Model-Free Reinforcement Learning

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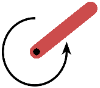
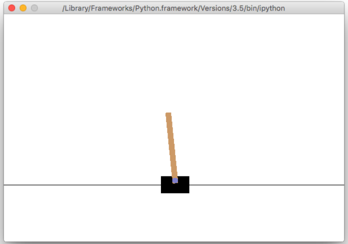
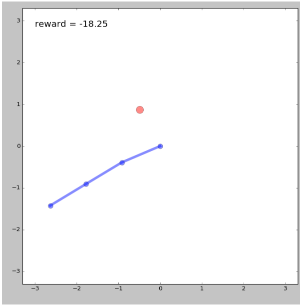
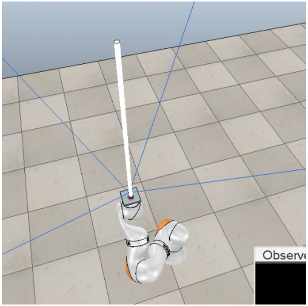
Policy Gradient Methods









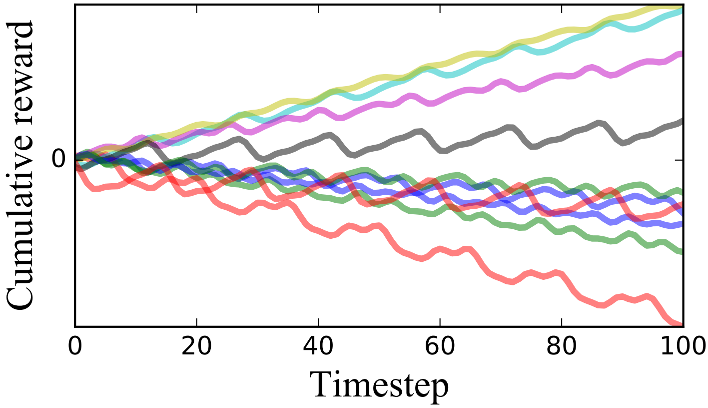
Policy Gradient Methods

The REINFORCE algorithm: Williams, "Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning" (*Machine Learning*, 1992)

