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Q1] pg 1

$x_1 \quad x_2 \longrightarrow$ No of items 1 & 2
$c_1 \quad c_2 \longrightarrow$ cost of items 1 & 2
$S_1 \quad S_2 \longrightarrow$ Selling price of — " —
$r_1 \quad r_2 \longrightarrow$ reduced price of — " —

$$H_{D_1D_2}(t_1 t_2) = P(D_1 \leq t_1 , D_2 \leq t_2)$$

Joint density is,

$$h_{D_1D_2}(t_1 t_2) = \begin{cases} k(\alpha t_1 + \beta t_2^2) & \text{if } 0 \leq t_1 \leq u_1 \text{ \& } 0 \leq t_2 \leq u_2 \\ 0 & \text{otherwise} \end{cases}$$

where α, β, k are given positive parameters.

probability of $D_1 \in [l_1, u_1]$ and $D_2 \in [l_2, u_2]$ is,

$\geq 0?$ $\geq 0?$

$$P(l_1 \leq D_1 \leq u_1 , l_2 \leq D_2 \leq u_2)$$

demand can't be negative

$$= \int_{t_1=l_1}^{u_1} \int_{t_2=l_2}^{u_2} h_{D_1,D_2}(t_1,t_2)\, dt_1\, dt_2 .$$

$$\int_{t_1=l_1}^{u_1} \int_{t_2=l_2}^{u_2} k(\alpha t_1 + \beta t_2^2)\, dt_1 dt_2$$ now lowerbound of demand can't be negative hence we can say $l_1 = l_2 = 0$

$$\therefore \int_0^{u_1} \int_0^{u_2} k(\alpha t_1 + \beta t_2^2)\, dt_1 dt_2 = 1 \quad \text{—— (1)}$$

Integral over its support for density func is 1 (normalized)

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pg 2

∴ We need to solve for

$$\int_0^{u_1}\int_0^{u_2} k(\alpha t_1 + \beta t_2^2)\, dt_1\, dt_2 = 1$$

→ Let solve for t_1, dt_2 will be treated like const.

$$k\int_0^{u_1} (\alpha t_1 + \beta t_2^2)\, dt_1 = k\left[\frac{\alpha t_1^2}{2} + \beta t_2^2 t_1\right]_0^{u_1}$$

$$= k\left(\frac{\alpha u_1^2}{2} + \beta t_2^2 u_1 - 0\right) = k\left(\frac{\alpha u_1^2}{2} + \beta t_2^2 u_1\right)$$

→ Now we consider for t_2 & solve while considering dt_2

$$k\int_0^{u_2}\left(\frac{\alpha u_1^2}{2} + \beta t_2^2 u_1\right) dt_2 = k\left[\frac{\alpha u_1^2 t_2}{2} + \beta u_1 \frac{t_2^3}{3}\right]_0^{u_2}$$

$$= k\left(\frac{\alpha u_1^2 u_2}{2} + \frac{\beta u_1 u_2^3}{3} - 0\right) = k\left(\frac{\alpha u_1^2 u_2}{2} + \frac{\beta u_1 u_2^3}{2}\right)$$

—— (2)

from eq^n (1) & (2) we get,

$$k\left(\frac{\alpha u_1^2 u_2}{2} + \frac{\beta u_1 u_2^3}{3}\right) = 1$$

$$\boxed{k = \frac{6}{u_1 u_2 (3\alpha u_1 + 2\beta u_2^2)}}$$

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Now let us Solve for the given region in probability

i.e (l_1, u_1) and (l_2, u_2) while later using value of 'K' which we obtain by using probability density func ie normalization

Why? → Our problem has nothing to do with 'K' (our ~~final~~ soln shouldnt have K)

$$\int_{l_1}^{u_1}\int_{l_2}^{u_2} K(\alpha t_1 + \beta t_2^2)\, dt_1 dt_2$$

Solving with respect to dt_1

$$K\int_{l_1}^{u_1} (\alpha t_1 + \beta t_2^2)\, dt_1$$

$$= K\left[\frac{\alpha t_1^2}{2} + \beta t_2^2 t_1\right]_{l_1}^{u_1}$$

$$= K\left(\frac{\alpha u_1^2}{2} + \beta t_2^2 u_1 - \frac{\alpha l_1^2}{2} - \beta t_2^2 l_1\right)$$

$$= K\left(\frac{\alpha(u_2^2 - l_1^2)}{2} + \beta t_2^2(u_1 - l_1)\right)$$

Now solving wrt to dt_2

$$K\int_{t_2}^{u_2}\left[\frac{\alpha(u_2^2 - l_1^2)}{2} + \beta t_2^2(u_1 - l_1)\right] dt_2$$

