CSC 280 Sichen Pan

2.30 a. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Sol'n: Assume to the contrary, that A is context free. By Pumping Lemma there exists a constant p such that every $w \in A$ of length $\geq P$ can be divided into $w = uvxyz$, such that $|vxy| \leq p$, $|vy| \geq 1$, and for every $i \geq 0$, $uv^ixy^iz \in A$.

Let $w = 0^p 1^p 0^p 1^p$

① if v and y are string that 0^k or 1^k.

then consider a string uv^2xy^2z

which will result in unequal length of 1's and 0's.

② if v or y is string that contains both 0 and 1, in either order, there exist a string uv^3xy^3z that is not in A,

1) $v = 1^k0^m$, $y = 0^s$ where $1 \leq k+m+s \leq p$

(2) $v = 0^k1^m$, $y = 1^s$

$uv^3xy^3z = u1^k0^m1^k0^m1^k0^m x 0^s0^s0^s z$.

where the string does not contain the symbol in correct order.

So the string is not a CFL.

2.30. d. $\{t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2$, each $t_i \in \{a,b\}^*$, and $t_i = t_j$ for some $i \neq j\}$

Soln: Assume to the contrary, that A is context free. By pumping Lemma, there exists a constant p such that every $w \in A$ of length $\geq p$ can be divided into $w = uvxyz$ such that $|vxy| \leq p$, $|vy| \geq 1$ and for every $i \geq 0$, $uv^ixy^iz \in A$

consider $k=2$, and a string $S = a^p b^p \# a^p b^p$

① if v and y are nonempty string that contains a hash tag. ~~(1) if v contains a hash tag~~ either v or y contains a hash will result in the string uv^2xy^2z not in ~~the~~ A.

if $v = b^k \#$, $y = a^m$, here $k \leq p$, in uv^2xy^2z

$a^p b^p \# b^k \# \ldots$ $t_1 \neq t_2$.

$v = b^k \# a^m$, $y = a^n$, in uv^2xy^2z ~~$a^p b^p$~~ $a^p b^p \# a^m b^k \# a$

$m+k \leq p$, so $t_1 \neq t_2$.

in the same way we could conclude that all string with a hash tag considering uv^2xy^2z is not in A.

② if v and y are non empty string that does not contain hashtag, then considering string $a^p b^p \# a^p b^p$.

Either if (vy) is to the right or to the left of "#", the string uv^2xy^2z will have different string of t_1 and t_2.

③. if v or y is empty, and the nonempty one contains a hash, considering a string uv^0xy^0z where the string has no hash tag and $k < 2$.

and $t_i = t_j$

④ if v and y are ~~both~~ both not containing "#" and one of them is ~~non~~ empty. then considering uv^2xy^2z where $t_1 \neq t_2$.

2.32. $\Sigma = \{1,2,3,4\}$, $C = \{w \in \Sigma^* \mid$ in w. number of 1 $\overset{\text{numb. 4}}{=2}$, number of 3 = number of 4 $\}$

Sol'n: Assume to the contrary, that C is context free. By pumping lemma, there exists a constant p such that every $w \in C$ of length $\geq p$ can be divided into $w = uvxyz$ such that $|vxy| \leq p$, $|vy| \geq 1$, and for every $i \geq 0$, $uv^ixy^iz \in A$.

let $w =$ ~~[illegible]~~ $1^p 3^p 2^p 4^p$

then if $|vxy|$ contains a symbol $k \in \{1,2,3,4\}$, then considering the string uv^2xy^2z,

Since $|vxy| \leq p$, if vxy contains 1, then vxy cannot contains 3. Vice versa.

if vxy contains 2, then vxy cannot contain 4. Vice versa.

So in uv^2xy^2z, the number of 1 and 2, or the number of 3 and 4 cannot be equal in at least one ~~each~~ of the case. So, C is not a CFL.