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数学建模国际赛

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Swimming Faster

Abstract:

In this article, we analyze the mechanism of four swimming strokes and study the most suitable stroke for swimming competitions.

We learn from Emest W. Maglischo's research that there is a certain correlation between the arm movement trajectory and athlete swimming speed. Therefore, we specifically analyze the arm as well as leg movement trajectory of the four strokes, then we establish the trajectory model after simplification. Based on the basic principles of fluid mechanics, we build the mechanical models of four strokes. Finally, we use Newton's second law to get the kinematic differential model of four strokes.

When solving, we first use Euler difference method to discretize the differential model. Then we apply iterative method and get the following results: The average speeds of freestyle, butterfly, backstroke, and breaststroke are $1.378m/s$, $1.047m/s$, $0.841m/s$, $0.634m/s$, and the average propulsion force is $12.25N$, $14.57N$, $17.07N$, $22.75N$. That is, freestyle has the fastest speed while breaststroke owns the most powerful propulsion. Then we use the TOPSIS method to comprehensively evaluate speed, propulsion, resistance, and distance. And we get scores for freestyle, butterfly, backstroke, and breaststroke, they are 0.360, 0.306, 0.210, 0.123. So we reasonably deduce that freestyle is the best stroke.

Furthermore, we take freestyle as an example to analyze the sensitivity of the swimming kinematics differential model, and find that increasing the arm swing frequency can increase the propulsion force. Then we build a model for the angle of attack during the palm stroke to find the optimal angle. Based on these two models, we put forward the following training recommendations: athletes should strengthen arm training to increase the frequency of arm strokes, and pay attention to being accustomed to the best angle of attack.

Keywords: kinematic differential model movement trajectory Euler difference method TOPSIS method angle of attack

Contents

I. Introduction

1.1 Background

In order to respond to the national strategy of ”building and sharing, health for all” ,and to achieve the goals set out in the ”Healthy China 2030” planning outline, China’s nationwide fitness is gradually transitioning from blindness to rationalization . Swimming is a physical activity carried out in the water. The particularity of the water environment causes it different from land sports, thus attracting more and more people to engage in swimming exercises. Adult swimmers are blind and confused in the choice of swimming styles, and they are not able to behave the swimming posture well. However, the current research on swimming is mainly based on real data for simulation analysis, and there is little research on swimming model. Therefore, modeling of swimming style based on fluid mechanics is particularly important.

1.2 The Task at Hand

- Establish mechanical models of different swimming strokes.
- Use the established mechanical model for kinematic analysis to obtain the performance of different swimming strokes.
- Establish mathematical models for swimming details and analyze its impact on swimming performance.
- Based on the established mathematical model, give training guidances to improve swimming performance.

II. Model Assumptions and Notations

2.1 Assumptions

In order to simplify the course of modeling and draw some reasonable conclusions from our model, we make assumptions as follows:

- Our model is established in still water。
- We assume that the athlete’s movement in the water is an uninterrupted periodic movement.
- During the whole process, the speed and frequency of the athlete’s limbs movements remain unchanged, and stay constant despite of physical exertion or other factors.
- Athletes always maintain the coordination of arms movements and legs movements throughout the exercise.
- We assume that the angle of attack of the athlete’s hand in the water remains constant at 90°.
- Based on the basic principles of fluid mechanics, we know that when the arms and legs move in the water, they will simultaneously receive resistance opposite to the direction of the limbs, and they will also receive lift, which together constitute the propulsion

force. According to R • E Schreichoff's[1] research, we find that when the angle of attack is 90°, the lift is very small, so we ignore the role of lift when modeling.

- To obtain a constant drag coefficient and lift coefficient,we assume that the sweep angle and the angle of attack remain the same during the movement of the hand .
- To make mechanical analysis easier,we simplify the trajectory of the limbs of different strokes.
- We ignore the shape of the limbs and the characteristics of the skin, and treat them as three-dimensional with known length, width and height.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

Table 1 Notations

Symbol	Definition
ω_h	Angular velocity of arm movement in freestyle
T	Cycle of movement
L	Straight line distance in the second stage of freestyle
$t_r o$	The time spent in the first stage of the freestyle
$t_s t$	The time spent in the second stage of the freestyle
$v_s t$	The velocity of arm in the second stage
ω_l	Angular velocity of leg movement in freestyle
ω_w	Angular velocity of arm movement in breaststroke
ω_{wl}	Angular velocity of leg movement in breaststroke
ω_d	Angular velocity of arm movement in butterfly
ω_{dl}	Angular velocity of leg movement in butterfly
C_L	Lift coefficient
C_D	Resistance coefficient
ρ	The density of water
R	Length of forearm
F	The force on the limbs
F_h	The force of arms in the forward direction
r	Length variable
W	Width of arm
F_l	The force of legs in the forward direction
f	Total resistance
V	The speed of swimming
W_l	Width of leg
S_p	The projected area of the body in the vertical direction of the water flow
S_l	The projected area of the leg in the vertical direction of the water flow
R_a	Length of the whole arm
R_D	Length of the thigh
$R_a l$	Length of the whole leg
t	Time variable

III. The Basic Mechanics Analysis in Swimming

3.1 Propulsion

When swimmers swim in the water, they mainly rely on the interaction between their limbs and the water to generate propulsion. The propulsion during swimming is divided into resistance propulsion and lift propulsion.

3.1.1 *Propulsion Provided by Resistance*

Since the arm stroke is not a straight stroke, it is complicated to analyze the driving force of swimming. According to Newton's third law of the definition of acting force and reaction force, using the characteristics of resistance of water, force is applied to the water through backward strokes, kicks or kicks of the limbs. The effect of limb movements is to impulse the human body. Acting on the water, the human body receives the amount of recoil given by the water to push the body forward. This swimming propulsion obtained by the resistance of water is called resistance propulsion.

3.1.2 *Propulsion Provided by Lift*

According to Bernoulli's theorem, when the velocity of the liquid[f]lowing through the surface of an object is fast, the pressure on the surface of the object will decrease; when the velocity of the liquid flowing through the surface of an object is slow, the pressure on the surface of the object will rise. The pressure in the high-pressure area on the palm of the hand is transmitted to the low-pressure area on the back of the hand to obtain lift. The direction of lift and drag always exists in the form of a right-angle relationship.

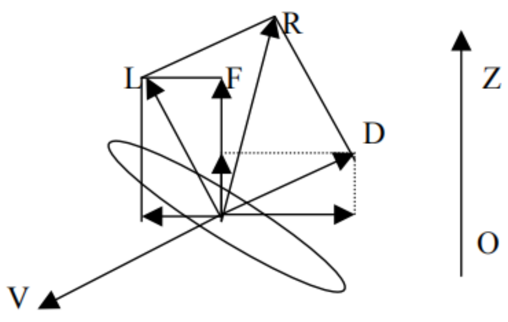


Figure 1 The relationship between stroke speed and lift,drag

Take the arm stroke as an example to analyze the force situation. As shown in Figure 1, the swimmer swims along the OZ direction, and the wing represents the cross-section of the hand. When the swimmer paddles along the V direction, the water force is R. the R is decomposed into the sum of the lift L and the resistance D.The sum of the projections of the lift L and the resistance D in the swimming direction F are the true swimming propulsion force. They can be expressed by the following two formulas:

$$D = \frac{1}{2}\rho C_D V^2 A$$

$$L = \frac{1}{2}\rho C_L V^2 A$$

C_D refers to the resistance coefficient, C_L refers to the lift coefficient, A refers to the frontal projection area along the direction of water flow, V refers to the stroke speed, ρrefers to the density of water.

3.2 Basic Model of Mechanics

3.2.1 *Swimming Resistance*

Water is a kind of fluid that is difficult to compress. Traveling in the water must remove the squeezing of the water from the body and pass through it. The resistance generated when swimming is caused by breaking the laminar flow of water and generating turbulence. The size of the turbulence area affects the vortex area and the speed of the object. The resistance that swimmers experience in swimming depends to a certain extent on the magnitude of turbulence.Common resistances include differential pressure resistance, wave resistance and friction resistance.

3.2.2 *Arms and Legs*

Athletes mainly rely on hands and legs to exert force on water to obtain propulsion.According to Newton's third law and the basic principles of fluid mechanics,we can get the following force analysis of the hands and legs.

