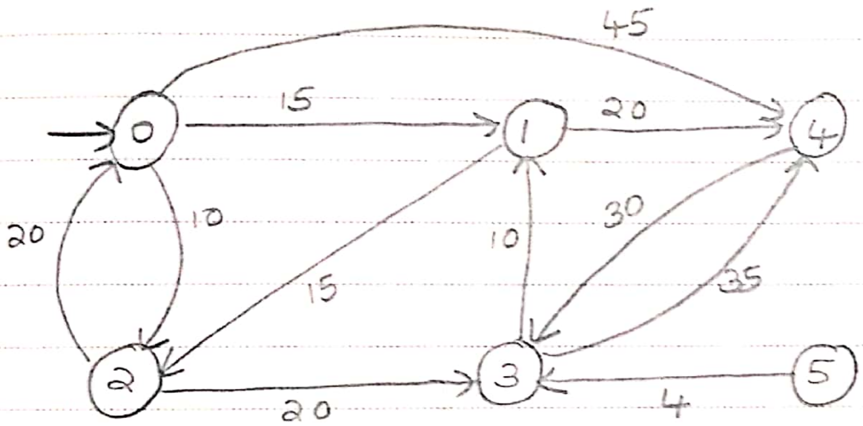
Dijkstra's Algorithm

Trace for Dijkstra's algorithm using 5 as the source vertex.



Cost matrix[6][6] =

$$\begin{array}{c|cccccc|}
 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 0 & 15 & 10 & \infty & 45 & \infty \\
1 & \infty & 0 & 15 & \infty & 20 & \infty \\
2 & 20 & \infty & 0 & 20 & \infty & \infty \\
3 & \infty & 10 & \infty & 0 & 35 & \infty \\
4 & \infty & \infty & \infty & 30 & 0 & \infty \\
5 & \infty & \infty & \infty & 4 & \infty & 0
\end{array}$$

Source = 5

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Initialization:

	dist		path
0	∞	0	5
1	∞	1	5
2	∞	2	5
3	4	3	5
4	∞	4	5
5	0	5	5

$V = \{0, 1, 2, 3, 4, 5\}$
$S = \{5\}$
$V - S = \{0, 1, 2, 3, 4\}$

Iteration 01:

$u = \cancel{4}\ 3$
$dist[u] = 4$

$S = \{5, 3\}$
$V - S = \{0, 1, 2, 4\}$

	dist		path
0	∞	0	5
1	14	1	3
2	∞	2	5
3	4	3	5
4	39	4	3
5	0	5	5

$\min(\infty, 4 + \infty) = \infty$
$\min(\infty, 4 + 10) = 14$
$\min(\infty, 4 + \infty) = \infty$

$\min(\infty, 4 + 35) = 39$

Find the <u>min</u> for all V−S set.

Eg:

$\min(\infty, 4 + 10)$

dist[1]   dist[3] + cost[3][1]

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Iteration 02:

$u = 1$
dist[u] = 14

$S = \{1, 3, 5\}$
$V - S = \{0, 2, 4\}$

	dist
0	∞
1	14
2	29
3	4
4	34
5	0

	path
0	5
1	3
2	1
3	5
4	1
5	5

$\min(\infty, 14+\infty) = \infty$

$\min(\infty, 14+15) = 29$

$\min(\cancel{\infty}39, 14+20) = 34$

Iteration 03:

$u = 2$
dist[u] = 29

$S = \{1, 2, 3, 5\}$
$V - S = \{0, 4\}$

	dist
0	49
1	14
2	29
3	4
4	34
5	0

	path
0	2
1	3
2	1
3	5
4	1
5	5

$\min(\infty, \cancel{\infty +} 29+20) = 49$

$\min(34, 29+\infty)$

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Iteration 04:

$u = 4$ $S = \{1, 2, 3, 4, 5\}$

$dist[u] = 34.$ $V - S = \{0\}$

dist

0	49
1	14
2	29
3	4
4	34
5	0

path

0	2
1	3
2	1
3	5
4	1
5	5

$\min(\overset{49}{\cancel{\infty}}, 34 + \infty) = 49$

Iteration 05:

$u = 0$ $S = \{0, 1, 2, 3, 4, 5\}$

$dist[u] = 49$ $V - S = \{\ \}$

dist

0	49
1	14
2	29
3	4
4	34
5	0

path

0	2
1	3
2	1
3	5
4	1
5	5

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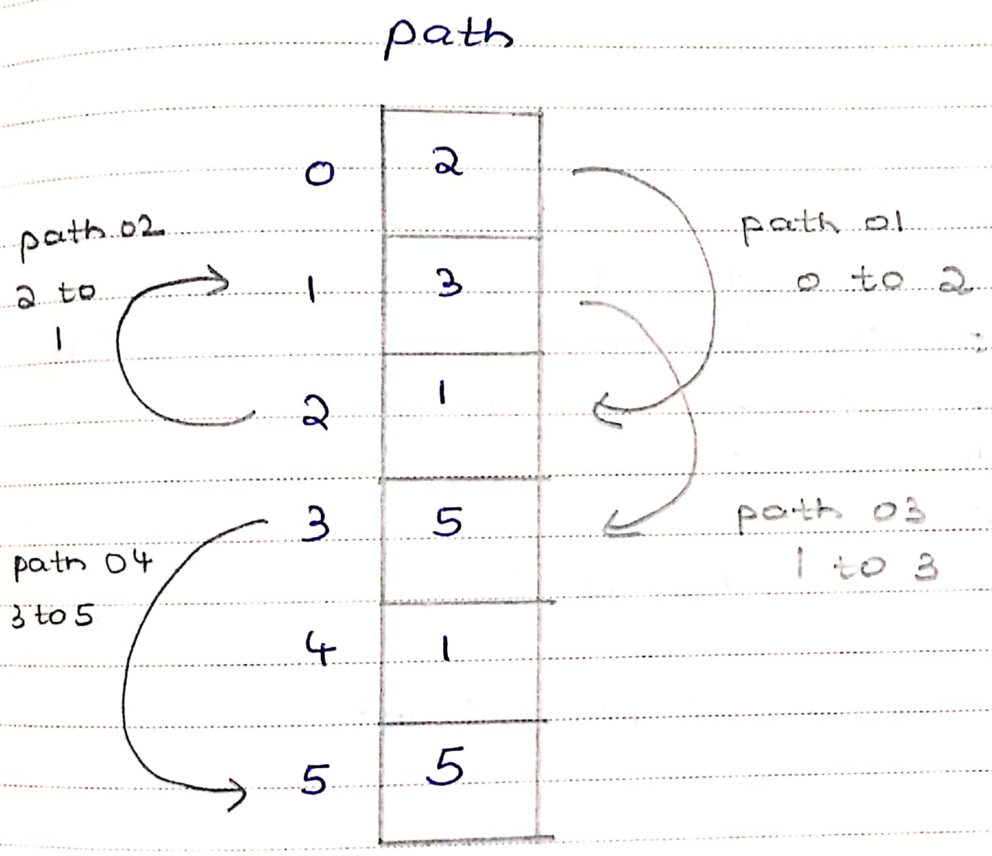
To Traverse Path:

To traverse path from 5 to 0, go in reverse order following the vertex path.



$0 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 5$

↓ Now reverse this & we have path from 5 to 0.

The algorithm is single souce shortest path.

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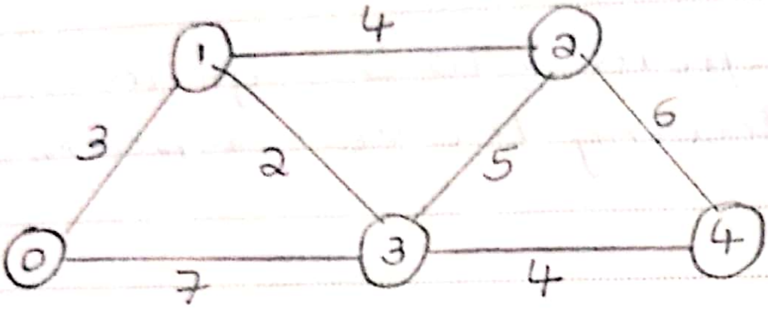
Example 02:



$$cost[5][5] = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 & \infty \\ 3 & 0 & 4 & 2 & \infty \\ \infty & 4 & 0 & 5 & 6 \\ 7 & 2 & 5 & 0 & 4 \\ \infty & \infty & 6 & 4 & 0 \end{bmatrix} \end{array}$$