Task	Random	REINFORCE	TNPG	RWR	REPS	TRPO	CEM	CMA-ES	DDPG
Cart-Pole Balancing	77.1 ± 0.0	4693.7 ± 14.0	**3986.4 ± 748.9**	**4861.5 ± 12.3**	565.6 ± 137.6	**4869.8 ± 37.6**	4815.4 ± 4.8	2440.4 ± 568.3	4634.4 ± 87.8
Inverted Pendulum*	−153.4 ± 0.2	13.4 ± 18.0	**209.7 ± 55.5**	84.7 ± 13.8	−113.3 ± 4.6	**247.2 ± 76.1**	38.2 ± 25.7	−40.1 ± 5.7	40.0 ± 244.6
Mountain Car	−415.4 ± 0.0	−67.1 ± 1.0	**-66.5 ± 4.5**	−79.4 ± 1.1	−275.6 ± 166.3	**-61.7 ± 0.9**	−66.0 ± 2.4	−85.0 ± 7.7	−288.4 ± 170.3
Acrobot	−1904.5 ± 1.0	−508.1 ± 91.0	−395.8 ± 121.2	−352.7 ± 35.9	−1001.5 ± 10.8	−326.0 ± 24.4	−436.8 ± 14.7	−785.6 ± 13.1	**-223.6 ± 5.8**
Double Inverted Pendulum*	149.7 ± 0.1	4116.5 ± 65.2	**4455.4 ± 37.6**	3614.8 ± 368.1	446.7 ± 114.8	**4412.4 ± 50.4**	2566.2 ± 178.9	1576.1 ± 51.3	2863.4 ± 154.0
Swimmer*	−1.7 ± 0.1	92.3 ± 0.1	**96.0 ± 0.2**	60.7 ± 5.5	3.8 ± 3.3	**96.0 ± 0.2**	68.8 ± 2.4	64.9 ± 1.4	85.8 ± 1.8
Hopper	8.4 ± 0.0	714.0 ± 29.3	**1155.1 ± 57.9**	553.2 ± 71.0	86.7 ± 17.6	**1183.3 ± 150.0**	63.1 ± 7.8	20.3 ± 14.3	267.1 ± 43.5
2D Walker	−1.7 ± 0.0	506.5 ± 78.8	**1382.6 ± 108.2**	136.0 ± 15.9	−37.0 ± 38.1	**1353.8 ± 85.0**	84.5 ± 19.2	77.1 ± 24.3	318.4 ± 181.6
Half-Cheetah	−90.8 ± 0.3	1183.1 ± 69.2	**1729.5 ± 184.6**	376.1 ± 28.2	34.5 ± 38.0	**1914.0 ± 120.1**	330.4 ± 274.8	441.3 ± 107.6	**2148.6 ± 702.7**
Ant*	13.4 ± 0.7	548.3 ± 55.5	**706.0 ± 127.7**	37.6 ± 3.1	39.0 ± 9.8	**730.2 ± 61.3**	49.2 ± 5.9	17.8 ± 15.5	326.2 ± 20.8
Simple Humanoid	41.5 ± 0.2	128.1 ± 34.0	**255.0 ± 24.5**	93.3 ± 17.4	28.3 ± 4.7	**269.7 ± 40.3**	60.6 ± 12.9	28.7 ± 3.9	99.4 ± 28.1
Full Humanoid	13.2 ± 0.1	262.2 ± 10.5	**288.4 ± 25.2**	46.7 ± 5.6	41.7 ± 6.1	**287.0 ± 23.4**	36.9 ± 2.9	N/A ± N/A	119.0 ± 31.2
Cart-Pole Balancing (LS)*	77.1 ± 0.0	420.9 ± 265.5	**945.1 ± 27.8**	68.9 ± 1.5	898.1 ± 22.1	**960.2 ± 46.0**	227.0 ± 223.0	68.0 ± 1.6	
Inverted Pendulum (LS)	−122.1 ± 0.1	−13.4 ± 3.2	**0.7 ± 6.1**	−107.4 ± 0.2	−87.2 ± 8.0	**4.5 ± 4.1**	−81.2 ± 33.2	−62.4 ± 3.4	
Mountain Car (LS)	−83.0 ± 0.0	−81.2 ± 0.6	**-65.7** ± 9.0	−81.7 ± 0.1	−82.6 ± 0.4	**-64.2 ± 9.5**	**-68.9 ± 1.3**	**-73.2 ± 0.6**	
Acrobot (LS)*	−393.2 ± 0.0	−128.9 ± 11.6	**-84.6 ± 2.9**	−235.9 ± 5.3	−379.5 ± 1.4	**-83.3 ± 9.9**	−149.5 ± 15.3	−159.9 ± 7.5	
Cart-Pole Balancing (NO)*	101.4 ± 0.1	616.0 ± 210.8	**916.3 ± 23.0**	93.8 ± 1.2	99.6 ± 7.2	606.2 ± 122.2	181.4 ± 32.1	104.4 ± 16.0	
Inverted Pendulum (NO)	−122.2 ± 0.1	6.5 ± 1.1	**11.5 ± 0.5**	−110.0 ± 1.4	−119.3 ± 4.2	**10.4 ± 2.2**	−55.6 ± 16.7	−80.3 ± 2.8	
Mountain Car (NO)	−83.0 ± 0.0	−74.7 ± 7.8	**-64.5 ± 8.6**	−81.7 ± 0.1	−82.9 ± 0.1	**-60.2 ± 2.0**	−67.4 ± 1.4	−73.5 ± 0.5	
Acrobot (NO)*	−393.5 ± 0.0	**-186.7 ± 31.3**	**-164.5 ± 13.4**	−233.1 ± 0.4	−258.5 ± 14.0	**-149.6 ± 8.6**	−213.4 ± 6.3	−236.6 ± 6.2	
Cart-Pole Balancing (SI)*	76.3 ± 0.1	431.7 ± 274.1	**980.5 ± 7.3**	69.0 ± 2.8	702.4 ± 196.4	**980.3 ± 5.1**	746.6 ± 93.2	71.6 ± 2.9	
Inverted Pendulum (SI)	−121.8 ± 0.2	−5.3 ± 5.6	**14.8 ± 1.7**	−108.7 ± 4.7	−92.8 ± 23.9	**14.1 ± 0.9**	−51.8 ± 10.6	−63.1 ± 4.8	
Mountain Car (SI)	−82.7 ± 0.0	−63.9 ± 0.2	**-61.8 ± 0.4**	−81.4 ± 0.1	−80.7 ± 2.3	**-61.6 ± 0.4**	−63.9 ± 1.0	−66.9 ± 0.6	
Acrobot (SI)*	−387.8 ± 1.0	**-169.1 ± 32.3**	**-156.6 ± 38.9**	−233.2 ± 2.6	−216.1 ± 7.7	**-170.9 ± 40.3**	−250.2 ± 13.7	−245.0 ± 5.5	
Swimmer + Gathering	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
Ant + Gathering	−5.8 ± 5.0	−0.1 ± 0.1	−0.4 ± 0.1	−5.5 ± 0.5	−6.7 ± 0.7	−0.4 ± 0.0	−4.7 ± 0.7	N/A ± N/A	−0.3 ± 0.3
Swimmer + Maze	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
Ant + Maze	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	N/A ± N/A	0.0 ± 0.0

