Internship Project Report

Maneuvering Target State Estimation using Extended Kalman Filter

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Introduction

This project implements an Extended Kalman filter to estimate the states and parameters of a target missile using a nonlinear point-mass model that includes gravitational and aerodynamic forces. Successful destruction of incoming enemy missile requires accurate estimation of its states. The EKF ensures accurate and reliable tracking, demonstrating its effectiveness in handling model non-linearities for target estimation. Two different point mass process models are used for ballistic and maneuvering target state estimatios. The effectiveness of estimator is shown with the help of simulation results.

Target state estimation is crucial for missile guidance, especially in hit-to-kill missiles, requiring accurate target information for successful interception and guidance command computation. State estimation algorithms are based on a certain model for the target behaviour. Since these free falling targets experience both gravitational and aerodynamic forces, they may exhibit complex maneuvers in the atmosphere. Outside the atmosphere the acceleration of the target varies very little only with respect to its altitude. However, once the target enters the atmosphere it experiences aerodynamic forces that are not only dependent on its altitude, but more significantly on its velocity, and aerodynamic characteristics..Therefore, the objective of the estimator is to obtain the position, velocity, acceleration, drag coefficient and aerodynamic lift coefficients of the target using the available sensor measurements.

In this project we are modelling the target using equations of motion as shown in 1(a) and 2(a). Subsequently a Measurement Model is used to model the Radar (Radio Detection and Ranging) measurements which are Range, Elevation Angle, Azimuth Angle and Range Rate using target kinematics with respect to radar location. Further the Extended Kalman Filter (EKF) is implemented to estimate position, velocity, acceleration, drag coefficient and aerodynamic lift coefficients of the target using the available radar measurements.

Extended Kalman Filter formulation is given in the next section. Non linear models of measurement obtained from Radar are presented in section Measurement Model. Dynamic modelling of target behaviour is presented in the section 1(a) and 2(a). Linearized System matrices are presented in the section 1(b) and 2(b). Simulation results are presented in 1(c) and 2(c) for both ballistic and maneuvering target. All the values in the plots in this section have been normalized between 0 and 1 in both X and Y axes.

Nomenclature

(x, y, z) = Position of Target in VEN Frame
(v_x, v_y, v_z) = Velocity of Target in VEN Frame
$(\hat{x}_k, \bar{x}_k)$ = Estimated and Predicted Target State Vector
$(\hat{P}_k, \bar{P}_k)$ = Estimated and Predicted State Covariance Matrix
u_k = Input Vector
w_k = Process Noise - Zero Mean White Gaussian Noise
y_k = Measurement Vector
v_k = Measurement Noise - Zero Mean White Gaussian Noise
Q = Process Noise Covariance Matrix
R = Measurement Noise Covariance Matrix
A = Jacobian matrix of system dynamics vector field
H = Jacobian matrix of the measurements vector
K = Kalman gain matrix
m = Mass of Target
s_{ref} = Reference Surface Area
c_D = Aerodynamic Drag Coefficient
c_L = Lift Coefficient
ρ = Air Density
ϕ = Roll Orientations
(γ_t, ψ_t) = Flight Path Angles in Elevation and Azimuth

*subscript k indicates discrete time sample

Extended Kalman Filter (EKF)

An Extended Kalman Filter implementation using the dynamic model is presented in this section.

State and measurement equations are as follows:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k) \tag{1}$$
$$\mathbf{y}_k = h(\mathbf{x}_k, \mathbf{v}_k) \tag{2}$$

The covariance matrix are as follows:

$$\mathbf{Q}_k = E[\mathbf{w}_k \mathbf{w}_k^T] \tag{3}$$
$$\mathbf{R}_k = E[\mathbf{v}_k \mathbf{v}_k^T] \tag{4}$$

State estimate propagation in between updates is carried out using the exact nonlinear equations of motion as shown below:

$$\bar{\mathbf{x}}_{k+1} = f(\hat{\mathbf{x}}_k, \mathbf{u}_k, \mathbf{w}_k) \tag{5}$$
$$\bar{\mathbf{P}}_{k+1} = \Psi_k \hat{\mathbf{P}}_k \Psi_k^T + \mathbf{Q}_k \tag{6}$$
$$\bar{\mathbf{y}}_{k+1} = \mathbf{H}_{k+1} \bar{\mathbf{x}}_{k+1} \tag{7}$$

The fundamental matrix Ψ can be found from system dynamic matrix(A) and sampling time of T:

$$\Psi = \mathbf{I} + \mathbf{A}T \tag{8}$$

Where:

$$\mathbf{A} = \left.\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}} \tag{9}$$

$$\mathbf{H} = \left.\frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}} \tag{10}$$

Gain computation using the linearized measurement model and the covariance update is given by:

$$\mathbf{K}_{k+1} = \bar{\mathbf{P}}_{k+1}\mathbf{H}_{k+1}^{T}(\mathbf{H}_{k+1}\bar{\mathbf{P}}_{k+1}\mathbf{H}_{k+1}^{T} + \mathbf{R}_k)^{-1} \tag{11}$$

$$\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}_{k+1}\mathbf{H}_{k+1})\bar{\mathbf{P}}_{k+1} \tag{12}$$

The state estimates are updated using the measurements as when they are available using the following equation:

$$\hat{\mathbf{x}}_{k+1} = \bar{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1}(\mathbf{y}_k - \bar{\mathbf{y}}_{k+1}) \tag{13}$$

Measurement Model

It is assumed that Range, Azimuth Angle, Elevation angle and Range Rate are available as radar measurements. Radar location which is known to us, here the radar is assumed to be placed at origin (0,0,0) of local inertial frame fixed at Earth Surface. The measurement model used for estimation as a function of states is as follows:

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{x}{\sqrt{y^2 + z^2}}\right)$$

$$\dot{R} = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{\sqrt{x^2 + y^2 + z^2}}$$

$$y = h(x) = \begin{bmatrix} R & \theta & \phi & \dot{R} \end{bmatrix}$$

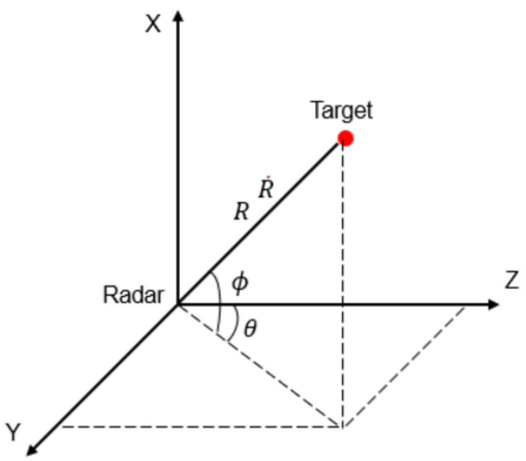

Figure 1: Local Earth Fixed inertial Coordinate System

Case Study 1 - Ballistic Target

1(a) Target Modelling

In this case target is assumed to follow ballistic path with gravity and aerodynamic drag force acting on it. Hence it's motion dynamics are modelled by gravity and ballistic coefficients as shown below:

$$A_x = -\frac{\rho \cdot v_t \cdot v_x}{2 \cdot \beta} - g$$
$$A_y = -\frac{\rho \cdot v_t \cdot v_y}{2 \cdot \beta}$$
$$A_z = -\frac{\rho \cdot v_t \cdot v_z}{2 \cdot \beta}$$

$$\rho = 0.3638 \times e^{k(x-11000)}, \;\; v_t = \sqrt{v_x^2 + v_y^2 + v_z^2}$$
$$g = 9.81 \text{ m/s}^2, \quad k = -0.0001516584, \quad \frac{1}{\dot{\beta}} = 0, \quad \beta = \frac{m}{s_{ref} c_D}$$

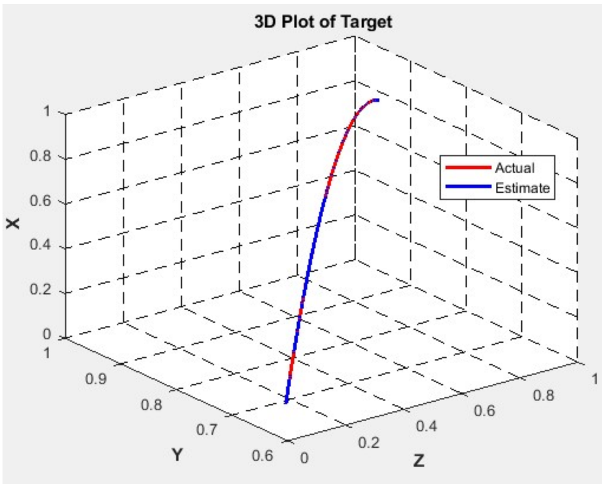

Figure 2: 3D Plot of Ballistic Target

1(b) Linearization

The following states are estimated by the EKF:

$$X = \begin{bmatrix} x & y & z & v_x & v_y & v_z & \frac{1}{\beta} \end{bmatrix}$$

The corresponding system dynamic matrix (A) for ballistic target is:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & 3X7 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A(4,1) = \frac{k\rho v_t v_x}{2\beta}, \quad A(4,4) = -\frac{\rho\left(\frac{v_x^2}{v_t} + v_t\right)}{2\beta}, \quad A(4,5) = -\frac{\rho v_x v_y}{2v_t\beta}, \quad A(4,6) = -\frac{\rho v_x v_z}{2v_t\beta}, \quad A(4,7) = -\frac{\rho v_x v_t}{2},$$

$$A(5,1) = \frac{k\rho v_t v_y}{2\beta}, \quad A(5,4) = -\frac{\rho v_x v_y}{2v_t\beta}, \quad A(5,5) = -\frac{\rho\left(\frac{v_y^2}{v_t} + v_t\right)}{2\beta}, \quad A(5,6) = -\frac{\rho v_y v_z}{2v_t\beta}, \quad A(5,7) = -\frac{\rho v_y v_t}{2},$$

$$A(6,1) = \frac{k\rho v_t v_z}{2\beta}, \quad A(6,4) = -\frac{\rho v_x v_z}{2v_t\beta}, \quad A(6,5) = -\frac{\rho v_y v_z}{2v_t\beta}, \quad A(6,6) = -\frac{\rho\left(\frac{v_z^2}{v_t} + v_t\right)}{2\beta}, \quad A(6,7) = -\frac{\rho v_z v_t}{2}$$

A(4,2) = A(4,3) = A(5,2) = A(5,3) = A(6,2) = A(6,3) = 0

The corresponding H matrix for ballistic target is:

$$H = \begin{bmatrix} \dots & \dots & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & 0 \end{bmatrix}$$

A complete list of the partial derivatives involved in the above expressions is given below:

$$H(1,1) = \frac{x}{R}, \quad H(1,2) = \frac{y}{R}, \quad H(1,3) = \frac{z}{R}, \quad H(2,2) = \frac{1}{\sqrt{y^2+z^2}}, \quad H(2,3) = -\frac{y}{z\sqrt{y^2+z^2}}$$

$$H(2,1) = 0 \quad H(3,1) = \frac{1}{R}, \quad H(3,2) = -\frac{xy}{R\sqrt{y^2+z^2}}, \quad H(3,3) = -\frac{xz}{R\sqrt{y^2+z^2}}$$

$$H(4,1) = \frac{(Rv_x - (xv_x + yv_y + zv_z)\frac{x}{R})}{R^2} \quad H(4,2) = \frac{(Rv_y - (xv_x + yv_y + zv_z)\frac{y}{R})}{R^2}$$

$$H(4,3) = \frac{(Rv_z - (xv_x + yv_y + zv_z)\frac{z}{R})}{R^2} \quad H(4,4) = \frac{x}{R}, \quad H(4,5) = \frac{y}{R}, \quad H(4,6) = \frac{z}{R}$$

1(c) Simulation Results

All the values in the plots shown below have been normalized between 0 and 1 in both X and Y axes.

