**Problem 1.**

Solution:

(1). State space: state of equipment – new.

Control space: 0 - wait; 1 – replace.

Reward function for one period:

Dynamic programming equation:

(2) If there is no salvage value, the reward function for one period is:

So the is non-increasing because in the case ; and is a constant in the case .

Further, and are constant and do not depend on . The function are non-increasing with respect to , by induction assumption.

Denote

We conclude that and are non-increasing because they are sums of non-increasing functions.

Thus, for

We conclude that is non-increasing.

(3) The reward function for one period is

The probabilities of deterioration by j steps in one period are given by Poisson distribution:

Value iteration:

MATLAB:

% [v\_lo,n\_it] = dne1\_value\_iteration\_revised (0.9,10000);

function [v\_lo,n\_it] = dne1\_value\_iteration\_revised (alpha,max\_it)

i = 0;

n\_it = max\_it;

v=[0,0,0,0,0];

vv=[0,0,0,0,0];

v\_lo=[0,0,0,0,0];

v\_up=[0,0,0,0,0];

while (i < n\_it)

vv(1) = max(2+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)),4+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)));

vv(2) = max(0.55+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)),5+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)));

vv(3) = max(-0.64+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)),6+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)));

vv(4) = max(-2.41+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)),7+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)));

vv(5) = max(-3.06+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)),8+alpha\*(0.37\*v(1)+0.37\*v(2)+0.18\*v(3)+0.06\*v(4)+0.02\*v(5)));

v\_lo = vv + min(vv-v)\*alpha/(1-alpha);

v\_up = vv + max(vv-v)\*alpha/(1-alpha);

if (isequal(v\_lo,v\_up))

n\_it=i;

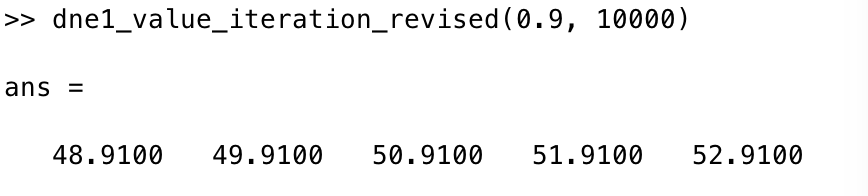
end

i=i+1;

