2 Задание

Собственные интегралы, зависящие от параметра

1 неделя

§13

(2.4) $\lim\limits_{\alpha\to 1}\int_0^1 \underbrace{x^2 e^{\alpha x^3}}_{f(x,\alpha)}dx \ominus$

$f(x,\alpha)$ непрерывна в $\Pi = \{(x,\alpha) : 0\le x\le 1 ; \alpha_1 \le \alpha \le \alpha_2\} \Rightarrow$

$\ominus \int_0^1 \lim\limits_{\alpha\to 1} x^2 e^{\alpha x^3} dx = \int_0^1 x^2 e^{x^3} dx = \frac{1}{3}\int_0^1 e^{x^3} d(x^3) =$

$= \frac{1}{3}\left(e^{x^3}\right)\Big|_0^1 = \frac{1}{3}\left(e^1 - e^0\right) = \frac{e-1}{3}$

(14.4) $\Phi(\alpha) = \int_{3\alpha}^{\alpha^2} e^{\alpha x^2}dx \qquad \Phi'(\alpha) - ?$

$f(x,\alpha) = e^{\alpha x^2}$ непрерывна в $\Pi = \{(x,\alpha) : 3\alpha \le x \le \alpha^2 ; \alpha_1 \le \alpha \le \alpha_2\}$

$\frac{\partial f}{\partial \alpha}(x,\alpha) = x^2 e^{\alpha x^2}$ непрерывна в Π

$\varphi(\alpha) = 3\alpha$ диф-ма на $[\alpha_1;\alpha_2]$

$\psi(\alpha) = \alpha^2$ диф-ма на $[\alpha_1;\alpha_2]$

$\Rightarrow \Phi'(\alpha) = f(\psi(\alpha);\alpha)\,\psi'(\alpha) - f(\varphi(\alpha);\alpha)\,\varphi'(\alpha) + \int_{\varphi(\alpha)}^{\psi(\alpha)} \frac{\partial f}{\partial \alpha}(x,\alpha)\,dx =$

$= e^{\alpha^5}\cdot 2\alpha - e^{9\alpha^3}\cdot 3 + \int_{3\alpha}^{\alpha^2} x^2 e^{\alpha x^2}dx$

Не совпадает с ответом в задачнике

(17) $I(\alpha) = \int_0^b \frac{dx}{x^2+\alpha^2} \quad \alpha > 0 \qquad \int_0^b \frac{dx}{(x^2+\alpha^2)^2}$

вып $f(x,\alpha) = \frac{1}{x^2+\alpha^2}$ непр в $\Pi = \{(x,\alpha) : 0\le x\le b ; \alpha_1 \le \alpha \le \alpha_2\}$ где $0<\alpha_1<\alpha_2$

$\frac{\partial f}{\partial \alpha}(x,\alpha) = \frac{-2\alpha}{(x^2+\alpha^2)^2}$ непр в Π $\Rightarrow I'(\alpha) = \int_0^b \frac{-2\alpha}{(x^2+\alpha^2)^2}dx \Rightarrow \int_0^b \frac{dx}{(x^2+\alpha^2)^2} = \frac{I'(\alpha)}{-2\alpha}$

$$I(\alpha) = \int_0^b \frac{dx}{x^2+\alpha^2} = \frac{1}{\alpha}\,\mathrm{arctg}\,\frac{x}{\alpha}\Big|_0^b = \frac{1}{\alpha}\,\mathrm{arctg}\,\frac{b}{\alpha}$$

$$\Rightarrow I'(\alpha) = -\frac{1}{\alpha^2}\,\mathrm{arctg}\,\frac{b}{\alpha} + \frac{1}{\alpha}\cdot\frac{1}{1+\frac{b^2}{\alpha^2}}\cdot\left(-\frac{b}{\alpha^2}\right) = -\frac{1}{\alpha^2}\,\mathrm{arctg}\,\frac{b}{\alpha} - \frac{b}{\alpha}\cdot\frac{1}{\alpha^2+b^2}$$

$$\Rightarrow \int_0^b \frac{dx}{(x^2+\alpha^2)^2} = \frac{I'(\alpha)}{-2\alpha} = \frac{1}{2\alpha^3}\,\mathrm{arctg}\,\frac{b}{\alpha} + \frac{b}{2\alpha^2(\alpha^2+b^2)}$$

