Problem 1.
MATLAB and Output:

```
load hw3.mat
who;

sample=[ones(1,50),2*ones(1,50)];

% estimate the initial mu and cov
mu1=sum(hw3_1(:,1:50),2)/50
mu2=sum(hw3_1(:,51:100),2)/50
a=0;
b=0;
for i = 1:50
    a= a +(hw3_1(:,i)-mu1)*(hw3_1(:,i)-mu1)';
end
cov1=a/50
for i = 51:100
    b= b +(hw3_1(:,i)-mu2)*(hw3_1(:,i)-mu2)';
end
cov2=b/50
label=[ones(1,50),2*ones(1,50)];

d1=[0;0];d2=[0;0];
for i=1:100
    if sample(i)==1
        d1=[d1,hw3_1(:,i)];
    else
        d2=[d2,hw3_1(:,i)];
    end
end

sd1=size(d1,2); sd2=size(d2,2);
d1=d1(:,2:sd1); d2=d2(:,2:sd2);
sd1=sd1-1; sd2=sd2-1;
rho1=sd1; rho2=sd2;

for i=1:100
    likelihood1=(1/(2*pi*sqrt(det(cov1))))*exp(-0.5*(hw3_1(:,i)-
mu1)'*inv(cov1)*(hw3_1(:,i)-mu1));
    likelihood2=(1/(2*pi*sqrt(det(cov2))))*exp(-0.5*(hw3_1(:,i)-
mu2)'*inv(cov2)*(hw3_1(:,i)-mu2));
    if likelihood1*rho1>likelihood2*rho2
        sample(i)=1;
    else
        sample(i)=2;
    end
end
```

```
mu1 =

    0.5656
   -0.6224

cov1 =

    3.3121   -0.9475
   -0.9475    2.2446

mu2 =

    1.9066
   -1.4085

cov2 =

    6.3584   -3.7961
   -3.7961    7.3799
```

```
% iteration 2
d1=[0;0];d2=[0;0];
for i=1:100
    if sample(i)==1
        d1=[d1,hw3_1(:,i)];
    else
        d2=[d2,hw3_1(:,i)];
    end
end

sd1=size(d1,2); sd2=size(d2,2);
d1=d1(:,2:sd1); d2=d2(:,2:sd2);
sd1=sd1-1; sd2=sd2-1;
rho1=sd1; rho2=sd2;

mu1=mean(d1,2), mu2=mean(d2,2),
dd1=d1-mu1*ones(1,sd1);
dd2=d2-mu2*ones(1,sd2);
cov1=(dd1*dd1')/sd1,
cov2=(dd2*dd2')/sd2,

for i=1:100
    likelihood1=(1/(2*pi*sqrt(det(cov1))))*exp(-0.5*(hw3_1(:,i)-
mu1)'*inv(cov1)*(hw3_1(:,i)-mu1));
    likelihood2=(1/(2*pi*sqrt(det(cov2))))*exp(-0.5*(hw3_1(:,i)-
mu2)'*inv(cov2)*(hw3_1(:,i)-mu2));
    if likelihood1*rho1>likelihood2*rho2
        sample(i)=1;
    else
        sample(i)=2;
    end
end
```

```
mu1 =

   -0.0263
   -0.1869

cov1 =

    0.9737    0.1068
    0.1068    0.7448

mu2 =

    4.0461
   -2.8595

cov2 =

    3.4376   -1.2289
   -1.2289    9.4352
```

```
% iteration 3
d1=[0;0];d2=[0;0];
for i=1:100
    if sample(i)==1
        d1=[d1,hw3_1(:,i)];
    else
        d2=[d2,hw3_1(:,i)];
    end
end

sd1=size(d1,2); sd2=size(d2,2);
d1=d1(:,2:sd1); d2=d2(:,2:sd2);
sd1=sd1-1; sd2=sd2-1;
rho1=sd1; rho2=sd2;

mu1=mean(d1,2), mu2=mean(d2,2),
dd1=d1-mu1*ones(1,sd1);
dd2=d2-mu2*ones(1,sd2);
cov1=(dd1*dd1')/sd1,
cov2=(dd2*dd2')/sd2,

for i=1:100