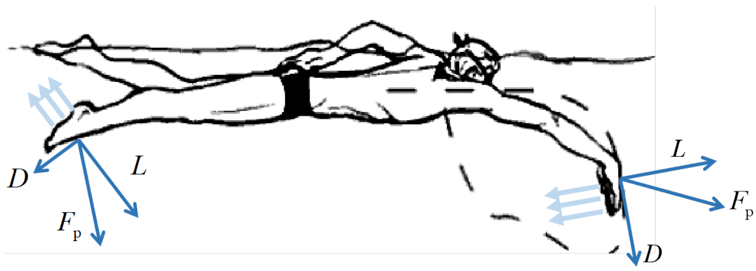


Figure 2 Force analysis

3.3 Basic Model of Newton's Second Law

According to Newton's second law, we can deduce the kinematics equation of the object from the force situation, and then find its motion state. Based on this, we can get the motion state of different strokes.

According to Newton's second law,we have

$$m\frac{d(v)}{dt} = F_{Propulsion} - F_{Resistance}$$

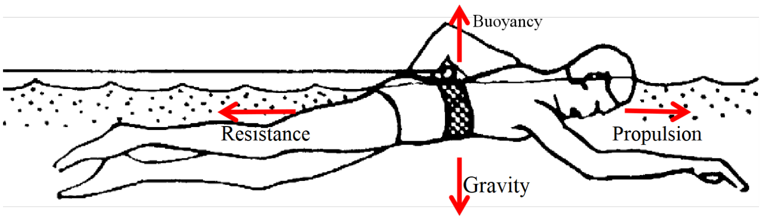


Figure 3 Force analysis

IV. Mechanical Models of Different Swimming Strokes

This chapter visually shows the movement process of freestyle, breaststroke, butterfly and backstroke, and simplifies their movements.Then decomposes them into several sub-actions.

Finally, to get the function of propulsion and resistance with time, we analyze each stroke separetely based on the basic principles of fluid dynamics and kinematics. In our model, we assume that the hand’ s angle of attack to the water is always 90°. According to R • E Schreichoff’ s[1] research, we know that the lift coefficient is approximately zero when angle of attack is 90° , so we ignore the influence of lift propulsion.

4.1 Freestyle Swimming

In the freestyle model, we take one hand movement(this period includes one hand movement and three leg movements) as a cycle.

4.1.1 *Movements Display*

First, we give an illustration of the freestyle legs movements:

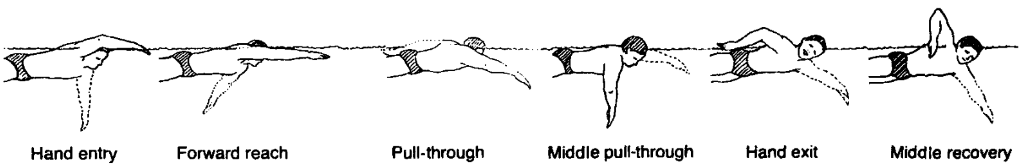


Figure 4 Freestyle movements

After observation, we can draw the trajectory of the arm movements:

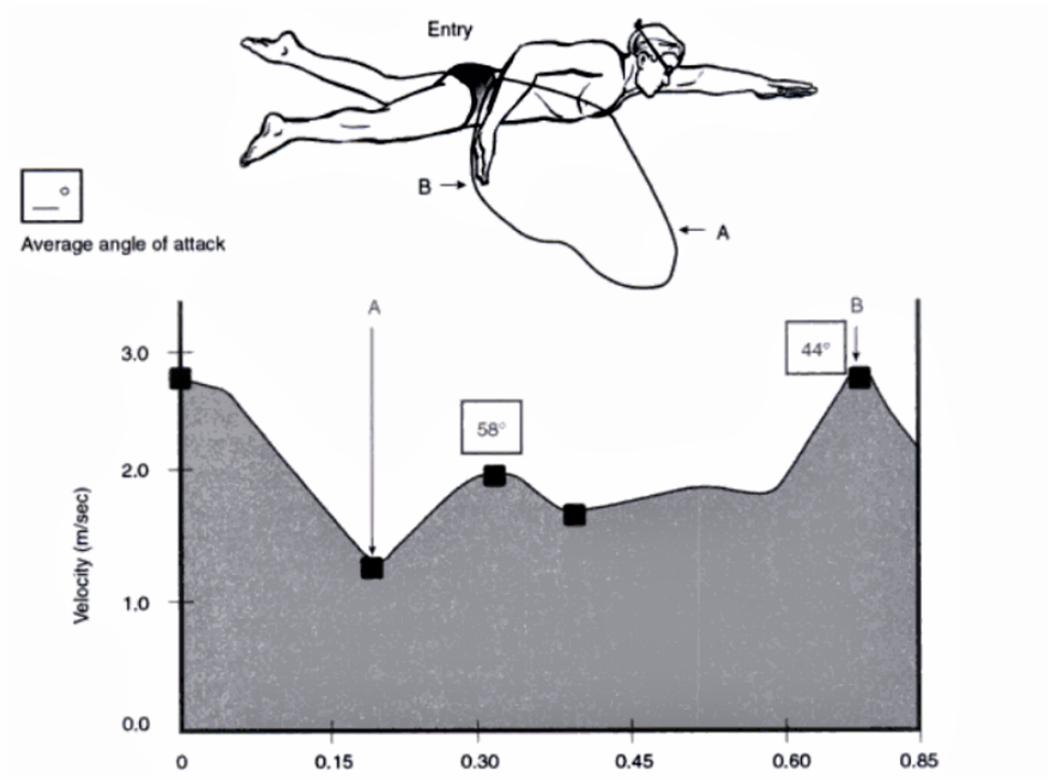


Figure 5 Trajectory of arm movements

Figure 5 is the research result from Emest W. Maglischo[2]. In addition to getting the trajectory of the arms, we can also find that the trajectory of the arms in the water is highly correlated with the swimming speed of the athlete during swimming. We reasonably conclude that the thrust of athlete in the water has a greater correlation with the trajectory of motion. Based on this, we launch the following modeling process.

4.1.2 ***Mechanical Model of Arm Movements***

Through observation, we reasonably simplify the freestyle arm movement and decompose it into the following three processes:

- Stage1: Quarter circular movement with elbow as axis, forearm as radius, ω_h as angular velocity.
- Stage2: Straight line pushing water movement with forearm as force part.
- Stage3: Quarter circular movement with elbow as axis, forearm as radius, ω_h as angular velocity.

we establish a coordinate system as shown in Figure 6,then we have trajectory equation of arm movements:

When $t \in (0, \frac{T}{3})$,we have

$$\begin{cases} x = R\cos(\omega_h t) \\ z = -R\sin(\omega_h t) \end{cases} \tag{1}$$

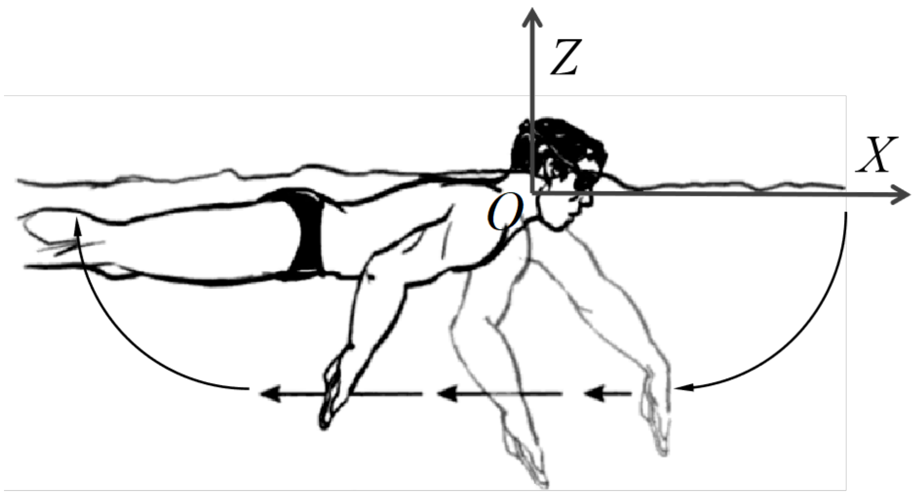


Figure 6 Arm movements

When $t \in (\frac{T}{3}, \frac{2T}{3})$,we have

$$\begin{cases} x = -v_{st}t \\ z = -R \end{cases} \tag{2}$$

When $t \in (\frac{2T}{3}, T)$,we have

$$\begin{cases} x = -L - \sin[\omega_h(t - t_{st} - t_{ro})] \\ z = -R + \cos[\omega_h(t - t_{st} - t_{ro})] \end{cases} \tag{3}$$

1. In step1: Take the micro-element segment of length dr, we have

$$dF = \frac{1}{2}\rho C_D \omega_w^2 W dr \tag{4}$$

We can get the component of force in the forward direction:

$$F_h = \int_0^R \frac{1}{2}\rho C_D(\omega_h r)^2 \sin(\omega_h t) W dr \tag{5}$$

2. In step2: Take the micro-element segment of length dr, we have

$$dF = \frac{1}{2}\rho C_D V_{st}^2 W dr \tag{6}$$

We can get the component of force of the entire forearm in the forward direction:

$$F_h = \int_0^R \frac{1}{2}\rho C_D V_{st}^2 W dr \tag{7}$$

3. In step3: Adopt the same infinitesimal method,we have the component of force of the entire forearm in the forward direction:

$$F_h = \int_0^R \frac{1}{2}\rho C_D(\omega_h r)^2 \sin[w_h(t - t_{st} - t_{ro})] W dr \tag{8}$$

4.1.3 *Mechanical Model of Leg Movements*

We simplify the leg movement to the following circular movement with an angular velocity of ω_l.