Table: Duan, Chen, Houthooft, Schulman and Abbeel, "Benchmarking Deep Reinforcement Learning for Continuous Control," *Proceedings of the ICML*, 2016.

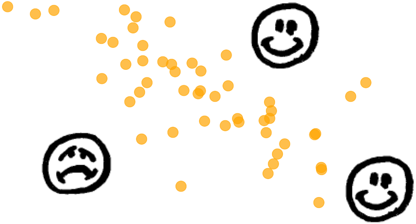
Policy Gradient Methods



One-Shot Games

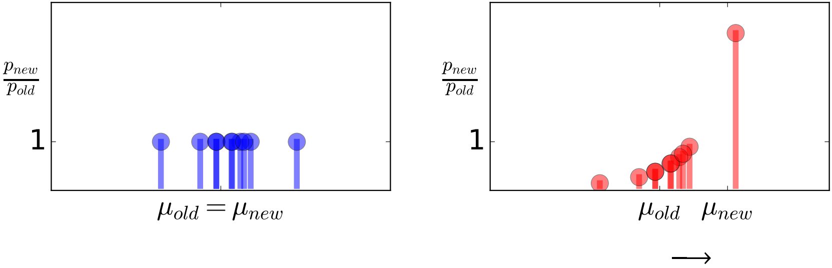
Problem statement:

$$W^* \quad = \quad \arg\max_{W} E_W[\,V\,]$$



Importance Weighting

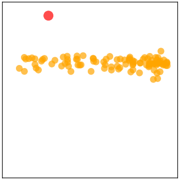
$$E_{new}[X] \quad = \quad E_{old}\left[X \cdot \frac{p_{new}}{p_{old}}\right]$$

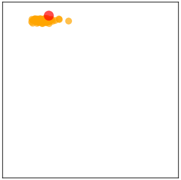


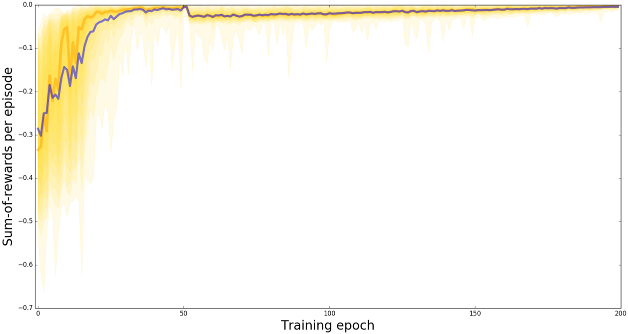
The Policy Gradient

$$\boxed{\nabla_W \, E_W\left[V\right] \quad = \quad E_{W_0}\left[\, V \cdot \frac{\nabla_W \, p_W}{p_{W_0}} \,\right]}$$

Dart-Throwing Game: $R(u) = -\|u - u^*\|^2$

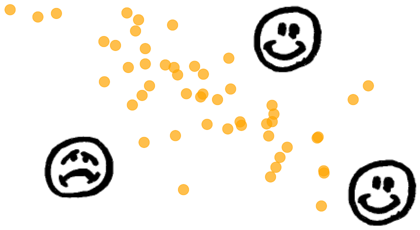






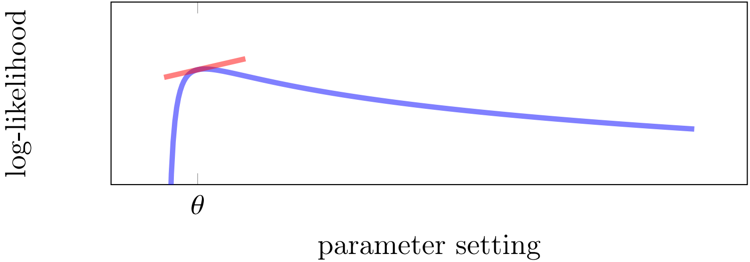
The Policy Gradient

$$\nabla_W E_W[V] = E_{W_0}\left[V \cdot \left(\frac{\nabla_W p}{p}\right)\right]$$



