

# Reproducible report III for ‘Biodiversity increases and decreases ecosystem stability’

Generalised conversion to overall ecosystem functioning / stability

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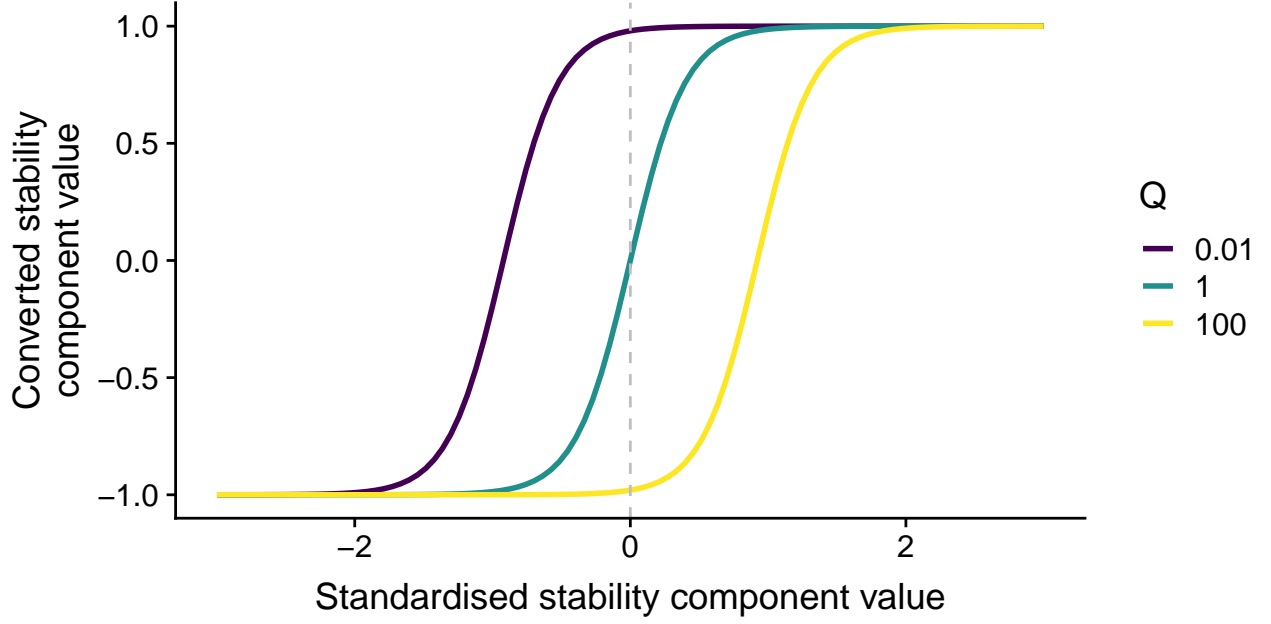
## Introduction

Calculating ecosystem multifunctionality (also known as overall ecosystem functioning) or overall ecosystem stability requires that values of an ecosystem function (e.g. biomass production) or of a stability component (e.g. resistance to temperature) be converted into a common currency. Mathematical functions used for these conversions include linear and threshold (step) functions. The generalised logistic function (also known as the Richard’s function) is flexible enough to give a wide range of conversion functions. If  $x$  is the measured variable, and  $y$  is the converted variable, the generalised logistic function is:

$$Y = A + \frac{K-A}{(C+Qe^{-Bx})^{1/v}}$$

- $A$  is the lower asymptote.
- $K$  is the upper asymptote.
- $B$  is the gradient.
- $v$  affects the symmetry, and also the value of  $y(0)$ .
- $Q$  affects the value of  $y(0)$ .
- $C$  is typically set to 1.
- $x$  is a variable, here the value of the measured ecosystem function or stability component.

The parameter  $Q$  controls the value of the function when  $x$  is zero; in other words, it shifts the curve horizontally:

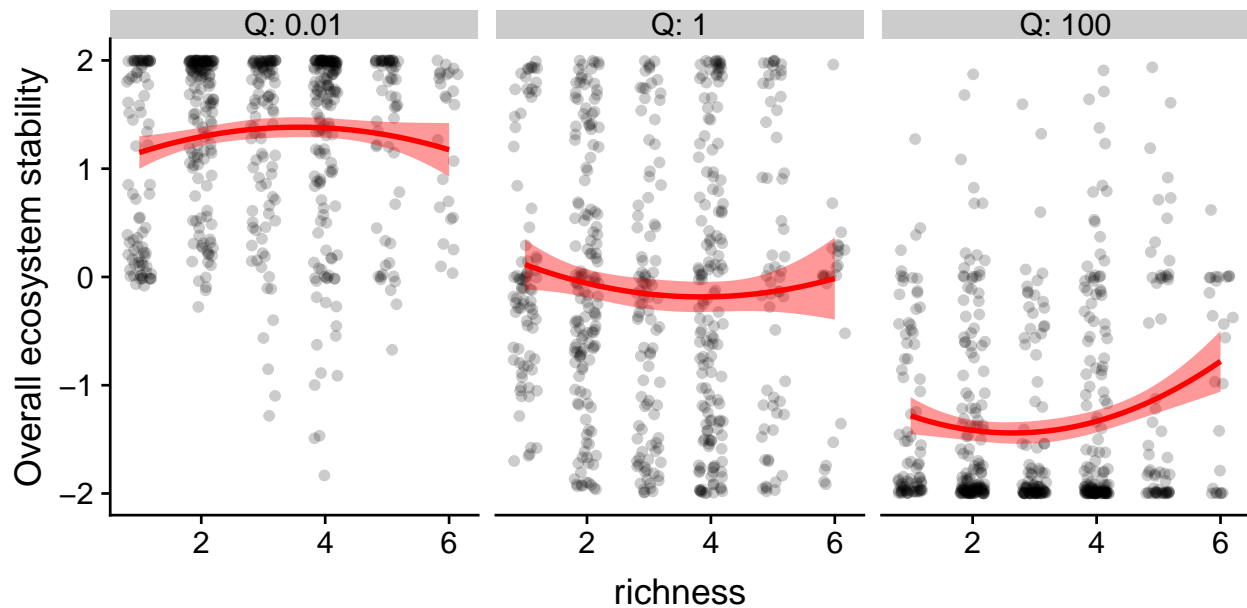


Overall ecosystem stability is then the sum of the standardised and converted stability components  $OES = c(z(r)) + c(z(t))$ , where  $r$  is the measured resistance,  $t$  is the measured temporal stability, the function  $z()$  subtracts the mean and divides by the standard deviation, and  $c()$  is the generalised logistic function. The parameters of  $c()$  were  $A = -1$ ,  $K = 1$ ,  $B = 5$ ,  $v = 1$ ,  $C = 1$  and  $Q$  was varied from  $10^{-2}$  to  $10^2$  (the same as in the figure above); these parameter values were chosen to produce converted stability measures that span the range  $A$  to  $K$  and to have a relatively threshold-like change from  $A$  to  $K$ . Standardisation prior to summation emphasises that the units of converted stability measures here are arbitrary (though generally need not be). Standardisation also implies equal weights for different stability components, while Gamfeldt et al (2008) point out that weighting of functions needs to be considered. Differential weightings, if desired and justified, can be incorporated by varying the parameters of the generalised logistic (or some other) conversion function.