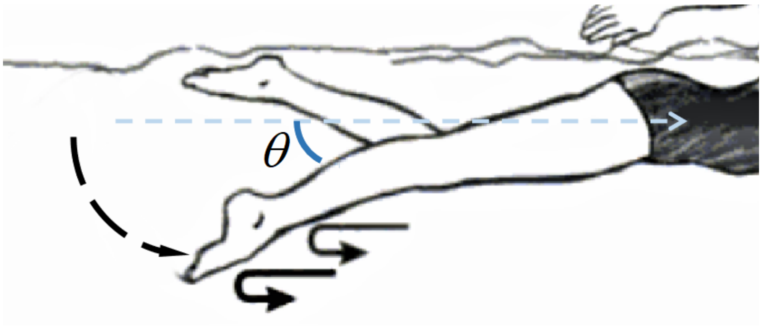


Figure 7 Leg movements

1. The analysis of propulsion:
   In a cycle, the positive and negative work of water on people takes up half a cycle, and the propulsion equation can be obtained:

$$F_l = \begin{cases} \int_0^R \frac{1}{2}\rho C_D(\omega_l r)^2 \sin(\theta - \omega_l t) W_l dr, & 0 < x \leq \frac{T}{6} \\ -\int_0^R \frac{1}{2}\rho C_D(\omega_l r)^2 \sin(\omega_l t - \theta) W_l dr, & \frac{T}{6} < x \leq \frac{T}{3} \\ \int_0^R \frac{1}{2}\rho C_D(\omega_l r)^2 \sin(3\theta - \omega_l t) W_l dr, & \frac{T}{3} < x \leq \frac{T}{2} \\ -\int_0^R \frac{1}{2}\rho C_D(\omega_l r)^2 \sin(\omega_l t - 3\theta) W_l dr, & \frac{T}{2} < x \leq \frac{2T}{3} \\ \int_0^R \frac{1}{2}\rho C_D(\omega_l r)^2 \sin(5\theta - \omega_l t) W_l dr, & \frac{2T}{3} < x \leq \frac{5T}{6} \\ -\int_0^R \frac{1}{2}\rho C_D(\omega_l r)^2 \sin(\omega_l t - 5\theta) W_l dr, & \frac{5T}{6} < x \leq T \end{cases} \tag{9}$$

2. The analysis of resistance:
   According to fluid mechanics, the total resistance received by the athlete is the sum of the resistance received by the body and the resistance received by the legs,we have:

$$f = \frac{1}{2}\rho C_D V^2 S_p + \frac{1}{2}\rho C_D V^2 S_l \tag{10}$$

4.2 Breaststroke

In the breaststroke model, we decompose a cycle action into four sub-actions: stroke, retracting leg, kick, and clamping. The trajectory is as follows:

Figure 8 The trajectory of breaststroke

1. In step1: Take the micro-element segment of length dr, we have

$$dF = \frac{1}{2}\rho C_D V_h^2 W dr \tag{11}$$

We can get the component of force in the forward direction:

$$F_h = \int_0^{R_a} \frac{1}{2}\rho C_D \omega_w^2 W \cos(\omega_w t) dr \tag{12}$$

2. In step2: There is no propulsion in the stage of leg retraction, but with the increase of the contact area between the leg and water, the greater resistance is produced. Add up the resistance to the body, we get the total resistance:

$$f = \frac{1}{2}\rho C_D V^2 S_p + \int_0^{R_D} \frac{1}{2}\rho C_D (\omega_{wl} r)^2 W_l \cos(\omega_{wl} t) dr \tag{13}$$

3. In step3: We have propulsion equation:

$$F_l = \frac{1}{2}\rho C_D V_D^2 S_l \tag{14}$$

4. In step4: We have propulsion equation:

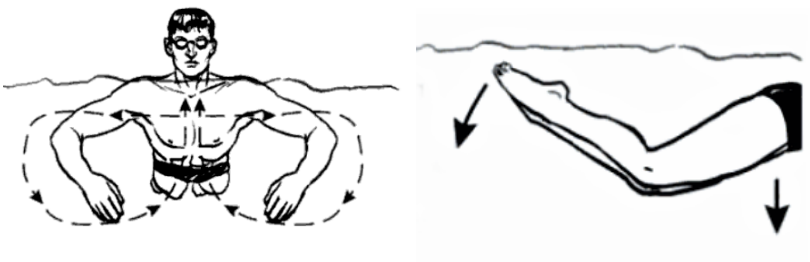
$$F_l = \int_0^{R_{al}} \frac{1}{2}\rho C_D \omega_{wl}^2 W_l \sin(\omega_{wl} t) dr \tag{15}$$

When the leg is open, the water has additional resistance to the leg. Add up the resistance to the body, we get the total resistance:

$$f = \frac{1}{2}\rho C_D V^2 S_p + \int_0^{Ral} \frac{1}{2}\rho C_D (\omega_{wl} r \sin(\omega_{wl} t))^2 W_l \sin(\omega_{wl} t) dt \tag{16}$$

4.3 Butterfly

In the butterfly model, the trajectories of the arms and legs are shown as follows:



(a) Arms (b) Legs

Figure 9 The trajectories of butterfly

After simplifying the butterfly model, we can establish the following mechanical model.

4.3.1 *Trajectory Equation*

Take the forward direction is the positive direction of the x-axis, the vertical upward is the positive direction of the z-axis, and the arm span is the y-axis.

When $0 < t \leq t_1$,the arms are in the water:

$$\begin{cases} x = R_a \cos(\omega_d t) \\ y = R_a \sin(\beta) - V_2 t \\ z = R_a - R_a * \sin(\omega_d t) \end{cases} \tag{17}$$

When $t_1 < t \leq T$,the arms are out of water:

$$\begin{cases} x = V_2 t \\ y = 0 \\ z = 0 \end{cases} \tag{18}$$

Trajectory equation of legs:

$$\begin{cases} y_f = sin(\omega_{dl} t) \\ V_f = \dfrac{dy_f}{dt} \\ \gamma = \arcsin(|\dfrac{y_f}{W_l|}|) \end{cases} \tag{19}$$

β refers to the angle between the projection of the arm on the xoy plane and the x-axis, y_f refers to the amplitude of leg movement, V_2 refers to the velocity component of the arm along the y-axis.

4.3.2 ***Mechanical Model***

Propulsion from arm movements:

$$F_h = \int_0^{R_a} \frac{1}{2}\rho C_D \omega_d r \cos[\arcsin(\frac{R_a \sin\beta - V_2 t}{R_a})dr \tag{20}$$

Propulsion from leg movements:

$$F_l = \begin{cases} \dfrac{1}{2} C_D (V_l sin\gamma - V)^2, & 0 < t \leq t_1 \\ -\dfrac{1}{2} C_D (V_l sin\gamma + V)^2, & t_1 < t \leq T \end{cases} \tag{21}$$

Total resistance:

$$f = \frac{1}{2}\rho C_D V^2 S_p \tag{22}$$

4.4 Backstroke

The force of the backstroke is similar to that of freestyle. The difference between it and freestyle is that the blocked area increases, and the stroke radius decreases slightly. Based on this, we can adjust the relevant parameters, and we do not repeat them.

Figure 10 The trajectory of backstroke

V. Performance Comparison of Different Strokes

Based on the mechanical model we have established, first we use Euler method to make a preliminary assessment of different strokes, and then we use the TOPSIS method to select the best stroke.