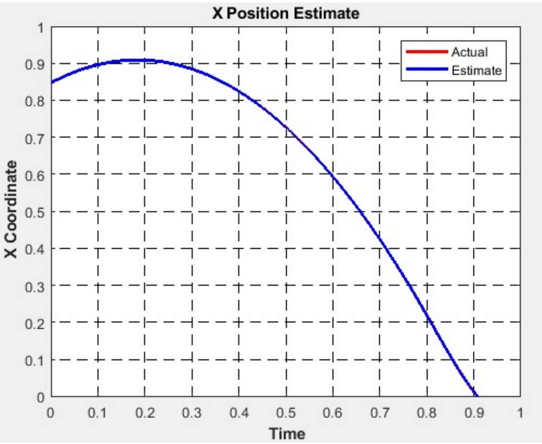

Figure 3: X Position Estimate

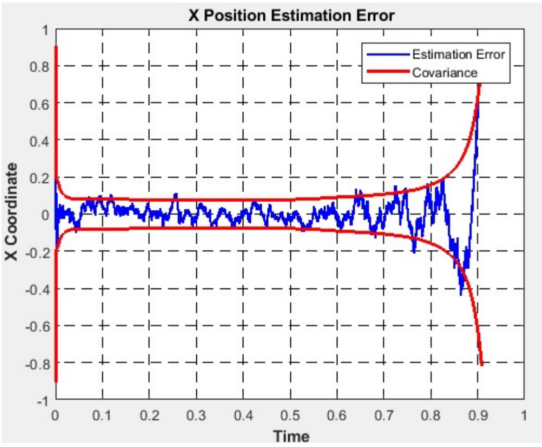

Figure 4: X Position Estimation Error

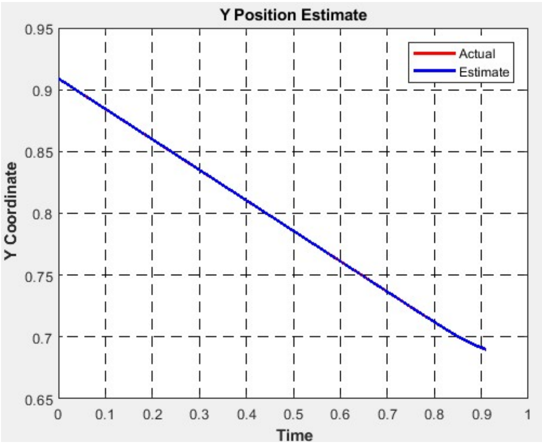

Figure 5: Y Position Estimate

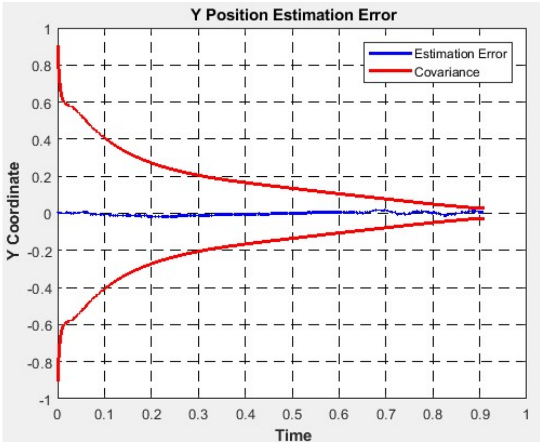

Figure 6: Y Position Estimation Error

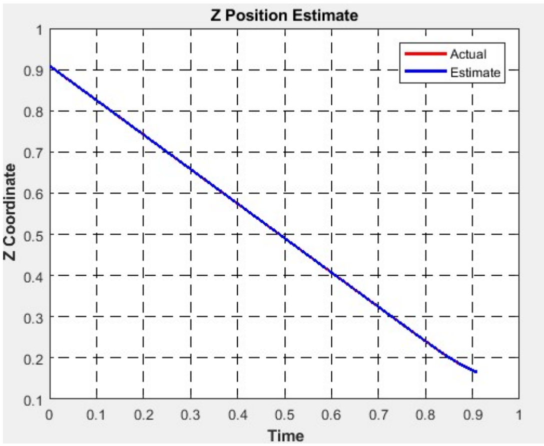

Figure 7: Z Position Estimate

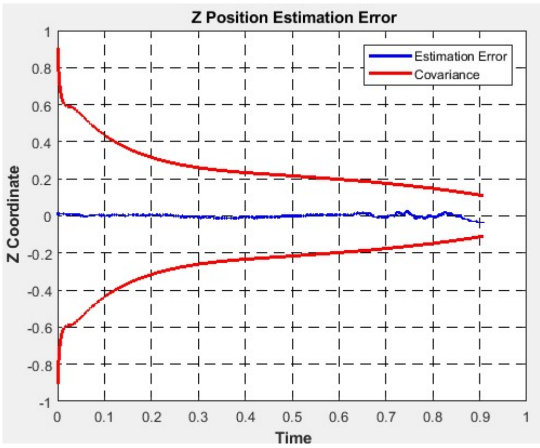

Figure 8: Z Position Estimation Error

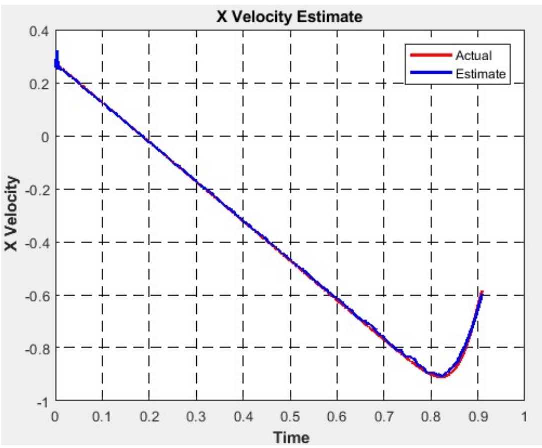

Figure 9: X Velocity Estimate

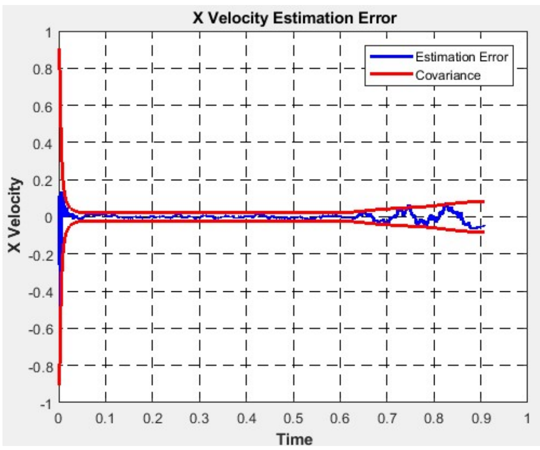

Figure 10: X Velocity Estimation Error

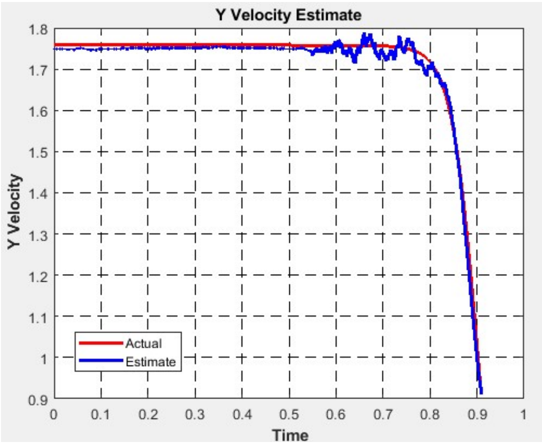

Figure 11: Y Velocity Estimate

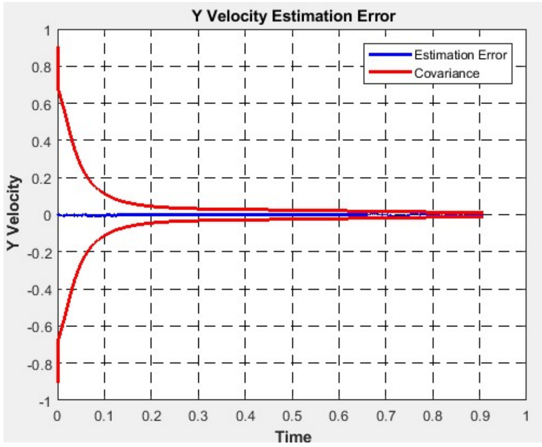

Figure 12: Y Velocity Estimation Error

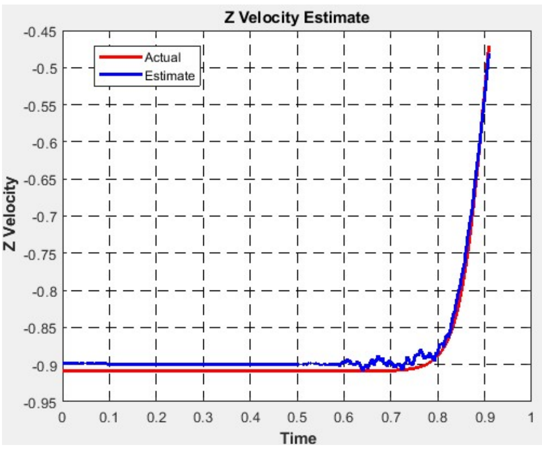

Figure 13: Z Velocity Estimate

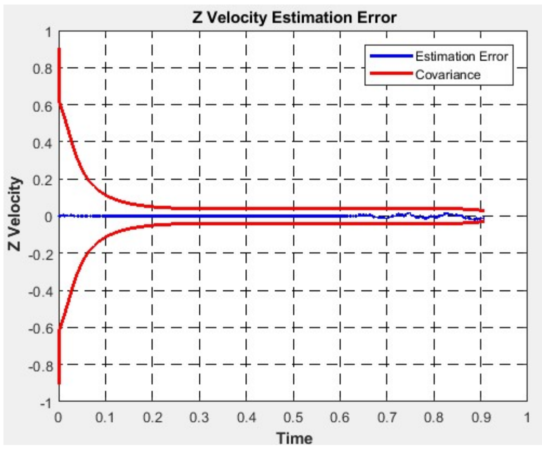

Figure 14: Z Velocity Estimation Error

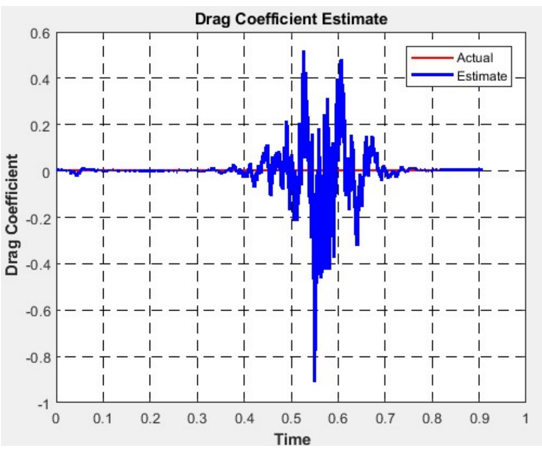

Figure 15: Drag Coefficient Estimate

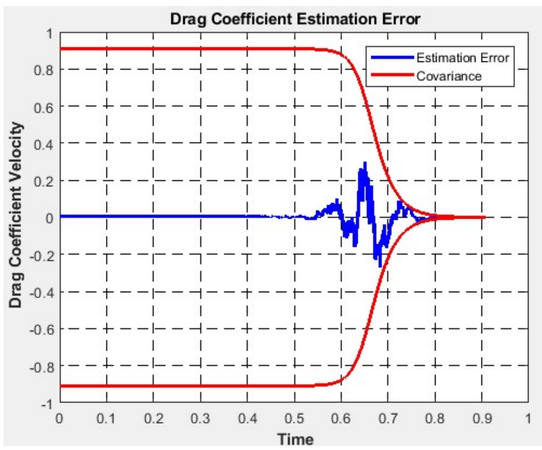

Figure 16: Drag Coeff. Estimation Error

Shown in Figure 15 and Figure 16, the estimator starts to estimate Drag coefficient once the system enters into the atmosphere because the ballistic coefficient becomes observable when effect of atmosphere is significant, before that the system is not observable and hence the estimator was not able to correctly estimate the Drag Coefficient at higher altitudes.

Case Study 2 - Maneuvering Target

2(a) Target Modelling