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$$= K\left[\frac{\alpha(u_1^2-l_1^2)}{2}t_2 + \beta(u_1-l_1)\frac{t_2^3}{3}\right]_{l_2}^{u_2}$$

Now we substitute value of K and solve this

$$= K\left[\frac{\alpha(u_1^2-l_1^2)}{2}u_2 + \beta(u_1-l_1)\frac{u_2^3}{3} - \frac{\alpha(u_1^2-l_1^2)}{2}l_2 - \beta(u_1-l_1)\frac{l_2^3}{3}\right]$$

$$\cancel{= \frac{6}{u_1u_2(3\alpha u_1+2\beta u_2^3)}\left(\text{'T'}\right)}$$

$$= \frac{6}{u_1u_2(3\alpha u_1+2\beta u_2^3)}\left[\frac{\alpha(u_1^2-l_1^2)u_2}{2} + \beta(u_1-l_1)\frac{u_2^3}{2} - \frac{\alpha(u_1^2-l_1^2)l_2}{2} - \beta(u_1-l_1)\frac{l_2^3}{3}\right]$$

$$= \frac{(u_1^2-l_1^2)(3\alpha u_2-3\alpha l_2)+(u_1-l_1)(2\beta u_2^3-2\beta l_2^3)}{3\alpha u_1^2u_2+2\beta u_1u_2^3} \quad \text{---} (*)$$

Now we solve for $P(D_1 \le x_1^*)$

$l_1 = 0$, $u_1 = x_1^*$, $l_2 = 0$, $u_2 = u_2$ substitute in $(*)$

and we know $P(D_i \le x_i^*) = \left(\frac{S_i - C_i}{S_i - r_i}\right)$ — (pg 25 chpt 1)

$$\cancel{(x_1^*)(3\alpha u_2)(S_1-r_1)+}$$

After substituting,

$$\frac{(x_i^*)(3\alpha u_2) + (x_i^*)(2\beta u_2^3)}{3\alpha u_i^2 u_2 + 2\beta u_1 u_2^3} = \frac{S_1 - c_1}{S_1 - r_1}$$

$$3\alpha u_2 x_i^{*2}(S_1 - r_1) + 2\beta u_2^3 x_i^*(S_1 - r_1)$$

$$= 3\alpha u_i^2 u_2 (S_1 - c_1) + 2\beta u_1 u_2^3 (S_1 - c_1)$$

~~Dividing group u_2~~

Dividing both sides by u_2 we get,

$$3\alpha x_i^{*2}(S_1 - r_1) + 2\beta u_2^2 x_i^*(S_1 - r_1) = 3\alpha u_i^2 (S_1 - c_1) + 2\beta u_2^2 u_1 (S_1 - c_1)$$

$$(S_1 - r_1)\left[3\alpha x_i^{*2} + 2\beta u_2^2 x_i^*\right] = (S_1 - c_1)\left[3\alpha u_i^2 + 2\beta_2^2 u_1\right]$$

$$\boxed{\therefore x_1^* = u_1 \sqrt{\frac{S_1 - c_1}{S_1 - r_1}}}$$

Similarly we can solve for x_2^*

$$\boxed{x_2^* = u_2 \sqrt{\frac{S_2 - c_2}{S_2 - r_2}}}$$

(2) Turbines

- Write two stage stochastic programming model

i) One stage model (first stage)

We keep fixed installation costs i.e we just install the turbines

ii) Second stage
↳ we minimize i.e consider the maintainence cost & outsourcing cost for scenario ω depending on time period

(Ans)

$$\min, \underbrace{\sum_{k\in K} f_k x_k}_{\text{first stage}} + \underbrace{E_\omega \left[\sum_{t\in T} \left(\sum_{k\in K} V_k z_{kt}(\omega) + O \cdot O_t(\omega) \right) \right]}_{\text{Recourse func}}$$

