) {
        a = find(a);
        b = find(b);
        if (a == b) return false;
        if (sz[a] > sz[b]) swap(a, b);
        changes.push_back(a);
        sz[b] += sz[a];
        p[a] = b;
        return true;
    }
};

```

**MatroidIntersection.cpp**  
Description: matroid intersection  
Time: ?

d2387f, 71 lines

```

template<typename T, typename A, typename B>
vector<T> matroid_intersection(const std::vector<T> &ground_set, const A
    &matroid1, const B &matroid2) {

```

```

    }
    void rollback(int t) {
        while (changes.size() > t) {
            int v = changes.back();
            sz[p[v]] -= sz[v];
            p[v] = v;
            changes.pop_back();
        }
    }
    struct Edge { int a, b; ll w; };
    struct Node {
        Edge key;
        Node *l, *r;
        ll delta;
        void prop() {
            key.w += delta;
            if (l) l->delta += delta;
            if (r) r->delta += delta;
            delta = 0;
        }
        Edge top() { prop(); return key; }
    };
    Node *merge(Node *a, Node *b) {
        if (!a || !b) return a ?: b;
        a->prop(), b->prop();
        if (a->key.w > b->key.w) swap(a, b);
        swap(a->l, (a->r = merge(b, a->r)));
        return a;
    }
    void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }
    pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
        RollbackUF uf(n);
        vector<Node*> heap(n);
        for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node(e));
        ll res = 0;
        vi seen(n, -1), path(n), par(n);
        seen[r] = r;
        vector<Edge> Q(n), in(n, {-1, -1}), comp;
        deque<tuple<int, int>, vector<Edge>> cycs;
        for (int s = 0; s < n; ++s) {
            int u = s, qi = 0, w;
            while (seen[u] < 0) {
                if (!heap[u]) return {-1, {}};
                Edge e = heap[u]->top();
                heap[u]->delta -= e.w, pop(heap[u]);
                Q[qi] = e, path[qi++] = u, seen[u] = s;
                res += e.w, u = uf.find(e.a);
                if (seen[u] == s) {
                    Node* cyc = 0;
                    int end = qi, time = uf.time();
                    do {
                        cyc = merge(cyc, heap[w = path[--qi]]);
                    } while (uf.join(u, w));
                    u = uf.find(u), heap[u] = cyc, seen[u] = -1;
                    cycs.push_front({u, time, {&Q[qi], &Q[qi]}});
                }
            }
            for (int i = 0; i < qi; ++i) {
                in[uf.find(Q[i].b)] = Q[i];
            }
        }
        for (auto& [u, t, comp] : cycs) { // restore so l ( optional )
            uf.rollback(t);
            Edge inEdge = in[u];
            for (auto& e : comp) in[uf.find(e.b)] = e;
            in[uf.find(inEdge.b)] = inEdge;
        }
        for (int i = 0; i < n; ++i) par[i] = in[i].a;
        return {res, par};
    }
}

```

```

//weighted - minimize (weight, cnt edges) with dijkstra
int n = ground_set.size();
vector<char> in_set(n), inm1(n), inm2(n);
vector<bool> used(n);
vi par(n), left, right;
while (true) {
    A m1 = matroid1;
    B m2 = matroid2;
    left.clear(); right.clear();
    for (int i = 0; i < n; i++)
        if (in_set[i]) {
            m1.add(ground_set[i]);
            m2.add(ground_set[i]);
            left.push_back(i);
        } else {
            right.push_back(i);
        }
    fill(all(inm1), 0); fill(all(inm2), 0);
    bool found = false;
    for (int i : right) if (inm1[i]) {
        inm1[i] = m1.independed_with(ground_set[i]);
        inm2[i] = m2.independed_with(ground_set[i]);
        if (inm1[i] && inm2[i]) {
            in_set[i] = 1;
            found = true;
            break;
        }
    }
    if (found) continue;
    fill(all(used), false); fill(all(par), -1);
    queue<int> que;
    for (int i : right) if (inm1[i]) {
        used[i] = true;
        que.push(i);
    }
    while (!que.empty() && !found) {
        int v = que.front();
        que.pop();
        if (in_set[v]) {
            A m = matroid1;
            for (int i : left) if (i != v) m.add(ground_set[i]);
            for (int u : right)
                if (!used[u] && m.independed_with(ground_set[u])) {
                    par[u] = v;
                    used[u] = true;
                    que.push(u);
                    if (inm2[u]) {
                        found = true;
                        for (; u != -1; u = par[u]) in_set[u] ^= 1;
                        break;
                    }
                }
        } else {
            B m = m2;
            m.add_extra(ground_set[v]);
            for (auto u : left)
                if (!used[u] && m.independed_without(ground_set[u])) {
                    par[u] = v;
                    used[u] = true;
                    que.push(u);
                }
            if (!found) break;
        }
    }
    vector<T> res;
    for (int i = 0; i < n; i++) if (in_set[i]) res.push_back(ground_set[i]);
    return res;
}

```

**MinMeanCycle.cpp**

**Description:****MINIMUM MEAN CYCLE ALGORITHM****Input:** A digraph  $G$ , weights  $c : E(G) \rightarrow \mathbb{R}$ .**Output:** A circuit  $C$  with minimum mean weight or the information that  $G$  is acyclic.

- ① Add a vertex  $s$  and edges  $(s, x)$  with  $c((s, x)) := 0$  for all  $x \in V(G)$  to  $G$ .
- ② Set  $n := |V(G)|$ ,  $F_0(s) := 0$ , and  $F_0(x) := \infty$  for all  $x \in V(G) \setminus \{s\}$ .
- ③ **For**  $k := 1$  to  $n$  **do**:
  - For** all  $x \in V(G)$  **do**:
  - Set  $F_k(x) := \infty$ .
  - For** all  $(w, x) \in \delta^-(x)$  **do**:
  - If  $F_{k-1}(w) + c((w, x)) < F_k(x)$  **then**:
  - Set  $F_k(x) := F_{k-1}(w) + c((w, x))$  and  $p_k(x) := w$ .
- ④ If  $F_n(x) = \infty$  for all  $x \in V(G)$  **then stop** ( $G$  is acyclic).
- ⑤ Let  $x$  be a vertex for which  $\max_{\substack{0 \leq k \leq n-1 \\ F_k(x) < \infty}} \frac{F_n(x) - F_k(x)}{n - k}$  is minimum.
- ⑥ Let  $C$  be any circuit in the edge progression given by  $p_n(x), p_{n-1}(p_n(x)), p_{n-2}(p_{n-1}(p_n(x))), \dots$

d41d8c, 1 lines

// ?

## Geometry (5)

**Point.cpp****Description:** struct Point

80dfd5, 80 lines

```
const ld EPS = 1e-7;

ld sq(ld x) {
    return x * x;
}

int sign(ld x) {
    if (x < -EPS) {
        return -1;
    }
    if (x > EPS) {
        return 1;
    }
    return 0;
}

#define vec point
struct point { //%- cross, * - dot
    ld x, y;
    auto operator<=(const point&) const = default;
};

ld operator*(const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
}

ld operator%(const point &a, const point &b) {
    return a.x * b.y - a.y * b.x;
}

point operator-(const point &a, const point &b) {
    return {a.x - b.x, a.y - b.y};
}

point operator+(const point &a, const point &b) {
    return {a.x + b.x, a.y + b.y};
}

point operator*(const point &a, ld b) {
    return {a.x * b, a.y * b};
}

point operator/(const point &a, ld b) {
    return {a.x / b, a.y / b};
}

bool operator<(const point &a, const point &b) {
    if (sign(a.y - b.y) != 0) {
        return a.y < b.y;
    } else if (sign(a.x - b.x) != 0) {

```

## Point Line Intersections Tangents

```
        return a.x < b.x;
    }
    return 0;
}
ld len2(const point &a) {
    return sq(a.x) + sq(a.y);
}
ld len(const point &a) {
    return sqrt(len2(a));
}
point norm(point a) {
    return a / len(a);
}
int half(point a) {
    return (sign(a.y) == -1 || (sign(a.y) == 0 && a.x < 0));
}
point ort(point a) {
    return {-a.y, a.x};
}
point turn(point a, ld ang) {
    return {a.x * cos(ang) - a.y * sin(ang), a.x * sin(ang) + a.y * cos(ang)};
}
ld getAngle(point &a, point &b) {
    return atan2(a % b, a * b);
}
bool cmpHalf(const point &a, const point &b) {
    if (half(a) != half(b)) {
        return half(b);
    } else {
        int sgn = sign(a % b);
        if (!sgn) {
            return len2(a) < len2(b);
        } else {
            return sgn == 1;
        }
    }
}

```

**Line.cpp****Description:** struct Line

887306, 26 lines

```
struct line {
    ld a, b, c;
    void norm() {
        // for half planes
        ld d = len({a, b});
        assert(sign(d) > 0);
        a /= d;
        b /= d;
        c /= d;
    }
    ld eval(point p) const { return a * p.x + b * p.y + c; }
    bool isIn(point p) const { return sign(eval(p)) >= 0; }
    bool operator==(const line &other) const {
        return sign(a * other.b - b * other.a) == 0 &&
               sign(a * other.c - c * other.a) == 0 &&
               sign(b * other.c - c * other.b) == 0;
    }
    line getln(point a, point b) {
        line res;
        res.a = a.y - b.y;
        res.b = b.x - a.x;
        res.c = -(res.a * a.x + res.b * a.y);
        res.norm();
        return res;
    }
}

```

**Intersections.cpp****Description:** Geometry intersections

45d7d9, 75 lines

```
bool isCrossed(ld lx, ld rx, ld ly, ld ry) {
    if (lx > rx)
        swap(lx, rx);
    if (ly > ry)
        swap(ly, ry);
    return sign(min(rx, ry) - max(lx, ly)) >= 0;
}

```

```
// if two segments [a, b] and [c, d] has AT LEAST one common point ->
// true
bool intersects(const point &a, const point &b, const point &c, const point &d) {
    if (!isCrossed(a.x, b.x, c.x, d.x))
        return false;
    if (!isCrossed(a.y, b.y, c.y, d.y))
        return false;
    if ((sign((b - a) % (c - a)) * sign((b - a) % (d - a)) == 1) || (sign((d - c) % (a - c)) * sign((d - c) % (b - c)) == 1))
        return 1;
}
//intersecting lines
bool intersect(line l1, line m, point &l1) {
    ld d = l1 * m.a - m.b * l1.a;
    if (sign(d) == 0) {
        return false;
    }
    ld dx = m.b * l1.c - m.c * l1.b;
    ld dy = m.c * l1.a - l1.c * m.a;
    l1 = {dx / d, dy / d};
    return true;
}
//intersecting circles
int intersect(point o1, ld r1, point o2, ld r2, point &i1, point &i2) {
    if (r1 < r2) {
        swap(o1, o2);
        swap(r1, r2);
    }
    if (sign(r1 - r2) == 0 && len2(o2 - o1) < EPS) {
        return 3;
    }
    ld ln = len(o1 - o2);
    if (sign(ln - r1 - r2) == 1 || sign(r1 - ln - r2) == 1) {
        return 0;
    }
    ld d = (sq(r1) - sq(r2) + sq(ln)) / 2 / ln;
    vec v = norm(o2 - o1);
    point a = o1 + v * d;
    if (sign(ln - r1 - r2) == 0 || sign(ln + r2 - r1) == 0) {
        i1 = a;
        return 1;
    }
    v = ort(v) * sqrt(sq(r1) - sq(d));
    i1 = a + v;
    i2 = a - v;
    return 2;
}
//intersecting line and circle, line should be normed
int intersect(point o, ld r, line l, point &i1, point &i2) {
    ld len = abs(l.eval(o));
    int sgn = sign(len - r);
    if (sgn == 1) {
        return 0;
    }
    vec v = norm(vec{l.a, l.b}) * len;
    if (sign(l.eval(o + v)) != 0) {
        v = vec{0, 0} - v;
    }
    point a = o + v;
    if (sgn == 0) {
        i1 = a;
        return 1;
    }
    v = norm({-l.b, l.a}) * sqrt(sq(r) - sq(len));
    i1 = a + v;
    i2 = a - v;
    return 2;
}

```

**Tangents.cpp****Description:** Tangents to circles.

c73373, 43 lines

```
// tangents from point to circle
int tangents(point &o, ld r, point &p, point &i1, point &i2) {
    ld ln = len(o - p);
    int sgn = sign(ln - r);
    if (sgn == -1) {

```

```

return 0;
} else if (sgn == 0) {
    il = p;
    return 1;
} else {
    ld x = sq(r) / ln;
    vec v = norm(p - o) * x;
    point a = o + v;
    v = ort(norm(p - o)) * sqrt(sq(r) - sq(x));
    il = a + v;
    i2 = a - v;
    return 2;
}

void _tangents(point c, ld r1, ld r2, vector<line> &ans) {
    ld r = r2 - r1;
    ld z = sq(c.x) + sq(c.y);
    ld d = z - sq(r);
    if (sign(d) == -1)
        return;
    d = sqrt(abs(d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}

// tangents between two circles
vector<line> tangents(point o1, ld r1, point o2, ld r2) {
    vector<line> ans;
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            _tangents(o2 - o1, r1 * i, r2 * j, ans);
    for (int i = 0; i < (int)ans.size(); ++i)
        ans[i].c -= ans[i].a * o1.x + ans[i].b * o1.y;
    return ans;
}

```

**Hull.cpp****Description:** Polygon functions

fc1928, 16 lines