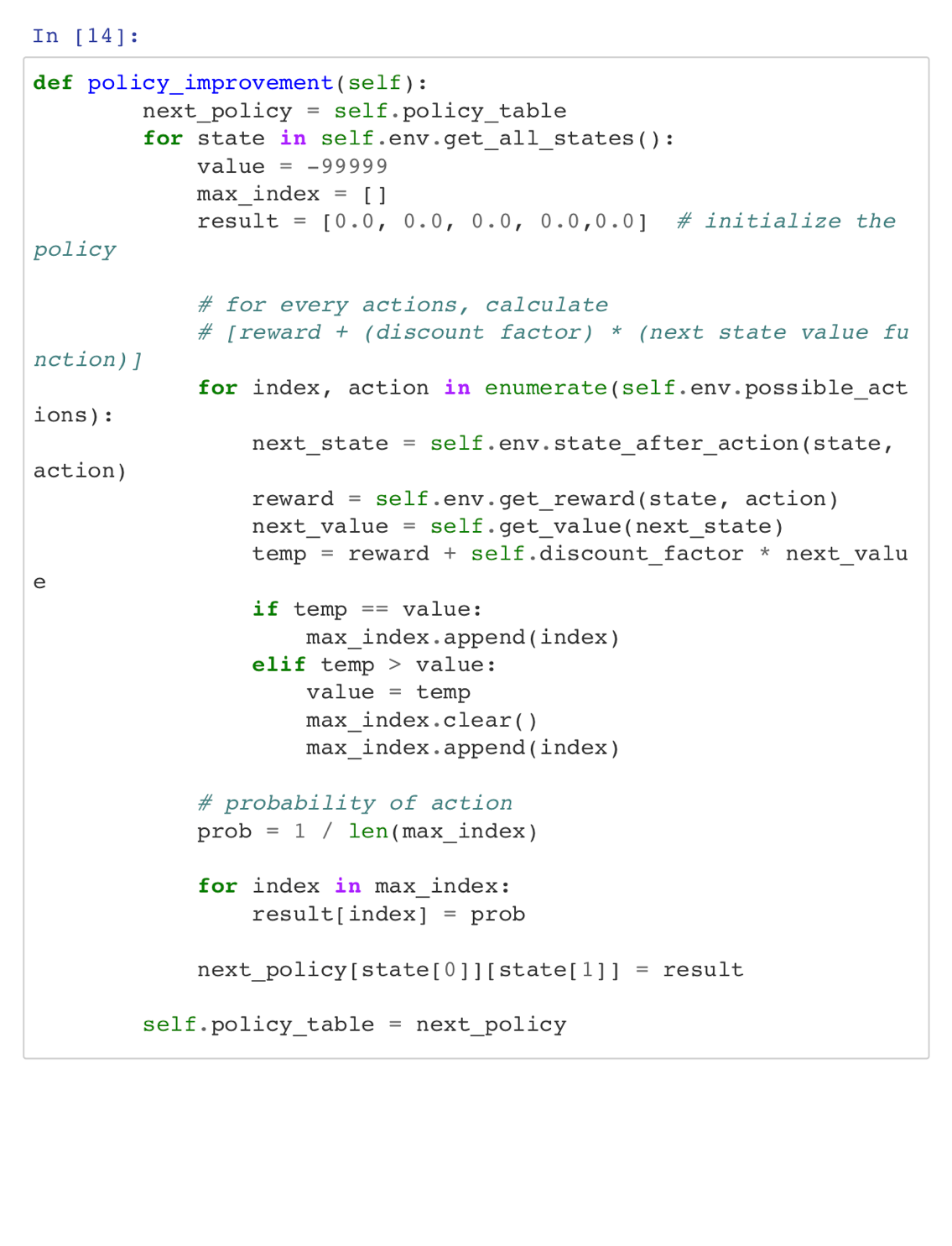
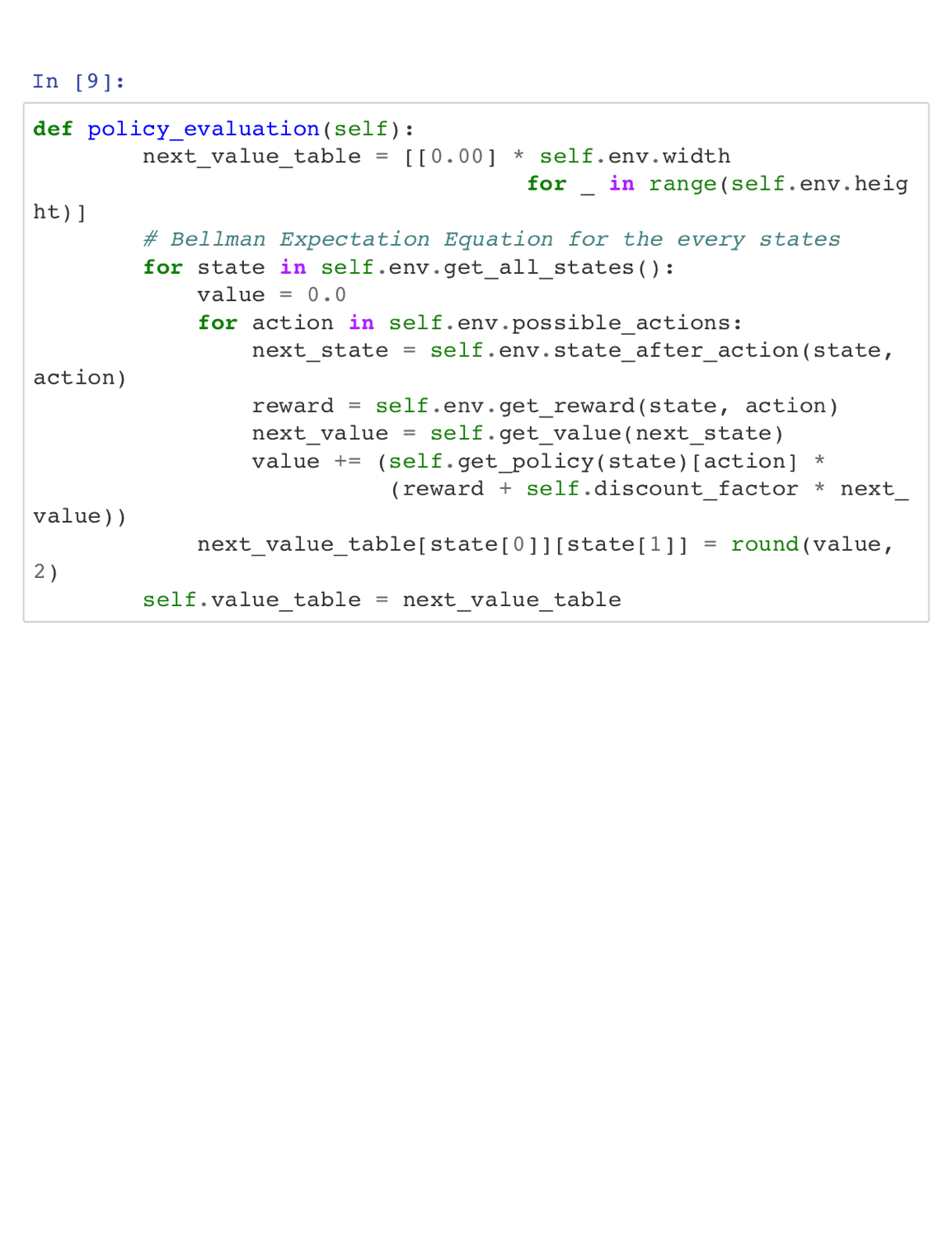
v=v\_lo;