In order to accommodate maneuvering targets, our model incorporates transformations to the acceleration terms, along with the introduction of aerodynamic lift coefficients. In this case apart from Drag Force along the direction of velocity vector target is experiencing additional force in plane perpendicular to velocity vector. This is called the lift force which enables the target to maneuver. This force component are modelled using two maneuvering coefficients z1 and z2 as shown below:

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}-g\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}\sin(\gamma_t) & 0 & -\cos(\gamma_t)\\ \cos(\gamma_t)\sin(\psi_t) & \cos(\psi_t) & \sin(\gamma_t)\sin(\psi_t)\\ \cos(\gamma_t)\cos(\psi_t) & -\sin(\psi_t) & \sin(\gamma_t)\cos(\psi_t)\end{bmatrix}\begin{bmatrix}-\frac{\rho v_t^2}{2\beta}\\ \frac{\rho v_t^2}{2}z_1\\ \frac{\rho v_t^2}{2}z_2\end{bmatrix}$$

$$\tan(\gamma_t) = \frac{v_x}{\sqrt{v_y^2 + v_z^2}}, \quad \tan(\psi_t) = \frac{v_y}{v_z}$$

$$\begin{aligned}
A_x &= \frac{1}{2}\rho v_t^2\left(\frac{-\sin(\gamma_t)}{\beta} + 0 - \cos(\gamma_t)z_2\right) - g\\
A_y &= \frac{1}{2}\rho v_t^2\left(\frac{-\cos(\gamma_t)\sin(\psi_t)}{\beta} + \cos(\psi_t)z_1 + \sin(\gamma_t)\sin(\psi_t)z_2\right)\\
A_z &= \frac{1}{2}\rho v_t^2\left(\frac{-\cos(\gamma_t)\cos(\psi_t)}{\beta} - \sin(\psi_t)z_1 + \sin(\gamma_t)\cos(\psi_t)z_2\right)
\end{aligned}$$

$$\frac{1}{\dot{\beta}} = 0, \quad \dot{z}_1 = 0, \quad \dot{z}_2 = 0, \quad \beta = \frac{m}{s_{ref}c_D}, \quad z_1 = \frac{s_{ref}c_L cos(\phi)}{m}, \quad z_2 = \frac{s_{ref}c_L sin(\phi)}{m}$$

