HOMEWORK 5

SOLUTIONS

3.* *a.* Estimate the roots of the equation

$$x^3 - 3x^2 + 5x \sin\left(\frac{\pi x}{4} - \frac{5\pi}{4}\right) + 3 = 0$$

by plotting the equation.

b. Use the estimates found in part *a* to nd the roots more accurately with the `fzero` function.

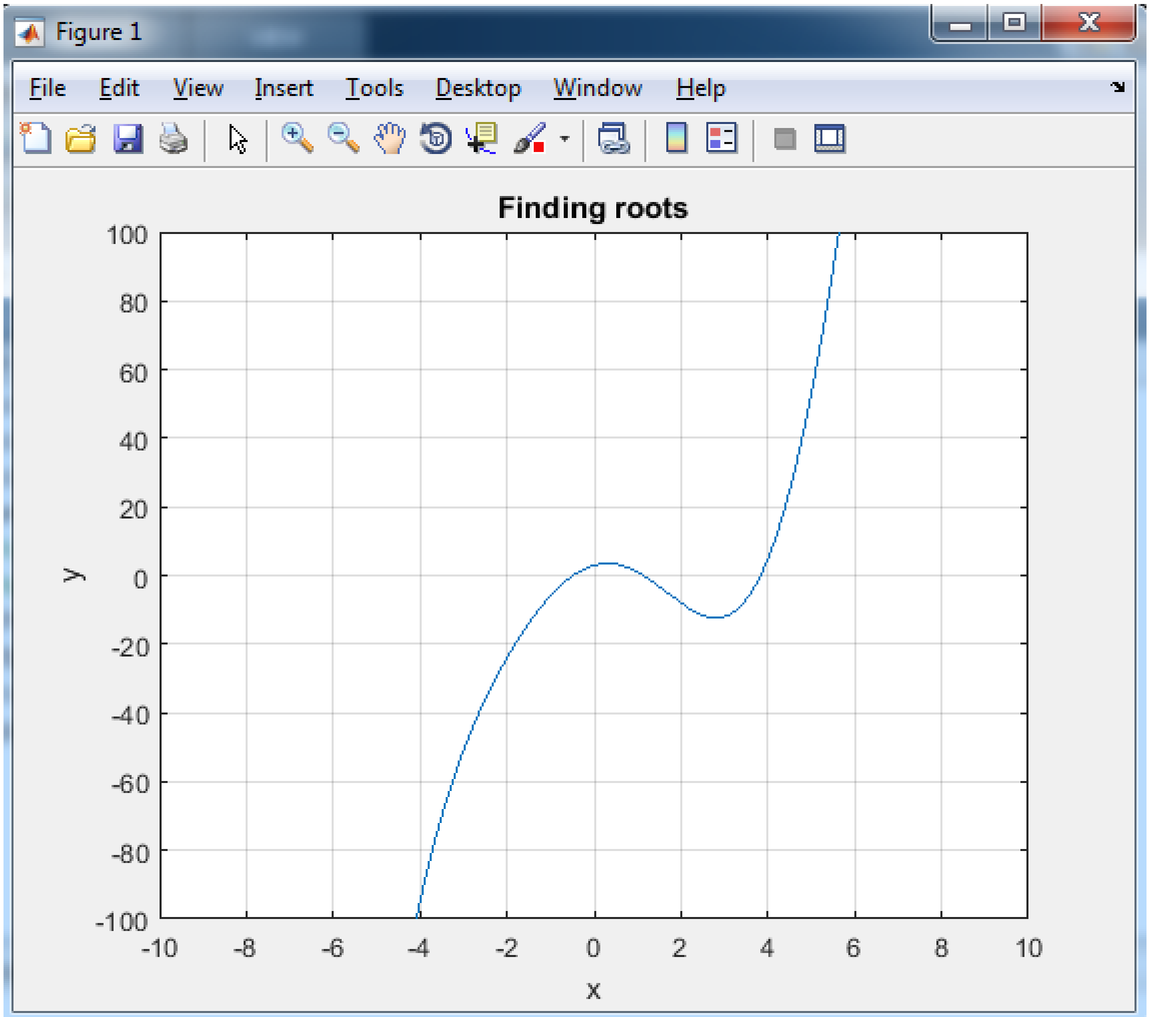
Solution

3a.

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_3a.m

prac5.m | que_3a.m | que_3b.m

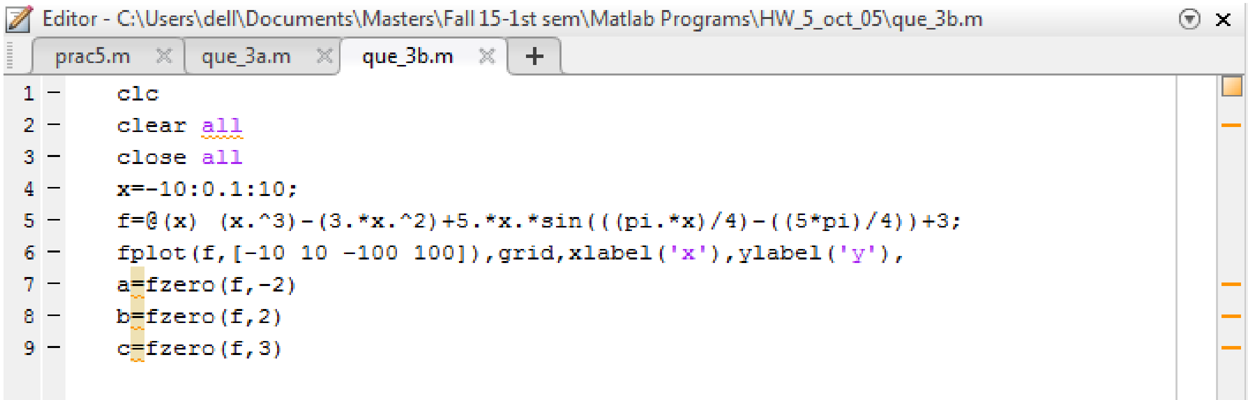
```
clc
clear all
close all
x=-10:0.1:10;
%y=(x.^3)-(3.*x.^2)+5.*x.*sin(((pi.*x)/4)-((5*pi)/4))+3;
%plot(x,y)
f=@(x) (x.^3)-(3.*x.^2)+5.*x.*sin(((pi.*x)/4)-((5*pi)/4))+3;
fplot(f,[-10 10 -100 100]),grid,xlabel('x'),ylabel('y'),
title('Finding roots')
ginput(1),ginput(1),ginput(1)
```