```

vector<point> hull(vector<point> p, bool need_all=false) {
    sort(all(p));
    p.erase(unique(all(p)), end(p));
    int n = p.size(), k = 0;
    if (n <= 2) return p;
    vector<point> ch(2 * n);
    ld th = need_all ? -EPS : +EPS; // 0 : 1 if int
    for (int i = 0; i < n; ch[k++] = p[i++]) {
        while (k >= 2 && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
            --k;
    }
    for (int i = n - 2, t = k + 1; i >= 0; ch[k++] = p[i--]) {
        while (k >= t && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
            --k;
    }
    ch.resize(k - 1);
    return ch;
}

```

**IsInPolygon.cpp****Description:** Is in polygon functions

f17b31, 65 lines

```

bool isOnSegment(point &a, point &b, point &x) {
    if (sign(len2(a - b)) == 0)
        return sign(len(a - x)) == 0;
    return sign((b - a) % (x - a)) == 0 && sign((b - x) * (a - x)) <= 0;
    // optional (slower, but works better if there are some precision
    // problems) return sign((b - a).len() - (x - a).len() - (x - b).len()
    // == 0;
}

int isIn(vector<point> &p, point &a) {
    int n = p.size();
    // depends on limitations(2*MAXC + 228)
    point b = a + point{2e9 + 228, 1};

```

```

    int cnt = 0;
    for (int i = 0; i < n; ++i) {
        point x = p[i];
        point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return 1;
        }
        cnt += intersects(x, y, a, b);
    }
    return 2 * (cnt % 2 == 1);
    /*optional (atan2 is VERY SLOW)!
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
        Point x = p[i];
        Point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return true;
        }
        x = x - a;
        y = y - a;
        ans += atan2(x ^ y, x * y);
    }
    return abs(ans) > 1;*/
}

bool isInTriangle(point &a, point &b, point &c, point &x) {
    return sign((b - a) % (x - a)) >= 0 && sign((c - b) % (x - b)) >= 0
        && sign((a - c) % (x - c)) >= 0;
}

// points should be in the counterclockwise order
bool isInConvex(vector<point> &p, point &a) {
    int n = p.size();
    assert(n >= 3);
    // assert(isConvex(p));
    // assert(isCounterclockwise(p));
    if (sign((p[1] - p[0]) % (a - p[0])) < 0)
        return 0;
    if (sign((p[n - 1] - p[0]) % (a - p[0])) > 0)
        return 0;
    int pos = lower_bound(p.begin() + 2, p.end(), a,
        [&](point a, point b) -> bool {
            return sign((a - p[0]) % (b - p[0])) > 0;
        }) -
        p.begin();
    assert(pos > 1 && pos < n);
    return isinTriangle(p[0], p[pos - 1], p[pos], a);
}

```

**Diameter.cpp****Description:** Rotating calipers.Time:  $\mathcal{O}(n)$ 

0f341c, 21 lines

```

ld diameter(vector<point> p) {
    p = hull(p);
    int n = p.size();
    if (n <= 1)
        return 0;
    if (n == 2)
        return len(p[0] - p[1]);
    ld ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (sign((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] - p[j])) >=
            0) {
            chkmax(ans, len(p[i] - p[j]));
            j = (j + 1) % n;
        }
        chkmax(ans, len(p[i] - p[j]));
        ++i;
    }
    return ans;
}

```

**TangentsAlex.cpp****Description:** Find both tangents to the convex polygon.  
(Zakaldovaly algos mozhet sgonyat za pivom tak zhe).Time:  $\mathcal{O}(\log(n))$ 

2eee8, 17 lines

```

pair<int, int> tangents_alex(vector<point> &p, point &a) {
    int n = p.size();
    int l = __lg(n);
    auto findWithSign = [&](int val) {
        int i = 0;
        for (int k = 1; k >= 0; --k) {
            int il = (i - (1 << k) + n) % n;
            int i2 = (i + (1 << k)) % n;
            if (sign((p[il] - a) % (p[i] - a)) == val)
                i = il;
            if (sign((p[i2] - a) % (p[i] - a)) == val)
                i = i2;
        }
        return i;
    };
    return {findWithSign(1), findWithSign(-1)};
}

```

**IsHpiEmpty.cpp****Description:** Determines is half plane intersectinos.Time:  $\mathcal{O}(n)$  (expected)

3b5e69, 42 lines

```

// all lines must be normed!!!!, sign > 0
bool isHpiEmpty(vector<line> lines) {
    // return hpi(lines).empty();
    // overflow/precision problems?
    shuffle(all(lines), rnd);
    const ld C = 1e9;
    point ans(C, C);
    vector<point> box = {{-C, -C}, {C, -C}, {C, C}, {-C, C}};
    for (int i = 0; i < 4; ++i)
        lines.push_back(getln(box[i], box[(i + 1) % 4]));
    int n = lines.size();
    for (int i = n - 4; i >= 0; --i) {
        if (lines[i].isIn(ans))
            continue;
        point up(0, C + 1), down(0, -C - 1), pi = {lines[i].b, -lines[i].a};
        for (int j = i + 1; j < n; ++j) {
            if (lines[i] == lines[j])
                continue;
            point p, pj = {lines[j].b, -lines[j].a};
            if (!intersect(lines[i], lines[j], p)) {
                if (sign(pi * pj) != -1)
                    continue;
                if (sign(lines[i].c + lines[j].c) *
                    (!sign(pi.y) ? sign(pi.x) : -1) == 1)
                    return true;
            } else {
                if (!sign(pi.y) ? sign(pi.x) : sign(pi.y) * (sign(pi % pj)) ==
                    1)
                    chkmin(up, p);
                else
                    chkmax(down, p);
            }
        }
        if ((ans = up) < down)
            return true;
    }
    // for (int i = 0; i < n; ++i) {
    //     assert(lines[i].eval(ans) < EPS);
    // }
    return false;
}

```

**HalfPlaneIntersection.cpp****Description:** Find the intersection of the half planes.Time:  $\mathcal{O}(n \log(n))$ 

fdf28f, 62 lines

```

vec getPoint(line l) { return {-l.b, l.a}; }

bool bad(line a, line b, line c) {

```

```

point x;
assert(intersect(b, c, x) == 1);
return a.eval(x) < 0;
}

// Do not forget about the bounding box
vector<point> hpi(vector<line> lines) {
    sort(all(lines), [](line al, line bl) -> bool {
        point a = getPoint(al);
        point b = getPoint(bl);
        if (half(a) != half(b)) {
            return half(a) < half(b);
        }
        return a % b > 0;
    });
    vector<pair<line, int>> st;
    for (int it = 0; it < 2; it++) {
        for (int i = 0; i < (int)lines.size(); i++) {
            bool flag = false;
            while (!st.empty()) {
                if (len(getPoint(st.back().first) - getPoint(lines[i])) < EPS) {
                    if (lines[i].c >= st.back().first.c) {
                        flag = true;
                        break;
                    } else {
                        st.pop_back();
                    }
                } else if (getPoint(st.back().first) % getPoint(lines[i]) < EPS / 2) {
                    return {};
                } else if (st.size() >= 2 && bad(st[st.size() - 2].first, st[st.size() - 1].first,
                                         lines[i])) {
                    st.pop_back();
                } else {
                    break;
                }
            }
            if (!flag)
                st.push_back({lines[i], i});
        }
    }
    vector<int> en(lines.size(), -1);
    vector<point> ans;
    for (int i = 0; i < (int)st.size(); i++) {
        if (en[st[i].second] == -1) {
            en[st[i].second] = i;
            continue;
        }
        for (int j = en[st[i].second]; j < i; j++) {
            point I;
            assert(intersect(st[j].first, st[j + 1].first, I) == 1);
            ans.push_back(I);
        }
        break;
    }
    return ans;
}

```

**CHT.cpp****Description:** CHT for minimum, k is decreasing, works for equal slopes

```

30B3ab, 34 lines
struct line {
    int k, b;
    int eval(int x) {
        return k * x + b;
    }
};

struct part {
    line a;
    ld x;
};

ld intersection(line a, line b) {
    return (ld) (a.b - b.b) / (b.k - a.k);
}

```

```

struct ConvexHullMin {
    vector<part> st;
    void add(line a) {
        if (!st.empty() && st.back().a.k == a.k) {
            if (st.back().a.b > a.b) st.pop_back();
            else return;
        }
        while (st.size() > 1 && intersection(st[st.size() - 2].a, a) <=
               st[st.size() - 2].x) st.pop_back();
        if (!st.empty()) st.back().x = intersection(st.back().a, a);
        st.push_back({a, INF});
    }
    int get_val(int x) {
        int l = -1, r = (int)st.size() - 1;
        while (r - l > 1) {
            int m = (l + r) / 2;
            if (st[m].x < x) l = m;
            else r = m;
        }
        return st[r].a.eval(x);
    }
};

DynamicCHT.cpp
Description: Dynamic CHT for maximum
8a0777, 30 lines
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {
        return Q ? p < o.p : k < o.k;
    }
};
struct LineContainer : multiset<Line> {
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
};

```

```

MinPlusConv.cpp
Description: Min-Plusconv, A is convex down
5d63d9, 28 lines
// Assumptions: 'a' is convex, 'opt' has size ' $n+m-1$ '
// 'opt[k]' will be equal to ' $\arg \min(a[k-i] + b[i])$ '
template<typename T>
void convex_arbitrary_min_plus_conv(T *a, int n, T *b, int m, int *opt) {
    auto rec = [&](auto &self, int lx, int rx, int ly, int ry) -> void {
        if (lx > rx) return;
        int mx = (lx + rx) >> 1;
        opt[mx] = ly;
        for (int i = ly; i <= ry; ++i)
            if (mx >= i && (mx - opt[mx]) >= n || a[mx - opt[mx]] + b[opt[mx]] > a[mx - i] + b[i])
                opt[mx] = i;
        self(self, lx, mx - 1, ly, opt[mx]);
        self(self, mx + 1, rx, opt[mx], ry);
    };
    rec(rec, 0, n + m - 2, 0, m - 1);
}

```

```

// Assumptions: 'a' is convex down
template<typename T>
std::vector<T> convex_arbitrary_min_plus_conv(const std::vector<T> &a,
const std::vector<T> &b) {
    int n = a.size(), m = b.size();
    int *opt = (int) malloc(sizeof(int) * (n + m - 1));
    convex_arbitrary_min_plus_conv(a.data(), n, b.data(), m, opt);
    std::vector<T> ans(n + m - 1);
    for (int i = 0; i < n + m - 1; ++i) ans[i] = a[i - opt[i]] + b[opt[i]];
    free(opt);
    return ans;
}

```

**Kinetic.cpp****Description:** kinetic segment tree  
**Time:**  $\mathcal{O}(hz)$ 

```

49b24c, 127 lines
//vnutrenniy functions — poluintervalli, vneshnie — otrezki. ishet min priamuy
struct line {
    ll k,b,temp;
    ll eval() const {
        return k * temp + b;
    }
    ll melting_point(const line& other) const {
        ll val1 = eval();
        ll val2 = other.eval();
        assert(val1 <= val2);
        if (other.k >= k) {
            return INF;
        }
        ll delta_val = val2 - val1;
        ll delta_k = k - other.k;
        assert(delta_val >= 0 && delta_k > 0);
        return (delta_val + delta_k - 1) / delta_k;
    }
};
struct kinetic_segtree {
    struct node {
        ll lazy_b = 0, lazy_temp = 0, melt = INF;
        line best;
        node(line best = line()) : best(best) {}
    };
    int n;
    vector<node> tree;
    void update(int v) {
        if (make_pair(tree[v < 1].best.eval(), tree[v < 1].best.k) <
            make_pair(tree[v < 1 | 1].best.eval(), tree[v < 1 | 1].best.k)) {
            tree[v].best = tree[v < 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v < 1 | 1].best);
        } else {
            tree[v].best = tree[v < 1 | 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v < 1 | 1].best);
        }
        tree[v].melt = min({tree[v].melt, tree[v < 1].melt, tree[v < 1 | 1].melt});
        assert(tree[v].melt > 0);
    }
    void apply(int v, int vl, int vr, ll delta_b, ll delta_temp) {
        tree[v].lazy_b += delta_b;
        tree[v].lazy_temp += delta_temp;

        tree[v].best.b += delta_b;
        tree[v].best.temp += delta_temp;

        tree[v].melt -= delta_temp;
        if (tree[v].melt <= 0) {
            push(v, vl, vr);
            update(v);
        }
    }
    void push(int v, int vl, int vr) {
        int vm = (vl + vr) / 2;
        apply(v < 1, vl, vm, tree[v].lazy_b, tree[v].lazy_temp);
        apply(v < 1 | 1, vm, vr, tree[v].lazy_b, tree[v].lazy_temp);
    }
}

```

