Kugle

Modelling and Control of a Ball-balancing Robot

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Agenda

1. Introduction
2. Problem Formulation
3. Model
4. Balance Controller
5. Path-following MPC
6. Conclusion

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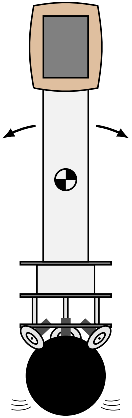
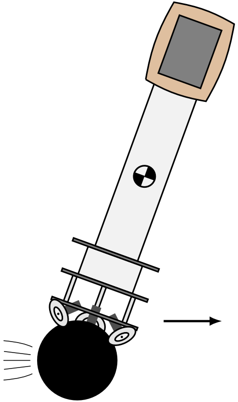
Introduction

- Robot Digital Signage project $\rightarrow$ Robot platform for app-developers
- Ballbot $\rightarrow$ Shape-accelerated underactuated balancing system
- Small wheelbase useful for Human-Robot Interaction applications



(a) Small movements required when standing still



(b) Inclination causes acceleration

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1.

Can a quaternion-based model of a ball-balancing robot, for use in controller design, be derived using Lagrangian mechanics?

2.

Can a non-linear Sliding mode controller be used to stabilize and control the ball-balancing robot and how does it compare to a traditional LQR?

3.

Can Model Predictive Control be used to control the motion of a ball-balancing robot and make it follow a given path?

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Model summary

State vector

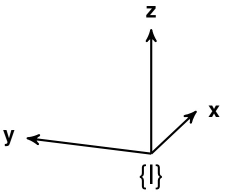
$$\boldsymbol{x} = \begin{bmatrix} {}^{\mathrm{I}}x \\ {}^{\mathrm{I}}y \\ {}^{\mathrm{K}}_{\mathrm{B}}\boldsymbol{q} \\ {}^{\mathrm{I}}\dot{x} \\ {}^{\mathrm{I}}\dot{y} \\ {}^{\mathrm{K}}_{\mathrm{B}}\dot{\boldsymbol{q}} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \chi \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \dot{\chi} \end{matrix}$$

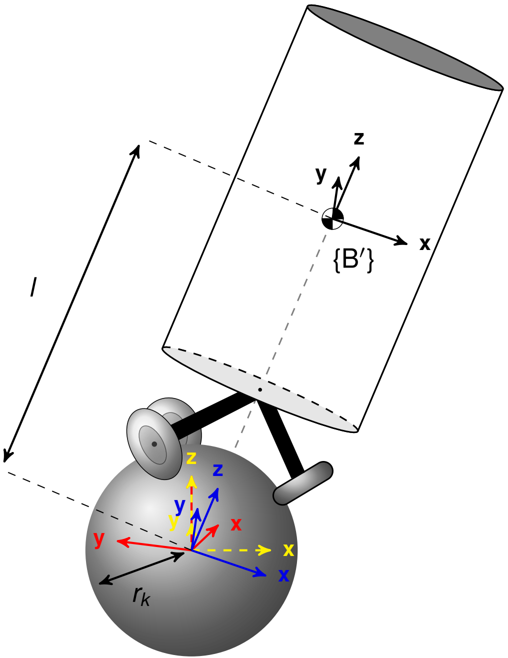
Frames

- $\{I\}$: inertial frame
- $\{K\}$: ball frame
- $\{K'\}$: rotating ball frame
- $\{H\}$: heading frame
- $\{B\}$: body frame
- $\{B'\}$: body COM frame

Energies

- T_k : ball kinetic
- T_b : body kinetic
- T_w : wheel kinetic
- V_b : body potential





Quaternion error

$$ {}^{\mathrm{B}}\boldsymbol{q}_e = {}^{\mathrm{K}}_{\tilde{\mathrm{B}}}\boldsymbol{q}^*_{\mathrm{ref}} \circ {}^{\mathrm{K}}_{\mathrm{B}}\boldsymbol{q} $$