↳ $Q(x, \omega)$

↳ $E_\omega[Q(x,\omega)]$

St,

1 → Demand fulfillment constraint:

$$\sum_{k\in K} g_{tk}(\omega) z_{kt}(\omega) + O_t(\omega) \geq d_t(\omega)$$

2 → Turbine working constraint:

$$z_{kt}(\omega) \leq x_k$$

3 → Non negativity constraint:

$$x_k \geq 0 \quad \forall k \in K$$

Decision Variables & parameters

→ $x_k \in \mathbb{Z}_{\geq 0}$: No of turbines of type $k \in K$ installed

→ f_k : Installation cost per turbine of type $k \in K$

→ $O_t(\omega)$: Amount of electricity outsourced in mwh

→ o : Unit cost of outsourced electricity

→ $z_{kt}(\omega) \in \{0, 1\}$: If turbine k operates in period t under scenario ω then 1

Otherwise it sets to 0

→ $\bar{q}_{tk} = E[q_{tk}(\omega)]$: Expected electricity generated by turbine k in period t for scenario (ω)

→ $\bar{d}_t : E[d_t(\omega)]$: Expected demand of electricity in period t for scenario ω

Constraint Explanation

(1) Total electricity generated + outsourced = demand

(2) Turbines can only be operated if installed

(3) Cannot install negative number of turbine.

Explanation & approach :

According to lecture notes from class 2, the first stage needs fixed things i.e objects that need to be fixed later the 2nd Stage will provide feedback.

Keeping this in mind, I first set up turbines since its illogical for the government to set up new turbines in every difference scenario which is dependent on time.

Hence keeping this & the question in mind the things that will vary is the demand for electricity, also due to weather the electricity generated varies.

If turbines cant produce sufficient electricity we need to outsource it. So time dependent factors were considered in the feedback funcⁿ (Recourse func) they were ~~elect~~ maintenance cost since it depends on turbines which we are using & outsourced energy. Also constraints for demand were varying so it was considered with time & scenario.

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(3) Wildfire (Robust opti)

We will Structure it as a two stage model where
first Stage : decision before disaster → plant location &
Chemical inventory

Second Stage: decision after disaster → Water treatment
Operations

We need to ensure water demand is met in worst
case scenario hence we will choose robust optimiz-
-ation.

Objective fun:

$$\min \underbrace{\sum_{i\in I}\sum_{k\in K} G_{ik} x_{ik} + \sum_{i\in I}\sum_{p\in P} C_p U_{ip}}_{\text{first stage}} + \underbrace{\max_{\xi\in \Xi} Q(x, U, \xi)}_{\text{Recourse}}$$

(4) Constraints

(1) Single plant type per location:

$$\sum_{k\in K} x_{ik} \le 1 \qquad \forall i \in I$$

(2) Chemical Storage capacity:

$$U_{ip} \le \sum_{k\in K} L_{kp} x_{ik} \qquad \forall i\in I,\ p\in P$$

→ Treatment Capacity :

$$w_i(\xi) \le m_i a_i(\xi) \sum_{k\in K} x_{ik} \quad \forall i \in I$$

→ Chemical usage balance :

$$\sum_{i\in I} r_p w_i(\xi) \le \sum_{i\in I} y_{ip}(\xi) \quad \forall p \in P$$

→ Chemical inventory :

$$y_{ip}(\xi) \le U_{ip} \quad \forall i \in I, \; p \in P$$

→ Demand satisfaction : (with penalty)

$$\sum_{i\in I} w_i(\xi) + z(\xi) \ge D$$

$$z(\xi) \ge 0$$

→ Uncertainity Set :

$$\Xi = \left\{ \xi : \sum_{i\in I} (1 - a_i(\xi)) \le \Gamma, \; a_i(\xi) \in \{0,1\} \right\}$$

Recourse func :

$$Q(x, U, \xi) = \min_{w, y, z} \left\{ \sum_{i\in I} \sum_{p\in P} C_p^{c} y_{ip}(\xi) + \sum_{i\in I} C_i^{t} w_i(\xi) + \pi z(\xi) \right.$$

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★ First Stage : (decision made before disaster)

Variables :

(1) $x_{ik} = \begin{cases} 1, \text{ if a plant of type } k \text{ is built at location } i \\ 0, \text{ otherwise} \end{cases}$

(2) U_{ip} = Inventory level of chemical p stored at location i

Parameters :

(1) G_k : fixed cost of installing plant of type k

(2) C_p : Unit storage cost of chemical p

(3) L_{kp} : Max storage capacity for chemical p in a plant of type k

★ Second stage : (After observing disruptions, cause of disaster)
↳ Scenario dependent

Variables :

(1) $w_i(\xi)$: Amount of water treatment done at plant i observed under Scenario ξ

(2) $y_{ip}(\xi)$: Amount of chemical p used at plant i under scenario ξ.