```

tree[v].lazy_b = 0;
tree[v].lazy_temp = 0;
}
void build(int v, int vl, int vr, const vector<line> &lines) {
    if (vl - vr == 1) {
        tree[v] = node(lines[vl]);
        return;
    }
    int vm = (vl + vr) / 2;
    build(v << 1, vl, vm, lines);
    build(v << 1 | 1, vm, vr, lines);
    update(v);
}
void add(int v, int vl, int vr, int l, int r, ll delta_b, ll
        delta_temp) {
    if (r <= vl || vr <= l) {
        return;
    }
    if (l <= vl && vr <= r) {
        apply(v, vl, vr, delta_b, delta_temp);
        return;
    }
    push(v, vl, vr);
    int vm = (vl + vr) / 2;
    add(v << 1, vl, vm, l, r, delta_b, delta_temp);
    add(v << 1 | 1, vm, vr, l, r, delta_b, delta_temp);
    update(v);
}
void change_line(int v, int vl, int vr, int pos, const line &new_line)
{
    if (vr - vl == 1) {
        tree[v].best = new_line;
        return;
    }
    push(v, vl, vr);
    int vm = (vl + vr) / 2;
    if (pos < vm) {
        change_line(v << 1, vl, vm, pos, new_line);
    } else {
        change_line(v << 1 | 1, vm, vr, pos, new_line);
    }
    update(v);
}
ll query(int v, int vl, int vr, int l, int r) {
    if (r <= vl || vr <= l) {
        return INF;
    }
    if (l <= vl && vr <= r) {
        return tree[v].best.eval();
    }
    push(v, vl, vr);
    int vm = (vl + vr) / 2;
    return min(query(v << 1, vl, vm, l, r), query(v << 1 | 1, vm, vr,
        l, r));
}
kinetic_segtree(const vector<line> &lines) : n(lines.size()), tree(4
    * n) {
    build(1, 0, n, lines);
}
kinetic_segtree(int n) : n(n), tree(4 * n) {
    vector<line> lines(n, {0, INF, 0});
    build(1, 0, n, lines);
}
void add(int l, int r, ll delta_b, ll delta_temp) {
    assert(delta_temp >= 0);
    add(1, 0, n, l, r + 1, delta_b, delta_temp);
}
void change_line(int pos, const line &new_line) {
    assert(0 <= pos && pos < n);
    change_line(1, 0, n, pos, new_line);
}
ll query(int l, int r) {
    return query(1, 0, n, l, r + 1);
}

```

## GoldenSearch.cpp

Description: Golden Search

31d45b, 14 lines

```

double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5) - 1) / 2, eps = 1e-7;
    double x1 = b - r * (b - a), x2 = a + r * (b - a);
    double f1 = f(x1), f2 = f(x2);
    while (b - a > eps) {
        if (f1 < f2) { // change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r * (b - a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r * (b - a); f2 = f(x2);
        }
    }
    return a;
}

3dBasic.cpp
Description: Basic 3d geom things
467773, 79 lines

```

```

const int inf = int(1e9) + int(1e5);
const ll infl = ll(2e18) + ll(1e10);
const ld eps = 1e-9;
bool ze(ld x) {
    return fabs(x) < eps;
}
struct pt {
    ld x, y, z;
    pt operator+(const pt &p) const {
        return pt{x + p.x, y + p.y, z + p.z};
    }
    pt operator-(const pt &p) const {
        return pt{x - p.x, y - p.y, z - p.z};
    }
    ld operator*(const pt &p) const {
        return x * p.x + y * p.y + z * p.z;
    }
    pt operator*(ld a) const {
        return pt{x * a, y * a, z * a};
    }
    pt operator%(const pt &p) const {
        return pt{y * p.z - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x};
    }
    ld abs() const {
        return sqrtl(*this * *this);
    }
    ld abs2() const {
        return *this * *this;
    }
    pt norm() const {
        ld d = abs();
        return pt{x / d, y / d, z / d};
    }
};

// BEGIN.CODE
struct Plane {
    pt v;
    ld c;
    Plane(pt a, pt b, pt c) {
        v = ((b - a) % (c - a)).norm();
        this->c = a * v;
    }
    ld dist(pt p) {
        return p * v - c;
    }
};
pt projection(pt p, pt a, pt b) {
    pt v = b - a;
    if (ze(v.abs2())) {
        // stub : bad line
        return a;
    }
    return a + v * (((p - a) * v) / (v * v));
}
pair<pt, pt> planesIntersection(Plane a, Plane b) {
    pt dir = a.v % b.v;
    if (ze(dir.abs2())) {
        // stub : parallel planes
    }
}
```

```

        return {pt{1e18, 1e18, 1e18}, pt{1e18, 1e18, 1e18}};
    }
    ld s = a.v * b.v;
    pt v3 = b.v - a.v * s;
    pt h = a.v * a.c + v3 * ((b.c - a.c * s) / (v3 * v3));
    return {h, h + dir};
}
pair<pt, pt> commonPerpendicular(pt a, pt b, pt c, pt d) {
    pt v = (b - a) % (d - c);
    ld S = v.abs();
    if (ze(S)) {
        // stub : parallel lines
        return {pt{1e18, 1e18, 1e18}, pt{1e18, 1e18, 1e18}};
    }
    v = v.norm();
    pt sh = v * (v * c - v * a);
    pt a2 = a + sh;
    ld s1 = ((c - a2) % (d - a2)) * v;
    pt p = a + (b - a) * (s1 / S);
    return {p, p + sh};
}

NDHull.cpp
Description: Hull in arbitrary number of dimensions
Time: O(N * Dim * Hull)
cf8067, 77 lines

```

```

const int DIM = 4;
typedef array<ll, DIM> pt;
pt operator-(const pt &a, const pt &b) {
    pt res;
    forn(i, DIM) res[i] = a[i] - b[i];
    return res;
}
typedef array<pt, DIM - 1> Edge;
typedef array<pt, DIM> Face;
vector<Face> faces;
ll det(pt *a) {
    int p[DIM];
    iota(p, p + DIM, 0);
    ll res = 0;
    do {
        ll x = 1;
        forn(i, DIM) {
            forn(j, i) if (p[j] > p[i]) x *= -1;
            x *= a[i][p[i]];
        }
        res += x;
    } while (next_permutation(p, p + DIM));
    return res;
}
ll V(Face f, pt pivot) {
    pt p[DIM];
    forn(i, DIM) p[i] = f[i] - pivot;
    return det(p);
}
void init(vector<pt> p) {
    forn(i, DIM + 1) {
        Face a;
        int q = 0;
        forn(j, DIM + 1) if (j != i) a[q++] = p[j];
        ll v = V(a, p[i]);
        assert(v != 0);
        if (v < 0) swap(a[0], a[1]);
        faces.push_back(a);
    }
}
void add(pt p) {
    vector<Face> newf, bad;
    for (auto f : faces) {
        if (V(f, p) < 0)
            bad.push_back(f);
        else
            newf.push_back(f);
    }
    if (bad.empty()) {
        cout << " Ignore \n";
        return;
    }
    cout << " Rebuild \n";
}
```

```

faces = newf;
vector<pair<Edge, pt>> edges;
for (auto f : bad) {
    sort(all(f));
    forn(i, DIM) {
        Edge e;
        int q = 0;
        forn(j, DIM) if (i != j) e[q++] = f[j];
        edges.emplace_back(e, f[i]);
    }
}
sort(all(edges));
forn(i, sz(edges)) {
    if (i + 1 < sz(edges) && edges[i + 1].first == edges[i].first) {
        ++i;
        continue;
    }
    Face f;
    forn(j, DIM - 1) f[j] = edges[i].first[j];
    f[DIM - 1] = p;
    if (V(f, edges[i].second) < 0) swap(f[0], f[1]);
    faces.push_back(f);
}

```

**GenerateNonConvex.cpp****Description:** Non convex polygon generation

2a7d37, 74 lines

```

vector<vec> pointsInGeneralPosition(int n, int maxC) {
    vector<vec> arr(n);
    for (int i = 0; i < n; ++i) {
        arr[i].x = randint(0, maxC);
        arr[i].y = randint(0, maxC);
    }
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < i; ++j) {
            for (int k = 0; k < j; ++k) {
                if (sign((arr[i] - arr[j]) % (arr[j] - arr[k])) == 0) {
                    return pointsInGeneralPosition(n, maxC);
                }
            }
        }
    }
    return arr;
}

vector<vec> pointsDifferent(int n, int maxC) {
    vector<vec> arr;
    while (arr.size() < n) {
        vec v;
        v.x = randint(0, maxC);
        v.y = randint(0, maxC);
        if (binary_search(all(arr), v)) {
            continue;
        }
        arr.pbc(v);
        sort(all(arr));
    }
    shuffle(all(arr), rnd);
    return arr;
}

vector<vec> generateNonconvex(int n, int maxC) {
    vector<vec> arr = pointsDifferent(n, maxC);
    bool was = 1;
    while (was) {
        was = 0;
        for (int i = 0; i < n; ++i) {
            for (int j = i + 2; j < n; ++j) {
                if ((j + 1) % n == i) continue;
                if (intersects(arr[i], arr[(i + 1)%n], arr[j], arr[(j + 1)%n])) {
                    reverse(arr.begin() + i + 1, arr.begin() + j + 1);
                    was = 1;
                }
            }
        }
        if (area(arr) < 0) {

```

**GenerateNonConvex MinDisk ClosestPair Faces**

```

        reverse(all(arr));
    }
    if (sign(area(arr)) == 0) {
        return generateNonconvex(n, maxC);
    }
    return arr;
}

template<typename T>
vector<vec<T>> polyRemoveOnOneLine(vector<vec<T>> arr) {
    int n = arr.size();
    for (int it = 0; it < 3; ++it) {
        vector<vec<T>> res;
        for (auto el : arr) {
            if (res.size() >= 2 && sign((res[res.size() - 2] - el) % (res.back() - el)) == 0) {
                res.pop_back();
            }
            res.pbc(el);
        }
        arr = res;
        rotate(arr.begin(), 1 + all(arr));
    }
    return arr;
}

```

**MinDisk.cpp****Description:** Computes the minimum circle that encloses a set of points.**Time:** expected  $\mathcal{O}(n)$ 

```

ld ccRadius(const vec& A, const vec& B, const vec& C) {
    return len(B-A)*len(C-B)*len(A-C)/abs((B-A)%(C-A))/2;
}
vec circumcenter(const vec& A, const vec& B, const vec& C) {
    vec b = C-A, c = B-A;
    return A + ort(b*(c*c)-c*(b*b))/(b*c)/2;
}

pair<vec, ld> mindisk(vector<vec> ps) {
    shuffle(all(ps), rnd);
    vec o = ps[0];
    ld r = 0, EPS = 1 + 1e-8;
    for (int i = 0; i < ps.size(); ++i) {
        if (len(o - ps[i]) > r * EPS) {
            o = ps[i], r = 0;
            for (int j = 0; j < i; ++j) {
                if (len(o - ps[j]) > r * EPS) {
                    o = (ps[i] + ps[j]) / 2;
                    r = len(o - ps[i]);
                    for (int k = 0; k < j; ++k) {
                        if (len(o - ps[k]) > r * EPS) {
                            o = circumcenter(ps[i], ps[j], ps[k]);
                            r = len(o - ps[i]);
                        }
                    }
                }
            }
        }
    }
    return {o, r};
}

```

**ClosestPair.cpp****Description:** Finds the closest pair of points.**Time:**  $\mathcal{O}(n \log n)$ 

```

// assumes points are long long, long double probably should work, but is
// slow
pair<vec, vec> closest(vector<vec> v) {
    assert(v.size() > 1);
    set<vec> S;
    sort(all(v), [](vec a, vec b) { return a.y < b.y; });
    pair<ll, pair<vec, vec>> ret(LLONG_MAX, {{0,0}, {0,0}});
    int j = 0;
    for (vec p : v) {
        vec d = (ll)sqrt(ret.first), 0);
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for ; lo != hi; ++lo)

```

```

            ret = min(ret, {len2(*lo - p), {*lo, p}});
            S.insert(p);
        }
        return ret.second;
    }
}

```

**Faces.cpp****Description:** dealing with planar graphs

```

// returns faces.size(), if v in the outer face
int find_face(const vector<vec> &pts, const vector<vector<int>> &faces,
              vec v) {
    int res = faces.size();
    ld resarea = 0;
    vector<vec> face;
    for (int i = 0; i < (int)faces.size(); ++i) {
        face.clear();
        for (int j : faces[i]) {
            face.push_back(pts[j]);
        }
        ld area = get_area(face);
        if (sign(area) > 0) {
            if (isIn(face, v)) {
                // return faces.size(); // if faces are connected
                if (res == (int)faces.size() || area < resarea) {
                    res = i;
                    resarea = area;
                }
            }
        }
    }
    return res;
}

// g.size() == pts.size() + 1, so that there is one new outer face
// all previously outer faces will have g[v].size() == 0
vector<vector<int>> build_faces_graph(const vector<vec> &pts, const
                                         vector<vector<int>> &faces) {
    vector<int> realface(faces.size());
    iota(all(realface), 0);
    vector<vec> qq;
    vector<int> ind;
    for (int i = 0; i < (int)faces.size(); ++i) {
        vector<vec> face;
        for (int j : faces[i]) {
            face.pbc(pts[j]);
        }
        ld a = get_area(face);
        if (a < 0) {
            // if only one outer face, then realface[i] = faces.size();
            // otherwise following code
            vec v = *min_element(all(face));
            v.x -= 10 * EPS;
            qq.pbc(v);
            ind.pbc(i);
            // realface[i] = find_face(pts, faces, v);
            // assert(realface[i] != i);
        }
    }
    if (1) { // slow, but easy to write
        for (int i = 0; i < (int)qq.size(); ++i) {
            int j = find_face(pts, faces, qq[i]);
            assert(j != ind[i]);
            realface[ind[i]] = j;
        }
    } else {
        vector<int> res = point_location(pts, faces, qq);
        for (int i = 0; i < (int)qq.size(); ++i) {
            int j = res[i];
            assert(j != ind[i]);
            realface[ind[i]] = j;
        }
    }
    map<pair<int, int>, int> edge2face;
    for (int i = 0; i < (int)faces.size(); ++i) {
        for (int j = 0; j < (int)faces[i].size(); ++j) {
            int a = faces[i][j];
            int b = faces[i][(j + 1) % faces[i].size()];
            edge2face[{a, b}] = realface[i];
        }
    }
}

```

