DAA ASSIGNMENT → 1

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Ques→ 1→ what do you understand by Asymptotic notations. Define different Asymptotic notations with example?

Ans → Asymptotic NOTATION

→ We use Asymptotic notation to represent order of growth of function.

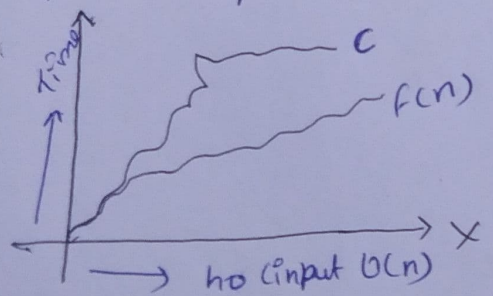
TYPES

→ O notation.
→ Θ notation.
→ Ω notation.

① BIG O notation

→ This is used to represent upper bound of a function.

→ Graphical representation

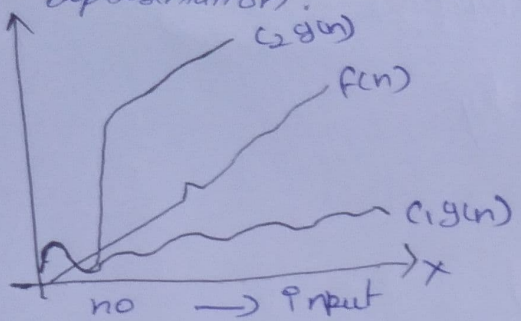


$O(gn) = \{ f(n) : 0 \le f(n) \le (g(n)$ where (is a Positive constant $\forall\ n \ge$ ~~number~~ $n_0 \}$.

② Θ notation

→ This is used to represent both upper as well as lower bound.

→ Graphical representation.



→ $\Theta(g(n)) = \{ f(n) : 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ where c_1 & c_2 are positive constant $\forall$ $n \geq n_0 \}$.

→ Big Ω

→ This is used to represent the lower bound of the function.

→ Graphical representation.

→ $\Omega(g(n)) = \{ f(n) : \leq (g(n) \leq f(n)$ where c is positive constant $\forall$ $n \geq n_0 \}$.

Ans→2→ For(i = 1 to n)

{
(i = i*2);
}

1, 2, 4, 8, ——— n

$T(n) = O(\log_2 n)$

Ans→3→ $T(n) = \begin{cases} 3T(n-1) & n>0 \\ 1 & n=0 \end{cases}$

$T(n) = 3T(n-1)$ — ①

$T(n-1) = 3T(n-2)$ —

$T(n) = 9T(n-2)$ — ②

$T(n) = 3^3 T(n-3)$ — ③

$T(n) = 3^k T(n-k)$ — ④

For $T(n-k) = T(0)$

$n-k = 0$

$n = k.$

$T(n) = 3^n T(0)$

$T(n) = 3^n.$

$T(n) = \theta(3^n).$

Ans→ 4→ $T(n) = \begin{cases} 2T(n-1)-1 & , n>0 \\ 1 & , n=0 \end{cases}$

$T(n) = 2T(n-1) - 1$ — ①

$T(n-1) = 2T(n-2) - 1.$

$T(n) = 4T(n-2) - 1 - 2$ — (2)

$T(n) = 8T(n-3) - (1+2+4)$ — (3)

$T(n) = 2^k T(n-k) - [1+2+4+ \cdots 2^{k-1}]$

$T(n-k) = T(0)$

$n = k.$

$T(n) = 2^n T(0) - [1+2+4+ \cdots]$

k terms.

Its an A.P.

$a = 1$

$r = 2$

$T(n) = 2^n - \left(\frac{1\,(2^{n+1}-1)}{2-1}\right)$

$T(n) = 2^n - 2^n + 1.$

$T(n) = 1.$

$T(n) = \Theta(1).$

Ans->5-> int i=1, s=1;

while (s <= n)

{

i++;

s = s+i;

printf ("#");

}

1, 3, 6, 10, 15, ---- n

(k terms)

Its kth term is $\frac{k(k+1)}{2} = n.$

$k = \sqrt{n}.$

$T(n) = O(\sqrt{n})$

Ans→6→
```
void function (int n) {
    Int i, count = 0;
    for(int i = 1; i * i < n; i++)
        count++
    }
```
$T(n) = O(\sqrt{n})$

