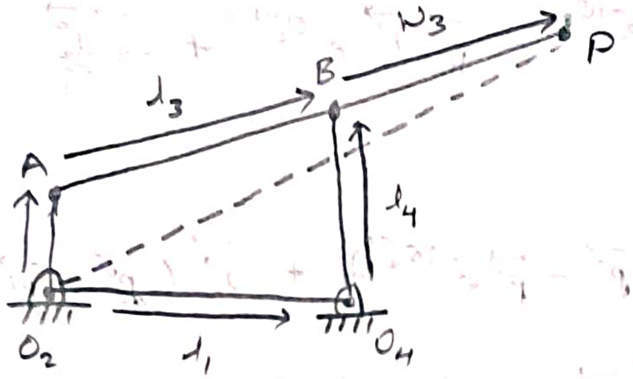
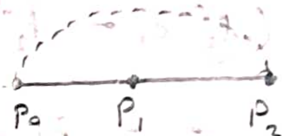
Synthesis Mechanism 1 :



Writing the loop closure equations :

$$\vec{l_2} + \vec{l_3} = \vec{l_1} + \vec{l_4}$$

Our required path is



Loop closure for Dyad 1,

$$\vec{l_2} + \vec{l_3} + \vec{w_3} = \vec{P}$$

Loop closure for Dyad 2,

$$\vec{l_1} + \vec{l_4} + \vec{w_3} = \vec{P}$$

Now, for the Dyad 1 equation, let $\vec{l_3} + \vec{w_3} = \vec{k}$.

$$\Rightarrow \vec{l_2^0} + \vec{k^0} = \vec{p^0}$$

$$e^{i\theta_2^{01}} \vec{l_2^0} + e^{i\theta_3^{01}} \vec{k^0} = \vec{p^1}$$

$$e^{i\theta_2^{02}} \vec{l_2^0} + e^{i\theta_3^{02}} \vec{k^0} = \vec{p^2}$$

From the above three equations,

$$\tilde{\vec{k^0}}\,\vec{k^0} = (\vec{p^0} - \vec{l_2^0})(\tilde{\vec{p^0}} - \tilde{\vec{l_2^0}}) \longrightarrow (1)$$

$$\tilde{\vec{k^0}}\,\vec{k^0} = (\vec{p^1} - e^{i\theta_2^{01}}\vec{l_2^0})(\tilde{\vec{p^1}} - e^{-i\theta_2^{01}}\tilde{\vec{l_2^0}}) \longrightarrow (2)$$

$$\tilde{\vec{k^0}}\,\vec{k^0} = (\vec{p^2} - e^{i\theta_2^{02}}\vec{l_2^0})(\tilde{\vec{p^2}} - e^{-i\theta_2^{02}}\tilde{\vec{l_2^0}}) \longrightarrow (3)$$

(1)* $\underset{\sim}{\vec{K^o}}\,\vec{K^o} = \vec{p^o}\,\underset{\sim}{\vec{p^o}} - \underset{\sim}{\vec{l_2^o}}\,\vec{p^o} - \vec{l_2^o}\,\underset{\sim}{\vec{p^o}} + \vec{l_2^o}\,\underset{\sim}{\vec{l_2^o}}$

(2)* $\underset{\sim}{\vec{K^o}}\,\vec{K^o} = \vec{p^1}\,\underset{\sim}{\vec{p^1}} - \vec{p^1}\,\underset{\sim}{\vec{l_2^o}}\,e^{-i\theta_2} - \underset{\sim}{\vec{p^1}}\,\vec{l_2^o}\,e^{i\theta_2^{01}} + \vec{l_2^o}\,\underset{\sim}{\vec{l_2^o}}$

(2)* $-$ (1)* gives,

$$0 = \vec{p^1}\,\underset{\sim}{\vec{p^1}} - \vec{p^o}\,\underset{\sim}{\vec{p^o}} + \underset{\sim}{\vec{l_2^o}}\left(\vec{p^o} - \vec{p^1}\,e^{-i\theta_2^{01}}\right) + \vec{l_2^o}\left(\underset{\sim}{\vec{p^o}} - \underset{\sim}{\vec{p^1}}\,e^{i\theta_2^{01}}\right)$$

(3)* $\underset{\sim}{\vec{K^o}}\,\vec{K^o} = \vec{p^2}\,\underset{\sim}{\vec{p^2}} - \vec{p^2}\,e^{-i\theta_2^{02}}\,\underset{\sim}{\vec{l_2^o}} - \underset{\sim}{\vec{p^2}}\,e^{i\theta_2^{02}}\,\vec{l_2^o} + \vec{l_2^o}\,\underset{\sim}{\vec{l_2^o}}$.