```

        }
    vector<vector<int>> g(faces.size() + 1);
    for (auto [pp, c] : edge2face) {
        g[c].pb(edge2face[{pp.second, pp.first}]);
    }
    for (auto &el : g) {
        sort(all(el));
        el.erase(unique(all(el)), el.end());
    }
    return g;
}

vector<vector<int>> get_faces(const vector<vec> &pts, const vector<vector<int>> &g) {
    int n = g.size();
    vector<vector<pair<int, int>>> g2(n);
    int cur_edge = 0;
    for (int i = 0; i < n; ++i) {
        for (int j : g[i]) {
            if (i < j) {
                g2[j].pb({i, cur_edge});
                g2[i].pb({j, cur_edge ^ 1});
                cur_edge += 2;
            }
        }
    }
    vector<int> ind(cur_edge), used(cur_edge);
    for (int i = 0; i < n; ++i) {
        sort(all(g2[i]), [&](auto a, auto b) {
            auto va = pts[a.first] - pts[i];
            auto vb = pts[b.first] - pts[i];
            return mp(half(va), (ld)0) < mp(half(vb), va % vb);
        });
        for (int j = 0; j < (int) g2[i].size(); ++j)
            ind[g2[i][j].second] = j;
    }
    vector<vector<int>> faces;
    for (int i = 0; i < n; ++i) {
        for (int ei = 0; ei < (int)g[i].size(); ++ei) {
            if (used[g[i][ei].second]) continue;
            vector<int> face;
            int v = i;
            int e = g2[v][ei].second;
            while (!used[e]) {
                used[e] = 1;
                face.pbc(v);
                int u = g2[v][ind[e]].first;
                int newe = g2[u][(ind[e] ^ 1) - 1 + g2[u].size()] % g2[u].size();
                v = u;
                e = newe;
            }
            faces.push_back(face);
        }
    }
    return faces;
}

pair<vector<vec>, vector<vector<int>>> build_graph(vector<pair<vec, vec>> segs) {
    vector<vec> p;
    vector<vector<int>> g;
    map<pair<ll, ll>, int> id;
    auto r = mp(ll(round(v.x * 1'000'000'000 + EPS * sign(v.x))), ll(
        round(v.y * 1'000'000'000 + EPS * sign(v.y))));
    if (!id.count(r)) {
        g.pbc({});
        int i = id.size();
        id[r] = i;
        p.pbc(v);
        return i;
    }
    return id[r];
}
for (int i = 0; i < (int)segs.size(); ++i) {
    vector<int> cur = {getid(segs[i].first), getid(segs[i].second)};
}

```

## PointLocation

```

for (int j = 0; j < (int)segs.size(); ++j) {
    if (i != j) {
        if (intersects(segs[i].first, segs[i].second, segs[j].first, segs[j].second)) {
            vec res;
            if (intersect(getln(segs[i].first, segs[i].second),
                          getln(segs[j].first, segs[j].second), res)) {
                cur.pbc(getid(res));
            } else {
                if (isOnSegment(segs[i].first, segs[i].second, segs[j].first))
                    cur.pbc(getid(segs[j].first));
                if (isOnSegment(segs[i].first, segs[i].second, segs[j].second))
                    cur.pbc(getid(segs[j].second));
            }
        }
    }
}
sort(all(cur), [&](int i, int j) { return p[i] < p[j]; });
cur.erase(unique(all(cur)), cur.end());
for (int j = 1; j < (int)cur.size(); ++j) {
    g[cur[j]].pb(cur[j - 1]);
    g[cur[j - 1]].pb(cur[j]);
}
for (auto &el : g) {
    sort(all(el));
    el.erase(unique(all(el)), el.end());
}
return {p, g};
}

PointLocation.cpp
Description: Point location xd
573c9d, 276 lines
const vec arb = {(int)1e9 + 228, (int)1e9 + 228}; // ne sopadajet s drygimi tochkami

bool ge(const ll& a, const ll& b) { return a >= b; }
bool le(const ll& a, const ll& b) { return a <= b; }
bool eq(const ll& a, const ll& b) { return a == b; }
bool gt(const ll& a, const ll& b) { return a > b; }
bool lt(const ll& a, const ll& b) { return a < b; }

ll vec::dot(const vec &a) const {
    return *this * a;
}
ll vec::cross(const vec &a) const {
    return *this % a;
}
ll vec::dot(const vec &a, const vec &b) const {
    return (a - *this) * (b - *this);
}
ll vec::cross(const vec &a, const vec &b) const {
    return (a - *this) % (b - *this);
}

struct Edge {
    vec l, r;
    auto operator<=>(const Edge &) const = default;
};

bool edge_cmp(const Edge& edge1, const Edge& edge2) {
    const vec a = edge1.l, b = edge1.r;
    const vec c = edge2.l, d = edge2.r;
    int val = sign(a.cross(b, c)) + sign(a.cross(b, d));
    if (val != 0)
        return val > 0;
    val = sign(c.cross(d, a)) + sign(c.cross(d, b));
    return val < 0;
}

enum EventType { DEL = 2, ADD = 3, GET = 1, VERT = 0 };

struct Event {
    EventType type;
    int pos;
}

```

```

bool operator<(const Event& event) const { return type < event.type;
}
};

vector<Edge> sweepline(vector<Edge> planar, vector<vec> queries) {
    using vec_type = decltype(vec::x);
    // collect all x-coordinates
    auto s =
        set<vec_type, std::function<bool(const vec_type&, const vec_type>>(
            &gt;>)(lt));
    for (vec p : queries)
        s.insert(p.x);
    for (auto e : planar) {
        s.insert(e.l.x);
        s.insert(e.r.x);
    }
    // map all x-coordinates to ids
    int cid = 0;
    auto id =
        map<vec_type, int, std::function<bool(const vec_type&, const vec_type&)>>(
            lt);
    for (auto x : s)
        id[x] = cid++;
    // create events
    auto t = set<Edge, decltype(*edge_cmp)>(edge_cmp);
    auto vert_cmp = [] (const pair<vec_type, int>& l,
                        const pair<vec_type, int>& r) {
        if (!eq(l.first, r.first))
            return lt(l.first, r.first);
        return l.second < r.second;
    };
    auto vert = set<pair<vec_type, int>, decltype(vert_cmp)>(vert_cmp);
    vector<vector<Event>> events(cid);
    for (int i = 0; i < (int)queries.size(); i++) {
        int x = id[queries[i].x];
        events[x].push_back(Event(GET, i));
    }
    for (int i = 0; i < (int)planar.size(); i++) {
        int lx = id[planar[i].l.x], rx = id[planar[i].r.x];
        if (lx > rx) {
            swap(lx, rx);
            swap(planar[i].l, planar[i].r);
        }
        if (lx == rx) {
            events[lx].push_back(Event(VERT, i));
        } else {
            events[lx].push_back(Event(ADD, i));
            events[rx].push_back(Event(DEL, i));
        }
    }
    // perform sweep line algorithm
    vector<Edge> ans(queries.size(), {arb, arb});
    for (int x = 0; x < cid; x++) {
        sort(events[x].begin(), events[x].end());
        vert.clear();
        for (Event event : events[x]) {
            if (event.type == DEL) {
                t.erase(planar[event.pos]);
            }
            if (event.type == VERT) {
                vert.insert(make_pair(
                    min(planar[event.pos].l.y, planar[event.pos].r.y),
                    event.pos));
            }
            if (event.type == ADD) {
                t.insert(planar[event.pos]);
            }
            if (event.type == GET) {
                auto jt = vert.upper_bound(
                    make_pair(queries[event.pos].y, planar.size()));
                if (jt != vert.begin()) {
                    --jt;
                    int i = jt->second;

```

```

        if (ge(max(planar[i].l.y, planar[i].r.y),
                 queries[event.pos].y)) {
            ans[event.pos] = planar[i];
            continue;
        }
    }
    Edge e;
    e.l = e.r = queries[event.pos];
    auto it = t.upper_bound(e);
    if (it != t.begin()) {
        ans[event.pos] = *(--it);
    }
}

for (Event event : events[x]) {
    if (event.type != GET)
        continue;
    if (ans[event.pos].l != arb &&
        eq(ans[event.pos].l.x, ans[event.pos].r.x))
        continue;

    Edge e;
    e.l = e.r = queries[event.pos];
    auto it = t.upper_bound(e);
    if (it == t.begin())
        e = {arb, arb};
    else
        e = *(--it);
    if (ans[event.pos].l == arb) {
        ans[event.pos] = e;
        continue;
    }
    if (e.l == arb)
        continue;
    if (e == ans[event.pos])
        continue;
    if (id[ans[event.pos].r.x] == x) {
        if (idle[e.l.x] == x) {
            if (gt(e.l.y, ans[event.pos].r.y))
                ans[event.pos] = e;
        }
    } else {
        ans[event.pos] = e;
    }
}
return ans;
}

struct DCEL {
    struct Edge {
        vec origin;
        int nxt;
        int twin;
        int face;
    };
    vector<Edge> body;
};

// outer face is -1, returns (1,i) is point is strictly inside face i,
// and (0,1) if point lies on the edge i
vector<pair<int, int>> point_location(DCEL planar, vector<vec> queries)
{
    vector<pair<int, int>> ans(queries.size());
    vector<Edge> planar2;
    map<Edge, int> pos;
    map<Edge, int> added_on;
    int n = planar.body.size();
    for (int i = 0; i < n; i++) {
        if (planar.body[i].face > planar.body[planar.body[i].twin].face)
            continue;
        Edge e;
        e.l = planar.body[i].origin;
        e.r = planar.body[planar.body[i].twin].origin;
        if (e.r.x < e.l.x) swap(e.l, e.r);
        added_on[e] = i;
        pos[e] =
            {planar.body[i].origin.x, planar.body[planar.body[i].twin].origin.x}
    }
}

```

```

        ? planar.body[i].face
        : planar.body[planar.body[i].twin].face;
    planar2.push_back(e);
}

auto res = sweepline(planar2, queries);
for (int i = 0; i < (int)queries.size(); i++) {
    if (res[i].l == arb) {
        ans[i] = make_pair(1, -1);
        continue;
    }
    vec p = queries[i];
    vec l = res[i].l, r = res[i].r;
    if (eq(p.cross(l, r), 0) && le(p.dot(l, r), 0)) {
        ans[i] = make_pair(0, added_on[res[i]]);
        continue;
    }
    ans[i] = make_pair(1, pos[res[i]]);
}
return ans;
}

DCEL buildDCEL(const vector<vec> &pts, const vector<vector<int>> &g) {
    int n = g.size();
    vector<vector<pair<int, int>>> g2(n);
    int cur_edge = 0;
    for (int i = 0; i < n; ++i) {
        for (int j : g[i]) {
            if (i < j) {
                g2[j].pb({i, cur_edge});
                g2[i].pb({j, cur_edge ^ 1});
                cur_edge += 2;
            }
        }
    }
    vector<int> ind(cur_edge), used(cur_edge);
    for (int i = 0; i < n; ++i) {
        sort(all(g2[i]), [&](auto a, auto b) {
            auto va = pts[a.first] - pts[i];
            auto vb = pts[b.first] - pts[i];
            return mp(half(va), OLL) < mp(half(vb), va % vb);
        });
        for (int j = 0; j < (int) g2[i].size(); ++j) {
            ind[g2[i][j].second] = j;
        }
    }
    using Edge = DCEL::Edge;
    vector<Edge> edges(cur_edge);
    for (int i = 0; i < cur_edge; ++i) {
        edges[i].twin = i ^ 1;
    }
    int cur_face = 0;
    for (int i = 0; i < n; ++i) {
        for (int ei = 0; ei < (int)g[i].size(); ++ei) {
            if (used[g2[i][ei].second]) continue;
            vector<vec> face;
            vector<int> inds;
            int v = i;
            int e = g2[v][ei].second;
            while (!used[e]) {
                edges[e].origin = pts[v];
                edges[e].face = cur_face;
                inds.pb(e);
                used[e] = 1;
                face.pbc(pts[v]);
                int u = g2[v][ind[e]].first;
                int newe = g2[u][(ind[e] ^ 1) - 1 + g2[u].size()] % g2[u].size().second;
                edges[e].nxt = newe;
                v = u;
                e = newe;
            }
            if (sign(get_area(face)) <= 0) {
                for (int i : inds) {
                    edges[i].face = -1;
                }
            } else {
                ++cur_face;
            }
        }
    }
}

```

```

    return {edges};
}

```

## Svg.cpp

Description: geometry visualizer

e9032a, 36 lines

```

struct SVG {
    FILE *out;
    ld sc = 50;

    void open() {
        out = fopen("image.svg", "w");
        fprintf(out, "<svg xmlns='http://www.w3.org/2000/svg' viewBox='-1000 -1000 2000 2000'>\n");
    }

    void line(vec a, vec b) {
        a = a * sc, b = b * sc;
        fprintf(out, "<line xl='%Lf' yl='%Lf' x2='%Lf' y2='%Lf' stroke='black' />\n", a.x, -a.y, b.x, -b.y);
    }

    void circle(vec a, ld r = -1, string col = "red") {
        r = (r == -1 ? 10 : sc * r);
        a = a * sc;
        fprintf(out, "<circle cx='%Lf' cy='%Lf' r='%Lf' fill='%s' />\n", a.x, -a.y, r, col.c_str());
    }

    void text(vec a, string s) {
        a = a * sc;
        fprintf(out, "<text x='%Lf' y='%Lf' font-size='10px'>%s</text>\n", a.x, -a.y, s.c_str());
    }

    void close() {
        fprintf(out, "</svg>\n");
        fclose(out);
        out = 0;
    }