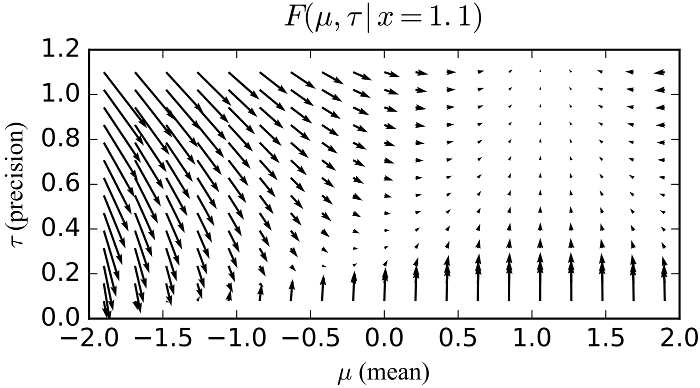
The Fisher Score

$$F(x) \quad = \quad \frac{\nabla_\theta \, d(x \,|\, \theta)}{d(x \,|\, \theta)} \quad = \quad \nabla_\theta \, \log d(x \,|\, \theta)$$

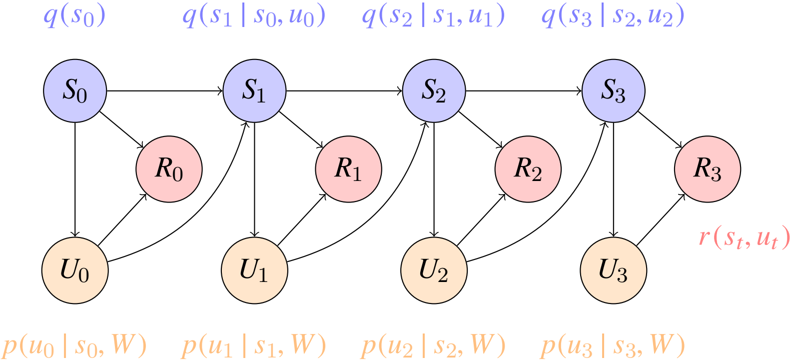


The Fisher Score

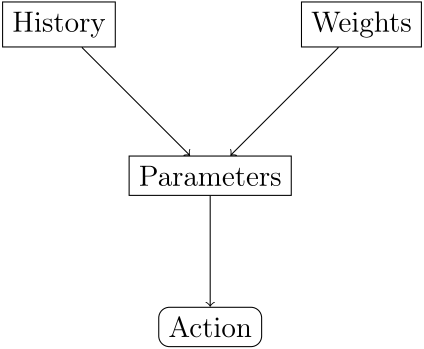
$$\nabla_{(\mu,\tau)} \log\left(\sqrt{\frac{\tau}{\pi}}\ \exp\left\{-\tau(x-\mu)^2\right\}\right) \quad = \quad \begin{pmatrix} 2\tau(x-\mu) \\ (2\tau)^{-1} - (x-\mu)^2 \end{pmatrix}$$



Extensive Games



Extensive Games



Action ~ Distribution(History, Weights)

Extensive Games

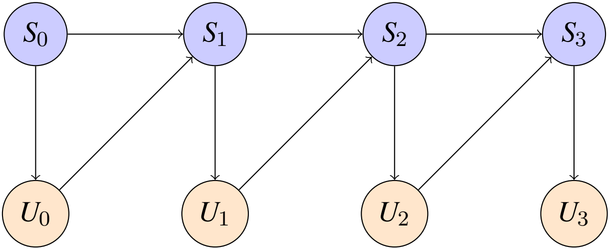
Problem statement:

$$W^* = \arg\max_{W} E_W [\, R_0 + R_1 + \cdots + R_{T-1} \,]$$

The Policy Gradient

$$\boxed{\nabla_W E_W[V] = E_W[V \cdot F]}$$

Episode Scores



$$q(S_0)\, p(U_0 \mid S_0, W)\, q(S_1 \mid S_0, U_0)\, p(U_1 \mid S_1, W)\, q(S_2 \mid S_1, U_1)\, \cdots$$