end

end



Policy iteration:



Linear programming:

% v = dne1\_LP

function v = dne1\_LP

clear();

f=[1;1;1;1;1];

A=[0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02;

0.37,0.37,0.18,0.06,0.02];

b=[2;4;0.55;5;-0.64;6;-2.41;7;-3.06;8];

v=linprog(f,A,b);

end

**Problem 2.**

Solution:

(1) State space:

Control space:

Dynamic programming equation:

(2) value iteration:

% [v\_lo,n\_it] = dne1\_value\_iteration\_revised (0.8,10000);

function p=p(~)

alphabet=[0,1,2,3,4];

prob= [0.1,0.2,0.3,0.2,0.2];

p=randsrc(1,1,[alphabet;prob]);

end

function [v\_lo,n\_it] = dne1\_value\_iteration\_revised (alpha,max\_it)

i = 0;

n\_it = max\_it;

v=[0,0,0,0,0,0,0,0,0,0,0,0,];

vv=[0,0,0,0,0,0,0,0,0,0,0,];

v\_lo=[0,0,0,0,0,0,0,0,0,0,0,];

v\_up=[0,0,0,0,0,0,0,0,0,0,0,];

while (i < n\_it)

vv(1) = max(2+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),4+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(2) = max(0.55+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),5+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(3) = max(2+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),4+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(4) = max(0.55+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),5+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(5) = max(2+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),4+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(6) = max(0.55+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),5+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(7) = max(2+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),4+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(8) = max(0.55+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),5+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(9) = max(2+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),4+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(10) = max(0.55+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),5+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(11) = max(2+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),4+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

vv(12) = max(0.55+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)),5+alpha\*(p\*v(1)+p\*v(2)+p\*v(3)+p\*v(4)+p\*v(5)+p\*v(6)+p\*v(7)+p\*v(8)+p\*v(9)+p\*v(10)+p\*v(11)+p\*v(12)));

v\_lo = vv + min(vv-v)\*alpha/(1-alpha);