Command Window

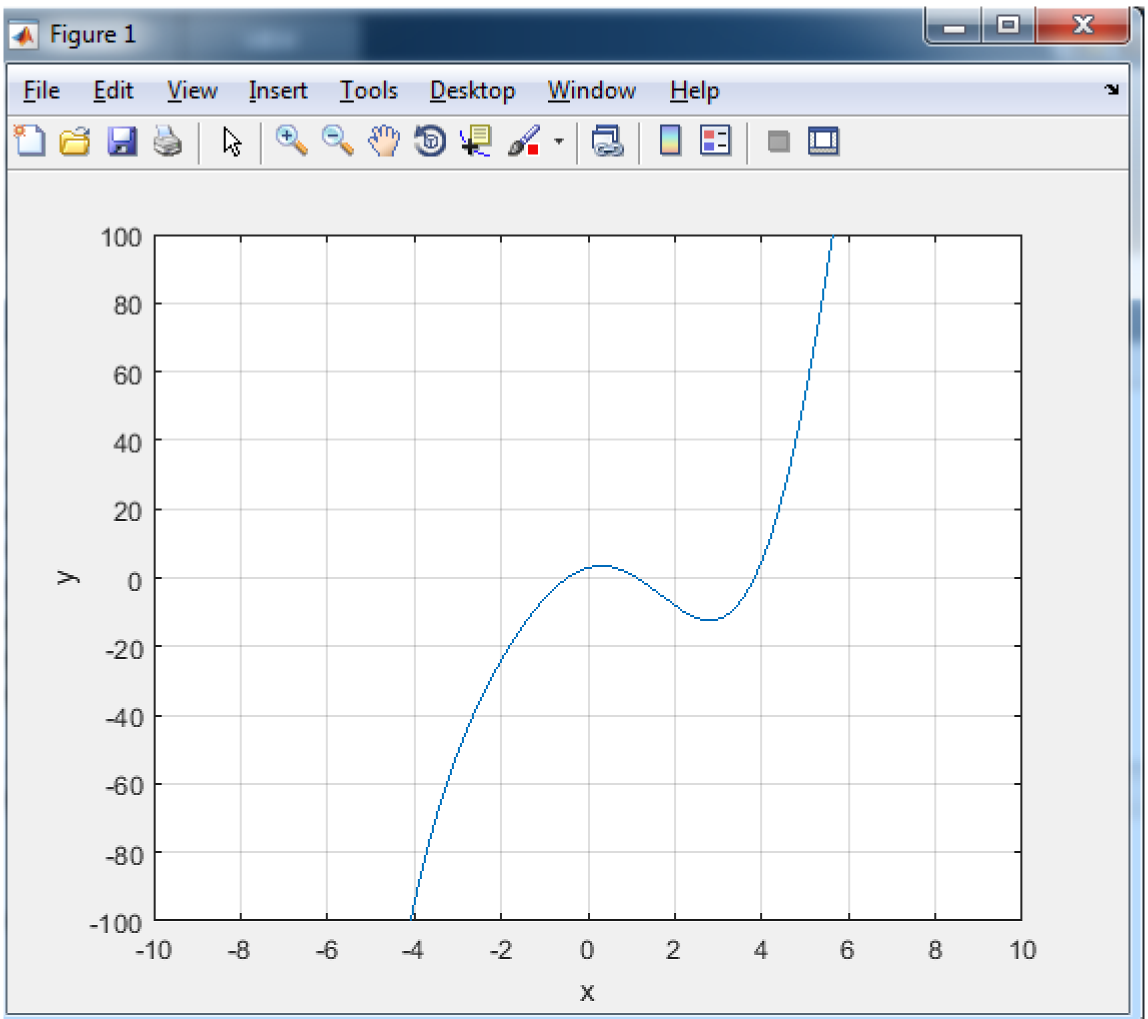
```
ans =

   -0.4378         0


ans =

    1.2212         0


ans =

    3.8479         0
```

3b.


Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_3b.m

prac5.m | que_3a.m | que_3b.m

```
clc
clear all
close all
x=-10:0.1:10;
f=@(x) (x.^3)-(3.*x.^2)+5.*x.*sin(((pi.*x)/4)-((5*pi)/4))+3;
fplot(f,[-10 10 -100 100]),grid,xlabel('x'),ylabel('y'),
a=fzero(f,-2)
b=fzero(f,2)
c=fzero(f,3)
```



```
Command Window
a =

  -0.4795


b =

    1.1346


c =

    3.8318
```

8. A certain shing vessel is initially located in a horizontal plane at $x = 0$ and $y = 10$ mi. It moves on a path for 10 hr such that $x = t$ and $y = 0.5t^2 + 10$, where t is in hours. An international shing boundary is described by the line $y = 2x + 6$.

a. Plot and label the path of the vessel and the boundary.

b. The perpendicular distance of the point (x_1, y_1) from the line $Ax + By + C = 0$ is given by

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

where the sign is chosen to make $d \geq 0$. Use this result to plot the distance of the shing vessel from the shing boundary as a function of time for $0 \leq t \leq 10$ hr.