    ~SVG() {
        if (out)
            close();
    }
} svg;

```

## Delauney.cpp

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0]}, ..., all counter-clockwise.

Time:  $\mathcal{O}(n \log n)$ 

9e818a, 97 lines

```

typedef vec P;
typedef struct Quad* Q;
// using lll = __int128_t; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point
#define rep(i,a,b) for (int i=a;i<b;++i)

lll vec::cross(const vec &b) const {
    return *this % b;
}

lll vec::cross(const vec &b, const vec &c) const {
    return (b - *this) % (c - *this);
}

struct Quad {
    Q rot, o; P p = arb; bool mark;
    P& F() { return r() ->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r() ->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = len2(p), A = len2(a - p2);
}
```

```

B = len2(b)-p2, C = len2(c)-p2;
return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{}}}};
    H = r->o; r->r() = r;
    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
}

void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}

pair<Q,Q> rec(const vector<P>& s) {
    if (s.size() <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (s.size() == 2) return {a, a->r()};
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r()};
    }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = s.size() / 2;
    tie(ra, A) = rec({all(s) - half});
    tie(B, rb) = rec({s.size() - half + all(s)});
    while ((B->p).cross(H(A)) < 0 && (A = A->next()) || (A->p).cross(H(B)) > 0 && (B = B->r()->o));
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    }
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC)))) {
            base = connect(RC, base->r());
        } else {
            base = connect(base->r(), LC->r());
        }
    }
    return {ra, rb};
}

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (pts.size() < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \ q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < q.size()) if (!(e = q[qi++])->mark) ADD;
    return pts;
}

```

## SegmentInPolygon.cpp

Description: length of longest segment inside polygon

<bits/stdc++.h>

## SegmentInPolygon BerlekampMassey GoncharFedor CRT

```

using namespace std;
const int N=210;
int n,w;
double s,res,ans;
int sgn(l1 x){return !x?0:(x>0?1:-1);}
struct point{
    int x,y;
    point operator-(point a){return {x-a.x,y-a.y};}
    ll operator*(point a){return l1l*x*a.y-l1l*y*a.x;}
    double len(){return sqrt(l1l*x*x+l1l*y*y);}
}p[N];
vector<pair<double,int>> v;
double isp(point x1,point y1,point x2,point y2){
    return 1.0*((x2-x1)*(y2-y1)-(y2-y1)*(x1-x2));
}

double calc(point a,point b){
    v.clear(),w=s=res=0;
    for(int i=1;i<n;i++){
        int x=sgn((b-a)|(p[i-1]-a)),y=sgn((b-a)|(p[i]-a));
        if(x==y) continue;
        v.push_back({isp(a,b,p[i-1],p[i]),(x<y?1:-1)*(x&y?2:1)} );
    }
    sort(v.begin(),v.end());
    for(int i=0;i<(int)v.size();i++){
        if(w==v[i].first) res=max(res,s),s=0;
        else res+=v[i].second;
    }
    return max(res,s)*(b-a).len();
}

signed main(){
    scanf("%d",&n);
    for(int i=1;i<=n;i++) scanf("%d%d",&p[i].x,&p[i].y);
    p[0]=p[n];
    for(int i=1;i<n;i++)
        for(int j=i+1;j<=n;j++) ans=max(ans,calc(p[i],p[j]));
    printf("%.9lf\n",ans);
    return 0;
}

```

## Math (6)

### BerlekampMassey.cpp

Description: Find the shortest linear-feedback shift register

Time:  $\mathcal{O}(n^2)$

```

08eddc, 86 lines
vector<int> berlekamp(vector<int> x) {
    vector<int> ls, cur;
    int lf = 0, d = 0;
    for (int i = 0; i < x.size(); ++i) {
        ll t = 0;
        for (int j = 0; j < cur.size(); ++j) {
            t = (t + (ll) x[i - j - 1] * cur[j]) % MOD;
        }
        if ((t - x[i]) % MOD == 0)
            continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            d = (t - x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * powmod(d, MOD - 2) % MOD;
        vector<int> c(i - lf - 1);
        c.push_back(k);
        for (auto &j : ls)
            c.push_back(-j * k % MOD);
        if (c.size() < cur.size())
            c.resize(cur.size());
        for (int j = 0; j < cur.size(); ++j) {
            c[j] = (c[j] + cur[j]) % MOD;
        }
        if (i - lf + (int)ls.size() >= (int)cur.size()) {
            tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % MOD);
        }
        cur = c;
    }
}
```

```

for (auto &i : cur)
    i = (i % MOD + MOD) % MOD;
return cur;
}
// for a_i = 2 * a_i-1 + a_i-2 returns {2, 1}

// kth element of p/q as fps
int getkfps(vector<int> p, vector<int> q, ll k) {
    assert(q[0] != 0);
    while (k) {
        auto f = q;
        for (int i = 1; i < (int) f.size(); i += 2) {
            f[i] = sub(0, f[i]);
        }
        auto p2 = conv(p, f);
        auto q2 = conv(q, f);
        p.clear(), q.clear();
        for (int i = k % 2; i < (int) p2.size(); i += 2) {
            p.pbc(p2[i]);
        }
        for (int i = 0; i < (int)q2.size(); i += 2) {
            q.pbc(q2[i]);
        }
        k >>= 1;
    }
    return mul(p[0], inv(q[0]));
}

// vals - initials values of recurrence, c - result of berlekamp on vals
int getk(const vector<int> &vals, vector<int> c, ll k) {
    int d = c.size();
    c.insert(c.begin(), MOD-1);
    while (c.back() == 0) {
        c.pop_back();
    }
    for (auto &el : c) {
        el = sub(0, el);
    }
    vector<int> p(d);
    copy(vals.begin(), vals.begin() + d, p.begin());
    p = conv(p, c);
    p.resize(d);
    return getkfps(p, c, k);
}

vector<int> getmod(vector<int> a, vector<int> md) {
    for (int i = a.size() - 1; i + 1 >= md.size(); --i) {
        int v = mul(a[i], inv(md.back()));
        for (int j = 0; j < md.size(); ++j) {
            a[i - md.size() + 1 + j] = sub(a[i - md.size() + 1 + j], mul(
                md[j], v));
        }
        a.pop_back();
    }
    return a;
}

```

```

vector<int> solve_triangle(ll A, ll B, ll C) { // / x,y >= 0, Ax+By <= C
    if (C < 0)
        return 0;
    if (A > B)
        swap(A, B);
    ll p = C / B;
    ll k = B / A;
    ll d = (C - p * B) / A;
    return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
        (p + 1) * (d + 1) + k * p * (p + 1) / 2;
}

vector<int> extgcd(int a, int b, int &x, int &y) { // define int ll
    if (a == 0) {
        x = 0, y = 1;
    }
    else {
        int t = a;
        a = b % a;
        b = t;
        extgcd(b, a, y, x);
        y -= b / a * x;
    }
    return {x, y};
}

```

### GoncharFedor.cpp

Description: Calculating number of points  $x, y \geq 0, Ax + By \leq C$

Time:  $\mathcal{O}(\log(C))$

```

0ef10e, 11 lines
ll solve_triangle(ll A, ll B, ll C) { // / x,y >= 0, Ax+By <= C
    if (C < 0)
        return 0;
    if (A > B)
        swap(A, B);
    ll p = C / B;
    ll k = B / A;
    ll d = (C - p * B) / A;
    return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
        (p + 1) * (d + 1) + k * p * (p + 1) / 2;
}

vector<int> CRT.cpp
Description: CRT for arbitrary modulus
28309e, 25 lines
int extgcd(int a, int b, int &x, int &y) { // define int ll
    if (a == 0) {
        x = 0, y = 1;
    }
    else {
        int t = a;
        a = b % a;
        b = t;
        extgcd(b, a, y, x);
        y -= b / a * x;
    }
    return {x, y};
}

```

```

    return b;
}
int xl, yl;
int g = extgcd(b % a, a, xl, yl);
x = yl - xl * (b / a);
y = xl;
return g;
}

int lcm(int a, int b) { return a / __gcd(a, b) * b; }
int crt(int mod1, int mod2, int rem1, int rem2) {
    int r = (rem2 - (rem1 % mod2) + mod2) % mod2;
    int x, y;
    int g = extgcd(mod1, mod2, x, y);
    if (r % g) return -1;
    x %= mod2;
    if (x < 0) x += mod2;
    int ans = (x * (r / g)) % mod2;
    ans = ans * mod1 + rem1;
    assert(ans % mod1 == rem1);
    assert(ans % mod2 == rem2);
    return ans % lcm(mod1, mod2);
}

```

**Fastmod.cpp****Description:** Fast multiplication by modulo(in [0;2b))

38ea39, 7 lines

```

struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(-ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    }
};

```

**ModularSqrt.cpp****Description:** Calculating sqrt modulo smthTime:  $\mathcal{O}(\log^2)$ 

19a793, 23 lines

```

ll sqrt(ll a, ll p) {
    a %= p;
    if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p - 1) / 2, p) == 1); // e lse no so lution
    if (p % 4 == 3) return modpow(a, (p + 1) / 4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works i f p % 8 == 5
    ll s = p - 1, n = 2;
    int r = 0, m;
    while (s % 2 == 0) ++r, s /= 2;
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p);
    ll b = modpow(a, s, p), g = modpow(n, s, p);
    for (; r = m) {
        ll t = b;
        for (m = 0; m < r && t != 1; ++m) t = t * t % p;
        if (m == 0) return x;
        ll gs = modpow(g, 1LL << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p;
    }
}

```

**DiscreteLog.cpp****Description:** Discrete logTime:  $\mathcal{O}(\sqrt{n})$ 

1cc247, 9 lines

```

ll modLog(ll a, ll b, ll m) {
    ll n = (ll)sqrt(m) + 1, e = 1, f = 1, j = 1;
    unordered_map<ll, ll> A;
    while (j <= n && (e = f = e * a % m) != b % m) A[e * b % m] = j++;
    if (e == b % m) return j;
    if (__gcd(m, e) == __gcd(m, b))
        for (int i = 2; i < n + 2; ++i)
            if (A.count(e = e * f % m)) return n * i - A[e];
    return -1;
}

```

**PrimalityTest.cpp****Description:** Checking primality of pTime:  $\mathcal{O}(\log(C))$ 

ad2714, 32 lines

```

const int iters = 8; // can change
bool isprime(ll p) {
    if (p == 1 || p == 4)
        return 0;
    if (p == 2 || p == 3)
        return 1;
    for (int it = 0; it < iters; ++it) {
        ll a = rnd() % (p - 2) + 2;
        ll nw = p - 1;
        while (nw % 2 == 0)
            nw /= 2;
        ll x = binpow(a, nw, p); // int128
        if (x == 1)
            continue;
        ll last = x;
        nw *= 2;
        while (nw <= p - 1) {
            x = (__int128_t)x * x % p;
            if (x == 1) {
                if (last != p - 1) {
                    return 0;
                }
                break;
            }
            last = x;
            nw *= 2;
        }
        if (x != 1)
            return 0;
    }
    return 1;
}

```

**XorConvolution.cpp****Description:** Calculating xor-convolution of 2 vectors modulo smthTime:  $\mathcal{O}(n \log(n))$ 

454afd, 23 lines

```

void fwht(vector<int> &a) {
    int n = a.size();
    for (int l = 1; l < n; l <= 1) {
        for (int i = 0; i < n; i += 2 * l) {
            for (int j = 0; j < l; ++j) {
                int u = a[i + j], v = a[i + j + 1];
                a[i + j] = add(u, v), a[i + j + 1] = sub(u, v);
            }
        }
    }
} // https://judge.yosupo.jp/problem/bitwise_xor_convolution
vector<int> xorconvo(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    fwht(a), fwht(b);
    int in = inv(n);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], mul(b[i], in));
    fwht(a);
    return a;
}

```

**Factorization.cpp****Description:** Factorizing a number real quickTime:  $\mathcal{O}\left(\frac{1}{\sqrt{n}}\right)$ 

f0d7c6, 51 lines

```

ll gcd(ll a, ll b) {
    while (b)
        a %= b, swap(a, b);
    return a;
}

ll f(ll a, ll n) { return ((__int128_t)a * a % n + 1) % n; }

vector<ll> factorize(ll n) {

```

**if** (n <= 1e6) { // can add primality check for speed?

