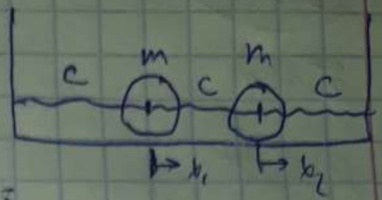
(19.48)

ур. Гам. — ?

Решение



$$T = \frac{3}{4} m \dot{x}_1^2 + \frac{3}{4} m \dot{x}_2^2$$

$$\Pi = \frac{c x_1^2}{2} + \frac{c(x_2 - x_1)^2}{2} + \frac{c x_2^2}{2} =$$

$$= c x_1^2 + c x_2^2 - c \cdot x_1 x_2 =$$

$$L = \frac{3}{4} m \dot{x}_1^2 + \frac{3}{4} m \dot{x}_2^2 - c x_1^2 - c x_2^2 + c x_1 x_2$$

$$p_1 = \frac{\partial L}{\partial \dot{x}_1} = \frac{3}{2} m \dot{x}_1 \Rightarrow \dot{x}_1 = \frac{2}{3m} \cdot p_1$$

$$p_2 = \frac{\partial L}{\partial \dot{x}_2} = \frac{3}{2} m \cdot \dot{x}_2 \Rightarrow \dot{x}_2 = \frac{2}{3m} p_2$$

Функция Гамильтона:

$$H = p_1 \dot{x}_1 + p_2 \dot{x}_2 - L = \frac{3}{2} m \dot{x}_1^2 + \frac{3}{2} m \dot{x}_2^2 - \frac{3}{4} m \dot{x}_1^2 - \frac{3}{4} m \dot{x}_2^2 +$$

$$+ c x_1^2 + c x_2^2 - c x_1 x_2 = \underbrace{\frac{3}{4} m \dot{x}_1^2}_{\frac{p_1^2}{3m}} + \underbrace{\frac{3}{4} m \dot{x}_2^2}_{\frac{p_2^2}{3m}} + c x_1^2 + c x_2 (x_2 - x_1)$$

Ур. Гамильтона:

$$\dot{x}_1 = \frac{\partial H}{\partial p_1} = \frac{2 p_1}{3m} \qquad \ddot{x}_2 = \frac{\partial H}{\partial p_2} = \frac{2 p_2}{3m}$$

$$\dot{p}_1 = -\frac{\partial H}{\partial x_1} = -2c x_1 + c x_2 = -c x_1 + c(x_2 - x_1) \qquad \dot{p}_2 = -\frac{\partial H}{\partial x_2} = -2c x_2 + c x_1 = -c x_2 + c(x_1 - x_2)$$

23.35в

$\tilde{q} = q$

$\tilde{p} = \frac{p^3}{6}$

$H = q e^{pt}$

$$H = q e^{pt}$$

$$\begin{cases} \tilde{q} = q \\ \tilde{p} = \frac{p^3}{6} \end{cases} \Leftrightarrow \begin{cases} q = \tilde{q} \\ p = \sqrt[3]{6\tilde{p}} \end{cases}$$

$$\begin{cases} \dot{\tilde{q}} = \frac{\partial H}{\partial p}(q, p) = qt e^{pt} = \tilde{q} \cdot e^{(\sqrt[3]{6\tilde{p}} \cdot t)} \\ \dot{\tilde{p}} = \frac{p^2}{2} \cdot \dot{p} = \frac{p^2}{2}\left(-\frac{\partial H}{\partial q}\right) = -\frac{p^2}{2} \cdot e^{pt} = \end{cases}$$

$$= -\sqrt[3]{\frac{9}{2}\tilde{p}^2} \cdot e^{(\sqrt[3]{6\tilde{p}} \cdot t)}$$

Проверим систему на Гамильтоновость:

$$\frac{\partial \tilde{H}}{\partial \tilde{p} \partial \tilde{q}} \stackrel{?}{=} \frac{\partial \tilde{H}}{\partial \tilde{q} \partial \tilde{p}}$$

$$-\frac{\partial \dot{\tilde{p}}}{\partial \tilde{p}} \neq \frac{\partial \dot{\tilde{q}}}{\partial \tilde{q}} = 1 \cdot e^{(\sqrt[3]{6\tilde{p}} \cdot t)}$$

$$-\left(\frac{2}{3} \cdot \frac{-\sqrt[3]{\frac{9}{2}}}{\sqrt[3]{\tilde{p}}} \cdot e^{(\sqrt[3]{6\tilde{p}} \cdot t)} + -\sqrt[3]{\frac{9}{2}\cancel{\tilde{p}^2}} \cdot \left(\frac{1}{3}\sqrt[3]{6} \cdot \frac{1}{\sqrt[3]{\cancel{\tilde{p}^2}}}\right) e^{(\sqrt[3]{6\tilde{p}} \cdot t)}\right) =$$