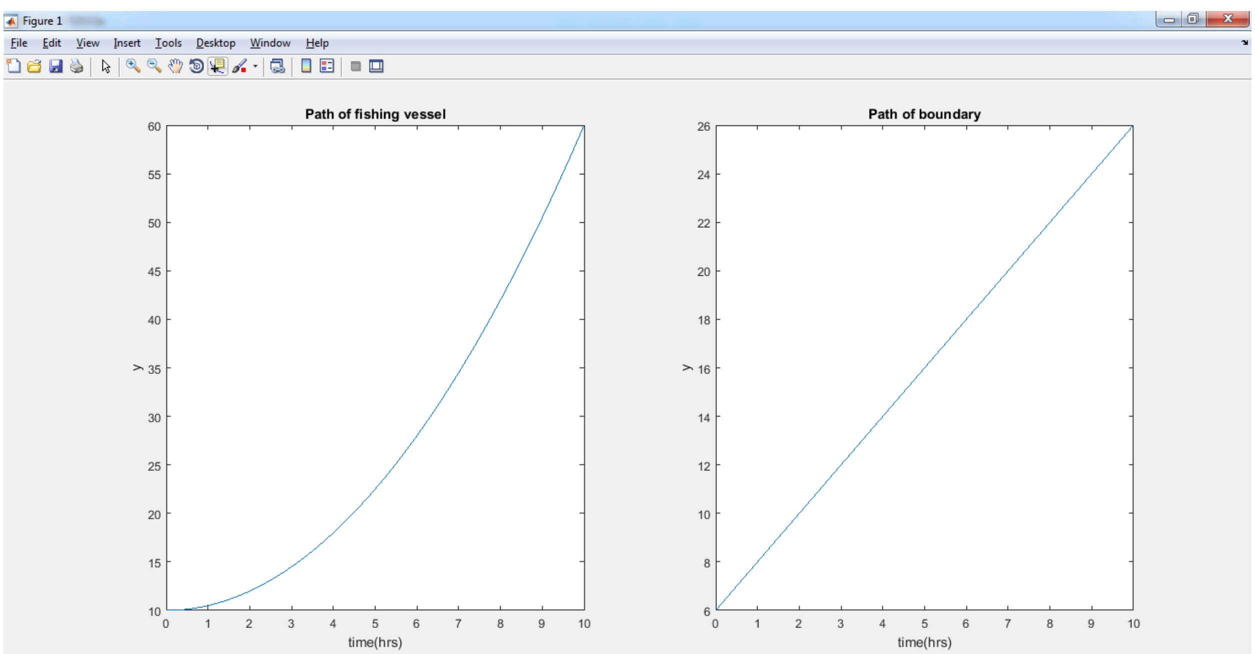
<u>**Solution**</u>

8a.

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_8a.m

que_8a.m | que_17.m | que_14.m

```
clc
clear all
close all
t=0:0.1:10;
%path of fishing vessel
for k=1:length(t)
    x(k)=t(k);
    y1(k)=(0.5*(t(k)^2))+10;
end
subplot(1,2,1)
plot(t,y1),xlabel('time(hrs)'),ylabel('y'),
title('Path of fishing vessel')
%path of boundary
for k=1:length(t)
    x(k)=t(k);
    y2(k)=(2*t(k))+6;
end
subplot(1,2,2)
plot(t,y2),xlabel('time(hrs)'),ylabel('y'),
title('Path of boundary')
```

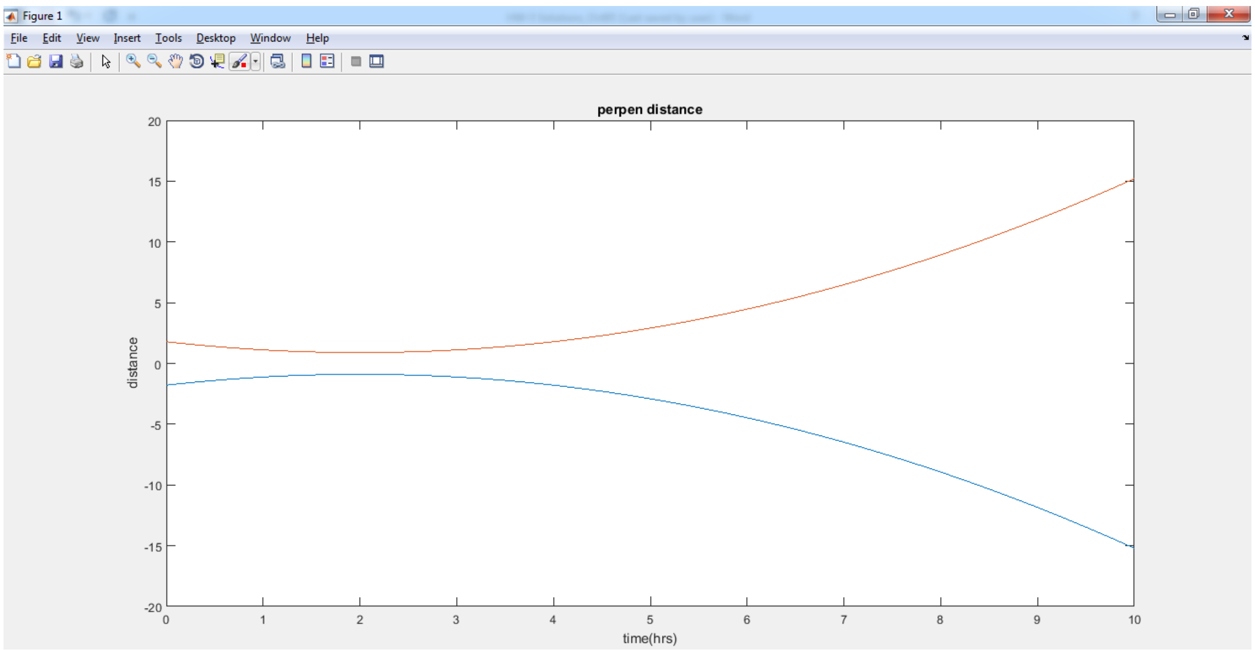


8b.

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_8b.m

que_36.m | que_8b.m

```
clc
clear all
close all
%2x-y+6=0; A=2,B=-1,C=6
A=2;B=-1;C=6;
t=0:0.1:10;
x1=t;
y1=(0.5.*(t.^2))+10;
d1=(A.*x1+B.*y1+C)./(sqrt(A^2+B^2));
d2=-(A.*x1+B.*y1+C)./(sqrt(A^2+B^2));
plot(t,d1,t,d2),xlabel('time(hrs)'),ylabel('distance'),title('perpen distance'
```