(3) $z(\xi)$: Penalty for not meeting water demand in scenario ξ.

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Parameters:

1. C_p^o : Operational cost / unit of chemical p used
2. C_i^t : Treatment cost of water at plant i
3. π : Penalty cost per unit of unmet water demand
4. r_p : Amount of chemical p required per unit of water treated
5. D : Total water demand
6. M_i : Maximum water treatment capacity of plant i

★ For uncertainty set Ξ

1. $a_i(\xi) = \begin{cases} 1, & \text{if plant } i \text{ is operational in } \xi \\ 0, & \text{otherwise} \end{cases}$
2. Γ : Max no. of plants that can fail at once

Explanation & reasoning:

→ Plant location & chemical inventory are fixed since we cant change them after disaster, they can only be 'utilised'. Hence they end up in first stage.

The first stage is divided into two parts: (i.e),

(1) $\sum_{i \in I} \sum_{k \in K} G_k x_{ik}$ representing the fixed installation cost

G_k → Cost of building plant k

x_{ik} → If plant k build in location i = {0,1}.

(2) $\sum_{i \in I} \sum_{p \in P} C_p U_{ip}$ representing inventory cost.

C_p → Unit storage cost of chemical p

U_{ip} → Inventory level of chemical p at location i.

→ The second stage (Recourse func) deals with the aftermath of disaster & scenarios. The aim is the minimise operational cost so,

$$Q(x, U, \xi) = \min \sum_{i \in I} \sum_{p \in P} \text{Operational cost.}$$

This includes water treatment cost, chemical usage cost, and penalty if demand not met!

$Q(x, v, \xi)$ is the recourse func^c

first stage decision x (plant location)

first stage decision v (Chemical inventory)

Scenario ξ picked from uncertainty set Ξ (plants that are functional)

★ Maximum No of Scenarios :

$$|\Xi| = \sum_{k=0}^{r} \binom{n}{k}$$

n = potential location
r = plant that can be affected at most

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Code

[1]:
```python
import numpy as np
from scipy import stats
```

[2]:
```python
np.random.seed(666)
N_SCENARIOS = 15000
```

[3]:
```python
l1 = stats.cauchy.rvs(loc=-2, scale=2, size=N_SCENARIOS)
l2 = stats.poisson.rvs(mu=5, size=N_SCENARIOS)
l3 = np.random.uniform(3, 10, N_SCENARIOS)
x_hat = 16
```

[4]:
```python
def calculate_F(x, l1_val, l2_val, l3_val):
    term1 = np.exp(-np.sin(abs(l1_val * x)))
    term2 = abs(x - l2_val) / np.sqrt(x + l3_val)
    return term1 + term2
```

[5]:
```python
F_values = np.zeros(N_SCENARIOS)
for i in range(N_SCENARIOS):
    F_values[i] = calculate_F(x_hat, l1[i], l2[i], l3[i])
```

[6]:
```python
expected_value = np.mean(F_values)
```

[7]:
```python
print(f"Estimated E[F(x̂,l1,l2,l3)] = {expected_value:.6f}")
```

```
Estimated E[F(x̂,l1,l2,l3)] = 3.588290
```

A Two-Stage Stochastic Programming Model for Assortment Planning Under Demand Uncertainty

1 Introduction and Problem Definition

In retail decision-making, one of the most crucial challenges is determining the right **assortment plan** while dealing with **uncertain customer demand**. Retailers must decide on:

- **Which products to display** in their assortment.
- **How much inventory to allocate** for each product.

However, customer preferences, demand fluctuations, and product substitutions introduce significant **uncertainty**. Making assortment decisions without considering this uncertainty can lead to:

- **Lost sales** if demand for a product is higher than expected.
- **Excess inventory holding costs** if demand turns out to be lower than anticipated.

To address this, we use a **two-stage stochastic programming (TSSP) approach**, where:

- The **first-stage** decisions involve selecting the product assortment and pre-allocating supplies before knowing actual demand.
- The **second-stage** decisions involve dynamically fulfilling demand based on observed customer behavior and inventory constraints.