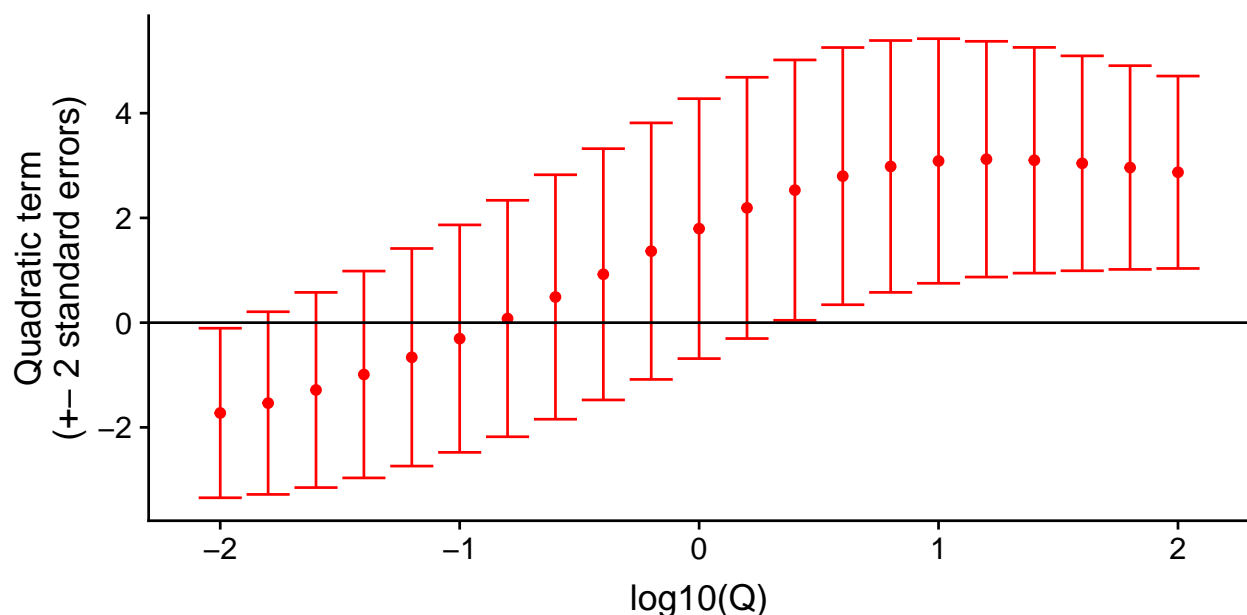
## Replicate level analysis

Replicate level analyses are with a data point for every replicate, i.e. a richness, temporal stability, and resistance for each replicate. (After this will come an analysis at the richness level, with one data point for each replicate created by aggregation across replicates.)

Below are relationships between overall ecosystem stability and species richness, one for each of three values of the  $Q$  parameter in the generalised logistic conversion function – the three values used in the graph above;  $Q = 0.01, 0, 100$ . The lines are linear models with quadratic terms; the shaded regions are confidence intervals of the regression.



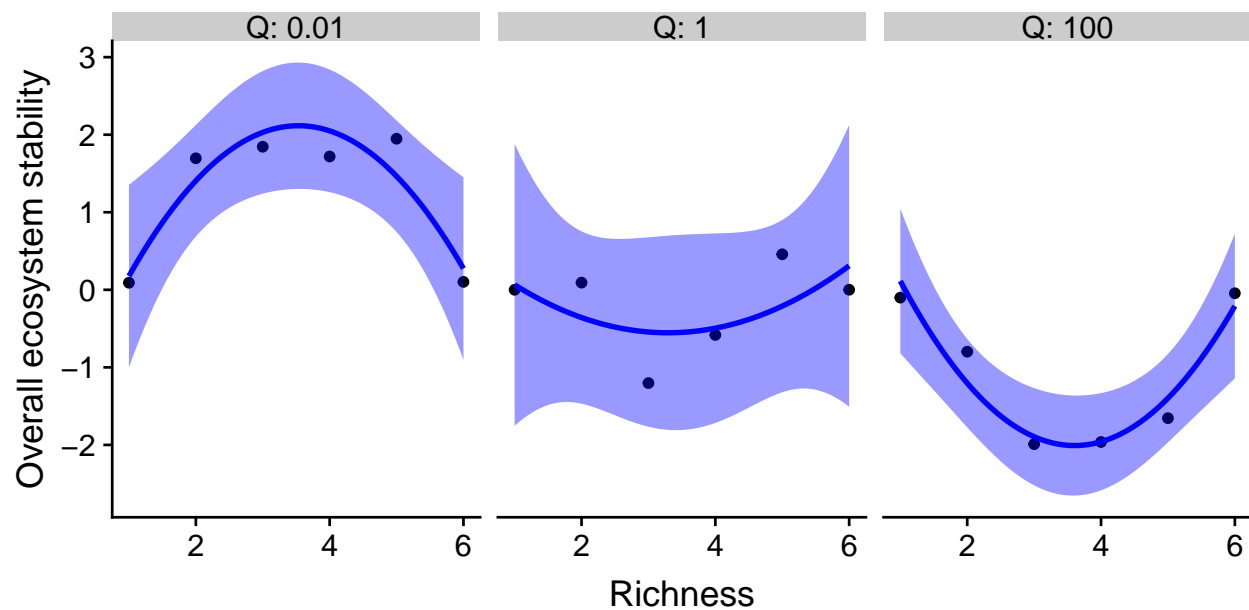
The graph above suggests that the curvature of the stability - richness relationship depends on  $Q$ . The graph below shows the quadratic term of the regression with error bars of two standard errors. It confirms that the curvature smoothly depends on  $Q$ , varying from negative (a hump-shaped relationship) to positive (a u-shaped relationship).