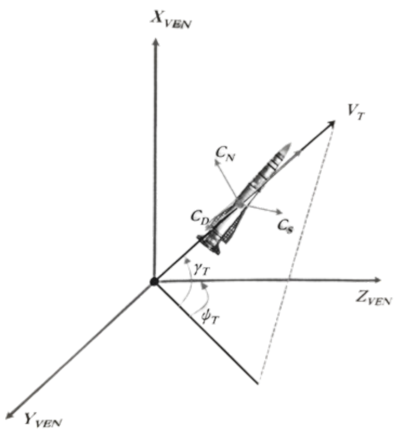

Figure 17: Convention of Local Vertical and Interceptor Velocity Frame

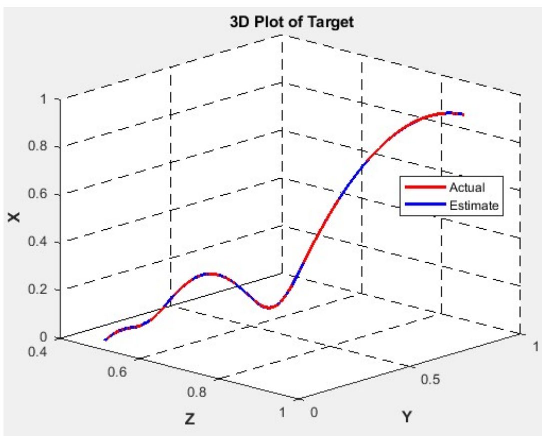

Figure 18: 3D Plot of Maneuvering Target

2(b) Linearization

The following states are estimated by the EKF:

$$X = \begin{bmatrix} x & y & z & v_x & v_y & v_z & \frac{1}{\beta} & \dot{z}_1 & \dot{z}_2 \end{bmatrix}$$

The corresponding A matrix for maneuvering target is:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 3X9 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A complete list of the partial derivatives involved in the above expressions is given below:

Aerodynamic Acceleration along X Axis:

$$A(4,1) = \frac{1}{2}k\rho v_t \left(-\frac{v_x}{\beta} - z_2\sqrt{v_y^2 + v_z^2} \right)$$

$$A(4,4) = -\frac{1}{2}\frac{\rho v_t}{\beta} - \frac{1}{2}\rho\frac{v_x^2}{\beta v_t} - \frac{1}{2}\frac{\rho v_x z_2 \sqrt{v_y^2 + v_z^2}}{v_t}$$

$$A(4,5) = -\frac{1}{2}\rho\frac{v_x v_y}{\beta v_t} - \frac{1}{2}\rho\frac{z_2 v_y \sqrt{v_y^2 + v_z^2}}{v_t} - \frac{1}{2}\rho\frac{v_t z_2 v_y}{\sqrt{v_y^2 + v_z^2}}$$

$$A(4,6) = -\frac{1}{2}\rho\frac{v_x v_z}{\beta v_t} - \frac{1}{2}\rho\frac{z_2 v_z \sqrt{v_y^2 + v_z^2}}{v_t} - \frac{1}{2}\rho\frac{v_t z_2 v_z}{\sqrt{v_y^2 + v_z^2}}$$

$$A(4,7) = -\frac{1}{2}\rho v_x v_t \quad A(4,9) = -\frac{1}{2}\rho v_t \sqrt{v_y^2 + v_z^2} \quad A(4,2) = A(4,3) = A(4,8) = 0$$

Aerodynamic Acceleration along Y Axis:

$$A(5,1) = \frac{1}{2}k\rho v_t^2 \left[-\frac{v_y}{\beta v_t} + \frac{v_z z_1}{\sqrt{v_y^2 + v_z^2}} + \frac{v_x v_y z_2}{v_t\sqrt{v_y^2 + v_z^2}} \right]$$

$$A(5,4) = \frac{\rho}{2}\left[-\frac{v_x v_y}{\beta v_t} + 2\frac{v_x v_z z_1}{\sqrt{v_y^2 + v_z^2}} + \frac{v_y v_t z_2}{\sqrt{v_y^2 + v_z^2}} + \frac{v_y z_2 v_x^2}{v_t\sqrt{v_y^2 + v_z^2}} \right]$$

$$A(5,5) = -\frac{1}{2}\rho\frac{v_t}{\beta} - \frac{1}{2}\rho\frac{v_y^2}{\beta v_t} + \frac{1}{2}\rho v_z z_1 \frac{2v_y(v_y^2 + v_z^2) - v_t^2 v_y}{(v_y^2 + v_z^2)^{3/2}} + \frac{1}{2}\rho z_2 v_x \frac{v_t v_z^2 + \frac{v_y^4}{v_t} + \frac{(v_y v_z)^2}{v_t}}{(v_y^2 + v_z^2)^{3/2}}$$

$$A(5,6) = \frac{\rho}{2}\left[-\frac{v_y v_z}{\beta v_t} + z_1\frac{(v_t v_y)^2 + 2v_z^2 v_y^2 + 2v_z^4}{(v_y^2 + v_z^2)^{3/2}} + v_x v_y z_2 \frac{(v_y^2 + v_z^2)\frac{v_z}{v_t} - v_t v_z}{(v_y^2 + v_z^2)^{3/2}} \right]$$

$$A(5,7) = -\frac{1}{2}\rho v_y v_t \quad A(5,8) = \frac{1}{2}\rho\frac{v_z v_t^2}{\sqrt{v_y^2 + v_z^2}} \quad A(5,9) = \frac{1}{2}\rho\frac{v_x v_y v_t}{\sqrt{v_y^2 + v_z^2}} \quad A(5,2) = A(5,3) = 0$$