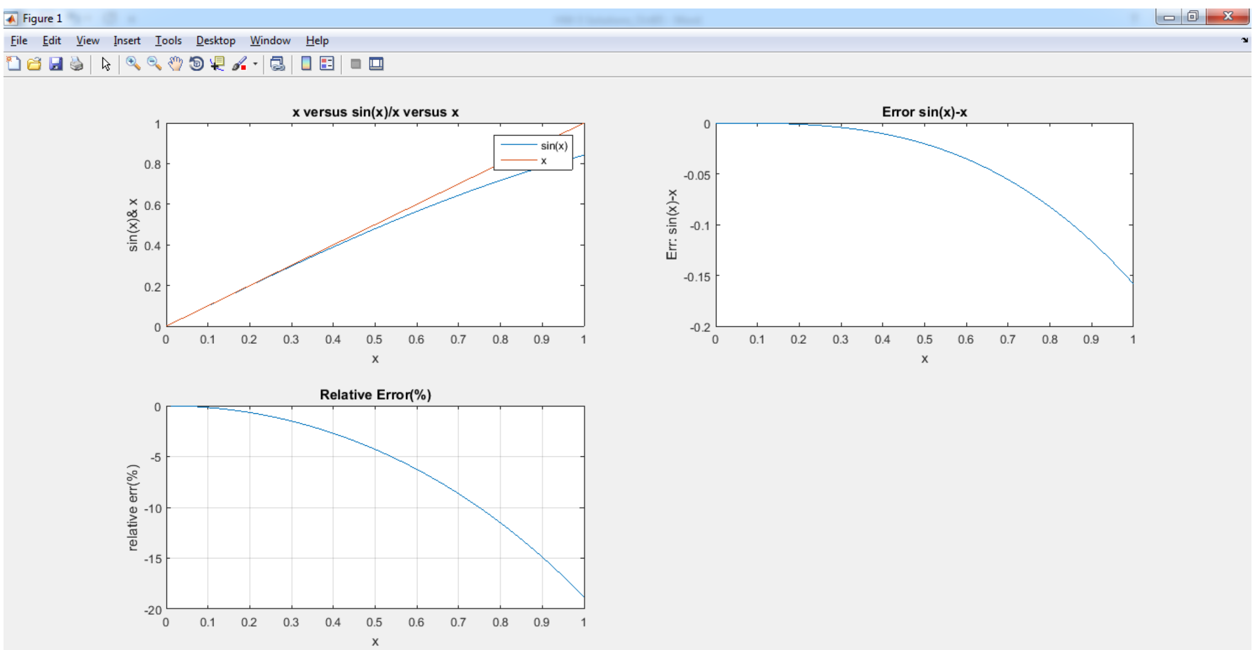
10.* Many applications use the following "small angle" approximation for the sine to obtain a simpler model that is easy to understand and analyze. This approximation states that $\sin x \approx x$, where x must be in radians. Investigate the accuracy of this approximation by creating three plots. For the rst, plot $\sin x$ and x versus x for $0 \le x \le 1$. For the second, plot the approximation error $\sin x - x$ versus x for $0 \le x \le 1$. For the third, plot the relative error $[\sin(x) - x]/\sin(x)$ versus x for $0 \le x \le 1$. How small must x be for the approximation to be accurate within 5 percent?

Solution

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_10.m

que_8a.m | que_17.m | que_14.m | que_10.m

```
clc
clear all
close all
x=0:0.01:1;
% to plot sin(x) and x versus x
subplot(2,2,1)
plot(x,sin(x),x,x),xlabel('x'),ylabel('sin(x)& x'),
legend('sin(x)','x')
title('x versus sin(x)/x versus x')
%to plot error sin(x)-x versus x
subplot(2,2,2)
plot(x,sin(x)-x),xlabel('x'),ylabel('Err: sin(x)-x'),
title('Error sin(x)-x')
%to plot relative error
d=((sin(x)-x)./sin(x))*100;
subplot(2,2,3)
plot(x,d),grid,xlabel('x'),ylabel('relative err(%)'),
title('Relative Error(%)')
```



14.* The function $y(t) = 1 - e^{-bt}$, where t is time and $b > 0$, describes many processes, such as the height of liquid in a tank as it is being lled and the temperature of an object being heated. Investigate the effect of the parameter b on $y(t)$. To do this, plot y versus t for several values of b on the same plot. How long will it take for $y(t)$ to reach 98 percent of its steady-state value?

Solution

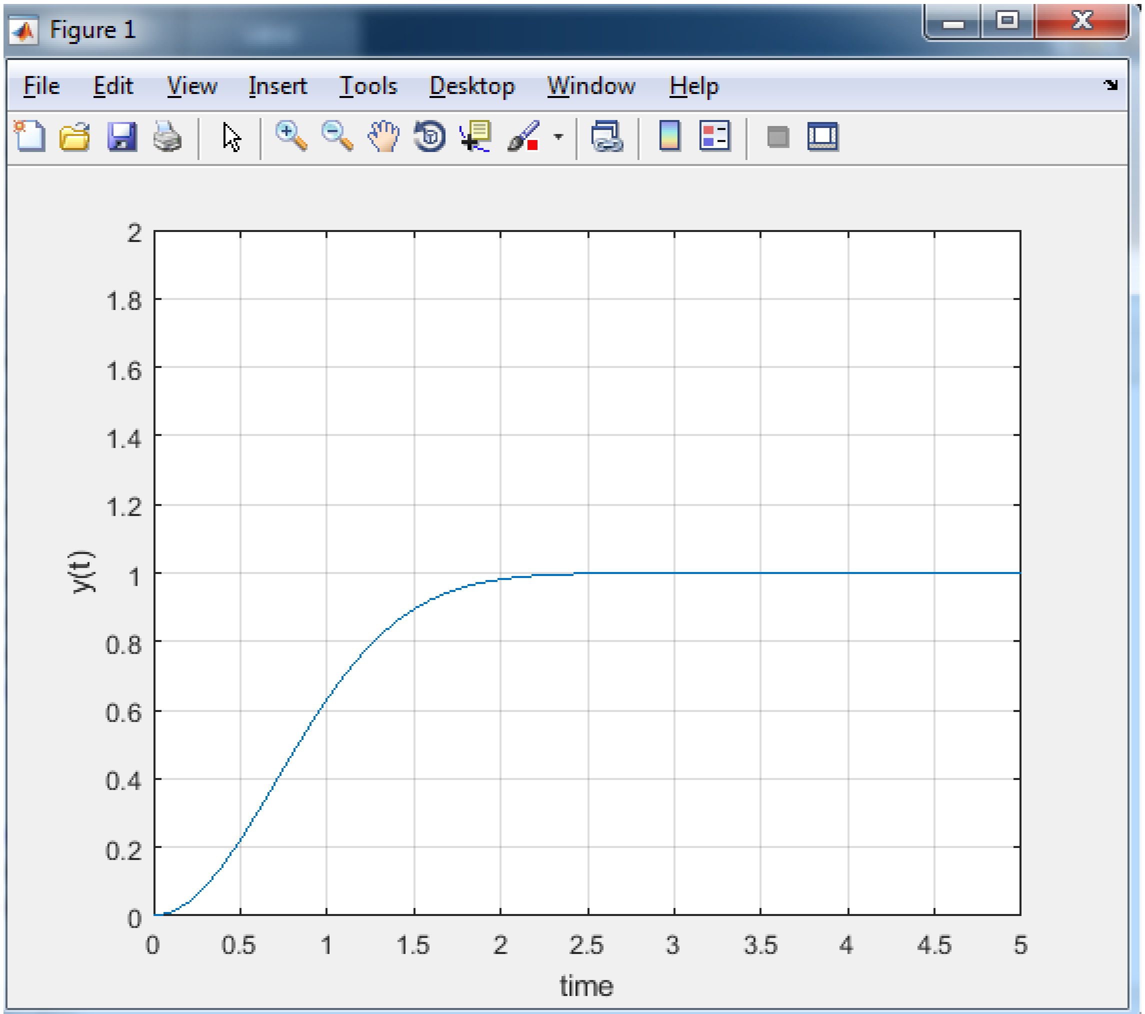
Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_14.m

que_14.m

```
clc
clear all
close all
b=0:0.1:10;
t=0:0.1:10;
y=(1-exp((-b).*t));
plot(t,y),grid,axis([0 5 0 2]),
xlabel('time'),ylabel('y(t)')
for k=1:length(t)
    if y(k)<=0.98
        ind=k
    end
end
disp('Time taken to reach 98% of steady-state: ')
disp(t(ind))
```