This framework ensures that the assortment plan is optimized while considering **uncertain demand patterns and customer substitution effects**.

2 Decision Stages and Evolution of Information

The problem is modeled as a two-stage stochastic optimization problem, where information about demand evolves over time. The decision-making process is divided into two stages:

2.1 First Stage: Pre-Decision Before Demand Realization

- **Decisions to be made:**
  - Which products to include in the assortment.
  - How much inventory to allocate for each product.
- **Uncertainty in this stage:**
  - The actual demand for each product is unknown.
  - The probability of customers substituting products if their first choice is unavailable.

At this stage, the retailer has access only to **historical data and probabilistic estimates** of demand but must finalize product assortment and inventory levels before demand is realized.

2.2 Second Stage: Adaptive Response After Demand Realization

- **Decisions to be made:**
  - Assigning available stock to fulfill primary demand.
  - Deciding how much of a given product can be used to fulfill substituted demand.
- **Uncertainty in this stage:**
  - Exact customer preferences are revealed.
  - Some products may experience excess demand, while others may have surplus stock.
  - Shelf space and inventory constraints may affect substitution choices.

At this point, **actual demand has been realized**, but decisions must adapt dynamically within inventory and space constraints.

3 Uncertainty Model Used

This study models uncertainty using **scenario-based stochastic programming**, where demand follows a probability distribution across multiple possible realizations. The key characteristics of the uncertainty model include:

3.1 Discrete Scenario-Based Representation

- Demand for each product is modeled as a set of discrete scenarios.
- Each scenario has a **probability of occurrence**, representing different customer behavior possibilities.

3.2 Demand Substitution Effects

- If a product is unavailable, customers may choose a **substitutable product**.
- A **substitution matrix** defines which products can replace others and at what rate.

3.3 Objective Function in Expectation Form

- The goal is to **maximize expected net profit**, balancing direct sales, substituted demand, and costs across all demand scenarios.

4 Mathematical Formulation

The problem is formulated as a **two-stage stochastic optimization model**.

4.1 First-Stage Decision (Assortment and Supply Allocation)

$$\max \sum_i \sum_s p_s \cdot \Pi_{is} - C(x) \tag{1}$$

where:

- p_s is the probability of demand scenario s.
- Π_{is} is the expected profit for product i under scenario s.
- $C(x)$ represents the cost of purchasing and holding inventory.

4.2 Second-Stage Decision (Demand Fulfillment)

$$\max \sum_i \sum_s p_s \cdot \Big(r_i d_{is} + r'_i y_{is} - h_i q_{is} - \beta_i b_{is} \Big) \tag{2}$$

where:

- r_i is the revenue per unit of product i.
- d_{is} is the actual demand satisfied for product i in scenario s.
- y_{is} is the amount of substituted demand fulfilled.
- h_i is the holding cost per unit.
- q_{is} is the remaining stock after demand realization.
- β_i is the penalty for unfulfilled demand.
- b_{is} is the quantity of unfulfilled demand in scenario s.

4.3 Constraints

- **Shelf Space Constraint:**

$$\sum_i x_i \cdot v_i \leq S \tag{3}$$

  Ensures that total space used does not exceed available shelf space S, where v_i is the space required for product i.

- **Inventory Balance Across Stages:**

$$x_i = d_{is} + q_{is}, \quad \forall i, s \tag{4}$$

- **Substitution Constraint:**

$$y_{is} \leq \sum_j \alpha_{ij} d_{js} \tag{5}$$

  Ensures that substituted demand does not exceed the predefined substitution matrix α_{ij}.

5 Conclusion

The **two-stage stochastic programming model** provides a robust and computationally efficient framework for **assortment planning under uncertain demand**. The key advantages include:

- **Optimized pre-decision assortment planning** while considering future uncertainties.
- **Dynamic response to realized demand**, improving customer satisfaction.
- **Explicit handling of product substitution effects**, reducing lost sales.
- **Computational feasibility** for large-scale retail applications.

This approach ensures that retailers make informed stocking decisions that balance **profitability, storage constraints, and customer preferences**, leading to a **resilient and adaptable retail strategy**.

A Robust Optimization Framework for Bioattack Response Supply Chains

1 Introduction: The Problem and Its Challenges

Dealing with **bioattacks** is one of the most serious challenges in public health planning. Unlike natural disasters, where historical data helps predict the scale of the event, **bioattacks are unpredictable**. We don't know **where** they will happen, **when**, or **how big** the impact will be.