    likelihood1=(1/(2*pi*sqrt(det(cov1))))*exp(-0.5*(hw3_1(:,i)-
mu1)'*inv(cov1)*(hw3_1(:,i)-mu1));
    likelihood2=(1/(2*pi*sqrt(det(cov2))))*exp(-0.5*(hw3_1(:,i)-
mu2)'*inv(cov2)*(hw3_1(:,i)-mu2));
    if likelihood1*rho1>likelihood2*rho2
        sample(i)=1;
    else
        sample(i)=2;
    end
end
```

```
mu1 =

   -0.0093
   -0.0906


mu2 =

    4.2852
   -3.2796
```

```
cov1 =

    0.9153    0.2282
    0.2282    0.9373


cov2 =

    2.8888    0.0774
    0.0774    7.6114
```

```
% iteration 4
d1=[0;0];d2=[0;0];
for i=1:100
    if sample(i)==1
        d1=[d1,hw3_1(:,i)];
    else
        d2=[d2,hw3_1(:,i)];
    end
end

sd1=size(d1,2); sd2=size(d2,2);
```

```
d1=d1(:,2:sd1); d2=d2(:,2:sd2);
sd1=sd1-1; sd2=sd2-1;
rho1=sd1; rho2=sd2;

mu1=mean(d1,2), mu2=mean(d2,2),
dd1=d1-mu1*ones(1,sd1);
dd2=d2-mu2*ones(1,sd2);
cov1=(dd1*dd1')/sd1,
cov2=(dd2*dd2')/sd2,

for i=1:100
    likelihood1=(1/(2*pi*sqrt(det(cov1))))*exp(-0.5*(hw3_1(:,i)-
mu1)'*inv(cov1)*(hw3_1(:,i)-mu1));
    likelihood2=(1/(2*pi*sqrt(det(cov2))))*exp(-0.5*(hw3_1(:,i)-
mu2)'*inv(cov2)*(hw3_1(:,i)-mu2));
    if likelihood1*rho1>likelihood2*rho2
        sample(i)=1;
    else
        sample(i)=2;
    end
end
```

```
mu1 =                          cov1 =

    0.0276                         0.9994    0.2856
   -0.0675                         0.2856    0.9622


mu2 =                          cov2 =

    4.3436                         2.8930    0.3738
   -3.4529                         0.3738    7.0120
```

```
% iteration 5
d1=[0;0];d2=[0;0];
for i=1:100
    if sample(i)==1
        d1=[d1,hw3_1(:,i)];
    else
        d2=[d2,hw3_1(:,i)];
    end
end

sd1=size(d1,2); sd2=size(d2,2);
d1=d1(:,2:sd1); d2=d2(:,2:sd2);
sd1=sd1-1; sd2=sd2-1;
rho1=sd1; rho2=sd2;

mu1=mean(d1,2), mu2=mean(d2,2),
dd1=d1-mu1*ones(1,sd1);
dd2=d2-mu2*ones(1,sd2);
cov1=(dd1*dd1')/sd1,
cov2=(dd2*dd2')/sd2,

for i=1:100
```

```
    likelihood1=(1/(2*pi*sqrt(det(cov1))))*exp(-0.5*(hw3_1(:,i)-
mu1)'*inv(cov1)*(hw3_1(:,i)-mu1));
    likelihood2=(1/(2*pi*sqrt(det(cov2))))*exp(-0.5*(hw3_1(:,i)-
mu2)'*inv(cov2)*(hw3_1(:,i)-mu2));
    if likelihood1*rho1>likelihood2*rho2
        sample(i)=1;
    else
        sample(i)=2;
    end
end
```

```
mu1 =

    0.0276
   -0.0675


mu2 =

    4.3436
   -3.4529
```

```
cov1 =

    0.9994    0.2856
    0.2856    0.9622


cov2 =

    2.8930    0.3738
    0.3738    7.0120
```