Episode Scores

$$\frac{\nabla\left(q_0\, p_1\, q_1\, p_1\, q_2\, p_2\, \cdots\, q_{T-1}\, p_{T-1}\right)}{\left(q_0\, p_1\, q_1\, p_1\, q_2\, p_2\, \cdots\, q_{T-1}\, p_{T-1}\right)} = \frac{\nabla\left(p_1\, p_1\, p_2\, \cdots\, p_{T-1}\right)}{\left(p_1\, p_1\, p_2\, \cdots\, p_{T-1}\right)}$$

Hence:

$$\boxed{F = \nabla \log p_0 + \nabla \log p_1 + \nabla \log p_2 + \cdots + \nabla \log p_{T-1}}$$

The Policy Gradient

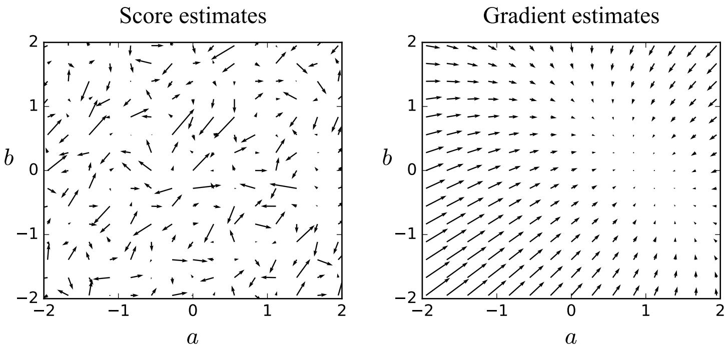
$$\nabla_W E_W\left[\sum_{t=0}^{T-1} R_t\right] = E_W\left[\underbrace{\sum_{t=0}^{T-1} R_t}_{V} \cdot \underbrace{\sum_{t=0}^{T-1} \nabla_W \log p(U_t \mid S_t, W)}_{F}\right]$$

Repeat-After-Me Game: $R(s, u) = -\|s - u\|^2$

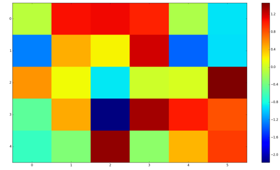
$$s \sim \mathcal{N}(1/2, 1)$$

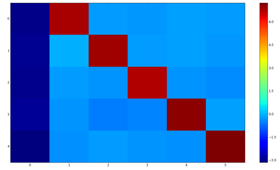
$$u \sim \mathcal{N}(as + b, 1)$$

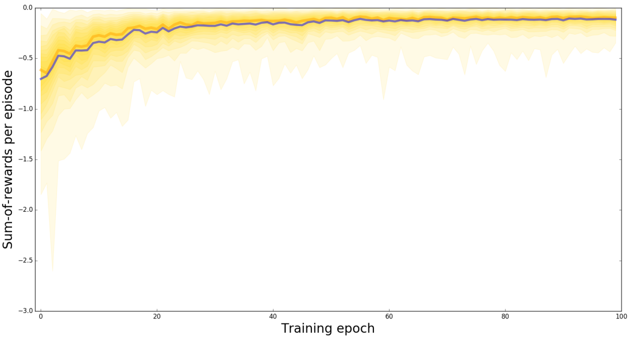
$$F\left(\begin{array}{c} a \\ b \end{array} \middle| s, u\right) = \left(\begin{array}{c} (as + b - u)s \\ (as + b - u) \end{array}\right)$$