(4)* $\left(\underset{\sim}{\vec{l_2^o}}\,c_1 + \vec{l_2^o}\,\underset{\sim}{c_1} = \vec{p^1}\,\underset{\sim}{\vec{p^1}} - \vec{p^o}\,\underset{\sim}{\vec{p^o}}\right) c_2,$

$$c_1 = \vec{p^1}\,e^{-i\theta_2^{01}} - \vec{p^o}$$

$$c_2 = \vec{p^2}\,e^{-i\theta_2^{02}} - \vec{p^o}$$

(3)* $-$ (1)* gives,

(5)* $\left(\underset{\sim}{\vec{l_2^o}}\,c_2 + \vec{l_2^o}\,\underset{\sim}{c_2} = \vec{p^2}\,\underset{\sim}{\vec{p^2}} - \vec{p^o}\,\underset{\sim}{\vec{p^o}}\right) c_1$

$c_2(4) - c_1(5)$,

$$l_2^o\left(c_2\,\underset{\sim}{c_1} - c_1\,\underset{\sim}{c_2}\right) = \vec{p^o}\,\underset{\sim}{\vec{p^o}}\,(c_1 - c_2) + \vec{p^1}\,\underset{\sim}{\vec{p^1}}\,c_2 - \vec{p^2}\,\underset{\sim}{\vec{p^2}}\,c_1$$

$$l_2^o = \frac{\vec{p^o}\,\underset{\sim}{\vec{p^o}}\,(c_1 - c_2) + c_2\,\vec{p^1}\,\underset{\sim}{\vec{p^1}} - c_1\,\vec{p^2}\,\underset{\sim}{\vec{p^2}}}{\underset{\sim}{c_1}\,c_2 - \underset{\sim}{c_2}\,c_1} \rightarrow (9)$$ [Solved in Matlab]

$$\vec{K_o} = \vec{p^o} - \vec{l_2^o}$$

$$e^{i\theta_3^{01}} = \frac{\vec{p^1} - \vec{l_2^o}\,e^{i\theta_2^{01}}}{\vec{k_3^o}}$$

$$e^{i\theta_3^{02}} = \frac{\vec{p^2} - \vec{l_2^o}\,e^{i\theta_2^{02}}}{\vec{k_3^o}}$$

From the Dyad 2 equation, we can write,

$$\vec{l_1} + \vec{l_4^0} + \vec{w_3^0} = \vec{p^0} \longrightarrow (6)$$

$$\vec{l_1} + \vec{l_4^1} + \vec{w_3^1} = \vec{p^1} \longrightarrow (7)$$

$$\vec{l_1} + \vec{l_4^2} + \vec{w_3^2} = \vec{p^2} \longrightarrow (8)$$

(7) - (6) gives,

$$(e^{i\theta_4^{01}} - 1)\vec{l_4^0} + (e^{i\theta_3^{01}} - 1)\vec{w_3^0} = \vec{p^1} - \vec{p^0}$$

(8) - (6) gives,

$$(e^{i\theta_4^{02}} - 1)\vec{l_4^0} + (e^{i\theta_3^{02}} - 1)\vec{w_3^0} = \vec{p^2} - \vec{p^0}$$

Let $A_2 = \begin{bmatrix} a_4^{01} & a_3^{01} \\ a_4^{02} & a_3^{02} \end{bmatrix}$, where $a_4^{01} = e^{i\theta_4^{01}} - 1$.

we can write the others similarly.