Aerodynamic Acceleration along Z Axis:

$$A(6,1) = \frac{1}{2}k\rho v_t^2 \left[-\frac{v_z}{\beta v_t} - \frac{v_y z_1}{\sqrt{v_y^2+v_z^2}} + \frac{v_x v_z z_2}{v_t\sqrt{v_y^2+v_z^2}} \right]$$

$$A(6,4) = \frac{\rho}{2} \left[-\frac{v_x v_z}{\beta v_t} - 2\frac{v_x v_y z_1}{\sqrt{v_y^2+v_z^2}} + \frac{v_z v_t z_2}{\sqrt{v_y^2+v_z^2}} + \frac{v_z z_2 v_x^2}{v_t\sqrt{v_y^2+v_z^2}} \right]$$

$$A(6,5) = \frac{\rho}{2} \left[-\frac{v_y v_z}{\beta v_t} - z_1 \frac{(v_t v_z)^2 + 2v_y^2 v_z^2 + 2v_y^4}{(v_y^2+v_z^2)^{3/2}} + v_x v_z z_2 \frac{(v_y^2+v_z^2)\frac{v_y}{v_t} - v_t v_y}{(v_y^2+v_z^2)^{3/2}} \right]$$

$$A(6,6) = -\frac{1}{2}\rho\frac{v_t}{\beta} - \frac{1}{2}\rho\frac{v_z^2}{\beta v_t} - \frac{1}{2}\rho v_y z_1 \frac{2v_z(v_y^2+v_z^2) - v_t^2 v_z}{(v_y^2+v_z^2)^{3/2}} + \frac{1}{2}\rho z_2 v_x \frac{v_t v_y^2 + \frac{v_z^4}{v_t} + \frac{(v_y v_z)^2}{v_t}}{(v_y^2+v_z^2)^{3/2}}$$

$$A(6,7) = -\frac{1}{2}\rho v_z v_t \quad A(6,8) = \frac{-1}{2}\rho\frac{v_y v_t^2}{\sqrt{v_y^2+v_z^2}} \quad A(6,9) = \frac{1}{2}\rho\frac{v_x v_z v_t}{\sqrt{v_y^2+v_z^2}} \quad A(6,2) = A(6,3) = 0$$

The corresponding H matrix for non-maneuvering target is:

$$H = \begin{bmatrix} \dots & \dots & \dots & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 \end{bmatrix}$$

A complete list of the partial derivatives involved in the above expressions is given below:

$$H(1,1) = \frac{x}{R}, \quad H(1,2) = \frac{y}{R}, \quad H(1,3) = \frac{z}{R}, \quad H(2,2) = \frac{1}{\sqrt{y^2+z^2}}, \quad H(2,3) = -\frac{y}{z\sqrt{y^2+z^2}}$$

$$H(2,1) = 0 \quad H(3,1) = \frac{1}{R}, \quad H(3,2) = -\frac{xy}{R\sqrt{y^2+z^2}}, \quad H(3,3) = -\frac{xz}{R\sqrt{y^2+z^2}}$$

$$H(4,1) = \frac{(Rv_x - (xv_x + yv_y + zv_z)\frac{x}{R})}{R^2} \quad H(4,2) = \frac{(Rv_y - (xv_x + yv_y + zv_z)\frac{y}{R})}{R^2}$$

$$H(4,3) = \frac{(Rv_z - (xv_x + yv_y + zv_z)\frac{z}{R})}{R^2} \quad H(4,4) = \frac{x}{R}, \quad H(4,5) = \frac{y}{R}, \quad H(4,6) = \frac{z}{R}$$