```

    vector<ll> res;
    for (ll i = 2; i * i <= n; ++i) {
        while (n % i == 0) {
            res.pbc(i);
            n /= i;
        }
    }
    if (n != 1)
        res.pbc(n);
    return res;
}

ll x = rnd() % (n - 1) + 1;
ll y = x;
ll tries = 10 * sqrt(sqrt(n));
const int C = 60;
for (ll i = 0; i < tries; i += C) {
    ll xs = x;
    ll ys = y;
    ll m = 1;
    for (int k = 0; k < C; ++k) {
        x = f(x, n);
        y = f(f(y, n), n);
        m = (__int128_t)m * abs(x - y) % n;
    }
    if (gcd(n, m) == 1)
        continue;
    x = xs, y = ys;
    for (int k = 0; k < C; ++k) {
        x = f(x, n);
        y = f(f(y, n), n);
        ll res = gcd(n, abs(x - y));
        if (res != 1 && res != n) {
            vector<ll> v1 = factorize(res), v2 = factorize(n / res);
            for (auto j : v2)
                v1.pbc(j);
            return v1;
        }
    }
}
return {n};
}

```

**PrimeCount.cpp****Description:** counting number of primes below NTime:  $\mathcal{O}(N^2/3)$ 

a8507c, 53 lines

```

ll prime_pi(const ll N) {
    if (N <= 1) return 0;
    if (N == 2) return 1;
    const int v = sqrt(N);
    int s = (v + 1) / 2;
    vector<int> smalls(s);
    for (int i = 1; i < s; i++) smalls[i] = i;
    vector<int> roughs(s);
    for (int i = 0; i < s; i++) roughs[i] = 2 * i + 1;
    vector<ll> larges(s);
    for (int i = 0; i < s; i++) larges[i] = (N / (2 * i + 1) - 1) / 2;
    vector<bool> skip(v + 1);
    const auto divide = [] (ll n, ll d) -> int { return n / d; };
    const auto half = [] (int n) -> int { return (n - 1) >> 1; };
    int pc = 0;
    for (int p = 3; p <= v; p += 2)
        if (!skip[p]) {
            int q = p * p;
            if ((ll)q * q > N) break;
            skip[p] = true;
            for (int i = q; i <= v; i += 2 * p) skip[i] = true;
            int ns = 0;
            for (int k = 0; k < s; k++) {
                int i = roughs[k];
                if (skip[i]) continue;
                ll d = (ll)i * p;
                larges[ns] = larges[k] -
                    (d <= v ? larges[smalls[d >> 1] - pc]
                    : smalls[half(divide(N, d))]) +
                    pc;
                roughs[ns++] = i;
            }
        }
}

```

```

}
s = ns;
for (int i = half(v), j = ((v / p) - 1) + 1; j >= p; j -= 2) {
    int c = smalls[j >> 1] - pc;
    for (int e = (j * p) >> 1; i >= e; i--) smalls[i] -= c;
}
pc++;
}

larges[0] += (ll)(s + 2 * (pc - 1)) * (s - 1) / 2;
for (int k = 1; k < s; k++) larges[0] -= larges[k];
for (int l = 1; l < s; l++) {
    ll q = roughs[l];
    ll M = N / q;
    int e = smalls[half(M / q)] - pc;
    if (e < l + 1) break;
    ll t = 0;
    for (int k = l + 1; k <= e; k++)
        t += smalls[half(divide(M, roughs[k]))];
    larges[0] += t - (ll)(e - 1) * (pc + l - 1);
}
return larges[0] + 1;
}

```

## NTT.cpp

Description: Fast FFT!  
Time:  $\mathcal{O}(n \log(n))$

e7ea21, 272 lines

// Don't use Ofast, potential slow down by 2x!  
// Write mint first!

```

int maxn, maxk;
vector<mint> rvi;
vector<mint> wpws;

void build_fft(int _maxk) {
    maxk = _maxk;
    maxn = (1 << maxk);
    rvi.resize(maxn);
    rvi[0] = 0;
    for (int i = 1; i < maxn; i += 1) {
        rvi[i] = (rvi[i >> 1] >> 1);
        if (i & 1)
            rvi[i] |= (1 << (maxk - 1));
    }
    mint w = mint(3).pow((mod - 1) / maxn);
    mint pw = 1;
    wpws.resize(maxn);
    rep(i, maxn) {
        wpws[rvi[i]] = pw;
        pw *= w;
    }
}

```

```

void fft(vector<mint>& a, int k) {
    int n = (1 << k);
    for (int ln = n / 2; ln >= 1; ln /= 2) {
        int ln2 = ln * 2;
        for (int i = 0; i < n; i += ln2) {
            auto w = wpws[i / ln];
            for (int j = i; j < i + ln; j += 1) {
                auto u = a[j];
                auto v = a[j + ln] * w;
                a[j] = u + v;
                a[j + ln] = u - v;
            }
        }
        rep(i, n) {
            int mriv = (rvi[i] >> (maxk - k));
            if (mriv < i)
                swap(a[i], a[mriv]);
        }
    }
}

void inv_fft(vector<mint>& a, int k) {
    fft(a, k);
    int n = (1 << k);

```

```

    mint invn = mint(n).inv();
    rep(i, n) {
        a[i] *= invn;
    }
    reverse(a.begin() + 1, a.end());
}

vector<mint> mul(vector<mint> a, vector<mint> b) {
    if (a.empty() || b.empty())
        return {};
    auto ca = a;
    auto cb = b;
    int lna = len(a);
    int lnb = len(b);
    int k = __lg(lna + lnb - 1);
    if (lna + lnb - 1 == (1 << k) + 1) {
        auto c = mul(vector<mint>(a.begin(), a.end() - 1), b);
        c.resize(lna + lnb - 1);
        rep(j, lnb) {
            c[lna - 1 + j] += a[lna - 1] * b[j];
        }
        return c;
    }
    if (lna + lnb - 1 > (1 << k)) {
        k += 1;
    }
    int n = (1 << k);
    a.resize(n);
    b.resize(n);
    fft(a, k);
    fft(b, k);
    rep(i, n) {
        a[i] *= b[i];
    }
    inv_fft(a, k);
    a.resize(lna + lnb - 1);
    return a;
}

vector<mint> operator+(vector<mint> a, vector<mint> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int)b.size(); ++i) {
        a[i] += b[i];
    }
    return a;
}

vector<mint> operator-(vector<mint> a, vector<mint> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int)b.size(); ++i) {
        a[i] -= b[i];
    }
    return a;
}

vector<mint> inv(const vector<mint>& a, int need) {
    vector<mint> b = { a[0].inv() };
    while ((int)b.size() < need) {
        vector<mint> al = a;
        int m = b.size();
        al.resize(min((int)al.size(), 2 * m));
        b = mul(b, vector<mint>(2) - mul(al, b));
        b.resize(2 * m);
    }
    b.resize(need);
    return b;
}

vector<mint> mul2(vector<mint> a, vector<mint> b) {
    int lna = len(a);
    int lnb = len(b);
    int k = 0;
    while ((1 << k) < lna) {
        ++k;
    }
    int n = (1 << k);
    a.resize(n);
    reverse(all(b));
}
```

```

    b.resize(n);
    fft(a, k);
    fft(b, k);
    rep(i, n) a[i] *= b[i];
    inv_fft(a, k);
    vector<mint> c(lna - lnb + 1);
    rep(i, len(c)) {
        c[i] = a[lnb - 1 + i];
    }
    return c;
}

vector<mint> multipoint(vector<mint> a, vector<mint> x) {
    int n = x.size();
    int m = len(a);
    vector<vector<mint>> tree(2 * n);
    for (int i = 0; i < n; ++i) {
        tree[i + n] = { 1, 0 - x[i] };
    }
    for (int i = n - 1; i; --i) {
        tree[i] = mul(tree[2 * i], tree[2 * i + 1]);
    }
    auto tinv = inv(tree[1], m);
    a.resize(n + m - 1);
    auto c = mul2(a, tinv);
    tree[1] = c;
    for (int i = 1; i < n; i += 1) {
        auto x = tree[i + 1];
        auto y = tree[i + i + 1];
        tree[i + i] = mul2(tree[i], y);
        tree[i + i + 1] = mul2(tree[i], x);
    }
    vector<mint> res(n);
    for (int i = 0; i < n; ++i) {
        res[i] = tree[i + n][0];
    }
    return res;
}

vector<mint> div(vector<mint> a, vector<mint> b) {
    int n = a.size() - 1;
    int m = b.size() - 1;
    if (n < m) return { 0 };
    reverse(all(a));
    reverse(all(b));
    a.resize(n - m + 1);
    b.resize(n - m + 1);
    vector<mint> c = inv(b, b.size());
    vector<mint> q = mul(a, c);
    q.resize(n - m + 1);
    reverse(all(q));
    return q;
}

vector<mint> mod_poly(vector<mint> a, vector<mint> b) {
    auto res = a - mul(b, div(a, b));
    res.resize(len(b) - 1);
    return res;
}

vector<mint> deriv(vector<mint> a) {
    for (int i = 1; i < (int)a.size(); ++i) {
        a[i - 1] = a[i] * i;
    }
    a.back() = 0;
    if (a.size() > 1) {
        a.pop_back();
    }
    return a;
}

vector<mint> integ(vector<mint> a) {
    a.push_back(0);
    for (int i = (int)a.size() - 1; i; --i) {
        a[i] = a[i - 1] * mint(i).inv();
    }
    a[0] = 0;
    return a;
}

```

```

vector<mint> log(vector<mint> a, int n) {
    auto res = integ(mul(deriv(a), inv(a, n)));
    res.resize(n);
    return res;
}

vector<mint> exp(vector<mint> a, int need) {
    vector<mint> b = { 1 };
    while ((int)b.size() < need) {
        vector<mint> a1 = a;
        int m = b.size();
        a1.resize(min((int)a1.size(), 2 * m));
        a1[0] += 1;
        b = mul(b, a1 - log(b, 2 * m));
        b.resize(2 * m);
    }
    b.resize(need);
    return b;
}

vector<mint> qf_projection(vector<mint> f) { // ensure that f[0]=0
    int lnf = len(f);
    int n = 1;
    while (n < len(f)) n *= 2;
    vector<mint> g(n);
    g[n - lnf] = 1;
    f.resize(n);
    rep(i, n) f[i] = 0 - f[i];
    int m = 1;
    while (n > 1) {
        f.resize(4 * n * m);
        f[2 * n * m] = i;
        g.resize(4 * n * m);
        fft(f);
        fft(g);
        auto q = f;
        rotate(q.begin(), q.begin() + 2 * n * m, q.end());
        vector<mint> gf(4 * n * m), ff(4 * n * m);
        rep(i, 4 * n * m) {
            gf[i] = g[i] * q[i];
            ff[i] = f[i] * q[i];
        }
        inv_fft(gf);
        inv_fft(ff);
        ff[0] -= 1;
        f.assign(2 * n * m, 0);
        g.assign(2 * n * m, 0);
        rep(i, n / 2) rep(j, 2 * m) {
            f[j * n + i] = ff[j * (2 * n) + 2 * i];
            g[j * n + i] = gf[j * (2 * n) + 2 * i + 1];
        }
        n /= 2; m *= 2;
    }
    vector<mint> res(m);
    rep(i, m) {
        res[i] = g[2 * i];
    }
    reverse(all(res));
    res.resize(lnf);
    return res;
}

```

**FFT.cpp**  
Description: Calculating product of two polynomials  
Time:  $\mathcal{O}(n \log(n))$

3adba5, 67 lines

```

const ld PI = acos(-1);
using cd = complex<ld>;
const int MAXLOG = 19, N = (1 << MAXLOG), MAXN = (1 << MAXLOG) + 228;
int rev[MAXN];
cd w[MAXN];
bool fftInit = false;

void initFFT() {
    for (int i = 0; i < N; ++i) {
        w[i] = cd(cos(2 * PI * i / N), sin(2 * PI * i / N));
    }
    rev[0] = 0;
}

```

## FFT AndConvolution SubsetConvolution Simplex

```

    for (int i = 1; i < N; ++i) {
        rev[i] = (rev[i >> 1] >> 1) ^ ((i & 1) << (MAXLOG - 1));
    }

    void FFT(int n, vector<cd> a, bool rv = false) {
        if (!fftInit) {
            initFFT();
            fftInit = 1;
        }
        int LOG = ceil(log2(n));
        for (int i = 0; i < n; ++i) {
            if (i < (rev[i] >> (MAXLOG - LOG))) {
                swap(a[i], a[(rev[i] >> (MAXLOG - LOG))]);
            }
        }
        for (int lvl = 0; lvl < LOG; ++lvl) {
            int len = 1 << lvl;
            for (int st = 0; st < n; st += len * 2) {
                for (int i = 0; i < len; ++i) {
                    cd x = a[st + i], y = a[st + len + i] * w[i << (MAXLOG -
                        1 - lvl)];
                    a[st + i] = x + y;
                    a[st + i + len] = x - y;
                }
            }
            if (rv) {
                reverse(a.begin() + 1, a.end());
                for (auto& i : a) i /= n;
            }
        }
    }

    vector<ll> mul(vector<ll> a, vector<ll> b) {
        int xd = max(a.size(), b.size()) * 2;
        int cur = 1;
        while (cur < xd) {
            cur *= 2;
        }
        a.resize(cur);
        b.resize(cur);
        vector<cd> ma(cur), mb(cur);
        for (int i = 0; i < cur; ++i) {
            ma[i] += a[i];
            mb[i] += b[i];
        }
        FFT(cur, ma);
        FFT(cur, mb);
        for (int i = 0; i < cur; ++i) ma[i] *= mb[i];
        FFT(cur, ma, true);
        vector<ll> ans(cur);
        for (int i = 0; i < cur; ++i) {
            ans[i] = (ll)(ma[i].real() + 0.5);
        }
        return ans;
    }
}

```

### AndConvolution.cpp

Description: Calculating and-convolution modulo smth  
Time:  $\mathcal{O}(n \log(n))$

5dedf4, 24 lines

```

void conv(vector<int> &a, bool x) {
    int n = a.size();
    for (int j = 0; (1 << j) < n; ++j) {
        for (int i = 0; i < n; ++i) {
            if (!(i & (1 << j))) {
                if (x)
                    a[i] = add(a[i], a[i | (1 << j)]);
                else
                    a[i] = sub(a[i], a[i | (1 << j)]);
            }
        }
    }
}

vector<int> andcon(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
}

```

```

    conv(a, 1), conv(b, 1);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], b[i]);
    conv(a, 0);
    return a;
}

```

### SubsetConvolution.cpp

Description: subset convolution

Time:  $\mathcal{O}(2^n * n^2)$  (500 ms n = 20 with pragms) a47122, 39 lines