5.1 Discretization of Differential Equations Based on Euler Method

We should find their numerical solutions since differential equations are complex and difficult to solve directly. Generally, there are Euler method and Runge-Kutta method for solving differential equations. In our model, the Euler method is used to discretize the differential equation. Its general form is as follows:

$$\begin{cases} \dfrac{dy}{dx} = f(x, y) \\ y(a) = y_0 \end{cases} \tag{23}$$

In this equation, $a \leq x \leq b$.

After discretization, we have difference equation:

$$\begin{cases} y_{n+1} = y_n + hf(x_n, y_n) \\ y_0 = y(a) \end{cases} \tag{24}$$

5.1.1 *Model Establishing*

The kinematic equation mentioned above is $m\dfrac{d(v)}{dt} = F_{Propulsion} - F_{Resistance}$. Based on Euler method, we discretize the differential equation and get an equation:

$$\begin{cases} v_{t+1} = \dfrac{h[F(t) - f(t)]}{m} + v(t) \\ v_0 = 0 \end{cases} \tag{25}$$

The same, we have distance difference equation:

$$\begin{cases} s_{t+1} = s(t) + hv(t) \\ s_0 = 0 \end{cases} \tag{26}$$

Based on the above difference equation, we use iterative method to solve the model. The solving algorithm steps are as follows:

1. Step1: Initialize the parameters: Set the number of cycles , and set parameter values such as arm angular velocity, body mass, limbs length, etc.
2. Step2: Initialize the matrixs: Initialize speed matrix, resistance matrix, propulsion matrix and distance matrix.
3. Step3: If $i < k$, we trun to step4, if $i > k$, we turn to step7.
4. Step4: If $t < tn$, we trun to step5, if $t > tn$, we turn to step3.
5. Step5: Calculate the speed, resistance, propulsion and distance at this moment, $t = t + 1$, then we turn to step4.
6. Step6: $i = i + 1$, then we turn to step3.
7. Step7: Over.

5.1.2 *Visualization of Results*

We realize the established model by programming, and we show the propulsion, resistance, speed distance change over time of different strokes in six seconds as follows.We also display average value of propulsion, resistance, speed.Apart from that, we give total distance of four strokes.