Command Window

```
ind =

    20

Time taken to reach 98% of steady-state:
    1.9000
```



17.* The height $h(t)$ and horizontal distance $x(t)$ traveled by a ball thrown at an angle A with a speed v are given by

$$h(t) = vt \sin A - \frac{1}{2}gt^2$$

$$x(t) = vt \cos A$$

At Earth's surface the acceleration due to gravity is $g = 9.81$ m/s^2.

a. Suppose the ball is thrown with a velocity $v = 10$ m/s at an angle of 35°. Use MATLAB to compute how high the ball will go, how far it will go, and how long it will take to hit the ground.

b. Use the values of v and A given in part *a* to plot the ball's *trajectory;* that is, plot h versus x for positive values of h.

c. Plot the trajectories for $v = 10$ m/s corresponding to ve values of the angle A: 20°, 30°, 45°, 60°, and 70°.

d. Plot the trajectories for $A = 45°$ corresponding to ve values of the initial velocity v: 10, 12, 14, 16, and 18 m/s.

Solution

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_17.m

que_17.m

```matlab
clc
clear all
close all
t=linspace(0,1.4,50);
g=9.81;v=10;
A=(pi/180)*35;
%a. to find height,distance,time

% to find height
h=((v.*t).*sin(A))-(0.5*g*t.^2);
subplot(2,2,1)
plot(t,h),grid,xlabel('time'),ylabel('height')
height=max(h);
disp('max height: '),disp(height)
```

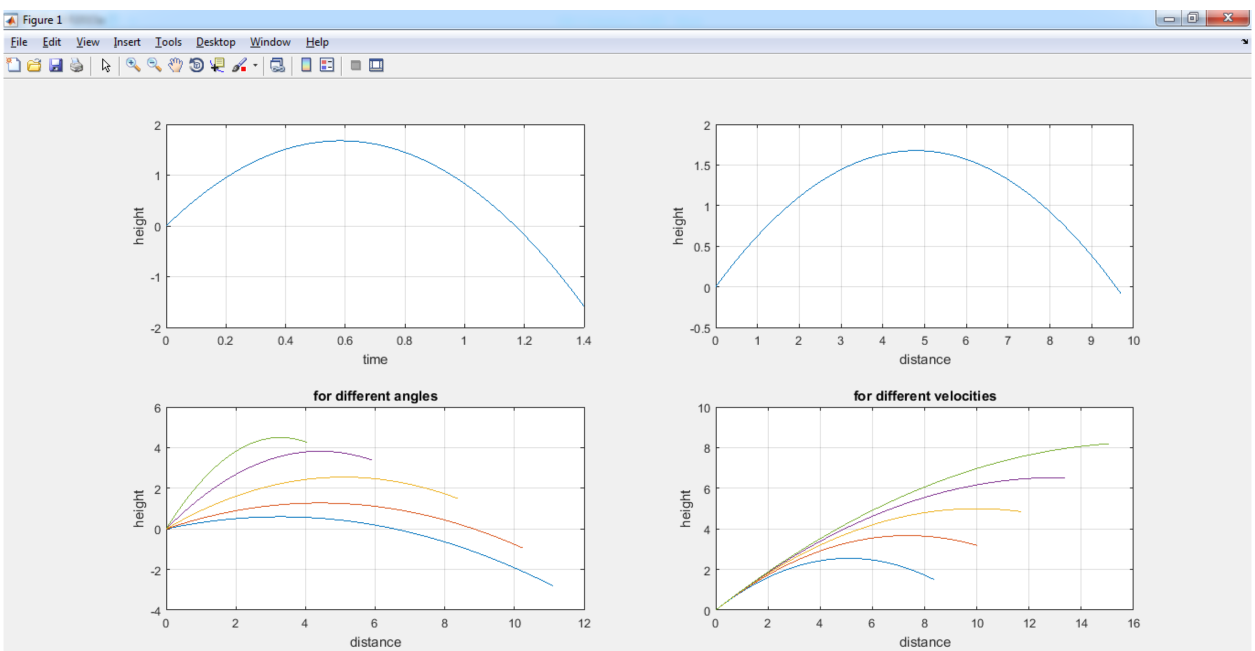
```matlab
% to find distance
[t1,t2]=ginput(1);
x=v.*t1.*cos(A);
disp('Distance travelled by ball: '),disp(x)
%to find time
disp('Time taken to hit the ground: '),disp(t1)

%b.to plot x versus h
tx=linspace(0,t1,50);
x1=v.*tx.*cos(A);
h1=((v.*tx).*sin(A))-(0.5*g*tx.^2);
subplot(2,2,2)
plot(x1,h1),grid,xlabel('distance'),ylabel('height')
```

```matlab
%c.to plot for different angle
B=[20 30 45 60 70];
C=(pi/180).*B;
for k=1:length(C);
h2=((v.*tx).*sin(C(k)))-(0.5.*g.*tx.^2);
x2=v.*tx.*cos(C(k));
subplot(2,2,3)
plot(x2,h2),grid,xlabel('distance'),ylabel('height'),title('trajectory')
title('for different angles')
hold on
end
```

```
%d.to plot for different velocities
ang=45;
D=(pi/180)*ang;
V=[10 12 14 16 18];
for k=1:length(V);
h3=((V(k).*tx).*sin(D))-(0.5.*g.*tx.^2);
x3=V(k).*tx.*cos(D);
subplot(2,2,4)
plot(x3,h3),grid,xlabel('distance'),ylabel('height'),title('trajectory')
title('for different velocities')
hold on
end
```

Command Window

```
max height:
    1.6759

Distance travelled by ball:
    9.6918

Time taken to hit the ground:
    1.1832
```



20. When a constant voltage was applied to a certain motor initially at rest, its rotational speed $s(t)$ versus time was measured. The data appear in the following table:

Time (sec)	1	2	3	4	5	6	7	8	10
Speed (rpm)	1210	1866	2301	2564	2724	2881	2879	2915	3010

Determine whether the following function can describe the data. If so, nd the values of the constants b and c.