Sliding surface

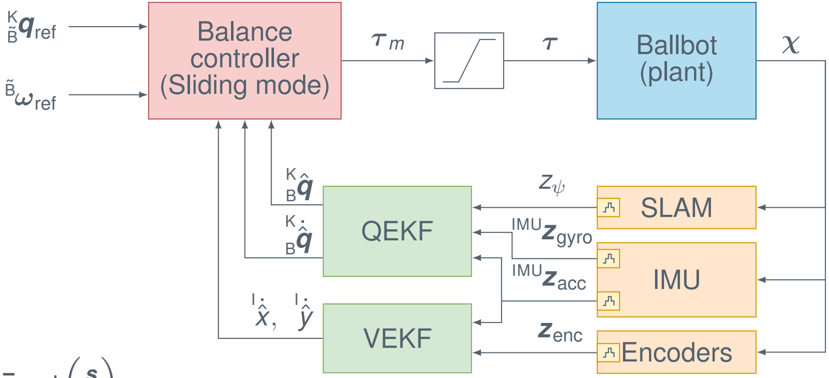
$$ \boldsymbol{s} = {}^{\mathrm{B}}\dot{\vec{q}}_e + \boldsymbol{K}\,{}^{\mathrm{B}}\vec{q}_e $$

Control law

$$ \tau_m = \tau_{\mathrm{equiv}} + \tau_{\mathrm{switch}} $$

Switching law

$$ \tau_{\mathrm{switch}} = -\tilde{\boldsymbol{g}}^{-1}\left({}^{\mathrm{K}}_{\mathrm{B}}\boldsymbol{q},\ {}^{\mathrm{K}}_{\tilde{\mathrm{B}}}\boldsymbol{q}_{\mathrm{ref}}\right)\bar{\eta}\,\mathrm{sat}\left(\frac{\boldsymbol{s}}{\epsilon}\right) $$



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Controller demonstration

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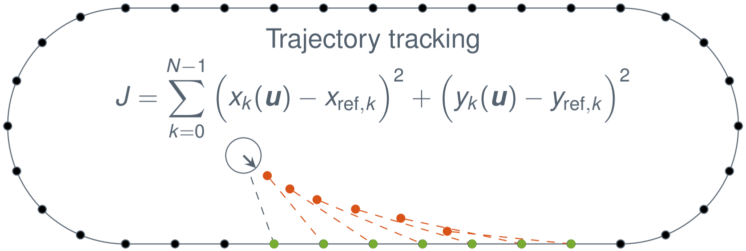
Trajectory tracking vs Path following

- Trajectory tracking
  $$\lim_{t\to\infty} \left\| \boldsymbol{x}(t) - \boldsymbol{x}_{\text{ref}}(t) \right\| = 0$$
  - Discrete reference points with assigned timing
  - Reference might run ahead of the system, leading to an infeasible tracking problem



- Reference trajectory
- Horizon, x_k, y_k
- Horizon reference points, $x_{\text{ref},k}$, $y_{\text{ref},k}$

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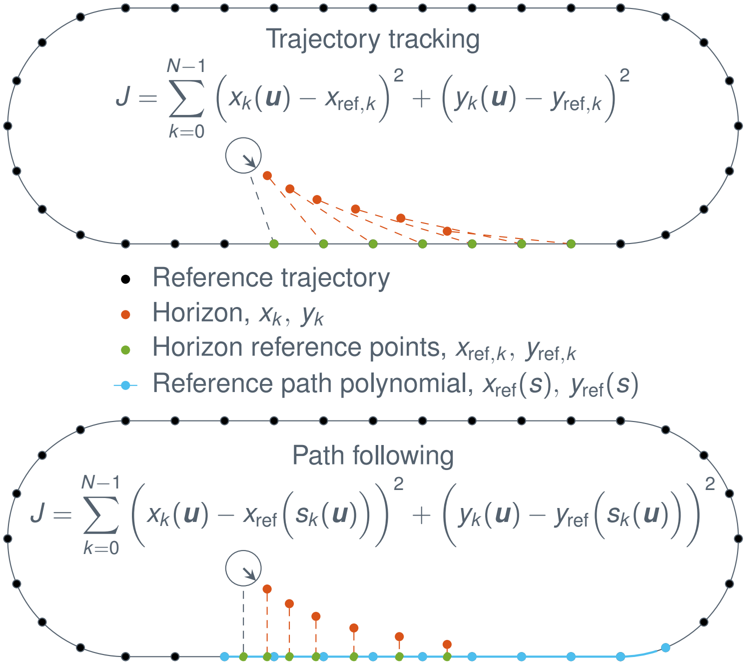
Trajectory tracking vs Path following

