```
import numpy as np
import pdb



def affine_forward(x, w, b):
  """
  Computes the forward pass for an affine (fully-connected) layer.

  The input x has shape (N, d_1, ..., d_k) and contains a minibatch of N
  examples, where each example x[i] has shape (d_1, ..., d_k). We will
  reshape each input into a vector of dimension D = d_1 * ... * d_k, and
  then transform it to an output vector of dimension M.

  Inputs:
  - x: A numpy array containing input data, of shape (N, d_1, ..., d_k)
  - w: A numpy array of weights, of shape (D, M)
  - b: A numpy array of biases, of shape (M,)

  Returns a tuple of:
  - out: output, of shape (N, M)
  - cache: (x, w, b)
  """

  # ================================================================ #
  # YOUR CODE HERE:
  #   Calculate the output of the forward pass.  Notice the dimensions
  #   of w are D x M, which is the transpose of what we did in earlier
  #   assignments.
  # ================================================================ #

  X = np.reshape(x, (x.shape[0], -1))
  out = X@w + b

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  cache = (x, w, b)
  return out, cache


def affine_backward(dout, cache):
  """
  Computes the backward pass for an affine layer.

  Inputs:
  - dout: Upstream derivative, of shape (N, M)
  - cache: Tuple of:
    - x: Input data, of shape (N, d_1, ... d_k)
    - w: Weights, of shape (D, M)

  Returns a tuple of:
  - dx: Gradient with respect to x, of shape (N, d1, ..., d_k)
  - dw: Gradient with respect to w, of shape (D, M)
  - db: Gradient with respect to b, of shape (M,)
  """
  x, w, b = cache
```

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  dx, dw, db = None, None, None

  # ================================================================ #
  # YOUR CODE HERE:
  #   Calculate the gradients for the backward pass.
  # ================================================================ #

  # dout is N x M
  # dx should be N x d1 x ... x dk; it relates to dout through multiplication with w, which is D x M
  # dw should be D x M; it relates to dout through multiplication with x, which is N x D after reshaping
  # db should be M; it is just the sum over dout examples

  N = x.shape[0]
  X = np.reshape(x, (N, -1))
  dx = np.reshape(dout@w.T, x.shape)
  dw = X.T@dout
  db = (dout.T@np.ones((N,)))

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  return dx, dw, db

def relu_forward(x):
  """
  Computes the forward pass for a layer of rectified linear units (ReLUs).

  Input:
  - x: Inputs, of any shape

  Returns a tuple of:
  - out: Output, of the same shape as x
  - cache: x
  """
  # ================================================================ #
  # YOUR CODE HERE:
  #   Implement the ReLU forward pass.
  # ================================================================ #
  out = np.maximum(x, 0)

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  cache = x
  return out, cache


def relu_backward(dout, cache):
  """
  Computes the backward pass for a layer of rectified linear units (ReLUs).

  Input:
  - dout: Upstream derivatives, of any shape
  - cache: Input x, of same shape as dout

  Returns:
```

```
    - dx: Gradient with respect to x
    """
    x = cache

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the ReLU backward pass
    # ================================================================ #

    # ReLU directs linearly to those > 0
    dx = dout * (x>0)

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    return dx


def softmax_loss(x, y):
    """
    Computes the loss and gradient for softmax classification.

    Inputs:
    - x: Input data, of shape (N, C) where x[i, j] is the score for the jth class
      for the ith input.
    - y: Vector of labels, of shape (N,) where y[i] is the label for x[i] and
      0 <= y[i] < C

    Returns a tuple of:
    - loss: Scalar giving the loss
    - dx: Gradient of the loss with respect to x
    """

    probs = np.exp(x - np.max(x, axis=1, keepdims=True))
    probs /= np.sum(probs, axis=1, keepdims=True)
    N = x.shape[0]
    loss = -np.sum(np.log(probs[np.arange(N), y])) / N
    dx = probs.copy()
    dx[np.arange(N), y] -= 1
    dx /= N
    return loss, dx
```