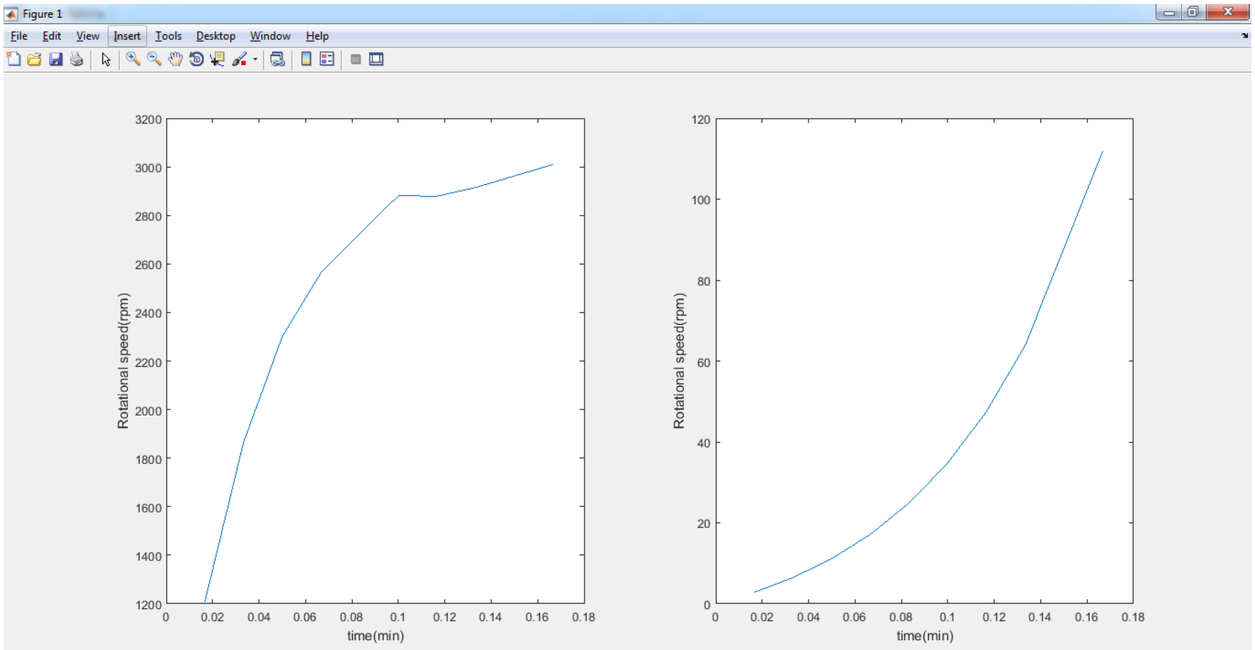
$$s(t) = b(1 - e^{ct})$$

Solution

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_20.m

que_17.m | que_27.m | que_31.m | que_36.m | que_20.m

```
clc
clear all
close all
tsec=[1 2 3 4 5 6 7 8 10];
tmin=tsec./60;
s=[1210 1866 2301 2564 2724 2881 2879 2915 3010];
% s1=0.016.*s-to convert rps to rpm;
subplot(1,2,1)
plot(tmin,s),xlabel('time(min)'),ylabel('Rotational speed(rpm)')
b=-10;c=15;
speed=b.*(1-exp(c.*tmin));
subplot(1,2,2)
plot(tmin,speed),xlabel('time(min)'),ylabel('Rotational speed(rpm)')
```



26. The circuit shown in Figure P26 consists of a resistor and a capacitor and is thus called an *RC* circuit. If we apply a sinusoidal voltage v_i, called the input voltage, to the circuit as shown, then eventually the output voltage v_o will be sinusoidal also, with the same frequency but with a different amplitude and shifted in time relative to the input voltage. Speci cally , if $v_i = A_i \sin \omega t$, then $v_o = A_o \sin(\omega t + \phi)$. The frequency–response plot is a plot of A_o/A_i versus frequency ω. It is usually plotted on logarithmic axes.