Ans→7→ $T(n) = O(n * \log_2 n * \log_2 n)$

$T(n) = O(n * (\log_2 n)^2)$

$T(n) = O(n (\log n)^2)$

Ans→8→
```
Function (int n) {                    T(n)
    if(n==1) return;
    for(i=1 to n)
    {
        for(j=1 to n){
            Printf("*");
        }
    }
    function(n-3);                    T(n-3)
}
```
$T(n) = T(n-3) + n^2$ —①

$T(n-1) = T(n-4) + (n-1)^2$

$T(n) = T(n-4) + n^2 + (n-1)^2$

$T(n) = T(n-5) + n^2 + (n-1)^2 + (n-2)^2$

$T(n) = T(n-k) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots)$
$(k-2)$ terms.

for $T(n-k) = 1$

$k = n-1$

$T(n) = T(1) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots)$

$(n-3)$ terms

$T(n) = T(1) + (1^2 + 2^2 + \cdots n^2)$

$T(n) = T(1) + \left(\frac{(n-3)(n-2)(2n-5)}{6}\right)$

$T(n) = 2 + \left(\frac{2n^3 + \cdots}{6}\right)$

$T(n) = n^3.$

$T(n) = O(n^3)$

Ans ⇒ 9⇒

```
void function(int n){
  for(i=1 to n)
  {
    for(j=1 ; j<=n ; j=j+i)
      printf("*");
  }
```

$i=1$	n times	
$i=2$	1, 3, 5, ---- n	n
$i=3$	1, 4, 7 --- n	$n/2$
⋮		
$i=n$	0	

$T(n) = [n + \frac{n}{2} + \frac{n}{3} \cdots \text{n times}]$

$T(n) = O(n \log n)$

Ans⇒10⇒ for the function n^k and a^n, what is the relation.

$k \geq 1$ & $a > 1$

relation is n^k is $O(c^n)$.

Ans→11→

```
void fun (int n)
{
  int j=1 ; i=0;
  while( i<n)
  {
    i = i+j ;
    j++ ;
  }
}
```

0, 3, 6, 10, 15 ------ n

so for this series is k terms

k^{th} term is $\frac{k(k+1)}{2}$.

$$n = \frac{k^2+k}{2}$$

$$k \simeq \sqrt{n}$$

$$T = O(\sqrt{n})$$

Ans→ 12→ Recurrence relation of faboracci series is

$$T(n) = T(n-1) + T(n-2) + 1$$

$$T(n) = 2T(n-2) + 1$$

$$T(n) = 4T(n-4) + 3$$

$$T(n) = 8T(n-6) + 7$$

$$T(n) = 16T(n-8) + 15$$

$$T(n) = 2^k T(n-2k) + (2^k - 1)$$

for $T(n-2k) = T(0)$

$$n = 2k$$

$$k = \frac{n}{2}.$$

$$T(n) = 2^{n/2} T(0) + (2^{n/2} - 1)$$

$$T(n) = 2^n - 1$$

$$T(n) = O(2^n)$$

Hence space complexity of fabinacci series is O(n) as it depends on height of recursive tree & it is equal to n in fabinacci series.

Ans 13 → On(log n)

```
void fun( for(int j=0; j<n; j++)
        {
          for( int i=0 ; i<n ; i=i*2)
          {
            printf("* ");
          }
        }
      }
void main ()
  {
    fun ();
  }.
```

(11) n3

```
# include <stdio.h>
void main()
  {
    int n;
    cin >> n;
    for (int i=0; i<n; i++)
      {
        for(int j=0; i<n ; j++)
        {
          for(int k=0; k<n ; k++)
          {
            k++
          }
        }
      }
  }
```

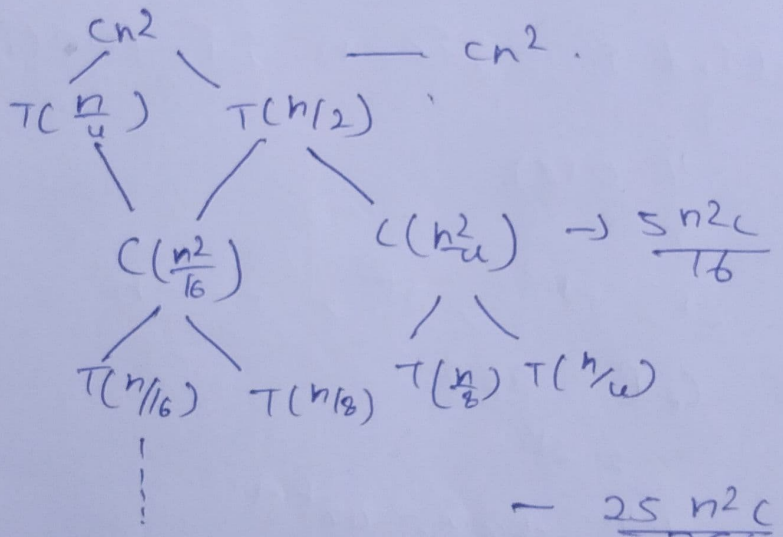
→ log (logn)

```
# include < bit / std c++.hs
void fun(int n)
{
if(n==2)
return 1;
else
fun(sqrt(n));
}
void main()
{
fun(100);
}
```