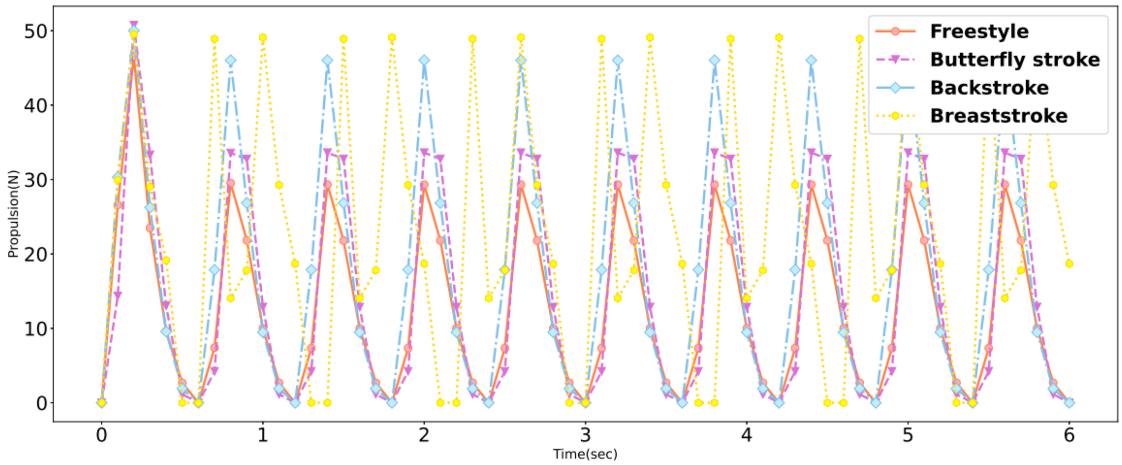


Figure 11 Propulsion change over time

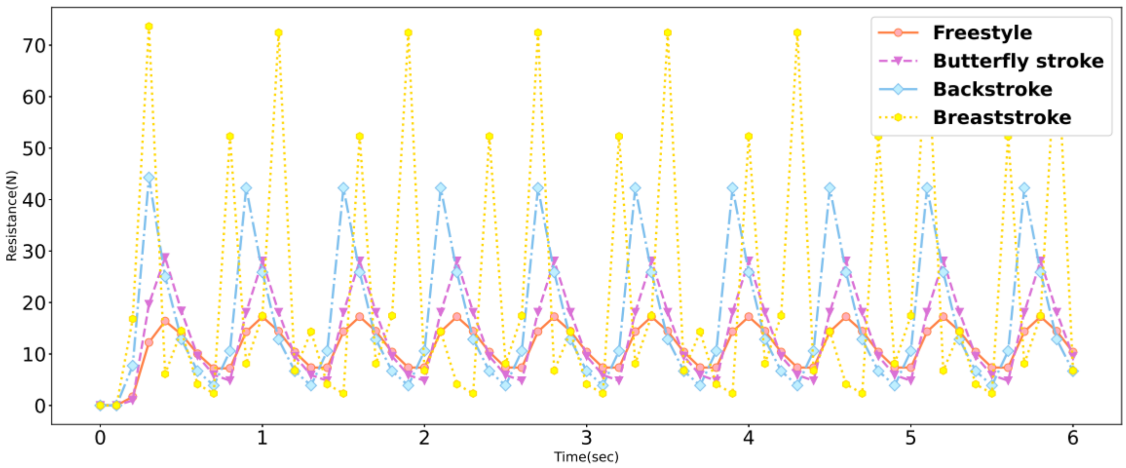


Figure 12 Resistance change over time

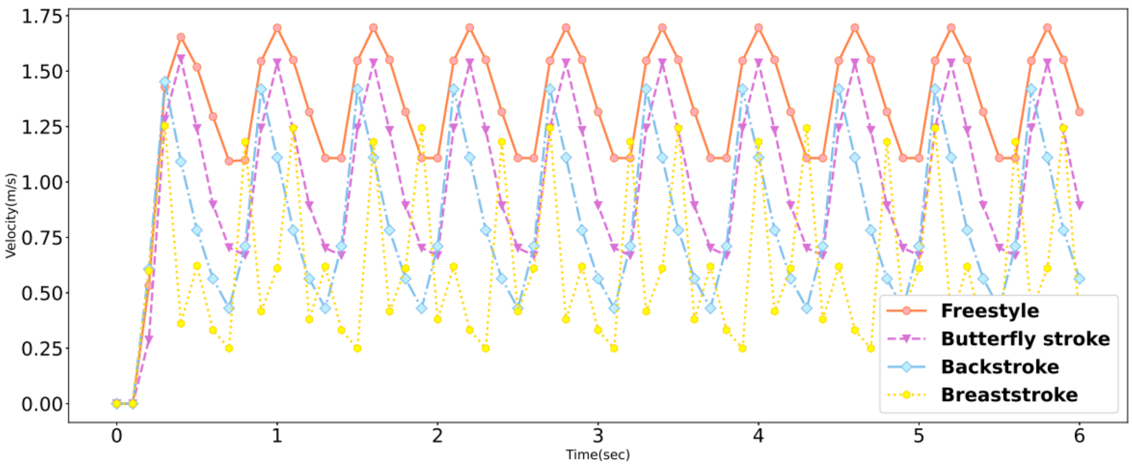


Figure 13 Velocity change over time

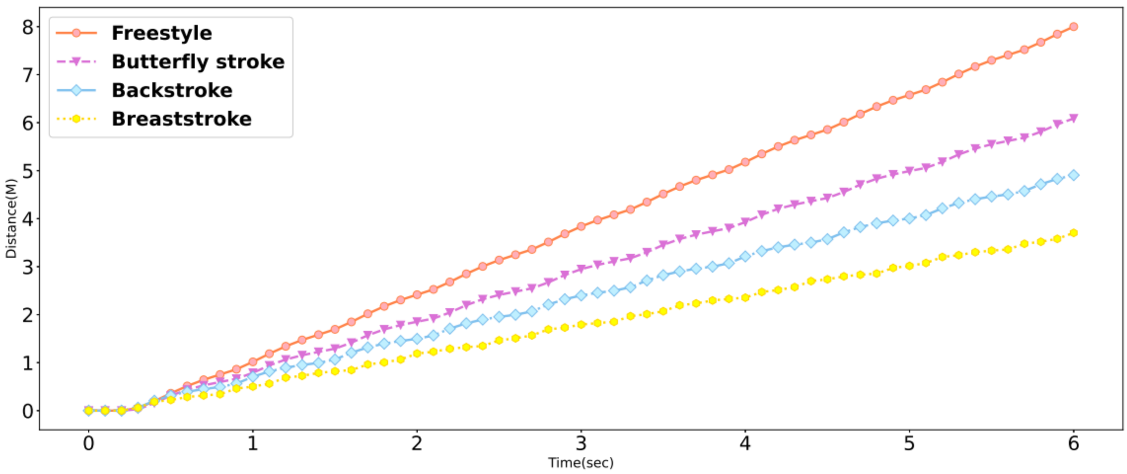


Figure 14 Distance change over time

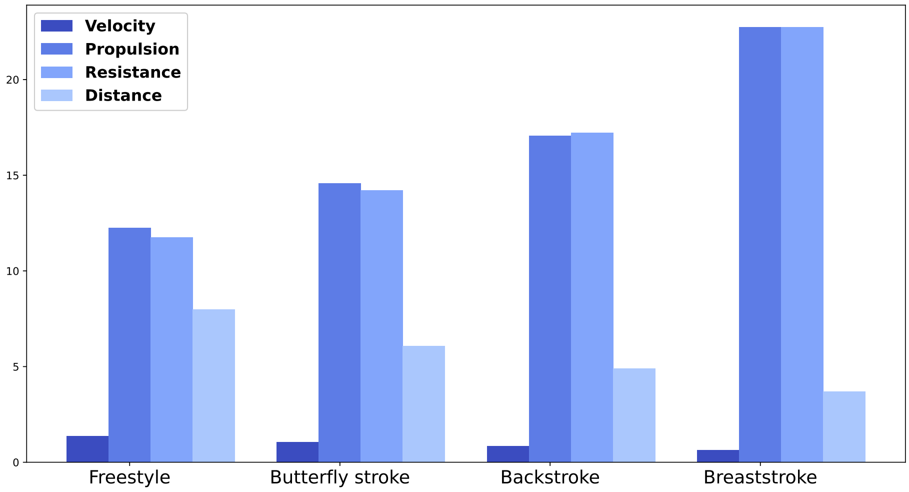


Figure 15 Summary

5.1.3 *Conclusion*

Table 2 Performance

	Propulsion(N)	Resistance(N)	Velocity(m/s)	Distance(m)
Freestyle	12.253	11.765	1.378	8.000
Butterfly	14.574	14.574	1.047	6.088
Backstoke	17.068	17.218	0.841	4.907
Breaststoke	22.754	22.738	0.634	3.704

From the results above, we can draw the following conclusions:

- The propulsion and resistance of freestyle have the smallest fluctuations among the four strokes. At the same time, the average resistance and propulsion are the smallest, too. It also has the largest speed.
- The propulsion of butterfly stroke is slightly greater than that of freestyle, while the resistance is also greater than that of freestyle, then the speed obtained is lower than freestyle.
- The propulsion of backstroke is greater than both freestyle and butterfly, but the resistance is also greater than the two, then the speed obtained is lower than that of the two.
- While breaststroke gains the greatest propulsion in the four strokes, its resistance is the largest, and the final speed is the lowest. In summary: the freestyle has the fastest speed and the breaststroke has the greatest thrust.

5 . 2 Determination of the Best Stroke Based on TOPSIS

Since energy consumption was not considered in the previous solution, it is possible that swimming speed is fast while energy consumption is also fast. In order to consider the overall situation and get the best stroke, we use the TOPSIS method to analyze the four swimming strokes.

5 . 2 . 1 *Establishment of Evaluation Matrix*

1. Step1, we establish the factor set U,$U = \{U_1, U_2, U_3, U_4\}$.
   U_1 refers to average value of velocity, U_2 refers to average value of propulsion ， U_3 refers to average value of resistance, U_4 refers to distance.
2. Step2,we forward the original matrix, which means convert all indicator types into large-scale. In our model, resistance is small-scale indicator, we use $max - x$ to convert it into large-scale.Then we get matrix X:
$$X = \begin{pmatrix} 1.378 & 12.253 & 10.973 & 8.000 \\ 1.047 & 14.574 & 8.516 & 6.088 \\ 0.841 & 17.068 & 5.520 & 4.907 \\ 0.634 & 22.754 & 0.000 & 3.704 \end{pmatrix} \tag{27}$$
3. Step3,To normalize the matrix$X$, we use $z_{ij} = \dfrac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}$, then we get normalization matrix Z:
$$Z = \begin{pmatrix} 0.680 & 0.358 & 0.734 & 0.679 \\ 0.517 & 0.426 & 0.570 & 0.517 \\ 0.415 & 0.499 & 0.369 & 0.416 \\ 0.313 & 0.665 & 0.000 & 0.314 \end{pmatrix} \tag{28}$$
4. Step4, we calculate the score and normalize We define maximum
   $Z^+ = (Z_1^+ + Z_2^+ + ...Z_m^+)$, minimum $Z^- = (Z_1^- + Z_2^- + ...Z_m^-)$
   The distance between the i-th evaluation object and the maximum value is
$$D_i^+ = \sqrt{\sum_{j=1}^{m} (Z_j^+ - z_{ij})^2} \tag{29}$$
   The distance between the i-th evaluation object and the minimum value is
$$D_i^- = \sqrt{\sum_{j=1}^{m} (Z_j^- - z_{ij})^2} \tag{30}$$
   Then, we can caculate the score of i-th evaluation object without normalization:
   $S_i = \dfrac{D_i^-}{D_i^- + D_i^+}$

5 . 2 . 2 *Conclusion*

Finally ,we get scores of different strokes shown in the following table.

We can conclude that in swimming competition, freestyle is the best stroke while breaststroke performs worst.

Table 3 Score

	Score	Rank
Freestyle	0.360	1
Butterfly	0.306	2
Backstroke	0.210	3
Breaststroke	0.123	4

VI. Sensitivity Analysis

In order to explore the influence of the angular velocity of the arm on the swimming speed of the athlete while swimming, we take the freestyle as an example to analyze the sensitivity of the model. After changing the angular velocity of the model by five percent, the results are as follows:

Table 4 Score

	ω_0	$(1+5\%)\omega_0$	$(1-5\%)\omega_0$
Velocity	1.378	1.417	1.292
Propulsion	12.253	13.639	11.466
Resisitance	11.765	12.745	10.560
Distance	78.000	8.531	7.775

We can get comparison graphs as follows.

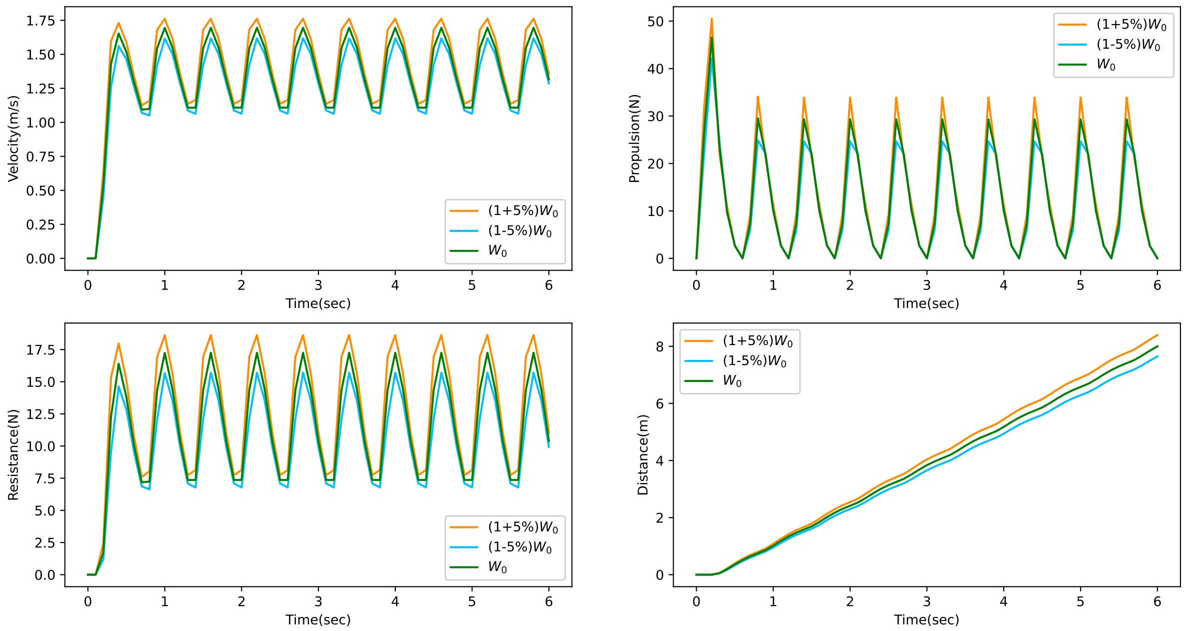


Figure 16 Performance comparison at different angular velocities

We can conclude that our model is sensitive to changes in angular velocity.