2(c) Simulation Results

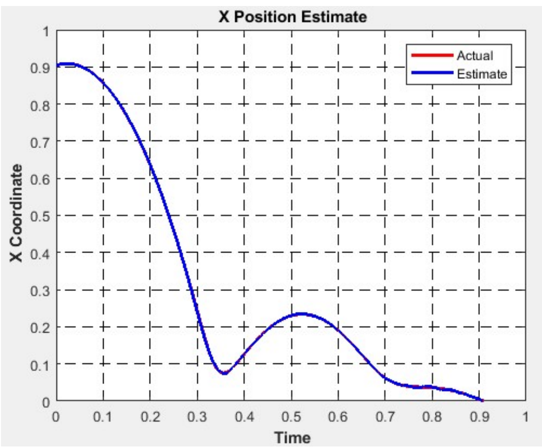

Figure 19: X Position Estimate

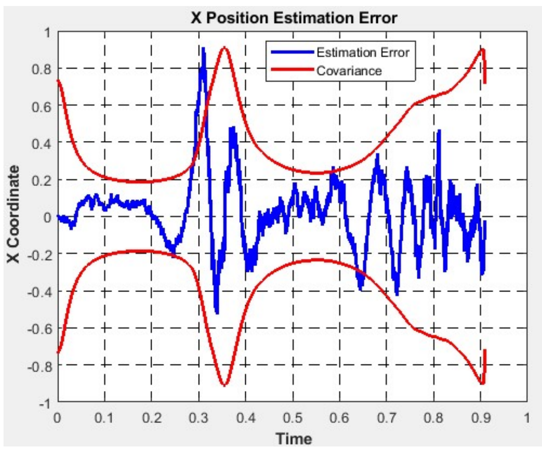

Figure 20: X Position Estimation Error

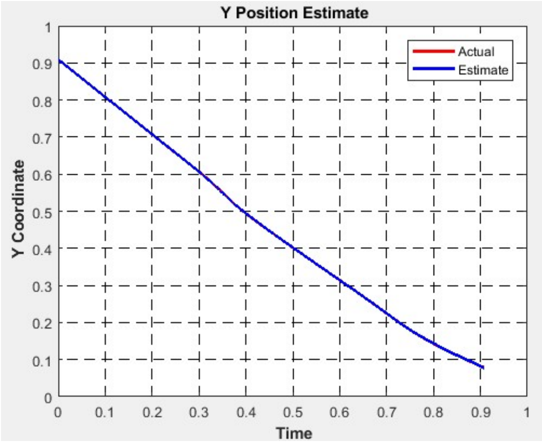

Figure 21: Y Position Estimate

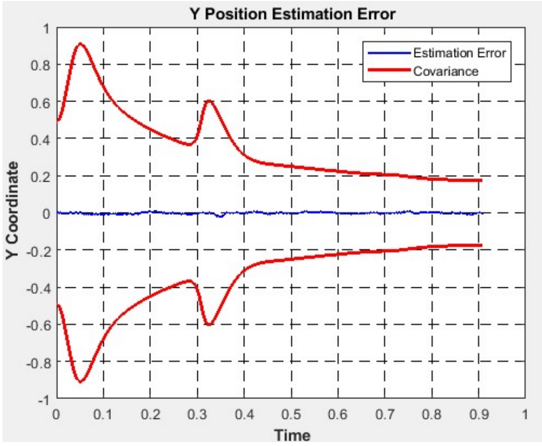

Figure 22: Y Position Estimation Error

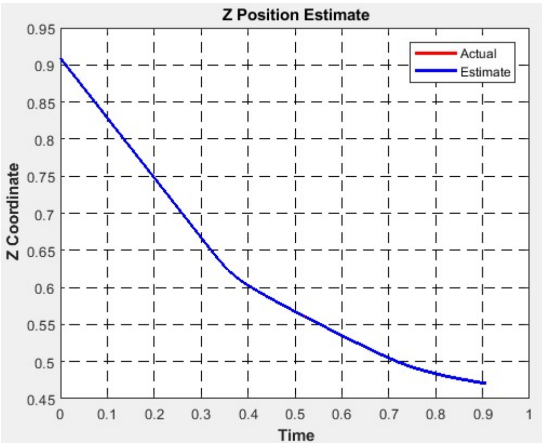

Figure 23: Z Position Estimate

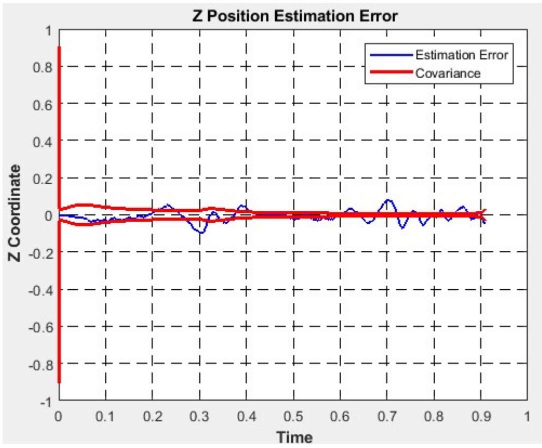

Figure 24: Z Position Estimation Error

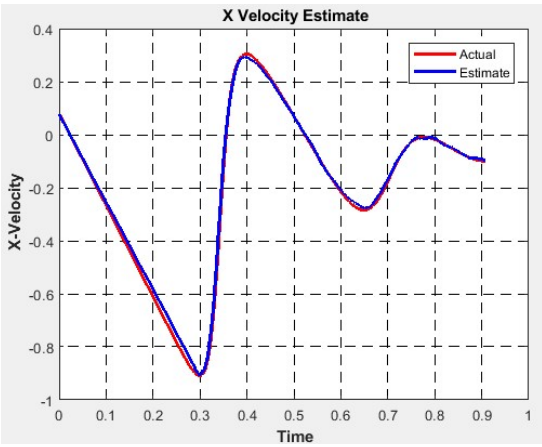

Figure 25: X Velocity Estimate

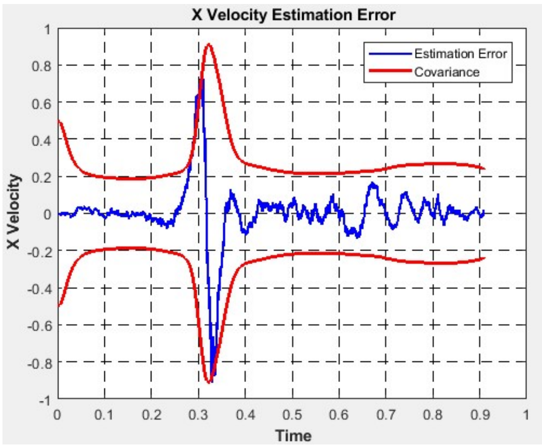

Figure 26: X Velocity Estimation Error

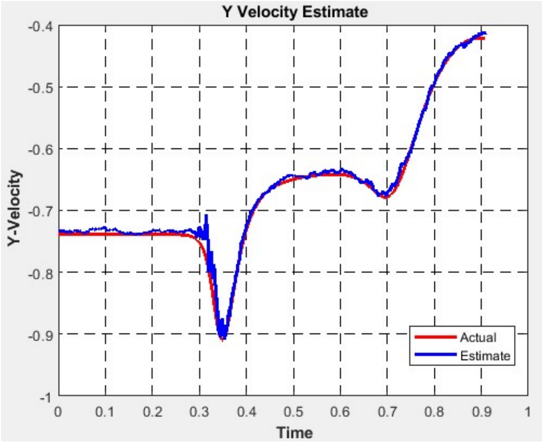

Figure 27: Y Velocity Estimate

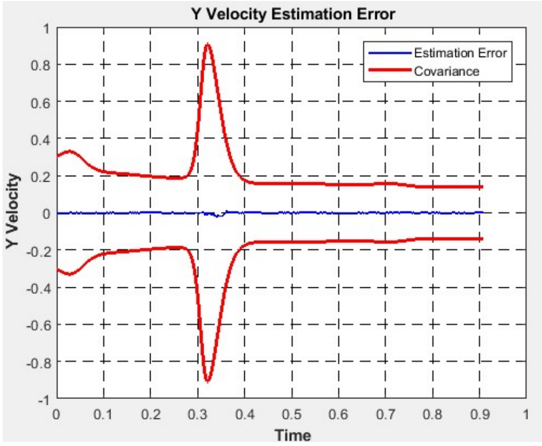

Figure 28: Y Velocity Estimation Error

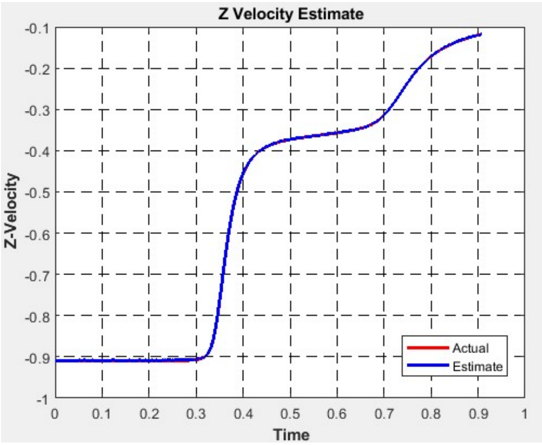

Figure 29: Z Velocity Estimate

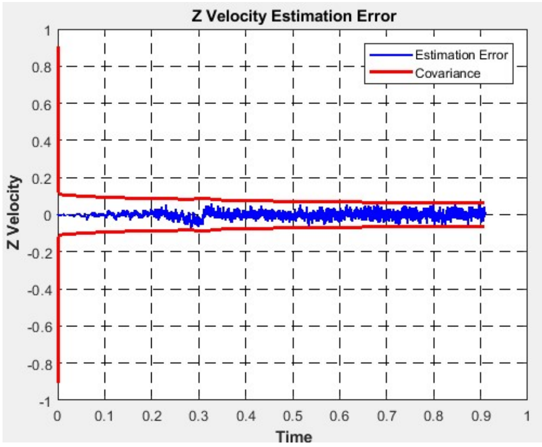

Figure 30: Z Velocity Estimation Error

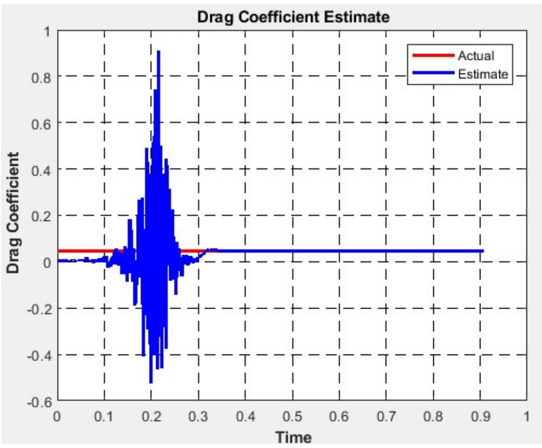

Figure 31: Drag Coefficient Estimate

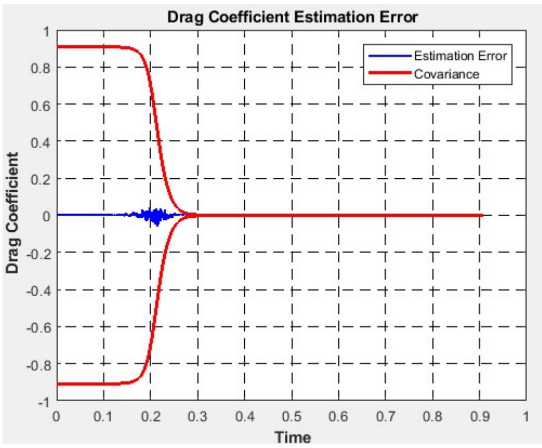

Figure 32: Drag Coeff. Estimation Error

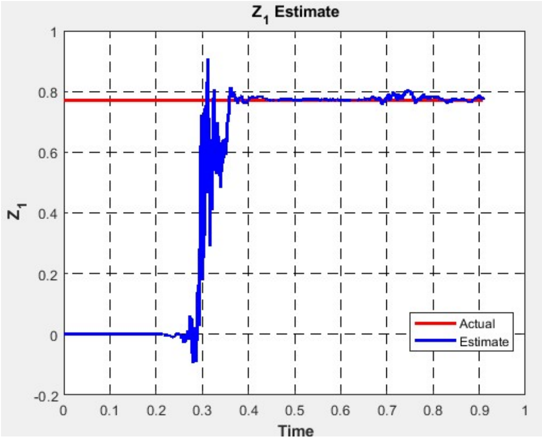

Figure 33: Z1 Estimate