```

void transform(int n, int N, vector<int> &b, const vector<int> &a,
              const vector<int> &pc, bool rev) {
    if (!rev) {
        b.assign(N << n, 0);
        for (int i = 0; i < (int)a.size(); ++i) b[pc[i] + i * N] = a[i];
    }
    for (int w = 1; w <= (1 << n); ++w) {
        for (int d = 0; !w & (1 << d); ++d) {
            int W = N * (w - (1 << d)), dd = N << d;
            for (int i = N * (w - (2 << d)); i < W; ++i) {
                if (!rev) b[i + dd] = add(b[i + dd], b[i]);
                else b[i + dd] = sub(b[i + dd], b[i]);
            }
        }
    }
}

vector<int> SubsetConvolution(const vector<int> &a, const vector<int> &b) {
    int n = 0;
    while ((1 << n) < max(a.size(), b.size())) n++;
    int N = n + 1;
    vector<int> pc(1 << n, 0);
    for (int i = 1; i < (1 << n); ++i) pc[i] = pc[i - (i & -i)] + 1;
    vector<int> bufA, bufB;
    transform(n, N, bufA, a, pc, false);
    transform(n, N, bufB, b, pc, false);
    for (int i = 0; i < (1 << n); i++) {
        int I = i * N;
        vector<int> Q(N);
        for (int ja = 0; ja <= pc[i]; ++ja) {
            for (int jb = pc[i] - ja; jb <= x; ++jb) {
                Q[ja + jb] = add(Q[ja + jb], mul(bufA[ja + I], bufB[jb + I]));
            }
        }
        copy(Q.begin(), Q.end(), bufA.begin() + I);
    }
    transform(n, N, bufA, a, pc, true);
    vector<int> res(1 << n);
    for (int i = 0; i < (1 << n); ++i) res[i] = bufA[pc[i] + i * N];
    return res;
}

```

### Simplex.cpp

Description: Simplex

Time: exponential XD(ok for 200-300 variables/bounds) 4dda3c, 99 lines

```

/* solver for linear programs of the form
maximize c^T x, subject to A x <= b, x >= 0
outputs target function for optimal solution and
the solution by reference
if unbounded above : returns inf, if infeasible : returns -inf
create Simplex_Steep<ld> LP(A, b, c), then call LP. Solve (x)
*/
template <typename DOUBLE>
struct Simplex_Steep {
    using VD = vector<DOUBLE>;
    using VVD = vector<VD>;
    using VI = vector<int>;
    DOUBLE EPS = 1e-12;
    int m, n;
    VI B, N;
    VVD D;
    Simplex_Steep(const VVD &A, const VD &b, const VD &c)
        : m(b.size()), n(c.size()), B(m), N(n + 1), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j) D[i][j] = A[i][j];
    }
}

```

```

    }
}

for (int j = 0; j < n; j++) {
    N[j] = j;
    D[m][j] = -c[j];
}
N[n] = -1;
D[m + 1][n] = 1;
}

void Pivot(int r, int s) {
    for (int i = 0; i < m + 2; i++)
        if (i != r)
            for (int j = 0; j < n + 2; j++)
                if (j != s) D[i][j] -= D[r][j] * D[i][s] / D[r][s];
    for (int j = 0; j < n + 2; j++)
        if (j != s) D[r][j] /= D[r][s];
    for (int i = 0; i < m + 2; i++)
        if (i != r) D[i][s] /= -D[r][s];
    D[r][s] = 1.0 / D[r][s];
    swap(B[r], N[s]);
}

bool Simplex(int phase) {
    int x = m + (int)(phase == 1);
    while (true) {
        int s = -1;
        DOUBLE c_val = -1;
        for (int j = 0; j <= n; j++) {
            if (phase == 2 && N[j] == -1) continue;
            DOUBLE norm_sq = 0;
            for (int k = 0; k <= m; k++) norm_sq += D[k][j] * D[k][j];
            norm_sq = max(norm_sq, EPS);
            DOUBLE c_val_j = D[x][j] / sqrtl(norm_sq);
            if (s == -1 || c_val_j < c_val || (c_val == c_val_j && N[j] < N[s])) {
                s = j;
                c_val = c_val_j;
            }
        }
        if (D[x][s] >= -EPS) return true;
        int r = -1;
        for (int i = 0; i < m; i++) {
            if (D[i][s] <= EPS) continue;
            if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                (D[i][n + 1] / D[i][s] == D[r][n + 1] / D[r][s] && B[i] < B[r]))
                r = i;
        }
        if (r == -1) return false;
        Pivot(r, s);
    }
}

DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++)
        if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] <= -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
            return numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++)
            if (B[i] == 0) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                    if (s == -1 || D[i][j] < D[i][s] ||
                        (D[i][j] == D[i][s] && N[j] < N[s]))
                        s = j;
                Pivot(i, s);
            }
        if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
        x = VD(n);
        for (int i = 0; i < m; i++)
            if (B[i] < n) x[B[i]] = D[i][n + 1];
        return D[m][n + 1];
    }
}

```

}

**DeterminantLd.cpp****Description:** Determinant in ld

1a6123, 18 lines

```

double det(vector<vector<double>>& a) {
    int n = sz(a);
    double res = 1;
    for (int i = 0; i < n; ++i) {
        int b = i;
        for (int j = i + 1; j < n; ++j)
            if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        for (int j = i + 1; j < n; ++j) {
            double v = a[j][i] / a[i][i];
            if (v != 0)
                for (int k = i + 1; k < n; ++k) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}

```

**DeterminantInt.cpp****Description:** Determinant in ints

c2ab5a, 19 lines

```

const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a);
    ll ans = 1;
    for (int i = 0; i < n; ++i) {
        for (int j = i + 1; j < n; ++j) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i];
                if (t)
                    for (int k = i; k < n; ++k)
                        a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            }
        }
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    }
    return (ans + mod) % mod;
}

```

**TridiagSLE.cpp****Description:** Tridiagonal SLE solver(didnt test yet)

532e1d, 16 lines

```

vector<ld> trisle(vector<ld> a, vector<ld> b, vector<ld> c) {
    // a[i] * x[i - 1] + c[i] * x[i] + b[i] * x[i + 1] = f[i]
    int n = a.size(); // a[0] == 0, b[n-1] == 0
    alpha[1] = -(ld)b[0] / c[0];
    beta[1] = (ld)f[0] / c[0];
    for (int i = 1; i < n - 1; i++) {
        ld zn = (ld)a[i] * alpha[i] + c[i];
        alpha[i + 1] = -(ld)b[i] / zn;
        beta[i + 1] = (f[i] - (ld)a[i] * beta[i]) / zn;
    }
    x[n - 1] = (f[n - 1] - a[n - 1] * beta[n - 1]) /
        (a[n - 1] * alpha[n - 1] + c[n - 1]);
    for (int i = n - 2; i >= 0; i--)
        x[i] = alpha[i + 1] * x[i + 1] + beta[i + 1];
    return x;
}

```

**SolveLinear.cpp****Description:** Solving linear systems

44c9ab, 35 lines

```

typedef vector<double> vd;
const double eps = 1e-12; // rep(i,a,b) = for(int i=a;i<b;++i)
int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n > assert(sz(A[0]) == m);
    vi col(m);

```

iota(all(col), 0);

rep(i, 0, n) {

double v, bv = 0;

rep(r, i, n) rep(c, i, m) if ((v = fabs(A[r][c])) &gt; bv) br = r, bc =

c,

bv = v;

if (bv &lt;= eps) {

rep(j, i, n) if (fabs(b[j]) &gt; eps) return -1;

break;

}

swap(A[i], A[br]);

swap(b[i], b[br]);

swap(col[i], col[bc]);

rep(j, 0, n) swap(A[j][i], A[j][bc]);

bv = 1 / A[i][i];

rep(j, i + 1, n) {

double fac = A[j][i] \* bv;

b[j] -= fac \* b[i];

rep(k, i + 1, m) A[j][k] -= fac \* A[i][k];

}

rank++;

x.assign(m, 0);

for (int i = rank; i--;) {

b[i] /= A[i][i];

x[col[i]] = b[i];

rep(j, 0, i) b[j] -= A[j][i] \* b[i];

}

return rank; // (multiple solutions if rank &lt; m)

}

**PolyInter.cpp****Description:** Interpolating polynomialsTime:  $\mathcal{O}(n^2)$ 

4edad5, 14 lines

```

typedefed vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    for (int k = 0; k < n - 1; ++k)
        for (int i = k + 1; i < n; ++i) y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0;
    temp[0] = 1;
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i) {
            res[i] += y[k] * temp[i];
            swap(last, temp[i]);
            temp[i] -= last * x[k];
        }
    return res;
}

```

**CharPoly.cpp****Description:** det(a - xI)

666c0e, 37 lines

```

vector<int> CharacteristicPolynomial(vector<vector<int>> a) {
    int n = a.size();
    for (int j = 0; j < n - 2; j++) {
        for (int i = j + 1; i < n; i++) {
            if (a[i][j] != 0) {
                swap(a[j + 1], a[i]);
                for (int k = 0; k < n; k++) swap(a[k][j + 1], a[k][i]);
                break;
            }
        }
        if (a[j + 1][j] != 0) {
            int flex = inv(a[j + 1][j]);
            for (int i = j + 2; i < n; i++) {
                if (a[i][j] == 0) continue;
                int coe = mul(flex, a[i][j]);
                for (int l = j; l < n; l++) a[i][l] = sub(a[i][l], mul(coe,
                    a[j + 1][l]));
                for (int k = 0; k < n; k++) a[k][j + 1] = add(a[k][j + 1],
                    mul(coe, a[k][i]));
            }
        }
    }
    vector<vector<int>> p(n + 1);
    p[0] = {1};
    for (int i = 1; i <= n; i++) {
        p[i].resize(i + 1);
        p[i].i
    }
}

```

```

for(int j = 0; j < i; j++) {
    p[i][j + 1] = sub(p[i][j + 1], p[i - 1][j]);
    p[i][j] = add(p[i][j], mul(p[i - 1][j], a[i - 1][i - 1]));
}
int x = 1;
for(int m = 1; m < i; m++) {
    x = mul(x, sub(0, a[i - m][i - m - 1]));
    int coe = mul(x, a[i - m - 1][i - 1]);
    for(int j = 0; j < i - m; j++) p[i][j] = add(p[i][j], mul(coe,
        p[i - m - 1][j]));
}
return p[n];
}

```

## FloorSum.cpp

Description: finds  $\sum_{x=0}^{n-1} \lfloor (kx + b)/m \rfloor$ . Require  $k \geq 0, b \geq 0, m \geq 0$ . 93210c 011 lines

```

template<typename T>
T floor_sum(T k, T b, T m, T n) {
    if (k == 0) {
        return (b / m) * n;
    }
    if (k >= m || b >= m) {
        return n * (n - 1) / 2 * (k / m) + n * (b / m) + floor_sum(k % m,
            b % m, m, n);
    }
    T ymax = (k * (n - 1) + b) / m;
    return n * ymax - floor_sum(m, m + k - b - 1, k, ymax);
}

```

## WaysCount.cpp

Description: Find number of right-up paths from (0, 0) to (x, y), not touching lines  $y=x+1$  and  $y=x+r$ . Time:  $O((x+y)/(r-1))$