- Trajectory tracking
  $$\lim_{t\to\infty} \left\| \boldsymbol{x}(t) - \boldsymbol{x}_{\text{ref}}(t) \right\| = 0$$
  - Discrete reference points with assigned timing
  - Reference might run ahead of the system, leading to an infeasible tracking problem
- Path following
  $$\lim_{t\to\infty} \left\| \boldsymbol{x}(t) - \boldsymbol{p}_{\text{ref}}\left(s(t)\right) \right\| = 0$$
  - Geometric reference path without any preassigned timing information
  - Strict forward motion, $\dot{s}(t) > 0$, ensures progress



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Optimization problem

MPC states

$$\chi = \begin{bmatrix} {}^{\mathrm{H}}_{\mathrm{B}}q_1 & {}^{\mathrm{H}}_{\mathrm{B}}q_2 & | & {}^{\mathrm{H}}x & {}^{\mathrm{H}}y & | & {}^{\mathrm{H}}\dot{x} & {}^{\mathrm{H}}\dot{y} & | & s & \dot{s} & | & {}^{\tilde{\mathrm{B}}}\omega_{\mathrm{ref,x}} & {}^{\tilde{\mathrm{B}}}\omega_{\mathrm{ref,y}} \end{bmatrix}^{\mathrm{T}}$$

Steady state shape-accelerated model

$$\begin{bmatrix} {}^{\mathrm{H}}\ddot{x} \\ {}^{\mathrm{H}}\ddot{y} \end{bmatrix} = c_q \begin{bmatrix} {}^{\mathrm{H}}_{\mathrm{B}}q_2 \\ -{}^{\mathrm{H}}_{\mathrm{B}}q_1 \end{bmatrix}$$

Control variables

$$\boldsymbol{u} = \begin{bmatrix} {}^{\tilde{\mathrm{B}}}\dot{\omega}_{\mathrm{ref,x}} & {}^{\tilde{\mathrm{B}}}\dot{\omega}_{\mathrm{ref,y}} & \ddot{s} & | & \gamma_v & \gamma_q & \gamma_o \end{bmatrix}^{\mathrm{T}}$$

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Steady state shape-accelerated model

$$\begin{bmatrix} {}^{\mathrm{H}}\ddot{x} \\ {}^{\mathrm{H}}\ddot{y} \end{bmatrix} = c_q \begin{bmatrix} {}^{\mathrm{H}}_{\mathrm{B}}q_2 \\ -{}^{\mathrm{H}}_{\mathrm{B}}q_1 \end{bmatrix}$$

Least-squares Cost function

$$J = \sum_{k=0}^{N-1} \left(\boldsymbol{h}^{\mathrm{T}} \boldsymbol{W} \boldsymbol{h} \right) + \boldsymbol{h}_n^{\mathrm{T}} \boldsymbol{W_N} \boldsymbol{h}_n$$

Control variables

$$\boldsymbol{u} = \begin{bmatrix} {}^{\tilde{\mathrm{B}}}\dot{\omega}_{\mathrm{ref,x}} & {}^{\tilde{\mathrm{B}}}\dot{\omega}_{\mathrm{ref,y}} & \ddot{s} & | & \gamma_v & \gamma_q & \gamma_o \end{bmatrix}^{\mathrm{T}}$$

