

### Homework 4

due Wednesday, March 25, 2020

**Problem 1.** Consider the equipment replacement problem of Assignment 2. Assume that we would like to identify the optimal replacement policy by solving an infinite-horizon discounted total reward problem.

- (1.1) Formulate the infinite-horizon Markov decision problem.
- (1.2) If there is no salvage value, then show that the optimal value function is non-increasing function of the state.
- (1.3) Solve the infinite horizon problem (with salvage value present) for the following values of the parameters:  $c_0 = 1$ ,  $c_1 = 1$ ,  $R = 5$ ,  $K = 10$ ,  $\gamma = 0.8$ ,  $\mu = 0.2$ ,  $\lambda = 1$  and discount factor  $\alpha = 0.9$ . Solve the problem in all three ways: value iteration method, policy iteration method and linear programming.

**Problem 2.** We consider an inventory model as discussed in class. The stock at the beginning of period  $t$  denoted by  $x_t$ , orders at the beginning of period  $t$  by  $u_t$ , and random demand in period  $t$  (observed only *after* the orders are placed) by  $d_t$ . We assume ordering cost 5, selling price 10 and holding cost 2. The demands in successive periods are i.i.d. with values  $(0, 1, 2, 3, 4)$  whose respective probabilities are  $0.1, 0.2, 0.3, 0.2, 0.2$ . The capacity of the inventory is 12.

- (2.1) Formulate an infinite horizon problem with discount factor 0.8 to determine the best re-order policy.
- (2.2) Solve the problem in (2.1) by value and policy iteration methods.

**Problem 3.** Fisher boat is sent to the waters of three connected lakes during one fishing season. Let  $x_i$   $i = 1, 2, 3$  be the (estimated) current amounts of fish in lake  $i$ . If we fish in lake  $i$ , then we harvest  $r_i x_i$  fish, provided the fishing conditions are good. The weather may change abruptly with probability  $p$  so that we end the fishing season. We assume that  $0 < r_i < 1$  for all  $i = 1, 2, 3$ . Identify the lake-selection policy that maximizes the amount of fish before the end of the season.