```

mint flex(ll x, ll y, ll l, ll r) {
    if (l >= 0 or r <= 0) {
        return 0;
    }
    ll n = x + y;
    mint res = 0;
    for (ll k = -(n / (r - 1)); k <= n / (r - 1); k += 1) {
        res += cnk(n, x + k * (r - 1));
        res -= cnk(n, y - r + k * (r - 1));
    }
    return res;
}

```

## 6.1 Fun things

$$\text{ClassesCount} = \frac{1}{|G|} \sum_{\pi \in G} I(\pi)$$

$$\text{ClassesCount} = \frac{1}{|G|} \sum_{\pi \in G} k^{C(\pi)}$$

Stirling 2kind - count of partitions of n objects into k nonempty sets:

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$$

$$S(n, k) = \sum_{j=0}^{n-1} \binom{n-1}{j} S(j, k - 1)$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k+j} \binom{k}{j} j^n$$

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

$\binom{n}{k} \equiv \prod_i \binom{n_i}{k_i}$ ,  $n_i, k_i$  - digits of  $n, k$  in p-adic system

$$\int_a^b f(x) dx \approx \frac{b-a}{6} \left( f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, O(\log \log)$$

$$G(n) = n \oplus (n \gg 1)$$

$$g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} g(d) \mu\left(\frac{n}{d}\right)$$

$$\sum_{d|n} \mu(d) = [n = 1], \mu(1) = 1, \mu(p) = -1, \mu(p^k) = 0$$

$$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\operatorname{tg}(a \pm b) = \frac{\operatorname{tg} a \pm \operatorname{tg} b}{1 \mp \operatorname{tg} a \operatorname{tg} b}$$

$$\operatorname{ctg}(a \pm b) = \frac{\operatorname{ctg} a \operatorname{ctg} b \mp 1}{\operatorname{ctg} b \pm \operatorname{ctg} a}$$

$$\sin \frac{a}{2} = \pm \sqrt{\frac{1 - \cos a}{2}}$$

$$\cos \frac{a}{2} = \pm \sqrt{\frac{1 + \cos a}{2}}$$

$$\operatorname{tg} \frac{a}{2} = \frac{\sin a}{1 - \cos a} = \frac{1 - \cos a}{\sin a}$$

$$\sin \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}$$

$$\cos \alpha = \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}$$

$$\operatorname{tg} \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 - \operatorname{tg}^2 \frac{\alpha}{2}}$$

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$\sin^3 \alpha = \frac{3 \sin \alpha - \sin 3\alpha}{4}$$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\cos^3 \alpha = \frac{3 \cos \alpha + \cos 3\alpha}{4}$$

$$\sin a \sin b = \frac{\cos(a-b) - \cos(a+b)}{2}$$

$$\sin a \cos b = \frac{\sin(a-b) + \sin(a+b)}{2}$$

$$\cos a \cos b = \frac{\cos(a-b) + \cos(a+b)}{2}$$

1 jan 2000 - saturday, 1 jan 1900 - monday, 14 apr 1961 - friday

Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751, 22:4506715738447323, 23:44152005855084346

Fibonacci: 45:1134903170, 46:1836311903(max int), 91: 4660046610375530309

Highly composite numbers:

$\leq 1000 : d(840) = 32, \leq 10^4 : d(9240) = 64, \leq 10^5 : d(83160) = 128, \leq 10^6 : d(720720) = 240, \leq 10^7 : d(8648640) = 448, \leq 10^8 : d(91891800) = 768, \leq 10^9 : d(931170240) = 1344, \leq 10^{11} : d(97772875200) = 4032, \leq 10^{15} : d(866421317361600) = 26880, \leq 10^{18} : d(897612484786617600) = 103680$

BEST Theorem:

$$ec(G) = \#\text{SpanningTrees}(G) \cdot \prod_{v \in V} (\deg(v) - 1)!$$

Erdos: Graph exists

$$\Leftrightarrow d_1 \geq \dots \geq d_n, \forall k \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

Pick: Area = Interior +  $\frac{\text{Bounds}}{2} - 1$

Euler:  $V - E + F = 1 + C$

Kirchhoff: put degree on diagonal, -1 for each edge, cut out first row + column, calc det - result is #SpanningTrees

Tree Hash: for vertex  $v$  calculate  $\prod_i (c_i + d_{h_i})$ , where  $c_i$  - hash of ith child,  $d_{h_i}$  - random number associated to depth of current child

Get position of Gray Code g: int n = 0; for (; g; g>>= 1) n xor= g; return n;

# Table of Basic Integrals (7)

## Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \quad (7.1)$$

$$\int \frac{1}{x} dx = \ln|x| \quad (7.2)$$

$$\int u dv = uv - \int v du \quad (7.3)$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| \quad (7.4)$$

## Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \quad (7.5)$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, \quad n \neq -1 \quad (7.6)$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \quad (7.7)$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \quad (7.8)$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (7.9)$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln|a^2+x^2| \quad (7.10)$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \quad (7.11)$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln|a^2+x^2| \quad (7.12)$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (7.13)$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \quad a \neq b \quad (7.14)$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|x+a| \quad (7.15)$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (7.16)$$

## Integrals with Roots

$$\int \sqrt{x-a} dx = \frac{2}{3}(x-a)^{3/2} \quad (7.17)$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a} \quad (7.18)$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} \quad (7.19)$$

$$\int x \sqrt{x-a} dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, & \text{or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, & \text{or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases} \quad (7.20)$$

$$\int \sqrt{ax+b} dx = \left( \frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b} \quad (7.21)$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a}(ax+b)^{5/2} \quad (7.22)$$

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a} \quad (7.23)$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \quad (7.24)$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln[\sqrt{x} + \sqrt{x+a}] \quad (7.25)$$

$$\int x \sqrt{ax+b} dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} \quad (7.26)$$

$$\int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}} \left[ (2ax+b)\sqrt{ax(ax+b)} - b^2 \ln|a\sqrt{x} + \sqrt{a(ax+b)}| \right] \quad (7.27)$$

$$\int \sqrt{x^3(ax+b)} dx = \left[ \frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln|a\sqrt{x} + \sqrt{a(ax+b)}| \quad (7.28)$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln|x + \sqrt{x^2 \pm a^2}| \quad (7.29)$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} \quad (7.30)$$

$$\int x \sqrt{x^2 \pm a^2} dx = \frac{1}{3}(x^2 \pm a^2)^{3/2} \quad (7.31)$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}| \quad (7.32)$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} \quad (7.33)$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} \quad (7.34)$$

$$\int \frac{x}{\sqrt{a^2-x^2}} dx = -\sqrt{a^2-x^2} \quad (7.35)$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln|x + \sqrt{x^2 \pm a^2}| \quad (7.36)$$

$$\int \sqrt{ax^2+bx+c} dx = \frac{b+2ax}{4a} \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}} \ln|2ax+b+2\sqrt{a(ax^2+bx+c)}| \quad (7.37)$$

$$\begin{aligned} \int x \sqrt{ax^2+bx+c} dx &= \frac{1}{48a^{5/2}} \left( 2\sqrt{a}\sqrt{ax^2+bx+c} (-3b^2 + 2abx + 8a(c+ax^2)) \right. \\ &\quad \left. + 3(b^3 - 4abc) \ln|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}| \right) \end{aligned} \quad (7.38)$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} dx = \frac{1}{\sqrt{a}} \ln|2ax+b+2\sqrt{a(ax^2+bx+c)}| \quad (7.39)$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}} dx = \frac{1}{a} \sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}} \ln|2ax+b+2\sqrt{a(ax^2+bx+c)}| \quad (7.40)$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \quad (7.41)$$

## Integrals with Logarithms

$$\int \ln ax dx = x \ln ax - x \quad (7.42)$$

$$\int x \ln x dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4} \quad (7.43)$$

$$\int x^2 \ln x dx = \frac{1}{3}x^3 \ln x - \frac{x^3}{9} \quad (7.44)$$

$$\int x^n \ln x dx = x^{n+1} \left( \frac{\ln x}{n+1} - \frac{1}{(n+1)^2} \right), \quad n \neq -1 \quad (7.45)$$

$$\int \frac{\ln ax}{x} dx = \frac{1}{2}(\ln ax)^2 \quad (7.46)$$

$$\int \frac{\ln x}{x^2} dx = -\frac{1}{x} - \frac{\ln x}{x} \quad (7.47)$$

$$\int \ln(ax+b) dx = \left( x + \frac{b}{a} \right) \ln(ax+b) - x, \quad a \neq 0 \quad (7.48)$$

$$\int \ln(x^2+a^2) dx = x \ln(x^2+a^2) + 2a \tan^{-1} \frac{x}{a} - 2x \quad (7.49)$$

$$\int \ln(x^2-a^2) dx = x \ln(x^2-a^2) + a \ln \frac{x+a}{x-a} - 2x \quad (7.50)$$

$$\int \ln(ax^2+bx+c) dx = \frac{1}{a} \sqrt{4ac-b^2} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left( \frac{b}{2a} + x \right) \ln(ax^2+bx+c) \quad (7.51)$$

$$\int x \ln(ax+b) dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2} \left( x^2 - \frac{b^2}{a^2} \right) \ln(ax+b) \quad (7.52)$$

$$\int x \ln(a^2 - b^2 x^2) dx = -\frac{1}{2}x^2 + \frac{1}{2} \left( x^2 - \frac{a^2}{b^2} \right) \ln(a^2 - b^2 x^2) \quad (7.53)$$

$$\int (\ln x)^2 dx = 2x - 2x \ln x + x(\ln x)^2 \quad (7.54)$$

$$\int (\ln x)^3 dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x \quad (7.55)$$

$$\int x(\ln x)^2 dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x \quad (7.56)$$

$$\int x^2(\ln x)^2 dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3 \ln x \quad (7.57)$$

## Integrals with Exponentials

$$\int e^{ax} dx = \frac{1}{a}e^{ax} \quad (7.58)$$

$$\int \sqrt{x}e^{ax} dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\operatorname{erf}(i\sqrt{ax}), \text{ where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (7.59)$$

$$\int xe^x dx = (x-1)e^x \quad (7.60)$$

$$\int xe^{ax} dx = \left( \frac{x}{a} - \frac{1}{a^2} \right) e^{ax} \quad (7.61)$$

$$\int x^2 e^x dx = (x^2 - 2x + 2) e^x \quad (7.62)$$

$$\int x^2 e^{ax} dx = \left( \frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax} \quad (7.63)$$

$$\int x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x \quad (7.64)$$

$$\int x^n e^x dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \quad (7.65)$$

$$\int x^n e^{ax} dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax], \text{ where } \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt \quad (7.66)$$

$$\int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(ix\sqrt{a}) \quad (7.67)$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a}) \quad (7.68)$$

$$\int xe^{-ax^2} dx = -\frac{1}{2a} e^{-ax^2} \quad (7.69)$$

$$\int x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2} \quad (7.70)$$

## Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a} \cos ax \quad (7.71)$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \quad (7.72)$$

$$\int \sin^3 ax dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a} \quad (7.73)$$

$$\int \sin^n ax dx = -\frac{1}{a} \cos ax \cdot {}_2F_1 \left[ \frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax \right] \quad (7.74)$$

$$\int \cos ax dx = \frac{1}{a} \sin ax \quad (7.75)$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \quad (7.76)$$

$$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a} \quad (7.77)$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1 \left[ \frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax \right] \quad (7.78)$$

$$\int \cos x \sin x dx = \frac{1}{2} \sin^2 x + c_1 = -\frac{1}{2} \cos^2 x + c_2 = -\frac{1}{4} \cos 2x + c_3 \quad (7.79)$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \quad (7.80)$$

$$\int \sin^2 ax \cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \quad (7.81)$$

$$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x \quad (7.82)$$

$$\int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \quad (7.83)$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax \quad (7.84)$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \quad (7.85)$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \quad (7.86)$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax \quad (7.87)$$

$$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax \quad (7.88)$$

$$\int \tan^n ax dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1 \left( \frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax \right) \quad (7.89)$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \quad (7.90)$$

$$\int \sec x dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left( \tan \frac{x}{2} \right) \quad (7.91)$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax \quad (7.92)$$

$$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| \quad (7.93)$$

$$\int \sec x \tan x dx = \sec x \quad (7.94)$$

$$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x \quad (7.95)$$

$$\int \sec^n x \tan x dx = \frac{1}{n} \sec^n x, n \neq 0 \quad (7.96)$$

$$\int \csc x dx = \ln |\tan \frac{x}{2}| = \ln |\csc x - \cot x| + C \quad (7.97)$$

$$\int \csc^2 ax dx = -\frac{1}{a} \cot ax \quad (7.98)$$

$$\int \csc^3 x dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x| \quad (7.99)$$

$$\int \csc^n x \cot x dx = -\frac{1}{n} \csc^n x, n \neq 0 \quad (7.100)$$

$$\int \sec x \csc x dx = \ln |\tan x| \quad (7.101)$$

## Products of Trigonometric Functions and Monomials

$$\int x \cos x \, dx = \cos x + x \sin x \quad (7.102)$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \quad (7.103)$$

$$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2) \sin x \quad (7.104)$$

$$\int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \quad (7.105)$$

$$\int x^n \cos x \, dx = -\frac{1}{2}(i)^{n+1} [\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)] \quad (7.106)$$

$$\int x^n \cos ax \, dx = \frac{1}{2}(ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)] \quad (7.107)$$

$$\int x \sin x \, dx = -x \cos x + \sin x \quad (7.108)$$

$$\int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (7.109)$$

$$\int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x \quad (7.110)$$

$$\int x^2 \sin ax \, dx = \frac{2-a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (7.111)$$

$$\int x^n \sin x \, dx = -\frac{1}{2}(i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)] \quad (7.112)$$

$$\int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x \quad (7.113)$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x \quad (7.114)$$

$$\int x \tan^2 x \, dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \quad (7.115)$$

$$\int x \sec^2 x \, dx = \ln \cos x + x \tan x \quad (7.116)$$

## Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (7.117)$$

$$\int e^{bx} \sin ax \, dx = \frac{1}{a^2+b^2} e^{bx} (b \sin ax - a \cos ax) \quad (7.118)$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (7.119)$$

$$\int e^{bx} \cos ax \, dx = \frac{1}{a^2+b^2} e^{bx} (a \sin ax + b \cos ax) \quad (7.120)$$

$$\int x e^x \sin x \, dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (7.121)$$

$$\int x e^x \cos x \, dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (7.122)$$

## Integrals of Hyperbolic Functions

$$\int \cosh ax \, dx = \frac{1}{a} \sinh ax \quad (7.123)$$

$$\int e^{ax} \cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \quad (7.124)$$

$$\int \sinh ax \, dx = \frac{1}{a} \cosh ax \quad (7.125)$$

$$\int e^{ax} \sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \quad (7.126)$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \quad (7.127)$$

$$\int e^{ax} \tanh bx \, dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1 \left[ 1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}; -e^{2bx} \right] \\ -\frac{1}{a} e^{ax} {}_2F_1 \left[ 1, \frac{a}{2b}, 1 + \frac{a}{2b}; -e^{2bx} \right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \quad (7.128)$$

$$\int \cos ax \cosh bx \, dx = \frac{1}{a^2+b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] \quad (7.129)$$

$$\int \cos ax \sinh bx \, dx = \frac{1}{a^2+b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] \quad (7.130)$$

$$\int \sin ax \cosh bx \, dx = \frac{1}{a^2+b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] \quad (7.131)$$

$$\int \sin ax \sinh bx \, dx = \frac{1}{a^2+b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \quad (7.132)$$

$$\int \sinh ax \cosh ax \, dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (7.133)$$

$$\int \sinh ax \cosh bx \, dx = \frac{1}{b^2-a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \quad (7.134)$$

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F - 6					
G - 7					
H - 8					
I - 9					
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