Sample rate

$f = 10$ Hz

Horizon

$N = 20$

Cost vectors

$$\boldsymbol{h} = \begin{bmatrix} e_{\mathrm{lon}} & e_{\mathrm{lat}} & e_{\mathrm{vel}} & e_{\mathrm{end}} & e_{\mathrm{obs}} & | & {}^{\mathrm{H}}_{\mathrm{B}}q_1 & {}^{\mathrm{H}}_{\mathrm{B}}q_2 & {}^{\tilde{\mathrm{B}}}\omega_{\mathrm{ref,x}} & {}^{\tilde{\mathrm{B}}}\omega_{\mathrm{ref,y}} & | & {}^{\tilde{\mathrm{B}}}\dot{\omega}_{\mathrm{ref,x}} & {}^{\tilde{\mathrm{B}}}\dot{\omega}_{\mathrm{ref,y}} & \gamma_v & \gamma_q & \gamma_o \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{h}_N = \begin{bmatrix} e_{\mathrm{lon}} & e_{\mathrm{lat}} & e_{\mathrm{vel}} & e_{\mathrm{end}} & e_{\mathrm{obs}} & | & {}^{\mathrm{H}}_{\mathrm{B}}q_1 & {}^{\mathrm{H}}_{\mathrm{B}}q_2 & {}^{\tilde{\mathrm{B}}}\omega_{\mathrm{ref,x}} & {}^{\tilde{\mathrm{B}}}\omega_{\mathrm{ref,y}} \end{bmatrix}^{\mathrm{T}}$$

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Longitudinal and Lateral errors

Reference trajectory direction

$$\psi_{\text{ref}} = \text{atan2}\left({}^{\text{H}}\dot{y}_{\text{ref}}(s),\ {}^{\text{H}}\dot{x}_{\text{ref}}(s) \right)$$

Tracking error

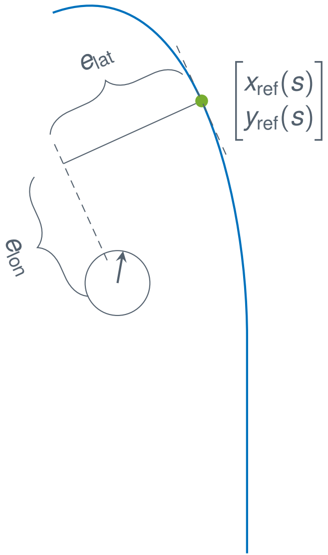
$$\begin{bmatrix} e_x \\ e_y \end{bmatrix} = \begin{bmatrix} {}^{\text{H}}x \\ {}^{\text{H}}y \end{bmatrix} - \begin{bmatrix} {}^{\text{H}}x_{\text{ref}}(s) \\ {}^{\text{H}}y_{\text{ref}}(s) \end{bmatrix}$$

Lateral error

$$e_{\text{lat}} = \begin{bmatrix} \sin(\psi_{\text{ref}}) \\ \cos(\psi_{\text{ref}}) \end{bmatrix}^{\text{T}} \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$

Longitudinal error

$$e_{\text{lon}} = \begin{bmatrix} \cos(\psi_{\text{ref}}) \\ \sin(\psi_{\text{ref}}) \end{bmatrix}^{\text{T}} \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$



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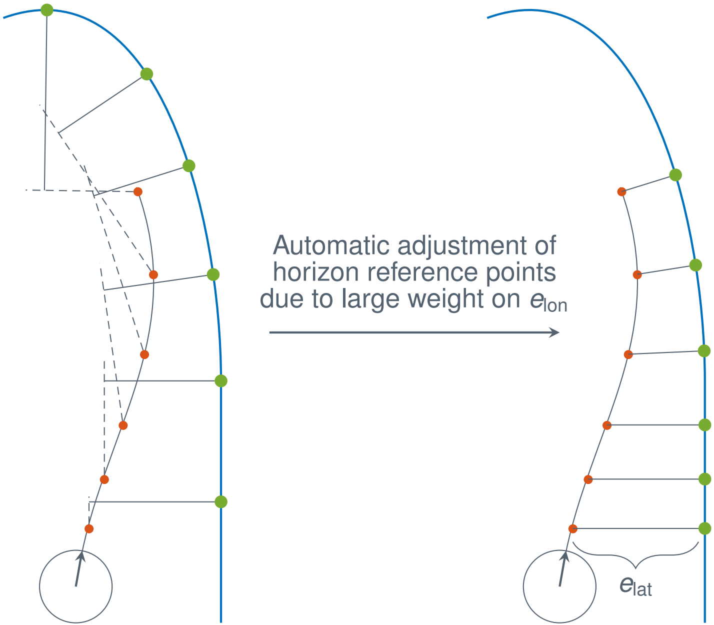
Longitudinal reference alignment