v\_up = vv + max(vv-v)\*alpha/(1-alpha);

if (isequal(v\_lo,v\_up))

n\_it=i;

end

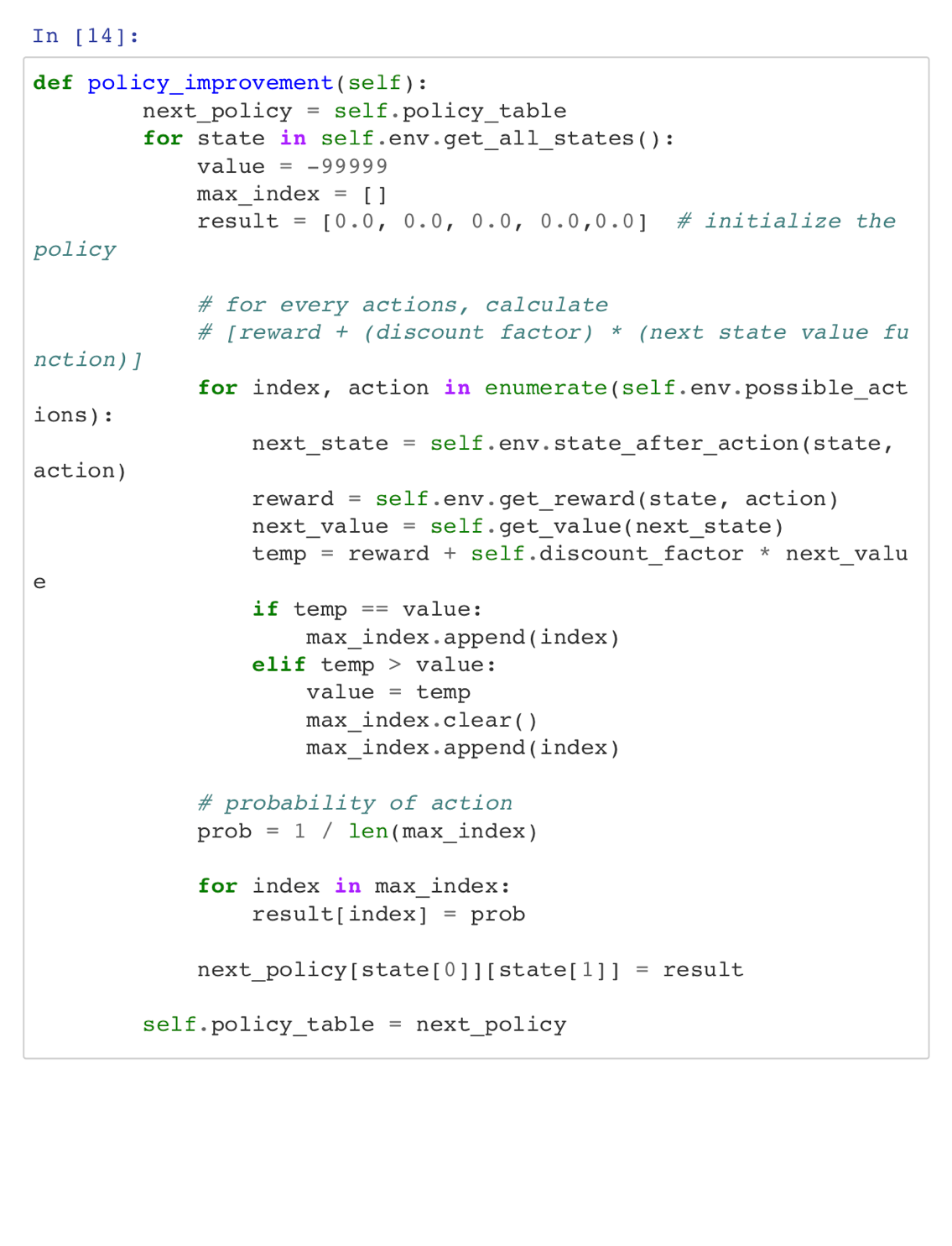
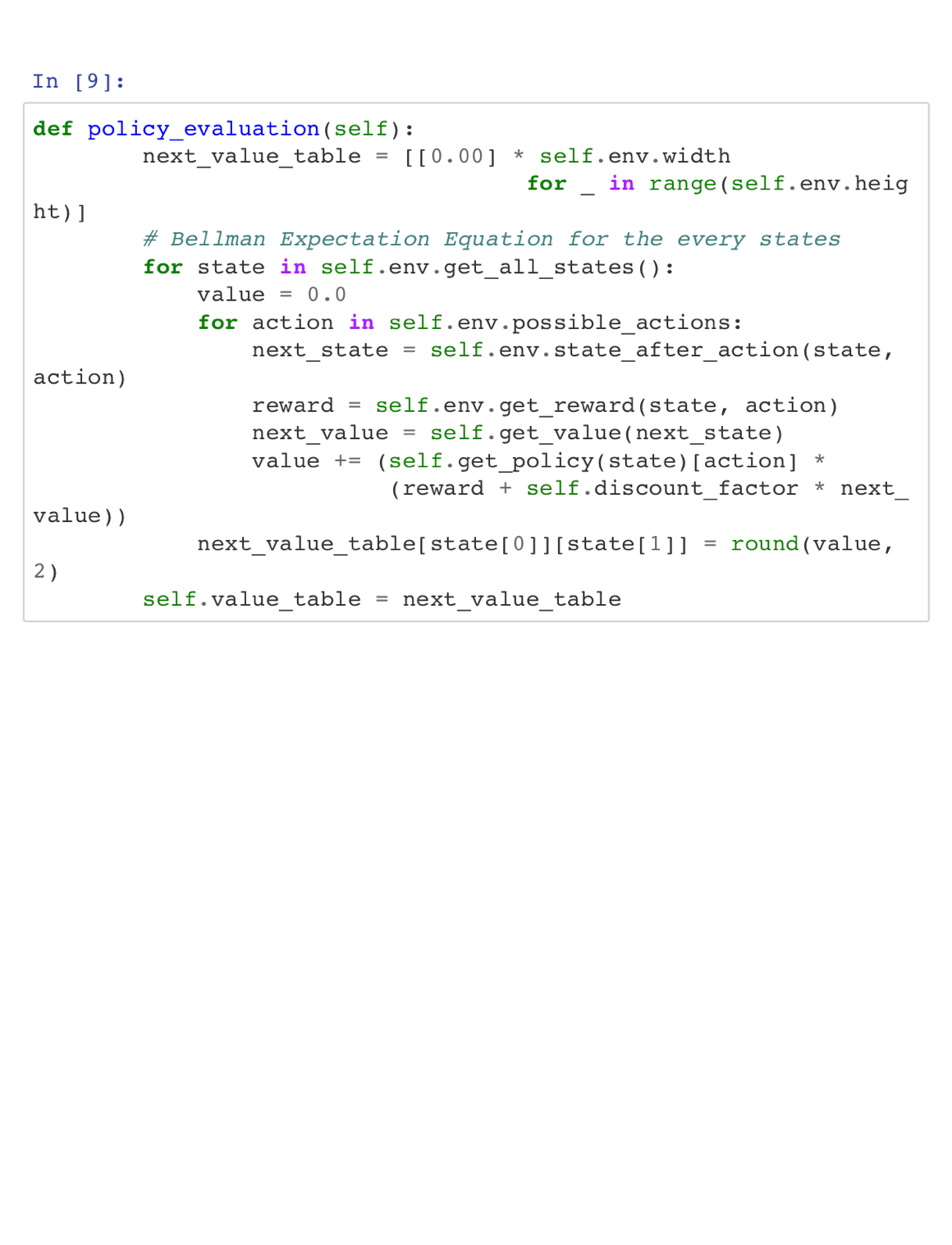
i=i+1;

v=v\_lo;

end

end

Policy iteration:



**Problem 3.**

Solution:

State space:

Control space:

Reward function is

Dynamic programming equation:

Policy: