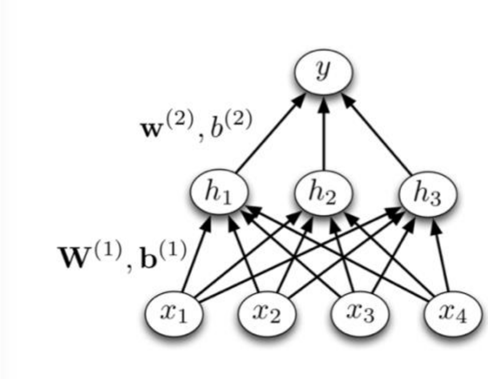
Hard-coding a Network

1/ It should be a multilayer perceptron as following:



I set weights and biases such that h_j activate if $x_j \geq x_{j+1}$.

If $x_1 \geq x_2$, $h_1 = 1$, $w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}x_4 + b_1 \geq 0$, $w_1 = (1, -1, 0, 0), b_1 = 0$

If $x_2 \geq x_3$, $h_2 = 1$, $w_{21}x_1 + w_{22}x_2 + w_{23}x_3 + w_{24}x_4 + b_2 \geq 0$, $w_1 = (0, 1, -1, 0), b_1 = 0$

If $x_3 \geq x_4$, $h_3 = 1$, $w_{31}x_1 + w_{32}x_2 + w_{33}x_3 + w_{34}x_4 + b_3 \geq 0$, $w_1 = (0, 0, 1, -1), b_1 = 0$

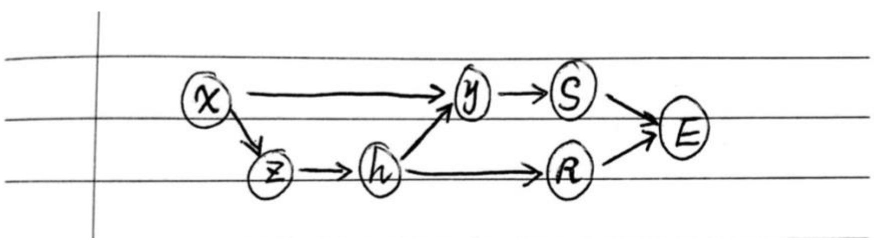
Thus, $W^{(1)} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \quad b^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Then I set weights and biases such that y activate if and only if no h activates.

If $h_1 = 0, h_2 = 0, h_3 = 0, y = 1, w_1h_1 + w_2h_2 + w_3h_3 + b \geq 0$,

For example, $w^{(2)} = (-1, -1, -1) \quad b^{(2)} = 0.1$

2/ The computation graph is as following:



Derive backprop equations:

$$\bar{E} = 1$$

$$\bar{R} = \bar{E}\frac{dE}{dR} = 1$$

$$\bar{S} = \bar{E}\frac{dE}{dS} = 1$$

$$\overline{y_i} = \bar{S}\frac{dS}{dy_i} = y_i - s_i$$

$$\overline{h_j} = \bar{R}\frac{dR}{dh_j} + \sum_{i=1}^{N} \overline{y_i}\frac{dy_i}{dh_i} = r_j + \sum_{i=1}^{N} \overline{y_i} w_{ij}^{(2)}$$

$$\overline{z_j} = \overline{h_j}\frac{dh_j}{dz_j} = \overline{h_j}\sigma'(z_j)$$

$$\overline{x_i} = \sum_{j=1}^{K} \overline{z_j}\frac{dz_j}{dx_i} + \overline{y_i}\frac{dy_i}{dx_i} = \sum_{j=1}^{K} \overline{x_j} w_{ji}^{(1)} + \overline{y_i}$$

The vector form is:

$$\bar{E} = 1$$
$$\bar{R} = 1$$
$$\bar{S} = 1$$
$$\overline{\boldsymbol{y}} = \boldsymbol{y} - \boldsymbol{s}$$
$$\overline{\boldsymbol{h}} = \boldsymbol{r} + \boldsymbol{W}^{(2)T}\overline{\boldsymbol{y}}$$
$$\overline{\boldsymbol{z}} = \overline{\boldsymbol{h}} \circ \sigma'(\boldsymbol{z})$$
$$\overline{\boldsymbol{x}} = \boldsymbol{W}^{(1)T}\overline{\boldsymbol{z}} + \overline{\boldsymbol{y}}$$