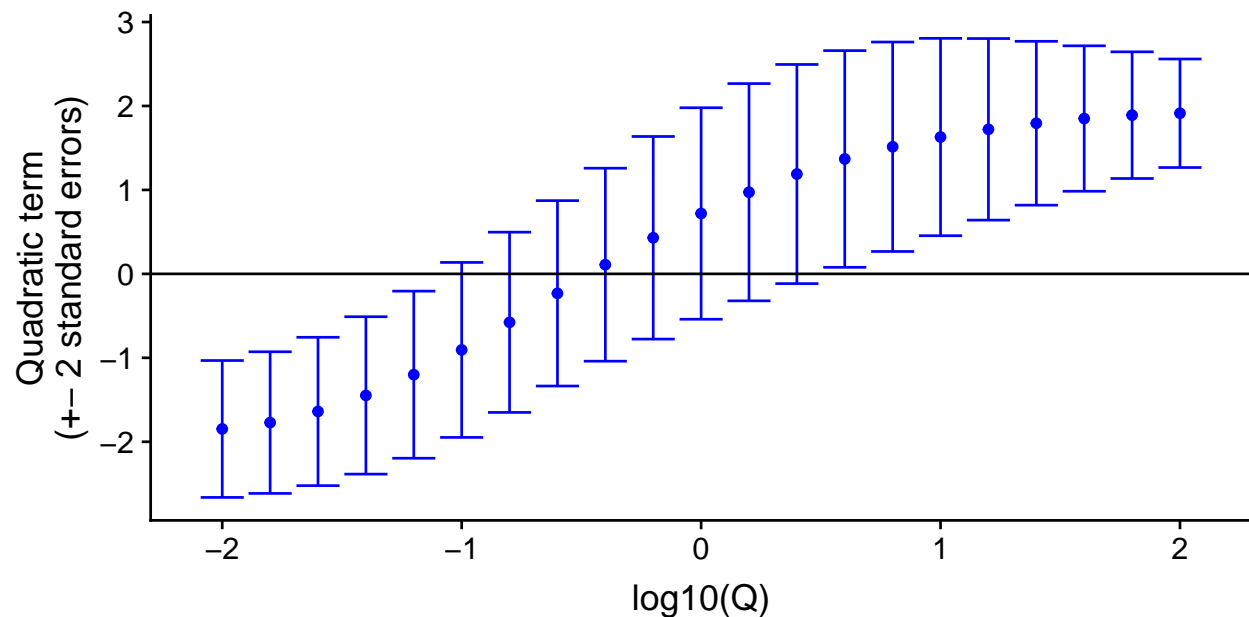
## Richness level analysis

This analysis results in only six data points, one for each richness levels, by averaging across all values within richness levels. (Text and graphs below are otherwise the same as those in the replicate level analysis above.)

Below are relationships between overall ecosystem stability and species richness, one for each of three values of the  $Q$  parameter in the generalised logistic conversion function – the three values used in the graph above;  $Q = (-2, 0, 2)$ . The lines are linear models with quadratic terms; the shaded regions are confidence intervals of the regression.