$S = \{0\}$ $\qquad$ $V = \{0, 1, 2, 3, 4\}$

Initialization:

$S = \{0\}$

$V - S = \{1, 2, 3, 4\}$

	dist	path
0	0	0
1	3	0
2	∞	0
3	7	0
4	∞	0

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Iteration 01:

$u = 1$

$dist[u] = 3$

$S = \{0, 1\}$

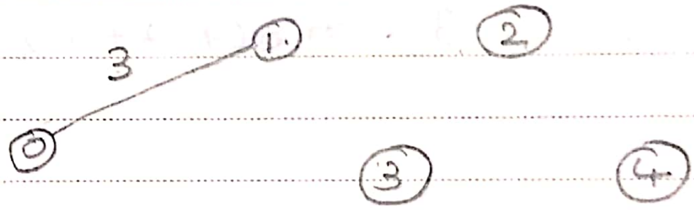
$V - S = \{2, 3, 4\}$

	dist	path
0	0	0
1	3	0
2	7	1
3	5	1
4	∞	0

$\min(\infty, 3 + 4) = 7$

$\min(7, 3 + 2) = 5$

$\min(\infty, 3 + \infty) = \infty$



Iteration 02:

$u = 3$

$dist[u] = 5$

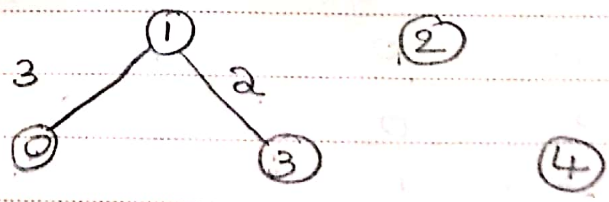
$S = \{0, 1, 3\}$

$V - S = \{2, 4\}$

	dist	path
0	0	0
1	3	0
2	7	1
3	5	1
4	9	3

$\min(7, 5 + 5) = 7$

$\min(\infty, 5 + 4) = 9$



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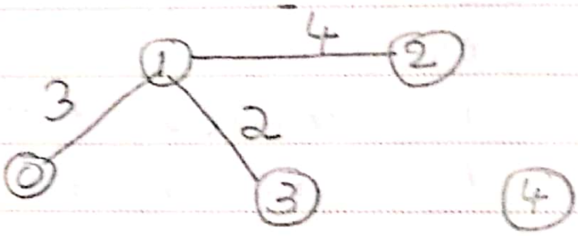
Itration 03:

$u = 2$ $\qquad$ $S = \{0, 1, 2, 3\}$

$dist[u] = 7$ $\qquad$ $V - S = \{4\}$

	dist	path
0	0	0
1	3	0
2	7	1
3	5	1
4	9	3

$min(9, 7+6) = 13$

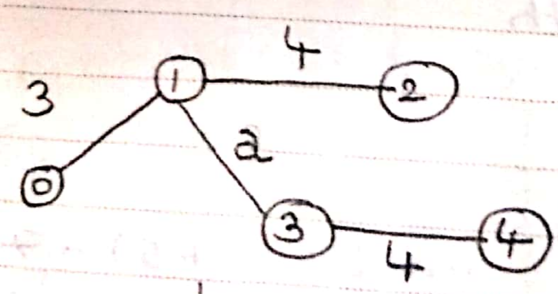


Itration 04:

$u = 4$ $\qquad$ $S = \{0, 1, 2, 3, 4\}$

$dist[u] = 9$ $\qquad$ $V - S = \{\}$

	dist	path
0	0	0
1	3	0
2	7	1
3	5	1
4	9	3



Final Single Source Shortest Path from 0.

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