- Reference trajectory points should align with predicted horizon states
- Enforce longitudinal alignment with large weight on e_{lon}
  - $W_{\text{lon}} \gg 100 W_{\text{lat}}$



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Obstacle avoidance

Obstacle proximity

$$p_{o,i} = \sqrt{\left({}^{H}x - {}^{H}x_{o,i}\right)^2 + \left({}^{H}y - {}^{H}y_{o,i}\right)^2} - r_{o,i}$$

Obstacle constraints

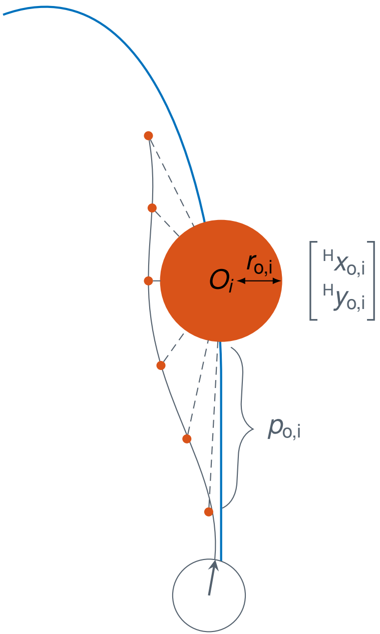
$$p_{o,i} \geq -\gamma_o$$
$$\gamma_o \geq 0$$

Obstacle avoidance error

Up to five obstacles considered

$$e_{obs} = \sum_{i=0}^{4} \mathrm{e}^{k_p(-p_{o,i}+c_p)}$$

Exponential barrier function pushes the robot away from the obstacles instead of driving on the constraint boundary



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Conclusion

1. Non-linear ballbot model derived using quaternions and Lagrangian mechanics

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1. Non-linear ballbot model derived using quaternions and Lagrangian mechanics
2. Non-linear model used for sliding mode controller design
3. Balance controller tested and verified in practice on Kugle V1 prototype
4. Sliding mode controller compared to LQR balance controller with almost identical performance

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1. Non-linear ballbot model derived using quaternions and Lagrangian mechanics
2. Non-linear model used for sliding mode controller design
3. Balance controller tested and verified in practice on Kugle V1 prototype
4. Sliding mode controller compared to LQR balance controller with almost identical performance
5. Path-following shape-accelerated MPC verified with closed-loop simulation