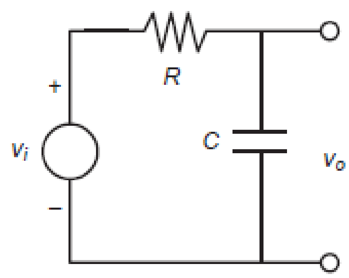


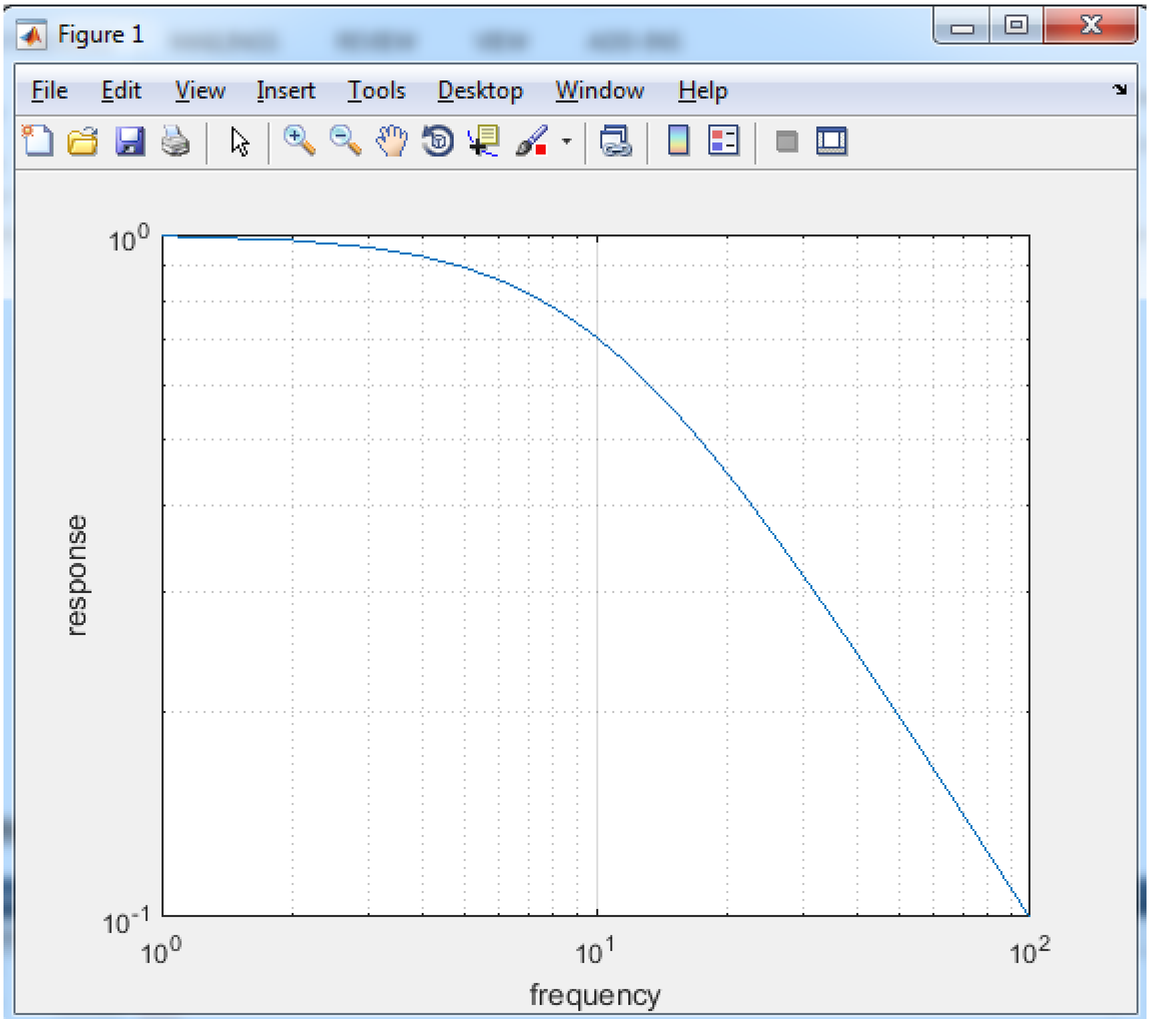
Figure P26

Solution

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_26.m

que_36.m | que_8b.m | que_14.m | que_26.m

```
clc
clear all
close all
RC=0.1;
s=[1:100]*i;
%Ao/Ai=Magitude
Mag=abs(1./((RC*s)+1));
loglog(imag(s),Mag),grid,
xlabel('frequency'),ylabel('response')
```



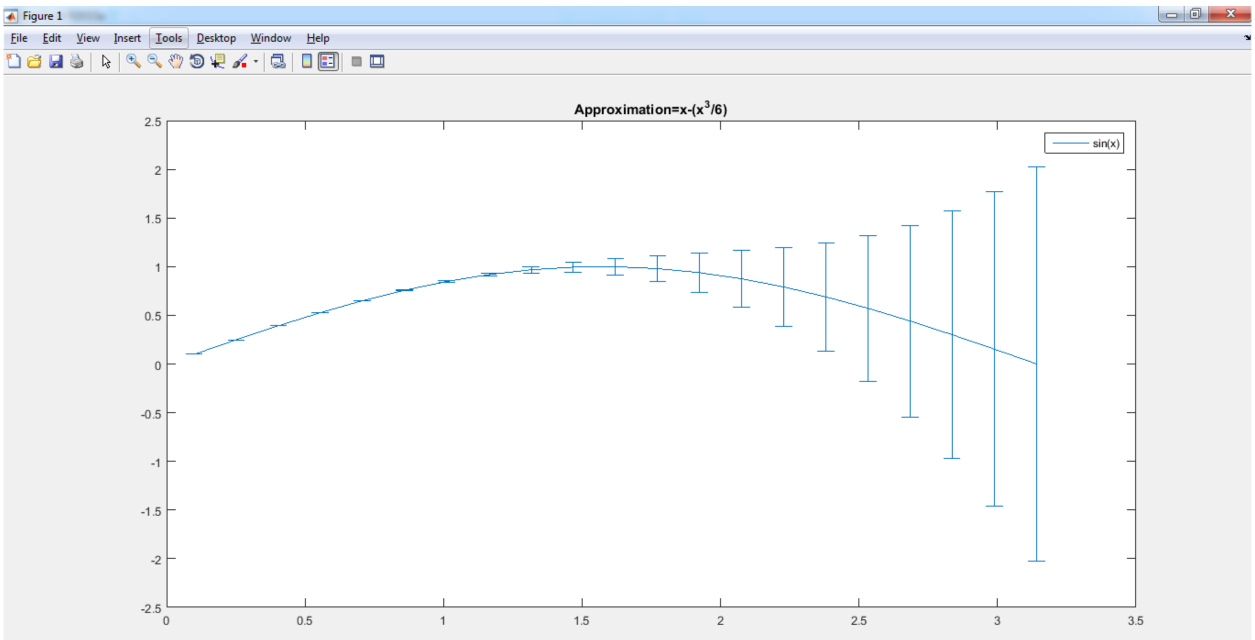
27. An approximation to the function sin x is sin $x \approx x - x^3/6$. Plot the sin x function and 20 evenly spaced error bars representing the error in the approximation.

Solution

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_27.m

que_17.m | que_27.m

```
clc
clear all
close all
x=linspace(0.1,pi,21);
approx=(x-(x.^3/6));
error=approx-sin(x);
errorbar(x,sin(x),error),legend('sin(x)'),
title('Approximation=x-(x^3/6)')
```



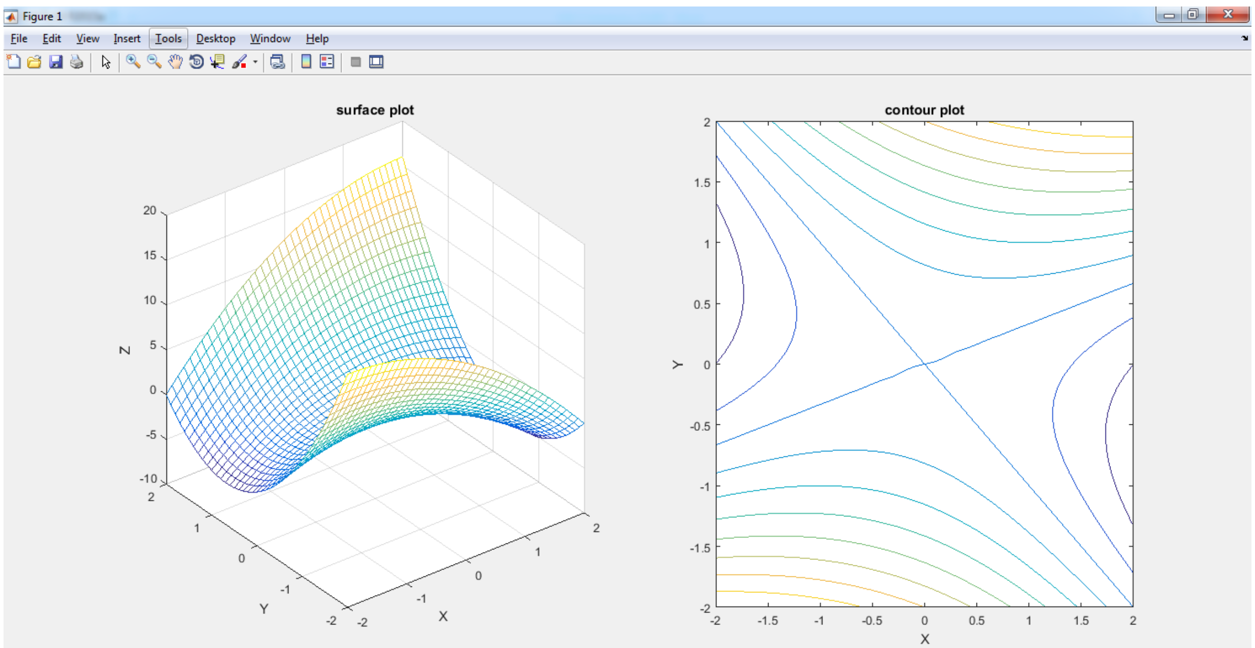
31. Obtain the surface and contour plots for the function $z = -x^2 + 2xy + 3y^2$. This surface has the shape of a saddle. At its saddlepoint at $x = y = 0$, the surface has zero slope, but this point does not correspond to either a minimum or a maximum. What type of contour lines corresponds to a saddlepoint?