VII. Discussion on the Angle of Attack in Freestyle Swimming

In this section, we use the hand model and sweep angle used by R • E Schreichoff in his research.

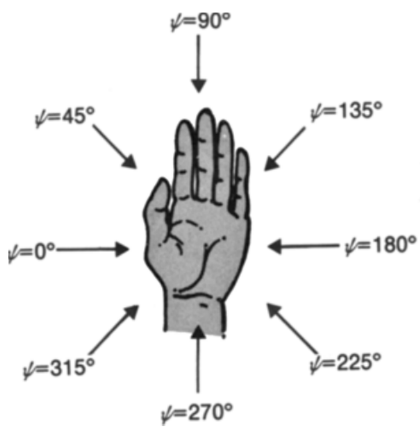


Figure 17 Sweep angle

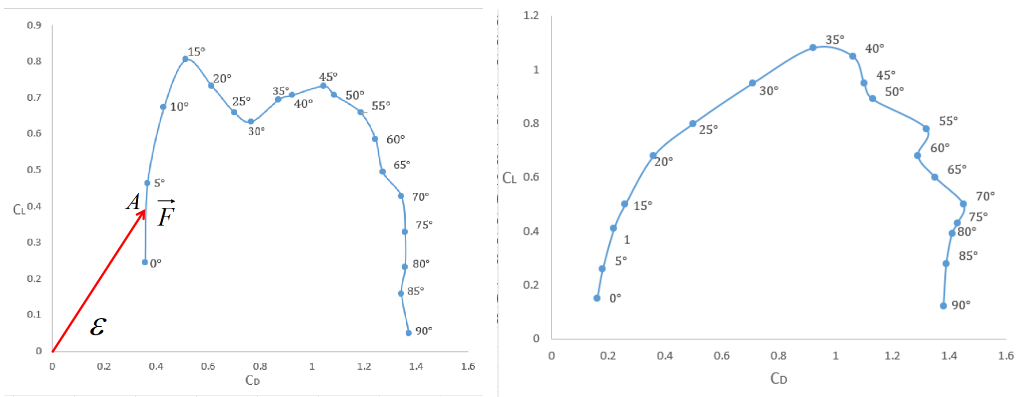
7.1 Calculation of Resultant Force

We know that the palm of the hand is stroked at a certain angle of attack, and it is affected by two forces at the same time, one is lift, the other is resistance.According to the principle of force synthesis, we have:

$$F = \sqrt{L^2 + D^2} = \sqrt{(\frac{1}{2}\rho C_L V^2 A)^2 + (\frac{1}{2}\rho C_D V^2 A)^2} = \sqrt{C_L^2 + C_D^2}R \tag{31}$$

7.2 Coefficients and Angels

According to R • E Schreichoff' s test results, the main stroke stages we will study are the inward stroke($\psi = 0°$) and the outward stroke($\psi = 270°$).We draw the resistance coefficient and lift coefficient in polar form.



(a) $\psi = 0°$ (b) $\psi = 270°$

Figure 18 Pole figure

According to the principle of fluid mechanics, for any point A on the pole figure curve, we make a connection OA, then the direction of OA is the direction of the resultant force provided by the water.And the tangent of the angle ε between OA and the abscissa is the lift-to-drag ratio:$\tan\varepsilon = \frac{C_L}{C_D}$.

It can be seen from the Figure 19 that ε is the angle between the resultant force and the resistance. The angle between the F and X axis (hand's chord) is equal to $\varepsilon + \alpha$. The Y axis is the central axis of pressure on the hand plane, which is parallel to the longitudinal axis of the body (forward direction). The deflection angle of the resultant force F from the forward direction β is equal to $|(\varepsilon + \alpha) - 90°|$.

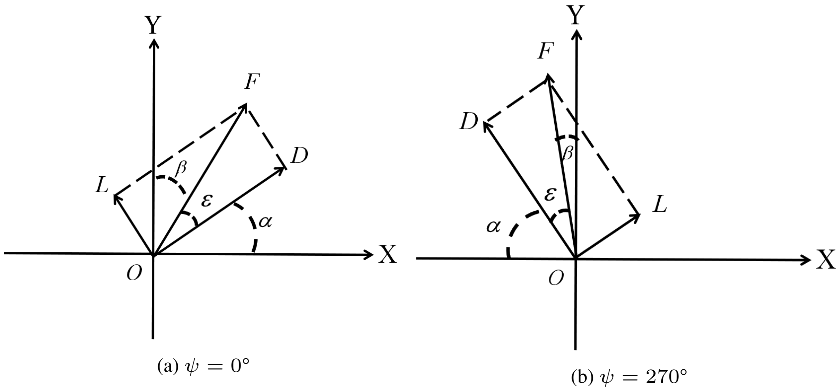


Figure 19 The direction of F

According to the above research, we calculate $\tan\varepsilon, \varepsilon, \beta, F$ when the palms take different angles of attack.(Here we only discuss the case when $\psi = 0°$)

Table 5 Performance when

α	$\tan\varepsilon$	ε	β	F
5°	1.314	52°42	34°18	$\sqrt{0.3341}R$
15°	1.6	58°	17°	$\sqrt{0.89}R$
25°	0.985	44°36	20°24	$\sqrt{0.8321}R$
35°	0.854	40°30	14°30	$\sqrt{1.1624}R$
45°	0.75	36°54	8°6	$\sqrt{1.5625}R$
55°	0.594	30°36	4°24	$\sqrt{1.7849}R$
65°	0.410	22°18	2°42	$\sqrt{1.7384}R$
75°	2.261	14°36	0°24	$\sqrt{1.9181}R$
85°	0.148	8°24	3°24	$\sqrt{1.8625}R$

7.3 Result Analysis

We know that to evaluate the effectiveness of the stroke, the size of the propulsion force is an important factor, we should also consider accumulation of the effect of the force over time. Therefore, the length of effective stroke trajectory should be considered in the evaluation.

It can be seen that the best angle of attack used in the inward stroke is between 45° and 65°. With this range of angle of attack, the value of the resultant force is larger, and the deflection angle β between the direction of the resultant force and the forward direction of the body is smaller, that is, the component force of the resultant force in the forward direction of the body is larger.Moreover, the use of the angle of attack within this range can not only ensure the curve stroke, but also have a longer stroke line.

When we small angle of attack($\alpha < 45°$) to stroke, the resultant force is relatively small, and the direction of the resultant force has a large deflection angle β with the body's forward direction. Although the angle of attack within this range can guarantee a curve stroke, because the propulsion force is too small, the stroke effect is not ideal.

When we large angle of attack($\alpha > 65°$) to stroke, the resultant force is large, and the direction of the resultant force has a small deflection angle β with the body's forward direction.However, from the analysis of the stroke trajectory, when the angle of attack is large, the trajectory is relatively straight and approximates a straight line. It can cause high energy consumption, and affect propulsion impulse.

In summary, to improve the effect of palm strokes, the best range of the palm stroke angle of attack during the inward stroke is $45° < \alpha < 65°$. The same, the best range of the palm stroke angle of attack during the outward stroke is $35° < \alpha < 75°$.

VIII. Guidance to the Training

We take freestyle as an example, and give the following two feasible training guidances based on our model.

8.1 Angle of Attack

Through the model in Chapter 7, we conclude that, the best range of the palm stroke angle of attack during the inward stroke is $45° < \alpha < 65°$. The same, the best range of the palm stroke angle of attack during the outward stroke is $35° < \alpha < 75°$. It is also required that the palm plane should always be parallel to the transverse plane, which can ensure a more ideal stroke effect. If the palm plane and the transverse plane have a certain deflection angle, it will inevitably cause a greater deflection angle between the resultant force and the forward direction of the body, which will affect the stroke. Because the angle of attack is difficult to control, it requires athletes to pay attention to the developing the sense of water during normal exercising. We hope that athletes can adjust the posture of the hand by using the sense.

8.2 The Frequency of Stroke

Through the model in Chapter 6, we can know that we can get a greater forward driving force when the frequency of swinging arms is increased, and thus a faster swimming speed can be obtained. When the swinging reduces, the average swimming speed will decrease. Therefore, arm swing frequency is a key factor for swimmers to obtain faster swimming speed. During training, it is necessary for athletes to strengthen the training of arm strength and master the arm swing technique.