$$= \left(\frac{2}{3}\sqrt[3]{\frac{9}{2}} \cdot \frac{1}{\sqrt[3]{\tilde{p}}} + \frac{1}{3}\sqrt[3]{6} \cdot \sqrt[3]{\frac{9}{2}}\right) e^{(\sqrt[3]{6\tilde{p}} \cdot t)}$$

$$\sqrt[3]{\frac{4}{6}} \quad \sqrt[3]{\frac{8 \cdot 9}{27 \cdot 2}} \cdot \frac{1}{\sqrt[3]{\tilde{p}}} \quad \sqrt[3]{\frac{6 \cdot 9}{27 \cdot 2}}$$

$$\left(\frac{1}{\sqrt[3]{6\tilde{p}}} + 1\right) e^{(\sqrt[3]{6\tilde{p}} \cdot t)} \neq 1 \cdot e^{(\sqrt[3]{6\tilde{p}} \cdot t)}$$

$$\frac{\partial \tilde{H}}{\partial \tilde{p} \partial \tilde{q}} \neq \frac{\partial \tilde{H}}{\partial \tilde{q} \partial \tilde{p}}$$

Ответ: $\dot{\tilde{q}} = \tilde{q} \cdot e^{(\sqrt[3]{6\tilde{p}}\, t)}$

$\dot{\tilde{p}} = -\sqrt[3]{\frac{9}{2}\tilde{p}^2} \cdot e^{(\sqrt[3]{6\tilde{p}} \cdot t)}$; система негамильтонова

Ур. Г-Я: $\frac{\partial S}{\partial t} + \frac{1}{2m}(p_x^2 + p_z^2)\left[\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2\right] +$

$+ \frac{6}{ml^2}\cdot\left(\frac{\partial S}{\partial \varphi}\right)^2 - \frac{m(x^2+z^2)}{2}\omega^2 + mgz = 0$

Найти полный интеграл $S = S(t, x, z, \varphi, \alpha_x, \alpha_z, \alpha_\varphi)$

$S = S_0(t, \alpha_x, \alpha_z, \alpha_\varphi) + S_1(x, \dots) + S_2(z, \dots) + S_3(\varphi, \dots)$

Подставим в Г-Я:

$$\frac{\partial S_0}{\partial t} + \frac{1}{2m}\left(\frac{\partial S_1}{\partial x}\right)^2 + \frac{1}{2m}\left(\frac{\partial S_2}{\partial z}\right)^2 + \frac{6}{ml^2}\left(\frac{\partial S_3}{\partial \varphi}\right)^2 - \frac{m(x^2+z^2)\omega^2}{2} + mgz$$

$$\frac{1}{2m}\left(\frac{\partial S_1}{\partial x}\right)^2 = \alpha_x + \frac{mx^2\omega^2}{2} \Rightarrow S_1 = \int\sqrt{2m\alpha_x - (mx\omega)^2}\,dx$$

$$\frac{1}{2m}\left(\frac{\partial S_1}{\partial z}\right)^2 = \alpha_z + \frac{mz^2\omega^2}{2} - mgz \Rightarrow S_2 = \int\sqrt{2m\alpha_z + (mz\omega)^2 - 2m^2gz}\,dz$$

$$\frac{6}{ml^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = \alpha_\varphi \Rightarrow S_3 = \sqrt{\frac{ml^2\alpha_\varphi}{6}}\cdot\varphi$$

$$\frac{\partial S_0}{\partial t} = \alpha - \alpha_x - \alpha_z - \alpha_\varphi \Rightarrow S_0 = -(\alpha_x + \alpha_z + \alpha_\varphi)t$$

$$\Rightarrow S = -(\alpha_x + \alpha_z + \alpha_\varphi)t + \int\sqrt{2m\alpha_x - (mx\omega)^2}\,dx + \int\sqrt{2m\alpha_z + (mz\omega)^2 - 2m^2gz}\,dz + \sqrt{\frac{\alpha_\varphi\cdot ml^2}{6}}$$

— полный интеграл движения

$$\beta_x = \frac{\partial S}{\partial \alpha_x} = -t + \int\frac{\sqrt{m}\cdot dx}{\sqrt{2m\alpha_x - (mx\omega)^2}} \Rightarrow \beta + t = \int\frac{\sqrt{m}\,dx}{\sqrt{2m\alpha_x - (mx\omega)^2}}$$

$$\beta_z = \frac{\partial S}{\partial \alpha_z} = -t + \int\frac{\sqrt{m}\,dz}{\sqrt{2m\alpha_z - (mz\omega)^2 - 2m^2gz}}$$

$$\beta_{\dot\varphi} = \frac{\partial S}{\partial \alpha_\varphi} = -t + \sqrt{\frac{ml^2}{2\alpha_\varphi}}$$