Repeat-After-Me Game: $R(s, u) = -\|s - u\|^2$



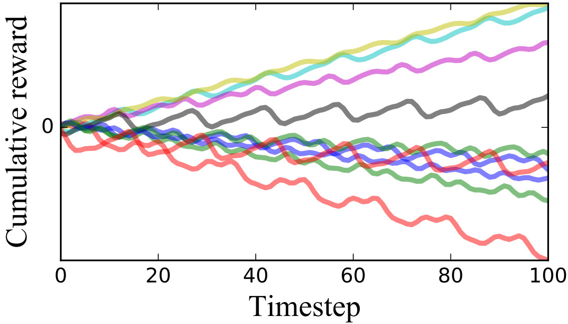




Apportioning Blame

$$V = R_0 + R_1 + R_2 + \cdots + R_{T-1}$$

$$F = F_0 + F_1 + F_2 + \cdots + F_{T-1}$$



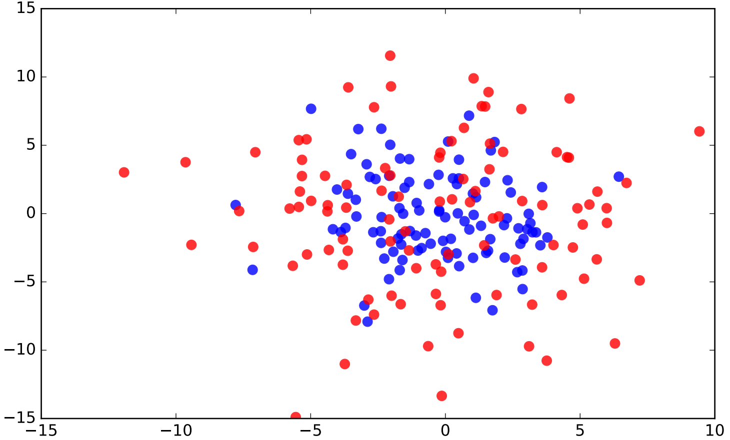
Apportioning Blame

$$E \begin{bmatrix} F_0 R_0 & F_0 R_1 & F_0 R_2 & \cdots & F_0 R_{T-1} \\ F_1 R_0 & F_1 R_1 & F_1 R_2 & \cdots & F_1 R_{T-1} \\ F_2 R_0 & F_2 R_1 & F_2 R_2 & \cdots & F_2 R_{T-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ F_{T-1} R_0 & F_{T-1} R_1 & F_{T-1} R_2 & \cdots & F_{T-1} R_{T-1} \end{bmatrix}$$

Apportioning Blame

$$E\begin{bmatrix} F_0 R_0 & F_0 R_1 & F_0 R_2 & \cdots & F_0 R_{T-1} \\ 0 & F_1 R_1 & F_1 R_2 & \cdots & F_1 R_{T-1} \\ 0 & 0 & F_2 R_2 & \cdots & F_2 R_{T-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & F_{T-1} R_{T-1} \end{bmatrix}$$

Apportioning Blame



Keeping or dropping
the zero-mean terms.

The Policy Gradient Method

```
For I epochs:
    For N episodes:
        Collect rollout, [(S_t,, U_t, R_t)]_{t=0}^{T-1}
        For each action U_t:
          Compute score, F_t = F(W | U_t);
          Compute tailsum, V_t = sum_{k=t}^{T-1} R_k ;
        Get gradient, G_n = sum_{t=0}^{T-1} V_t F_t ;
    Average gradients, G = 1/N sum_{n=0}^{N-1} G_n ;
    Adjust weights, W <- W + alpha G;
```

For I epochs:

For N episodes:

Collect rollout, $\left[(S_t,, U_t, R_t)\right]_{t=0}^{T-1}$

For each action U_t:

Compute score, $F_t = F(W \,|\, U_t)$;
Compute tailsum, $V_t = \sum_{k=t}^{T-1} R_k$;

Get gradient, $G_n = \sum_{t=0}^{T-1} V_t F_t$;

Average gradients, $G = \frac{1}{N} \sum_{n=0}^{N-1} G_n$;
Adjust weights, $W \leftarrow W + \alpha G$;

Future Complications

1. Short-lived dependencies:

$$V_t \;=\; R_t \;+\; \gamma R_{t+1} \;+\; \gamma^2 R_{t+2} \;+\; \gamma^3 R_{t+3} \;+\; \cdots$$

2. Baselines:

$$V_t \;\leftarrow\; V_t \;-\; \hat{V}_t$$

3. "Natural" directions:

$$G \;\leftarrow\; E[\,-\nabla F\,]^{-1}.\,G$$

4. Line searches:

$$\alpha \;=\; \arg\max_{\alpha} J(W + \alpha G)$$

References

- Fisher: “On the Mathematical Foundations of Theoretical Statistics” (*Proceedings of the Royal Society A*, 1922).
- Williams: “Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning” (*Machine Learning*, 1992).
- Amari: “Natural Gradient Works Efficiently in Learning” (*Neural Computation*, 1998).
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`github.com/mathias-madsen/reinforce_tutorial`