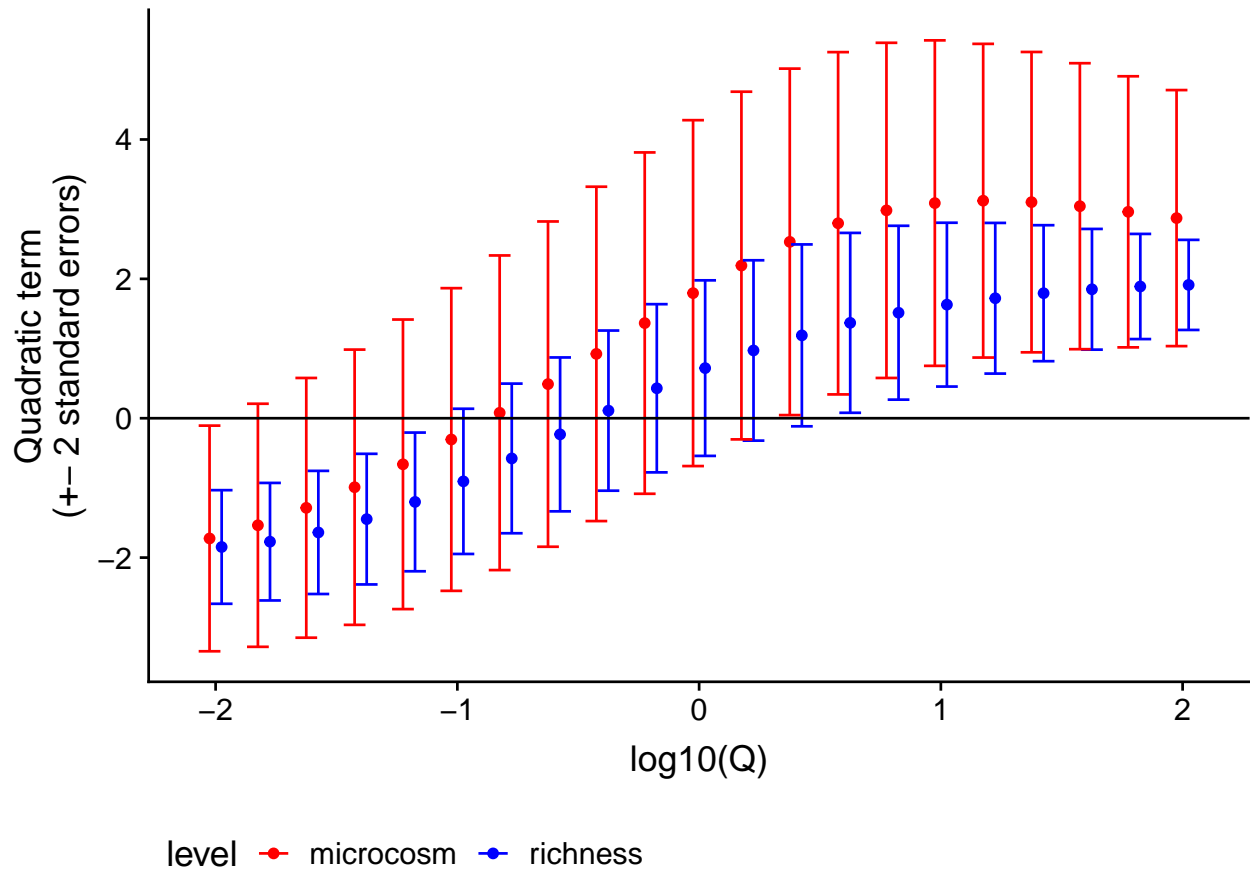


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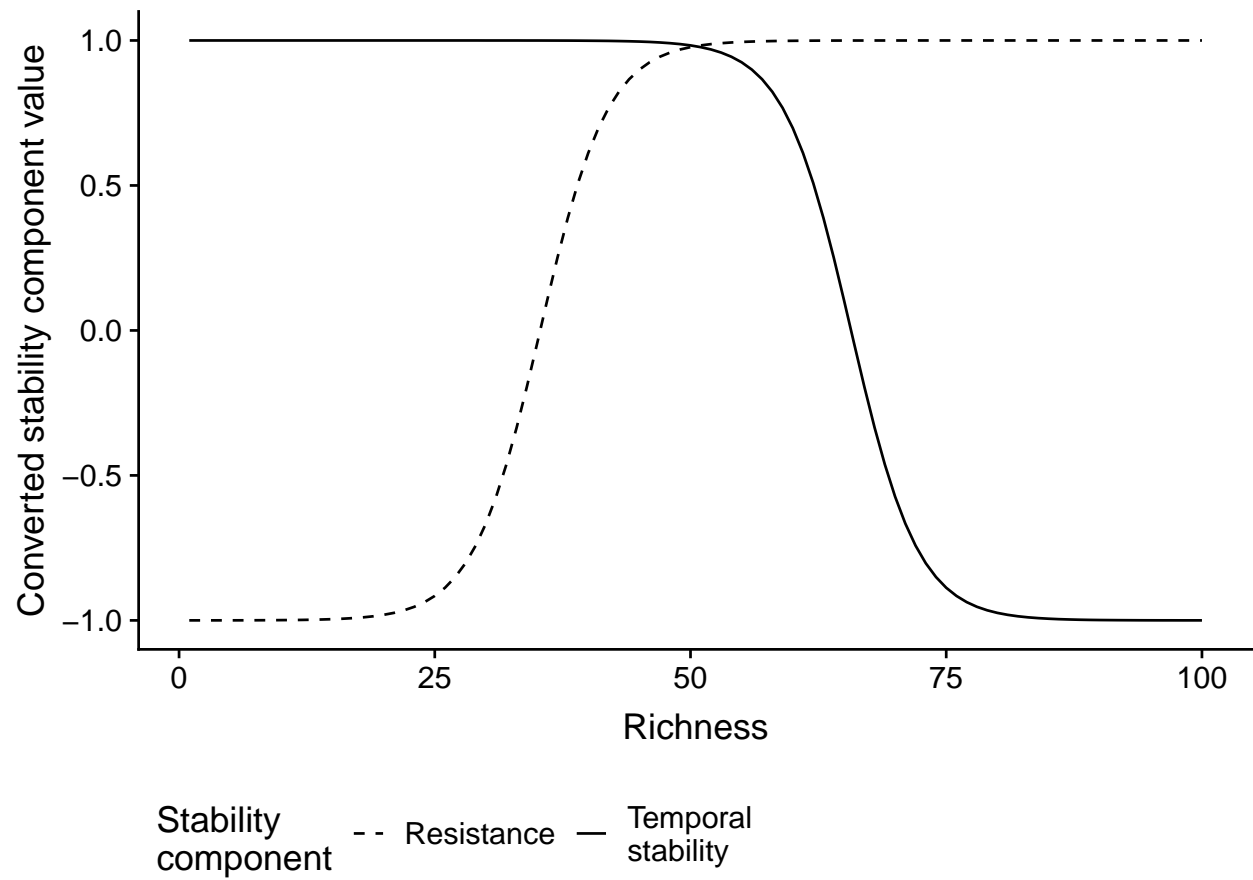
## Replicate and richness level results