Problem 2.

1)

```
clear
load hw3.mat

data1 = hw3_2_1;
data2 = hw3_2_2;
x1 = -4:0.1:8;
x2 = -4:0.1:8;
h1 = 2;
n = size(data1, 2);
hn = h1 / sqrt(n);
l=0;

for i = -4:0.1:8
    l = l+1;
    m=0;
    for j = -4:0.1:8
        m=m+1;
        p1(l,m) = 0;
        p2(l,m) = 0;
        for k = 1:n
            xd1 = [i;j] - data1(:,k);
            xd2 = [i;j] - data2(:,k);
            xd1 = xd1(1)^2+xd1(2)^2;
            xd2 = xd2(1)^2+xd2(2)^2;
            p1(l,m) = p1(l,m) + 1/(n*hn*sqrt(2*pi))*exp(-xd1/(2*hn^2));
            p2(l,m) = p2(l,m) + 1/(n*hn*sqrt(2*pi))*exp(-xd2/(2*hn^2));
        end
        if i==1 && j==-2
            p1x = p1(l,m)
            p2x = p2(l,m)
        end
    end
end
```

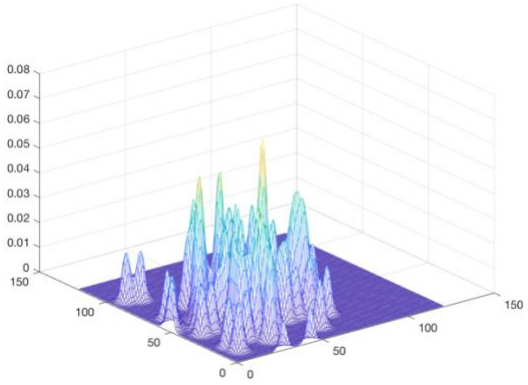
```
figure(1);
mesh(p1);
figure(2);
mesh(p2);
```

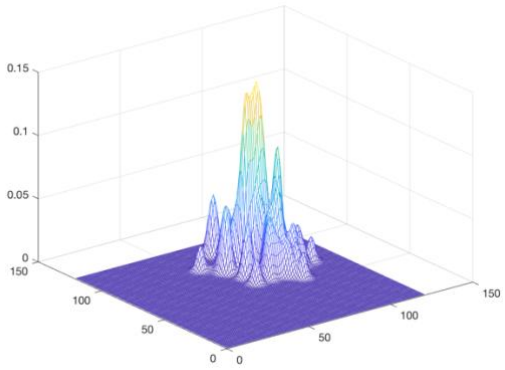
```
p1x =

    0.0062


p2x =

  1.4097e-119
```





Then given a point $x = [1, -2]^t$, we can get the class conditional probability from the results we got:

$$p(x \mid w\_1) = 0.0062$$
$$p(x|w_2) = 1.4097 \times 10^{-119}$$

Due to $p(w_1) = p(w_2) = 0.5$,

$$\mathrm{p}(w_2|\mathrm{x}) \propto \mathrm{p}(\mathrm{x}|w_2) \times p(w_2)$$
$$\mathrm{p}(w_1|\mathrm{x}) \propto \mathrm{p}(\mathrm{x}|w_1) \times p(w_1)$$
$$p(x \mid w_1) > p(x \mid w_2)$$

Thus, we classify the test data to w_1.

2)
```
for j = 1:n
    data1_new(1,j) = data1(1,j) / sqrt(data1(1,j)^2 + data1(2,j)^2);
    data1_new(2,j) = data1(2,j) / sqrt(data1(1,j)^2 + data1(2,j)^2);
    data2_new(1,j) = data2(1,j) / sqrt(data2(1,j)^2 + data2(2,j)^2);
    data2_new(2,j) = data2(2,j) / sqrt(data2(1,j)^2 + data2(2,j)^2);
end
wk1 = data1_new;
wk2 = data2_new;
x = [1;-2];
x = [x(1)/sqrt(x(1)^2+x(2)^2);x(2)/sqrt(x(1)^2+x(2)^2)];

sigma = 0.2;
net1 = x' * wk1;
```

```
net2 = x' * wk2;

g1 = sum(exp((net1-1)/sigma^2))
g2 = sum(exp((net2-1)/sigma^2))
```

```
g1 =

    8.9007


g2 =

   1.9525e-09
```