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Discussion

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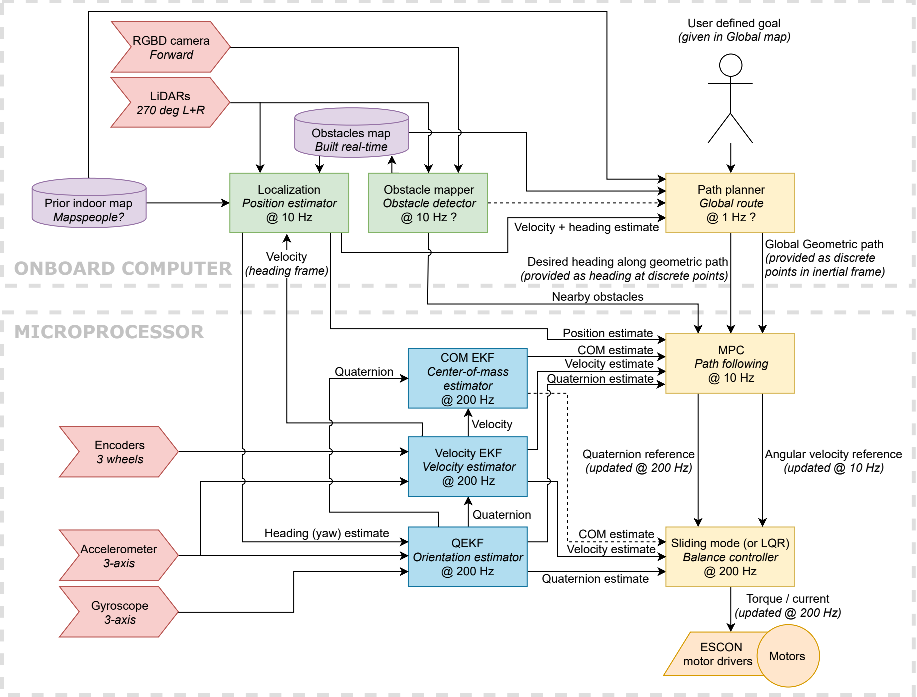
System architecture



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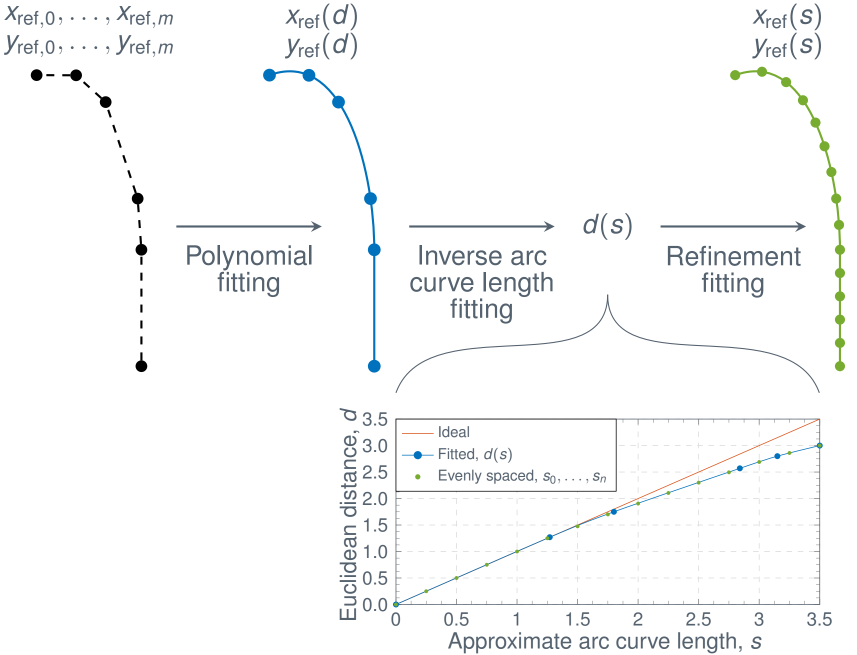
MPC Polynomial fitting



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MPC Cost function

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Cost vectors

$$\boldsymbol{h} = \left[e_{\text{lon}} \quad e_{\text{lat}} \quad e_{\text{vel}} \quad e_{\text{end}} \quad e_{\text{obs}} \;\middle|\; {}^{\text{H}}_{\text{B}}q_1 \quad {}^{\text{H}}_{\text{B}}q_2 \quad {}^{\tilde{\text{B}}}\omega_{\text{ref,x}} \quad {}^{\tilde{\text{B}}}\omega_{\text{ref,y}} \;\middle|\; {}^{\tilde{\text{B}}}\dot{\omega}_{\text{ref,x}} \quad {}^{\tilde{\text{B}}}\dot{\omega}_{\text{ref,y}} \quad \gamma_v \quad \gamma_q \quad \gamma_o \right]^{\text{T}}$$

$$\boldsymbol{h}_N = \left[e_{\text{lon}} \quad e_{\text{lat}} \quad e_{\text{vel}} \quad e_{\text{end}} \quad e_{\text{obs}} \;\middle|\; {}^{\text{H}}_{\text{B}}q_1 \quad {}^{\text{H}}_{\text{B}}q_2 \quad {}^{\tilde{\text{B}}}\omega_{\text{ref,x}} \quad {}^{\tilde{\text{B}}}\omega_{\text{ref,y}} \right]^{\text{T}}$$