Now, we can combine the ~~other~~ above eq^{n}s and write them as,

$$\begin{bmatrix} \vec{l_4^0} \\ \vec{w_3^0} \end{bmatrix} = A_2^{-1} \begin{bmatrix} \vec{p^1} - \vec{p^0} \\ \vec{p^2} - \vec{p^0} \end{bmatrix} \longrightarrow (10) \quad \begin{bmatrix} \text{Solved in} \\ \text{Matlab} \end{bmatrix}$$

Assumptions:

$P_0 = (13, 12)$, $P^1 = (6, 12)$, $P^2 = (-1, 12)$

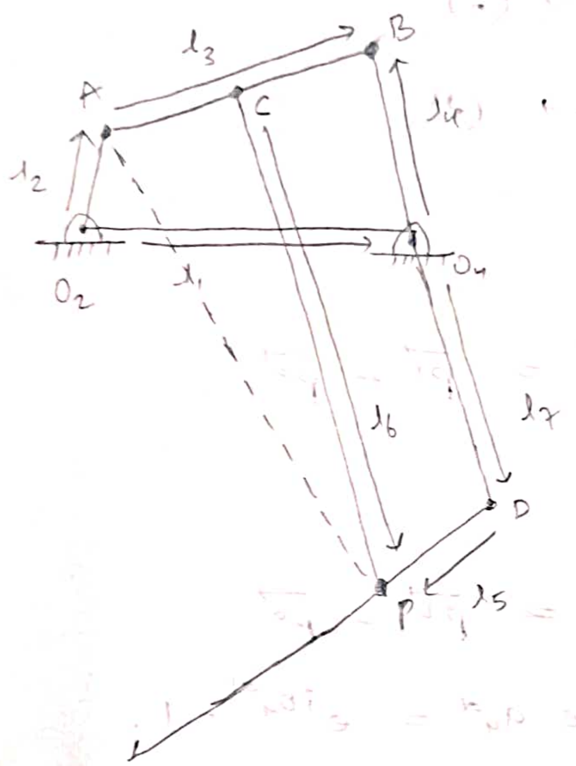
$\theta_2^0 = 59°$, $\theta_2^1 = 180°$, $\theta_2^2 = 294°$

$\theta_4^0 = 80.05°$, $\theta_4^1 = 120.86°$, $\theta_4^2 = 141.37°$

$\Rightarrow \theta_2^{01} = 121°$, $\theta_2^{02} = 235°$

$\theta_4^{01} = 40.81°$, $\theta_4^{02} = 61.32°$

Synthesis Mechanism 2:



Writing loop closure eqn,

$$\vec{l_2} + \vec{l_3} = \vec{l_1} + \vec{l_4}$$

Loop closure for Dyad 1,

$$\vec{l_2} + \frac{\vec{l_3}}{2} + \vec{l_6} = \vec{P}$$

Loop closure for Dyad 2,

$$\vec{l_1} + \vec{l_7} + \vec{l_5} = \vec{P}.$$

from diyad-1

$$\vec{l_2}^{\,0} + \frac{\vec{l_3}^{\,0}}{2} + \vec{l_6}^{\,0} = \vec{P}^{\,0} \longrightarrow (1)$$

$$\vec{l_2^{\,1}} + \frac{\vec{l_3^{\,1}}}{2} + \vec{l_6^{\,1}} = \vec{P^{1}} \longrightarrow (2)$$

$$\vec{l_2^{\,2}} + \frac{\vec{l_3^{\,2}}}{2} + \vec{l_6^{\,2}} = \vec{P^{2}} \longrightarrow (3)$$

$$\vec{l_2^{\,3}} + \frac{\vec{l_3^{\,3}}}{2} + \vec{l_6^{\,3}} = \vec{P^{3}} \longrightarrow (4)$$

(2) - (1) gives,

$$(e^{i\theta_2^{01}} - 1)\,\vec{l_2^{\,0}} + (e^{i\theta_3^{01}} - 1)\,\frac{\vec{l_3^{\,0}}}{2} + (e^{i\theta_6^{01}} - 1)\,\vec{l_6} = \vec{P^{1}} - \vec{P^{0}}$$

$\hookrightarrow (5)$

similarly, we can write

$$(e^{i\theta_2^{02}}-1)\,\vec{l_2^{0}} + (e^{i\theta_3^{02}}-1)\,\frac{\vec{l_3^{0}}}{2} + (e^{i\theta_6^{02}}-1)\,\vec{l_6^{0}} = \vec{p^2} - \vec{p^0} \longrightarrow (6)$$

$$(e^{i\theta_2^{03}}-1)\,\vec{l_2^{0}} + (e^{i\theta_3^{03}}-1)\,\frac{\vec{l_3^{0}}}{2} + (e^{i\theta_6^{03}}-1)\,\vec{l_6^{0}} = \vec{p^3} - \vec{p^0} \longrightarrow (7)$$

Let $A_1 = \begin{bmatrix} a_2^{01} & a_3^{01} & a_6^{01} \\ a_2^{02} & a_3^{02} & a_6^{02} \\ a_2^{03} & a_3^{03} & a_6^{03} \end{bmatrix}$, where $a_2^{01} = e^{i\theta_2^{01}} - 1$.