In addition, increasing the arm swing frequency will also increase the resistance. For people with weaker physical strength, they should not challenge their limits in order to achieve their goals. They should increase training intensity step by step.

IX. Strength and Weakness

9.1 Strength

We start from Newton's second law and Newton's third law, and cleverly use the movement trajectory to establish swimming kinematics model, which is reliable.

In addition to basic differential equation solution, we consider the average speed, propulsion, resistance and swimming distance in a period of time and use the TOPSIS method to improve the our models. Then we get a more reliable result of optimal swimming stroke.

Finally, we use angular velocity of arm to analyze the sensitivity of our model. We also establish a model for the palm stroke angle of attack.Based on the two, we put forward effective suggestions for swimmers.

9.2 Weakness

When modeling,we do not consider human physiological factors, so the model has some errors.

We simplify the movement trajectory, so there is a certain deviation from the actual trajectory.

X. References

[1] R • E Schreichoff. Hydrodynamic Analysis of Swimming Propulsion. The Third International Swimming Biomechanics Symposium,1981

[2] Emest W. Maglischo. Swimming Fatest.

[3] Daniel A. Marinho. Hydrodynamic Drag During Gliding in Swimming. Journal of Applied Biomechanics, 2009, 25, 253-257.

[4] Timothy Wei, Russell Mark, and Sean Hutchison. The Fluid Dynamics of Competitive Swimming. Annu. Rev. Fluid Mech. 2014. 46:547–65.

XI. Appendix

Listing 1: Description of Attachment

```
Our folder includes some attachments, the description is as follows:
 ‘code’ : This folder contains the executable programs in this article.
 ‘data_sensitivity_analysis.xlsx’ : This excel contains the solution result.
 ‘data_solution_result.xlsx’ : This excel contains sensitivity analysis results.
```

Listing 2: Butterfly Solving Code

```
%dieyong
w_hand = 2*pi;
ts = pi/w_hand;
td=0.1;
ns = ts/td;
ns2 = ns/3;
ns2 = floor(ns2);
ks = 10;

ddl = 0.4;
vdd = ddl/ts;

l_hand = 0.7;
theta = 30;

F = zeros(1,ks*(ns+ns2));
f = zeros(1,ks*(ns+ns2));
v_p = zeros(1,ks*(ns+ns2));

fsum = zeros(1,ks*(ns+ns2));
Fsum = zeros(1,ks*(ns+ns2));

m = 50;
s = zeros(1,(ns+ns2));
k = 0;

n=0;
kk = 0;
while(kk<ks)
for t = 1+k : 1:ns+k
syms ld;
F(t) =
    10*int(theta*0.08*(ld*cos(asin((ddl-vdd*mod(t,ns+ns2)*td)/ld))*w_hand*abs(sin(w_hand*(mod(t,ns+ns2))*td
f(t) = 0.4*theta*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;

if 5*sin(pi/4*mod(t,ns+ns2)*td)<0
```

```
fsum(t) = f(t)-5*sin(pi/4*mod(t,ns+ns2)*td);
Fsum(t) = F(t)-5*sin(pi/4*mod(t,ns+ns2)*td);
else
fsum(t) = f(t);
Fsum(t) = F(t);
end

end

for t = ns+k:1:ns+k+ns2
F(t)=3*sin(pi/4*mod(t,ns+ns2)*td);
f(t) = 0.4*theta*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;
if 3*sin(pi/4*mod(t,ns+ns2)*td)<0
fsum(t) = f(t)-3*sin(pi/4*mod(t,ns+ns2)*td);
Fsum(t) = F(t)-3*sin(pi/4*mod(t,ns+ns2)*td);
else
fsum(t) = f(t);
Fsum(t) = F(t);
end

end
k = k+ns+ns2;
kk = kk+1;
end
```

Listing 3: Breaststroke Solving Code

```
w_hand = 2*pi;
ts = pi/w_hand;
td=0.1;
ns = ts/td;
ns2 =1;
ns3 = 1;
ns4 = 1;
nsn = ns+ns2+ns3+ns4;
ks = 10;


l_hand = 0.7;
theta = 30;

F = zeros(1,ks*(nsn));
f = zeros(1,ks*(nsn));
v_p = zeros(1,ks*(nsn));

m = 50;
s = zeros(1,(nsn));
```

```
k = 0;

n=0;
kk = 0;


while(kk<ks)
for t = 1+k : 1:ns+k
syms ld;
F(t) =
    8*int(theta*0.08*(ld*w_hand*abs(sin(w_hand*(mod(t,nsn))*td))-v_p(t))^2,ld,0,l_hand);
f(t) = 1.25*theta*1.25*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;
end

for t = ns+k:1:ns+k+ns2
F(t)=0;
f(t) = 1.25*theta*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;
end

for t = ns+k+ns2:1:ns+k+ns2+ns3
f(t) = 1.25*theta*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;
end

for t = ns+k+ns2+ns3:1:ns+k+ns2+ns3+ns4
F(t)=6*int(theta*0.08*(ld*w_hand*abs(sin(w_hand*(mod(t,nsn))*td))-v_p(t))^2,ld,0,l_hand);
f(t) = 1.25*theta*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;
end

k = k+nsn;
kk = kk+1;
end
```

Listing 4: Backstroke Solving Code

```
w_hand = 2*pi;
ts = pi/w_hand;

td=0.1;
ns = ts/td;
ns2 = ns/4;
ns2 = floor(ns2);
```

```
ks = 10;
l_hand = 0.7;
theta = 30;

F = zeros(1,ks*(ns+ns2));
f = zeros(1,ks*(ns+ns2));
v_p = zeros(1,ks*(ns+ns2));
fsum = zeros(1,ks*(ns+ns2));
Fsum = zeros(1,ks*(ns+ns2));

m = 50;
s = zeros(1,(ns+ns2));
k = 0;

n=0;
kk = 0;
while(kk<ks)
for t = 1+k : 1:ns+k
syms ld;
F(t) =
    8*int(theta*0.08*(ld*w_hand*abs(sin(w_hand*(mod(t,ns+ns2))*td))-v_p(t))^2,ld,0,l_hand)+5*sin(pi/4*mod(t
f(t) = 0.7*theta*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;

if 5*sin(pi/4*mod(t,ns+ns2)*td)<0
fsum(t) = f(t)-5*sin(pi/4*mod(t,ns+ns2)*td);
Fsum(t) = F(t)-5*sin(pi/4*mod(t,ns+ns2)*td);
else
fsum(t) = f(t);
Fsum(t) = F(t);
end

end

for t = ns+k:1:ns+k+ns2
F(t)=5*sin(pi/4*mod(t,ns+ns2)*td);
f(t) = 0.7*theta*v_p(t)^2;
v_p(t+1)=v_p(t)+(F(t)-f(t))/m;
s(t+1)=s(t)+v_p(t)*td;

if 5*sin(pi/4*mod(t,ns+ns2)*td)<0
fsum(t) = f(t)-5*sin(pi/4*mod(t,ns+ns2)*td);
Fsum(t) = F(t)-5*sin(pi/4*mod(t,ns+ns2)*td);
else
fsum(t) = f(t);
Fsum(t) = F(t);
end
```

```
end
k = k+ns+ns2;
kk = kk+1;
end
```

