Ma 661 Dynamic Programming and Reinforcement Learning

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Homework 1

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Problem 1. Consider the optimization problem

$$\begin{aligned} \max\ & 4z_1 + 5z_2 + 4z_3 + 8z_4 + 6z_5 \\ & 4z_1 + 6z_2 + 5z_3 + 9z_4 + 7z_5 \leq 19 \\ & z_j \geq 0, \quad j = 1, \ldots, 5. \end{aligned} \tag{1}$$

Assume that all z_j may take only nonnegative integer values.

(a) Formulate a dynamic programming problem for solving problem (1). Describe the state space, the action space, the feasible action mapping, and the dynamic programming equation.

(b) Use the dynamic programming equations to obtain the optimal solution.

Problem 2. Let $A_1, \ldots, A_n$ be matrices with A_i having k_{i-1} rows and k_i columns for $i = 0, 1, 2, \ldots, n$. and some positive integers $k_0, k_1, \ldots, k_n$. The problem is to choose the order of multiplying the matrices that will minimize the number of multiplications needed to compute the product $A_1 A_2 \cdots A_n$.

(a) Formulate a dynamic programming problem to determine the optimal order in which to multiply the matrices. Clearly formulate the dynamic programming equation.

(b) Solve the problem numerically, when $n = 4$ and $(k_0, k_1, \ldots, k_4) = (10, 30, 70, 2, 100)$.

Problem 3. A directed graph has node set $\mathcal{N} = \{1, \ldots, 10\}$ and the arc set given in the table on the next page. Determine the shortest path from node 1 to node 10 using dynamic programming.

For all problems, you can use any software to code the dynamic programming equation and solve. However, you cannot use an optimization solver to solve the problems. Please, provide printouts.

Arc	From	To	Cost
1	1	2	4
2	1	4	7
3	1	6	8
4	1	8	9
5	2	4	7
6	4	6	12
7	8	6	6
8	2	3	11
9	4	3	5
10	4	5	10
11	6	5	16
12	6	7	15
13	7	8	11
14	8	9	12
15	3	5	10
16	9	7	9
17	3	10	16
18	5	10	8
19	7	10	4
20	9	10	14