The graph below shows the overlaid data from two of the graphs above: those showing the quadratic term of the regression with error bars of two standard errors.



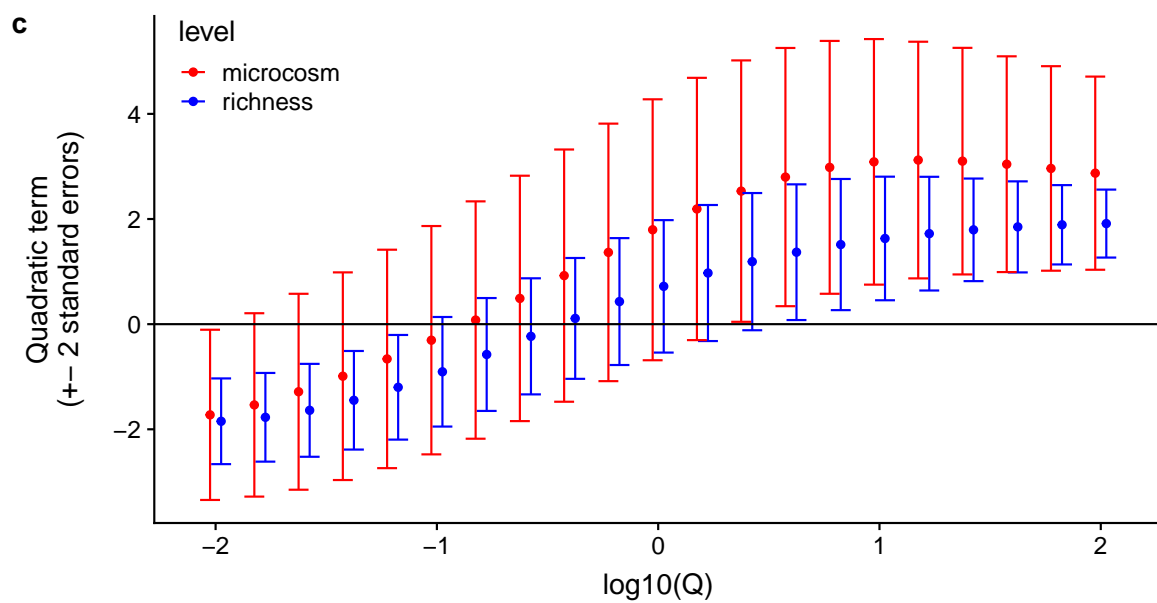
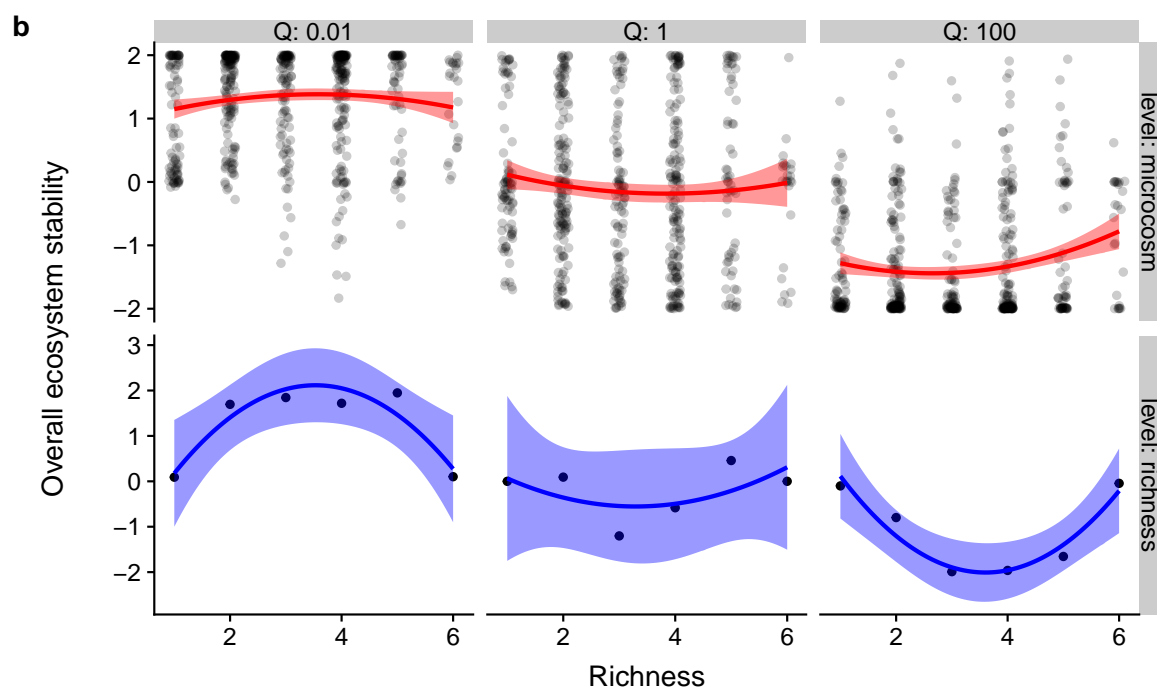
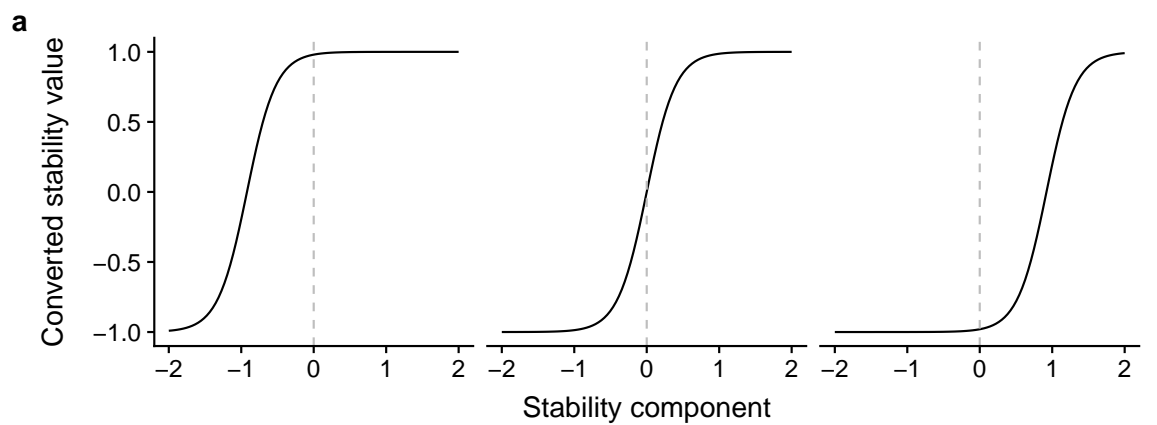
## Explanation

Why does the hump-shaped (or u-shaped) relationship occur? First consider temporal stability, which increases linearly with richness, and so progresses from left to right along the conversion function. Consequently, with low values of  $Q$ , temporal stability crosses the threshold of the conversion function at low species richness (see graph below). Then consider resistance, which decreases linearly with richness, and so progresses along the conversion function in the opposite direction (i.e. from right to left). Hence, resistance crosses the threshold at high species richness. The result is that only at intermediate richness are there high values of both stability components, such that overall ecosystem stability is high.