Ans) $T(n) = T(n/4) + T(n/2) + cn^2$

$T(1) = c$

$T(0) = 0$



$T(n)$ = constant for each level

$T(n) = cn^2 + \frac{5cn^2}{16} + \frac{25cn^2}{256} + \cdots$

It is a G.P with $a = n^2$, $r = \frac{5}{16}$

so sum of G.P

$T(n) = cn^2 / (1 - \frac{5}{16}) = \frac{16cn^2}{11} = \frac{16cn^2}{11}$

$T(n) = O(n^2)$

Ans→15→ for (int i to n)

```
{
for (int j = 1 ; j < n ; j += i)
 {
   // O(1).
 }
}
```

$n, \frac{n}{2}, \frac{n}{3}, \frac{n}{4}, \frac{n}{5}, \ldots\ldots 1$

k times

$k = \log_2 n$

$n \cdot (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \frac{1}{n})$

$(n (\log n))$

$T(n) = O(n \log n)$.

Ans→16→ for(int i = 2 ; i <= n ; i = pow(i, k))

```
{
 // O(1)
}
```

$2, 2^k, 2^{k^2}, 2^{k^3}, \ldots n$

At G.P $a = 2$

$r = 2^k$

k^{th} term $= ar^{k-1}$

$n = 2(2^k)^{k-1}$

let $k^{(k-1)} = x$

$k \log_k k = \log x$ —

$k = \log x$ —①

$n = 2^x$

$\log_2 n = x \log_2 2$

$x = \log_2 n$

$\log x = \log(\log n)$

from —①.

$k = \log(\log(n))$

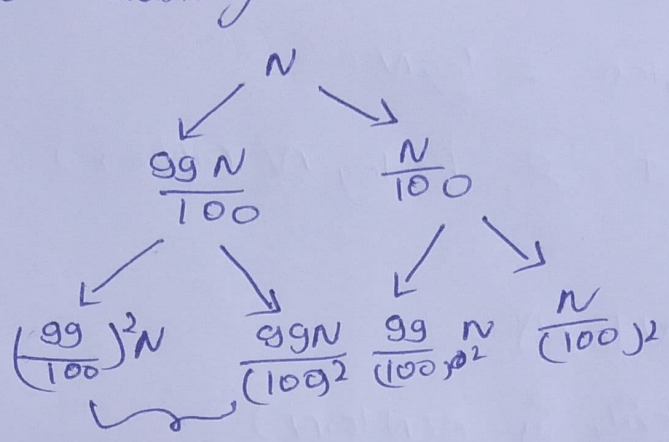
$T(n) = O(\log(\log n))$.

Ans-17 → hence pivot is divided in 99% & 1%

so

$$T(n) = T\left(\frac{99}{100}N\right) + T\left(\frac{N}{100}\right) + N$$

Now as here we can use 2 extreme of a tree where starting point is N



$$N\left(\frac{99\times99}{100\times100} + \frac{99(1)}{100\times100}\right) + \frac{100}{100\times100}N$$

$$= \frac{99}{100}N + \frac{N}{100}$$

$$= N$$

$$= N$$

so cost of each level is N only.

Total cost = height * cost of each level

so for 1st stream — $N, \frac{99}{100}N, \left(\frac{99}{100}\right)^2 N, \ldots$

$$\left(\frac{99}{100}\right)^{h-1} N = 1$$

$$\left(\frac{99}{100}\right)^{h-1} = \frac{1}{N}$$

$$N = \left(\frac{100}{99}\right)^{h-1}$$

$$\log N = h \log(1)$$

$$h = \log N \quad \text{or}$$

$$h = \frac{\log N}{\log\left(\frac{100}{99}\right)} + 1$$

height of 2nd stream

$N, \frac{N}{100}, \frac{N}{100^2}, \frac{N}{(100)^3}, \text{---} 1$

$N\left(\frac{1}{100}\right)^{h-1} = 1$

$N = (100)^{h-1}$

$(h-1)\log 100 = \log N$

$h = \frac{\log N}{\log 100} + 1$ & $h = \log N$ (approx)

$T(n) = O(N \log N)$

so time complexity is $O(N \log N)$

height of both extra is $\frac{\log N}{\log 100} + 1$ of $\left(\frac{1}{100}\right)$

and $\frac{\log N}{\log\left(\frac{100}{99}\right)} + 1$ of $\left(\frac{99}{100}\right)$

so we can conclude that if division is done more than height of tree will be more & when division ratio is less then height is less.