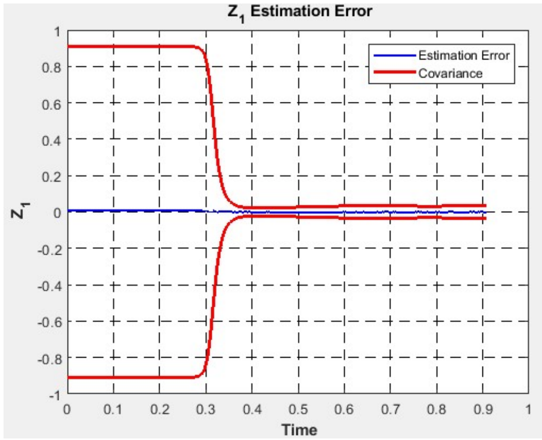

Figure 34: Z1 Estimation Error

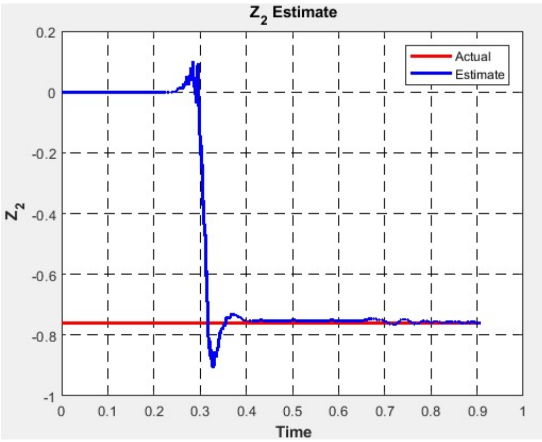

Figure 35: Z2 Estimate

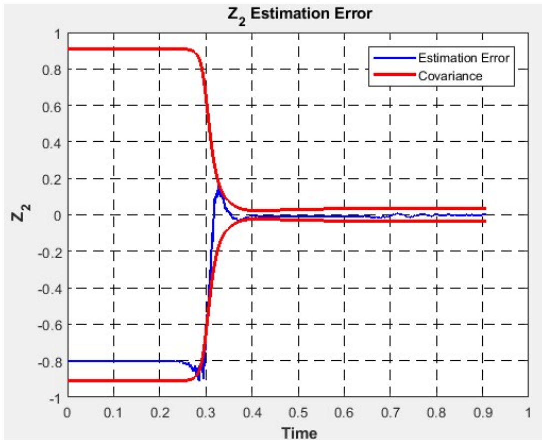

Figure 36: Z2 Estimation Error

Shown in Figure 31, Figure 32, Figure 33, Figure 34, Figure 35, Figure 36, the estimator starts to estimate Drag coefficient and Aerodynamic lift coefficients once the system enters into the atmosphere because the system becomes observable, before that the system was not observable and hence the estimator was not able to correctly estimate the Drag Coefficient and the Aerodynamic lift coefficients.

Conclusion

An extended Kalman Filter based state estimation algorithm was formulated using line-of-sight angles, Range, Range Rate measurements. The filter derivation employed non linear equations of motion for target dynamics including gravitational and aerodynamic acceleration terms . A set of states and parameters required to estimate the acceleration of target were employed in the filter formulation. Constant aerodynamic lift coefficients based approach is used to model maneuvering target.

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