Longitudinal velocity

$$v_{\text{lon}} = \begin{bmatrix} \cos(\psi_{\text{ref}}) \\ \sin(\psi_{\text{ref}}) \end{bmatrix}^{\text{T}} \begin{bmatrix} {}^{\text{H}}\dot{x} \\ {}^{\text{H}}\dot{y} \end{bmatrix}$$

Velocity error

$e_{\text{vel}} = v_{\text{lon}} - v_{\text{ref}}$

- Ensures velocity tracking of a desired velocity
- Ensures progress indirectly

Away from end error

$e_{\text{end}} = s - s_{\text{max}}$

- Ensures progress
- Affects velocity tracking

Weight considerations

- W_{lat}, W_{obs} and W_v limits how big avoidable obstacles can be
- $W_{\dot{\omega}}$ and W_{ω} control aggressiveness
- W_v, $W_{\dot{\omega}}$, W_{ω} and W_q defines how fast the velocity tracking converges

Weights

$W_{\text{lon}} = 9999$
$W_{\text{lat}} = 25$
$W_{\text{vel}} = 80$
$W_{\text{end}} = 0$
$W_{\text{obs}} = 20$
$W_q = 10$
$W_{\omega} = 20$
$W_{\dot{\omega}} = 100$
$W_{\gamma_v} = 99999$
$W_{\gamma_q} = 9999$
$W_{\gamma_o} = 9999$

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MPC Constraints

Constraints

$$
\begin{aligned}
-q_{max} &\leq {}^{H}_{B}q_1 \leq q_{max} \\
-q_{max} &\leq {}^{H}_{B}q_2 \leq q_{max} \\
-\omega_{max} &\leq {}^{\tilde{B}}\omega_{ref,x} \leq \omega_{max} \\
-\omega_{max} &\leq {}^{\tilde{B}}\omega_{ref,y} \leq \omega_{max} \\
-\dot{\omega}_{max} &\leq {}^{\tilde{B}}\dot{\omega}_{ref,x} \leq \dot{\omega}_{max} \\
-\dot{\omega}_{max} &\leq {}^{\tilde{B}}\dot{\omega}_{ref,y} \leq \dot{\omega}_{max} \\
0 &\leq s \leq s_{max} \\
\dot{s} &\geq 0 \\
v &\leq v_{ref} + \gamma_v \\
p_{o,i} &\geq -\gamma_o \\
\gamma_v &\geq 0 \\
\gamma_q &\geq 0 \\
\gamma_o &\geq 0
\end{aligned}
$$

Terminal Constraints

$$
\begin{aligned}
{}^{H}_{B}q_1[N] &= 0 \\
{}^{H}_{B}q_2[N] &= 0 \\
{}^{\tilde{B}}\omega_{ref,x}[N] &= 0 \\
{}^{\tilde{B}}\omega_{ref,y}[N] &= 0 \\
v[N] &\leq v_{max} \\
s[N] &\leq s_{max}
\end{aligned}
$$

Maximum angle

$$q_{max} = \sin\left(\frac{1}{2}\theta_{max}\right) + \gamma_q$$

Velocity norm

$$v = \sqrt{{}^{H}\dot{x}^2 + {}^{H}\dot{y}^2}$$

Constraint considerations

- $\dot{\omega}_{max}$ limits the maximum angular acceleration and is thus related to torque saturations
- γ_v, γ_q and γ_o ensures feasibility even though the problem is initialized in or goes through an infeasible region

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