Solution

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_31.m

que_17.m | que_27.m | que_31.m

```
clc
clear all
close all
%surface plot
[X,Y]=meshgrid(-2:0.1:2);
Z=((-X.^2)+(2.*X.*Y)+(3.*(Y.^2)));
subplot(1,2,1)
mesh(X,Y,Z),xlabel('X'),ylabel('Y'),zlabel('Z'),title('surface plot')
%contour plot
[X,Y]=meshgrid(-2:0.1:2);
Z=((-X.^2)+(2.*X.*Y)+(3.*(Y.^2)));
subplot(1,2,2)
contour(X,Y,Z),xlabel('X'),ylabel('Y'),title('contour plot')
```



36. The electric potential eld V at a point, due to two charged particles, is given by

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

where q_1 and q_2 are the charges of the particles in coulombs (C), r_1 and r_2 are the distances of the charges from the point (in meters), and ϵ_0 is the permittivity of free space, whose value is

$$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$

Suppose the charges are $q_1 = 2 \times 10^{-10}$ C and $q_2 = 4 \times 10^{-10}$ C. Their respective locations in the xy plane are (0.3, 0) and (−0.3, 0) m. Plot the electric potential eld on a three-dimensional surface plot with V plotted on the z axis over the ranges $-0.25 \le x \le 0.25$ and $-0.25 \le y \le 0.25$. Create the plot in two ways: (*a*) by using the `surf` function and (*b*) by using the `meshc` function.

Solution

Editor - C:\Users\dell\Documents\Masters\Fall 15-1st sem\Matlab Programs\HW_5_oct_05\que_36.m

que_17.m | que_27.m | que_31.m | que_36.m

```
clc
clear all
close all
x1=0.3;x2=-0.3;
y1=0;y2=0;
q1=2*10^-10;
q2=4*10^-10;
e=8.854*10^-12;
%V=(1/(4*pi*e))*((q1/r1)+(q2/r2))
%by using surf function
[X,Y]=meshgrid(-0.25:0.01:0.25);
r1=sqrt((X-x1).^2+(Y-y1).^2);
r2=sqrt((X-x2).^2+(Y-y2).^2);
V1=(1/(4*pi*e))*((q1./r1)+(q2./r2));
subplot(1,2,1)
surf(X,Y,V1),xlabel('X'),ylabel('Y'),zlabel('V'),title('surf')
%by using meshc function
[X,Y]=meshgrid(-0.25:0.01:0.25);
V2=(1/(4*pi*e))*((q1./r1)+(q2./r2));
subplot(1,2,2)
meshc(X,Y,V2),xlabel('X'),ylabel('Y'),zlabel('V'),title('meshc')
```

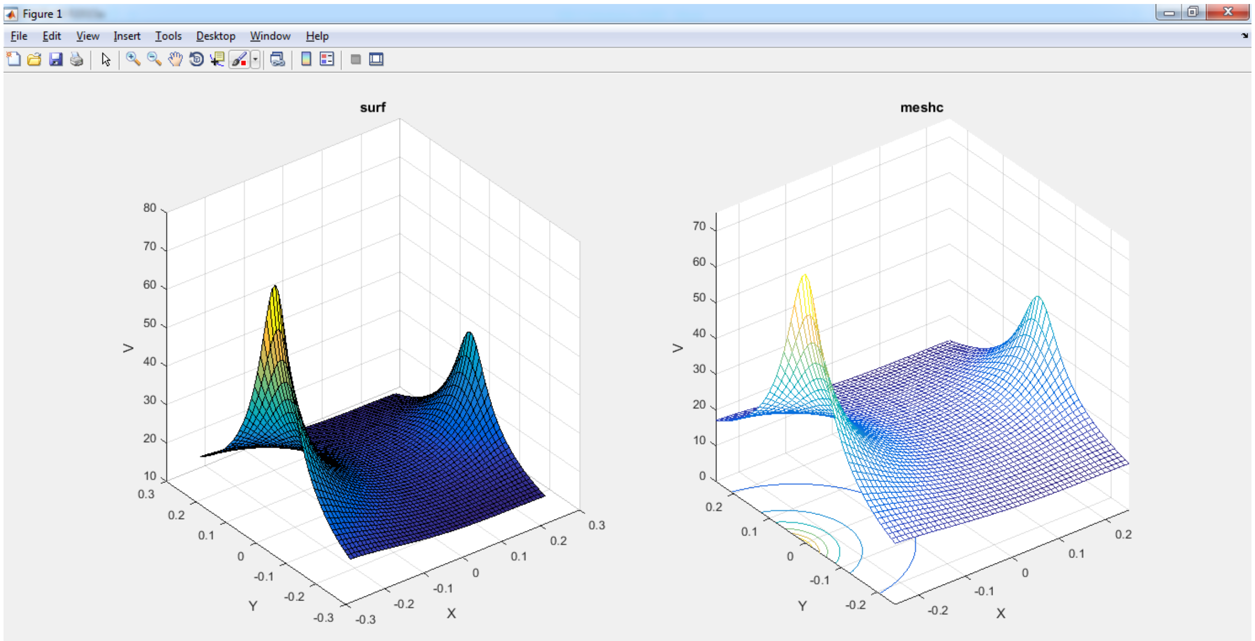


Figure 1
File Edit View Insert Tools Desktop Window Help
surf
meshc
V
X
Y