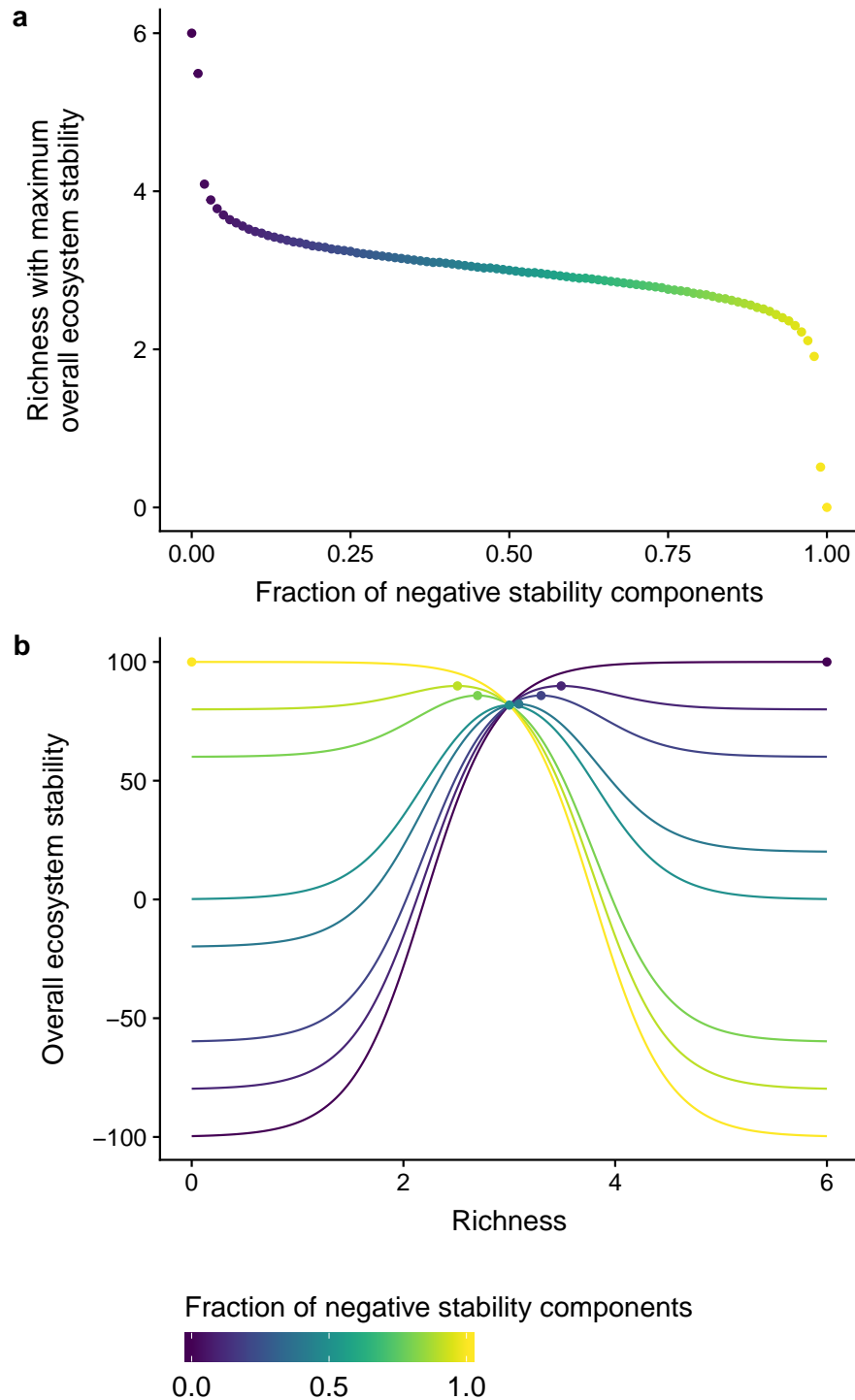
**Figure 3**

Putting it all together to produce figure 3:



## How does consideration of more than two components affect the unimodal pattern?

While the unimodal relationship is the most pronounced when equal numbers of positive and negative relationships are considered, a unimodal relationship will persist as long as there is at least one opposing stability component. Let's look at different fractions of negative stability components with a total of 100 different functions.





## References

Gamfeldt, L. et al. 2008. Multiple functions increase the importance of biodiversity for overall ecosystem functioning. - *Ecology* 89: 1223–1231.