[solved in matlab]

$$\Rightarrow \begin{bmatrix} \vec{l_2^{0}} \\ \vec{l_3^{0}}/2 \\ \vec{l_6^{0}} \end{bmatrix} = A_1^{-1} \begin{bmatrix} \vec{p^1} - \vec{p^0} \\ \vec{p^2} - \vec{p^0} \\ \vec{p^3} - \vec{p^0} \end{bmatrix} \longrightarrow (8)$$

From the Dyad 2 eq^n, we can write,

$$\vec{l_1^0} + \vec{l_7^0} + \vec{l_5^0} = \vec{p^0} \longrightarrow (9)$$

$$\vec{l_1^0} + \vec{l_7^1} + \vec{l_5^1} = \vec{p^1} \longrightarrow (10)$$

$$\vec{l_1} + \vec{l_7^2} + \vec{l_5^2} = \vec{p^2} \longrightarrow (11)$$

$$\vec{l_1} + \vec{l_7^3} + \vec{l_5^3} = \vec{p^3} \longrightarrow (12)$$

(10) − (9) gives,

$$\vec{l_7^0}\left(e^{i\theta_7^{01}} - 1\right) + \vec{l_5^0}\left(e^{i\theta_5^{01}} - 1\right) = \vec{p^1} - \vec{p^0} \longrightarrow (13)$$

(11) − (9) gives,

$$\vec{l_7^0}\left(e^{i\theta_7^{02}} - 1\right) + \vec{l_5^0}\left(e^{i\theta_5^{02}} - 1\right) = \vec{p^2} - \vec{p^0} \longrightarrow (14)$$

(12) − (9) gives,

$$\vec{l_7^0}\left(e^{i\theta_7^{03}} - 1\right) + \vec{l_5^0}\left(e^{i\theta_5^{03}} - 1\right) = \vec{p^3} - \vec{p^0} \longrightarrow (15)$$

Let $A_3 = \begin{bmatrix} a_7^{01} & a_5^{01} \\ a_7^{02} & a_5^{02} \end{bmatrix}$, where $a_7^{01} = e^{i\theta_7^{01}} - 1$.

$$\Rightarrow \begin{bmatrix} \vec{l_7^0} \\ \vec{l_5^0} \end{bmatrix} = A_3^{-1} \begin{bmatrix} \vec{p^1} - \vec{p^0} \\ \vec{p^2} - \vec{p^0} \end{bmatrix} \longrightarrow (\ ) \quad \begin{bmatrix} \text{Solved in} \\ \text{Matlab} \end{bmatrix}$$

Similarly, let $A_4 = \begin{bmatrix} a_7^{01} & a_5^{01} \\ a_7^{03} & a_5^{03} \end{bmatrix}$, ~~where~~

$$\Rightarrow \begin{bmatrix} \vec{l_7^0} \\ \vec{l_5^0} \end{bmatrix} = A_4^{-1} \begin{bmatrix} \vec{p^1} - \vec{p^0} \\ \vec{p^3} - \vec{p^0} \end{bmatrix} \longrightarrow (\ ) \quad \begin{bmatrix} \text{Solved in} \\ \text{Matlab} \end{bmatrix}$$

Assumptions :

$p^0 = (7.248, -20.341)$, $p^1 = (14.209, -17.698)$

$p^2 = (20.067, -15.26)$, $p^3 = (15.256, -20.039)$

$\theta_7^{01} = 0.539°$, $\theta_7^{02} = 0.883°$, $\theta_7^{03} = 0.372°$.

$\theta_5^{01} = 0.1°$, $\theta_5^{02} = 0.459°$, $\theta_5^{03} = 0.418°$.

$\theta_2^{01} = 1.65°$, $\theta_2^{02} = 3.29°$, $\theta_2^{03} = 5.13°$

$\theta_3^{01} = -0.273°$, $\theta_3^{02} = 0.099°$, $\theta_3^{03} = 0.454°$

$\theta_6^{01} = 0.461°$, $\theta_6^{02} = 0.809°$, $\theta_6^{03} = 0.378°$.