The government has to make **two key decisions** to prepare for such events:

1. **Where to store medical countermeasures (MCMs)** like vaccines, antibiotics, or antidotes **before an attack** happens.
2. **How to distribute those MCMs after an attack** to minimize casualties.

The problem is, without knowing the **exact attack location and demand**, how do we decide where to stockpile and how much? If we store too much, it's expensive. If we store too little, people might not get treatment in time.

That's where **robust optimization (RO)** comes in—it helps make **the best possible decisions under worst-case uncertainty** so that the response plan is **resilient no matter what happens**.

2 Decision Stages and How Information Evolves

The whole planning process unfolds in **two key stages**, with **different decisions and levels of uncertainty in each**.

2.1 First Stage: Prepositioning Inventory (Before the Attack)

- **Decisions to be made:**
  - Where do we place stockpiles?
  - How much MCM do we store in each location?
- **What's uncertain?**

- We don't know **where** the attack will happen.
- We don't know **how many people** will need treatment.

- **Random Variables:**
  - $\tilde{d}_i$: The actual number of infected people at location i (depends on attack size).
  - $\tilde{x}_i$: The amount of MCM actually needed at site i.

At this stage, **no attack has happened yet**, so we are making **blind decisions** with just an estimate of potential threats.

2.2 Second Stage: Shipment Response (After the Attack)

Once an attack happens, we finally get some **new information**. We now **know which areas are affected**, but some uncertainty still remains.

- **Decisions to be made:**
  - How much MCM should be shipped from storage to attack zones?
  - How do we prioritize areas with severe cases?
- **What's uncertain?**
  - The **exact number of people infected** (there could be delays in reporting cases).
  - **Transportation times** (some routes may be blocked, causing delivery delays).
- **Random Variables:**
  - $\tilde{\tau}_{ij}$: The actual transportation time from storage site i to demand location j.
  - $\tilde{s}_i$: The remaining unmet demand at location i.

At this stage, we know **more than before**, but we **still** don't have perfect information—meaning our plan needs to be **flexible and adaptive**.

3 Mathematical Model and Parameters

3.1 Sets and Indices

- $\mathcal{N}$: Set of all locations (storage sites and demand locations).
- $\mathcal{A}$: Set of all transportation links.
- $\mathbb{T}$: Set of discrete time periods in the response horizon.

3.2 Decision Variables

- x_i: MCM stockpile at location i.
- f_{ij}: MCM shipments from location i to j after an attack.
- s_i: Shortfall (unmet demand) at location i.

3.3 Parameters

- $\bar{S}_i$: Estimated mean demand at node i.
- σ_i^2: Variance of demand at node i.
- C_i: Storage capacity at location i.
- T_{ij}: Transportation capacity from i to j.
- h_i: Inventory holding cost at site i.
- b: Cost per life lost.

4 Why Robust Optimization Works Best

- **Handles Worst-Case Scenarios**: Ensures the plan works even in extreme attacks.
- **Doesn't Need Probability Estimates**: Works with just an uncertainty range.
- **Computationally Efficient**

5 Conclusion: A Resilient and Practical Planning Approach

The **robust optimization framework** ensures that **stockpiles and shipment plans remain effective**, no matter **where or when an attack happens**.

Instead of relying on **guesswork** or **fixed probabilities**, this approach **plans for the worst** while **keeping costs manageable**. It allows decision-makers to:

- **Preposition inventory intelligently.**
- **Respond quickly and dynamically after an attack.**
- **Guarantee medical countermeasure availability in high-risk scenarios.**

A Robust Chance-Constrained Approach to Evacuation Planning Under Uncertain Demand Distribution

1 Introduction

Evacuation planning is essential in disaster management, aimed at effectively moving evacuees from dangerous areas to safety. A significant issue in this planning is the uncertainty regarding the number of evacuees (demand) at each source location. This unpredictability in demand stems from various factors, such as the severity of the disaster, public perception, and limitations in infrastructure. Conventional deterministic models rely on fixed estimates.

To tackle this issue, **Chance-Constrained Programming (CCP)** is utilized to maintain demand constraints with a specified probability. However, CCP necessitates comprehensive knowledge of the entire probability distribution of demand, which is seldom accessible in practical situations. This research introduces a **Robust Chance-Constrained Programming (RCCP)** method that accommodates demand uncertainty using just **partial distributional information** like **mean, variance, and symmetry**.