$= \int_0^{\pi}$

18.3 $$I(\alpha) = \int_0^{\pi} \underbrace{\ln\frac{1+\alpha\cos x}{1-\alpha\cos x}\cdot\frac{dx}{\cos x}}_{f(x,\alpha)},\quad |\alpha|<1$$

$f(x,\alpha)$ имеет разрыв в $x=\frac{\pi}{2}$. Исследуем его

$$\lim_{x\to\frac{\pi}{2}} \ln\frac{1+\alpha\cos x}{1-\alpha\cos x}\cdot\frac{1}{\cos x} = \lim_{x\to\frac{\pi}{2}}\left[\ln\left(1+\frac{2\alpha\cos x}{1-\alpha\cos x}\right)\cdot\frac{1}{\cos x}\right] = \lim_{x\to\frac{\pi}{2}} \frac{2\alpha\cancel{\cos x}}{(1-\alpha\cos x)\cancel{\cos x}} = 2\alpha$$

$\ln(1)=0$

разрыв 1го рода $\Rightarrow$ устраним его доопределив ф-цию $f(x,\alpha)$ в $x=\frac{\pi}{2}$ 2α

Теперь $f(x)$ непр. в $\Pi = \{(x,\alpha): 0\le x\le\pi;\ \alpha_1\le\alpha\le\alpha_2\}$ $-1<\alpha_1<\alpha_2<1$

$\frac{\partial f}{\partial\alpha}(x,\alpha) \textcircled{=}$ $\cancel{\frac{1-\alpha\cos x}{1+\alpha\cos x}\cdot\left(\frac{\cos x}{1-\alpha\cos x}+\frac{(1+\alpha\cos x)\cos x}{(1-\alpha\cos x)^2}\right)}$

$\cancel{\frac{\cos^2 x}{(1-\alpha\cos x)^2}}$ непрерывна в Π

$$\textcircled{=}\ \frac{1}{\cos x}\,\frac{1-\alpha\cos x}{1+\alpha\cos x}\cdot\left[\frac{\cos x}{1-\alpha\cos x} + \frac{(1+\alpha\cos x)\cdot\cos x}{(1-\alpha\cos x)^2}\right] =$$

$$= \frac{\cos x}{1+\alpha\cos x} + \frac{\cos x}{1-\alpha\cos x} = \frac{2\cancel{\cos x}}{1-\alpha^2\cos^2 x}\cdot\frac{1}{\cancel{\cos x}} = \frac{2}{1-\alpha^2\cos^2 x}$$

$$\Rightarrow I'(\alpha) = \int_0^{\pi}\frac{\partial f}{\partial\alpha}(x,\alpha)\,dx = \int_0^{\pi}\frac{2}{1-\alpha^2\cos^2 x}\,dx = \left|\begin{array}{l} u = \mathrm{tg}\,x \\ du = \frac{1}{\cos^2 x}dx \Rightarrow dx = \cos^2 x\,du \\ \cos^2 x = \frac{1}{u^2+1}\end{array}\right.$$

$$= \int_0^{\infty} \frac{2\cancel{(u^2+1)}}{u^2-\alpha^2+1}\cdot\frac{1}{\cancel{u^2+1}}\,du = \frac{1}{2}\cdot 4\int_0^{\infty} \frac{du}{u^2+(1-\alpha^2)} = 4\,\frac{1}{\sqrt{1-\alpha^2}}\left(\operatorname{arctg}\frac{u}{\sqrt{1-\alpha^2}}\right)\Bigg|_0^{\infty} = \frac{2\pi}{\sqrt{1-\alpha^2}}$$

$$\Rightarrow\ I(\alpha) = \int \frac{2\pi}{\sqrt{1-\alpha^2}}\,d\alpha = 2\pi \arcsin\alpha + C$$

$$I(0) = 0 \Rightarrow C = 0 \Rightarrow I(\alpha) = 2\pi\cdot\arcsin\alpha$$