Ans → (a) n, n, $\log n$, $\log\log n$, root(n), $n\log n$, $2n$, 2^n, 4^n, n^2, 100

Ans → $O[100] < (\log\log n) < O(\log n) < O(\sqrt{n}) < O(n) < O(n\log n) < O(n^2) < O(2^n) < O(2^{2n}) < O(4^n)$

(b) $2(2^n)$, $4n$, $2n$, 1, $\log(n)$, $\log(\log(n))$, $\sqrt{\log(n)}$, $\log 2n$, $2\log n$, n, $\log(n!)$, $n!$, n^2, $n\log(n)$

Ans → $O(1) < O(\log(\log n)) < O(\log(n!)) < O(2n) < O(\log(2n)) < O(2\log n) < O(n) < O(n\log(n)) < O(\log(n!)) < O(2n) < O(4n) < O(n^2) < O(n!) < O(2(2^n))$.

© 8^{2n}, $\log_2 n$, $n \log_6(n)$, $n \log_2(n)$, $\log(n!)$, $\log_8(n)$, 96, $8n^2$, $7n^3$, $5n$.

Ans→ $O(96) < O(\log_8(n)) < O(\log_2 n) < O(\log(n)) < O(n \log_8(n)) < O(n \log_2(n)) < O(5n) < O(8n^3) < O(7n^3) < O(n!) < O(8^{n^2 n})$.

Ans→ (9→)
```
void linearSearch(int arr[], int n, int key)
{
for(i=0  to i=n)
 if arr[i] == key
   count << found;
 else
   continue
}
```

Ans→ 20→) Iterative Insertion sort

→
```
void InsertionSort(arr, n) {
 int i, temp, j
 for(i=1 to n
  {
  temp = arr[i]
  j = i - 1
  while j >= 0 && arr[j] > temp
   {
```

```
        arr[j+1] = arr[j]
        j--
    }
    arr[j+1] = temp
  }
}
```

RECURSIVE INSERTION SORT

```
→ insertion sort(arr, n)
  {
    if n<=1
      return;
      insertion sort(arr, n-1);
      last = arr[n-1];
      j = n-2
      while(j>=0 and arr[j] > last)
        {
          arr[j+1] = arr[j]
          j--
        }
      arr[j+1] = last;
  }
```

Insertion sort is called online sorting because it don't know the whole input, it might make decision that later turn out to be not optimal.

Other algorithm are off-line algorithms ~~that~~ are discussed in lectures.

Ans-> 21-> TIME COMPLEXITIES

	BEST	AVG	worst	Space
~~Buffe~~ Bubble Sort	O(n2)	O(n2)	O(n2)	O(1)
selection sort	O(n2)	O(n2)	O(n2)	O(1)
Insertion sort	(n)	O(n2)	O(n2)	O(1)
merge sort	O(nlogn)	O(nlogn)	O(nlogn)	O(n) & due to recursion.
Quick sort	O(nlogn)	O(nlogn)	O(n2)	O(n)
Heap sort	O(nlogn)	O(nlogn)	O(nlogn)	O(1)

Ans→ 22→

	Inplace	stable	online sorting
Bubble sort	yes	yes	NO
selection sort	yes	NO	NO
Insertion sort	yes	yes	yes
MERGE sort	NO	yes	No
QUICK SORT	yes	NO	NO
HEAP sort	yes	NO	NO

Ans→23→ Binary search (arr, int n, key)

```
{
 beg =0
 end =n-1
 while ( beg <=end )
  {
   mid= ( beg+ end) /2
   if [arr [mid]  == key]
     found
   elseif arr[mid] < key
     beg = mid +1
   else
     end=  mid-1
  }
}
```

Time complexity of linear search - O(n)
Space complexity of linear search - O(1)

Time complexity of Binary search = O(log n)
space complexity of Binary search - O(n)

Ans→24→ $T(n) = T(n/2) + 1$.