2 Problem Formulation

2.1 Decision Stages

The decision-making process in this problem consists of multiple stages:

1. **Evacuation Planning (Pre-disaster Phase)**
    - **Decisions:** Selection of evacuation paths, flow allocation, scheduling of departures.
    - **Objective:** Minimize congestion and ensure the timely evacuation of all individuals.
    - **Uncertainty:** The number of evacuees is unknown but estimated using statistical methods.
2. **Clearance Time Optimization (Operational Phase)**

- **Decisions:** Optimization of vehicle dispatch and traffic control measures.
- **Objective:** Ensure that evacuation is completed within the shortest possible time.
- **Uncertainty:** Traffic congestion and unforeseen bottlenecks may delay evacuees.

3. **Evacuation Execution (Real-time Phase)**
    - **Decisions:** Adjustments in the plan based on real-time demand realization.
    - **Objective:** Adapt to unforeseen circumstances such as changing traffic conditions.
    - **Uncertainty:** Unpredictable behavioral responses from evacuees may alter the planned schedule.

2.2 Random Variables

In evacuation planning under uncertainty, several parameters are subject to randomness. These variables are not deterministic and require robust modeling to ensure plan feasibility.

- **Random Demand** ($\tilde{S}_i$): The number of evacuees at source node i is not known in advance but follows an uncertain distribution.
- **Traffic Flow** ($\tilde{Q}_a$): The actual capacity of an arc (road/link) may vary due to congestion or unforeseen disruptions.
- **Travel Time** ($\tilde{\tau}_p$): The time taken to traverse a path is influenced by real-time road conditions, vehicle availability, and accidents.
- **Evacuation Participation Rate**: The fraction of the population that decides to evacuate may be uncertain, depending on warning systems and risk perception.

3 Mathematical Model

The problem is defined on a **directed network** $G = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ represents nodes (sources, intermediate intersections, and destinations) and $\mathcal{A}$ represents arcs (roads). The decision variables, random variables, and parameters used in the model are as follows:

3.1 Notation

3.1.1 Sets and Indices

- $\mathcal{N}$: Set of all nodes in the network.
- $\mathcal{A}$: Set of all arcs (roads/links).
- $\mathcal{N}_s$: Set of all **source nodes** (locations where evacuees originate).
- $\mathcal{N}_d$: Set of all **destination nodes** (safe locations).
- $\mathcal{P}$: Set of all pre-specified evacuation paths.
- $\mathbb{T}$: Set of discrete time periods in the evacuation horizon.

3.1.2 Decision Variables

- f_{pt}: Flow (number of evacuees) on path p at time t.
- β_i: Number of **unsatisfied evacuees** at source node i.

3.1.3 Random Variables

- $\tilde{S}_i$: **Random demand** at source node i.

3.1.4 Parameters

- $\bar{S}_i$: **Mean demand** at node i.
- σ_i^2: **Variance of demand** at node i.
- l_i^-, l_i^+: **Lower and upper support bounds** of demand.
- ϵ_i: **Confidence level parameter** (probability of constraint violation).
- C_a: **Capacity of arc** a.
- ℓ_j: **Capacity of destination node** j.
- τ_p: **Travel time** on path p.

4 Optimization Models

4.1 Deterministic Model (DPBM)

The **deterministic model** assumes a fixed demand estimate S_i at each source node. The objective is to minimize the number of evacuees left behind:

$$\min \sum_{i \in \mathcal{N}_s} \beta_i \tag{1}$$

4.2 Chance-Constrained Model (CCP)

$$P\left(\sum_{p\in\mathcal{P}_i^+}\sum_{t\in\mathbb{T}} f_{pt} + \beta_i \geq \tilde{S}_i\right) \geq 1 - \epsilon_i, \quad \forall i \in \mathcal{N}_s \tag{2}$$

4.3 Robust Chance-Constrained Model (RCCP)

- **RCCP1 (Moment-Based Approximation)**
- **RCCP2 (Symmetry-Based Approximation)**
- **RCCP3 (Support-Based Approximation)**

5 Conclusion

The proposed **RCCP model** ensures that evacuation plans remain feasible even under **highly uncertain demand scenarios**.

References:

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