Listing 5: TOPSIS

```
clear;clc
X = [1.378260271 1.04695903 0.841168464 0.634221047
12.2534991 14.5736829 17.0683564 22.75418101
11.76520289 14.22196616 17.21798263 22.73773404
8.000160218 6.087734791 4.906548256 3.703919876
];
X = X';

[n,m] = size(X);
disp(['' num2str(n) ', ' num2str(m) ''])
Judge = input(['' num2str(m) ' ']);

if Judge == 1
Position = input(':  '); %[2,3,4]
disp(' ')
Type = input('[1,3,2]:  '); %[2,1,3]

for i = 1 : size(Position,2)
X(:,Position(i)) = Positivization(X(:,Position(i)),Type(i),Position(i));

end
disp(' ')
disp(X)
end
disp("")
Judge = input(' ');
if Judge == 1
disp(['0.25,0.25,0.5, [0.25,0.25,0.5]']);
weigh = input(['' num2str(m) '。' '' num2str(m) ': ']);
OK = 0;
while OK == 0
if abs(sum(weigh) - 1)<0.000001 && size(weigh,1) == 1 && size(weigh,2) == m
OK =1;
else
weigh = input(': ');
end
end
else
weigh = ones(1,m) ./ m ;
end
```

```
Z = X ./ repmat(sum(X.*X) .^ 0.5, n, 1);
disp(' Z = ')
disp(Z)


D_P = sum([(Z - repmat(max(Z),n,1)) .^ 2 ] .* repmat(weigh,n,1) ,2) .^ 0.5;
D_N = sum([(Z - repmat(min(Z),n,1)) .^ 2 ] .* repmat(weigh,n,1) ,2) .^ 0.5;
S = D_N ./ (D_P+D_N);
stand_S = S / sum(S)
[sorted_S,index] = sort(stand_S ,'descend')


function [posit_x] = Inter2Max(x,a,b)
r_x = size(x,1); % row of x
M = max([a-min(x),max(x)-b]);
posit_x = zeros(r_x,1);
for i = 1: r_x
if x(i) < a
posit_x(i) = 1-(a-x(i))/M;
elseif x(i) > b
posit_x(i) = 1-(x(i)-b)/M;
else
posit_x(i) = 1;
end
end
end

function [posit_x] = Mid2Max(x,best)
M = max(abs(x-best));
posit_x = 1 - abs(x-best) / M;
end


function [posit_x] = Min2Max(x)
posit_x = max(x) - x;
%posit_x = 1 ./ x;
end

function [posit_x] = Positivization(x,type,i)

disp(['' num2str(i) ''] )
posit_x = Min2Max(x); %
disp(['' num2str(i) ''] )
disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
elseif type == 2 %
disp(['' num2str(i) ''] )
best = input(':  ');
posit_x = Mid2Max(x,best);
```

```
disp(['' num2str(i) ''] )
disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
elseif type == 3 %
disp(['' num2str(i) ''] )
a = input(':  ');
b = input(':  ');
posit_x = Inter2Max(x,a,b);
disp(['' num2str(i) ''] )
disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
else
disp('')
end
end
```

Listing 6: Drawing Graphs

```
#!/usr/bin/env python
# coding: utf-8
import pandas as pd
import time
import numpy as np
import matplotlib.pyplot as plt
x=[]
for i in range(61):
x.append(i)
for i in range(len(x)):
x[i]=x[i]/10
import pandas as pd
data=pd.read_excel('C:/Users/bo/Desktop/data_1_a.xlsx')
vziyou=list(data.iloc[1,2:])
vziyou.insert(0,0)
tziyou=list(data.iloc[2,2:])
tziyou.insert(0,0)
zziyou=list(data.iloc[3,2:])
zziyou.insert(0,0)
jziyou=list(data.iloc[4,2:])
jziyou.insert(0,0)
vdie=list(data.iloc[5,2:])
vdie.insert(0,0)
tdie=list(data.iloc[6,2:])
tdie.insert(0,0)
zdie=list(data.iloc[7,2:])
zdie.insert(0,0)
jdie=list(data.iloc[8,2:])
jdie.insert(0,0)
vyang=list(data.iloc[9,2:])
vyang.insert(0,0)
tyang=list(data.iloc[10,2:])
tyang.insert(0,0)
```

```
zyang=list(data.iloc[11,2:])
zyang.insert(0,0)
jyang=list(data.iloc[12,2:])
jyang.insert(0,0)
vwa=list(data.iloc[13,2:])
vwa.insert(0,0)
twa=list(data.iloc[14,2:])
twa.insert(0,0)
zwa=list(data.iloc[15,2:])
zwa.insert(0,0)
jwa=list(data.iloc[16,2:])
jwa.insert(0,0)
plt.figure(figsize=(25, 10))
plt.plot(x, vziyou, label='Freestyle', linewidth=3,linestyle='-', color='#FF8247',
    marker='o',
markerfacecolor='#FFAEB9', markersize=10)
plt.plot(x, vdie, label='Butterfly stroke', linewidth=3,linestyle='--',
    color='#da70d6', marker='v',
markerfacecolor='#da70d6', markersize=10)
plt.plot(x, vyang, label='Backstroke', linewidth=3,linestyle='-.', color='#7ec0ee',
    marker='D',
markerfacecolor='#bfefff', markersize=10)
plt.plot(x, vwa, label='Breaststroke', linewidth=3,linestyle=':', color='#ffd700',
    marker='h',
markerfacecolor='#ffff00', markersize=10)
plt.xlabel("Time(sec)",fontsize='xx-large')
plt.ylabel("Velocity(m/s)",fontsize='xx-large')
plt.tick_params(labelsize=25)
plt.legend(prop={"size":25,"weight":"black"})
plt.savefig('C:/Users/bo/Desktop/velocity.eps')
plt.show()
import matplotlib.pyplot as plt
import numpy as np
labels = ['Freestyle', 'Butterfly stroke', 'Backstroke', 'Breaststroke']
v=[1.378260271,1.04695903,0.841168464,0.634221047]
t=[12.2534991,14.5736829,17.0683564,22.75418101]
z=[11.76520289,14.22196616,17.21798263,22.73773404]
j=[8.000160218,6.087734791,4.906548256,3.703919876]
x = np.arange(len(labels))
width = 0.2
plt.bar(x, v, width=width, label='Velocity')
plt.bar(x + width, t, width=width, label='Propulsion', tick_label=labels)
plt.bar(x + 2 * width, z, width=width, label='Resistance')
plt.bar(x + 3 * width, j, width=width, label='Distance')
plt.xticks(fontsize='xx-large')
plt.legend(loc="upper left")
plt.legend(prop={"size":15,"weight":"black"})
plt.rcParams['savefig.dpi'] = 300
plt.rcParams['figure.dpi'] = 300
```

```
plt.rcParams['figure.figsize'] = (15.0, 8.0)
plt.savefig('C:/Users/bo/Desktop/Summary.eps')
plt.show()
x=[]
for i in range(61):
x.append(i)
for i in range(len(x)):
x[i]=x[i]/10
import pandas as pd
data=pd.read_excel('E:/TIM/tim_document/data.xlsx')
vjia=list(data.iloc[0,1:])
vjia.insert(0,0)
tjia=list(data.iloc[1,1:])
tjia.insert(0,0)
zjia=list(data.iloc[2,1:])
zjia.insert(0,0)
jjia=list(data.iloc[3,1:])
jjia.insert(0,0)
vjian=list(data.iloc[4,1:])
vjian.insert(0,0)
tjian=list(data.iloc[5,1:])
tjian.insert(0,0)
zjian=list(data.iloc[6,1:])
zjian.insert(0,0)
jjian=list(data.iloc[7,1:])
jjian.insert(0,0)
vyuan=list(data.iloc[8,1:])
vyuan.insert(0,0)
tyuan=list(data.iloc[9,1:])
tyuan.insert(0,0)
zyuan=list(data.iloc[10,1:])
zyuan.insert(0,0)
jyuan=list(data.iloc[11,1:])
jyuan.insert(0,0)
ax1 = plt.subplot(221)
plt.xlabel("Time(sec)")
plt.ylabel("Velocity(m/s)")
plt.plot(x, vjia,color='darkorange',label='(1+5%)$W_0$')
plt.plot(x, vjian, color='deepskyblue',label='(1-5%)$W_0$')
plt.plot(x, vyuan, color='green',label='$W_0$')
plt.legend()
ax2 = plt.subplot(222)
plt.xlabel("Time(sec)")
plt.ylabel("Propulsion(N)")
plt.plot(x, tjia,color='darkorange',label='(1+5%)$W_0$')
plt.plot(x, tjian, color='deepskyblue',label='(1-5%)$W_0$')
plt.plot(x, tyuan, color='green',label='$W_0$')
plt.legend()
ax3 = plt.subplot(223)
```

```
plt.xlabel("Time(sec)")
plt.ylabel("Resistance(N)")
plt.plot(x, zjia,color='darkorange',label='(1+5%)$W_0$')
plt.plot(x, zjian, color='deepskyblue',label='(1-5%)$W_0$')
plt.plot(x, zyuan, color='green',label='$W_0$')
plt.legend()
ax4 = plt.subplot(224)
plt.xlabel("Time(sec)")
plt.ylabel("Distance(m)")
plt.plot(x, jjia,color='darkorange',label='(1+5%)$W_0$')
plt.plot(x, jjian, color='deepskyblue',label='(1-5%)$W_0$')
plt.plot(x, jyuan, color='green',label='$W_0$')
plt.legend()
plt.savefig('C:/Users/bo/Desktop/data.eps')
plt.show()
```