p8104_hw1_ps3194

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Problem 1

Prove the second half of the DeMorgan's Laws. i.e.
$(A \cap B)^c = A^c \cup B^c$

solution: we need to prove that $(A \cap B)^c \subset (A^c \cup B^c)$ and $A^c \cup B^c \subset (A \cap B)^c$

(i) :
$\forall x \in (A \cap B)^c$ definition of complement
$\Rightarrow x \notin (A \cap B)$ proof by contradiction
$\Rightarrow x \notin A$ or $x \notin B$ definition of complement
$\Rightarrow x \in A^c$ or $x \in B^c$ definition of union
$\Rightarrow x \in A^c$ or $x \in B^c$ we get $(A \cap B)^c \subset (A^c \cup B^c)$

(ii) :
$\forall x \in A^c \cup B^c$ definition of union
$\Rightarrow x \in A^c$ or $x \in B^c$ definition of complement
$\Rightarrow x \notin A$ or $x \notin B$ proof by contradiction
$\Rightarrow x \notin (A \cap B)$ definition of complement
$\Rightarrow x \in (A \cap B)^c$ we get $A^c \cup B^c \subset (A \cap B)^c$

$(i) + (ii)$ by definition of set equality
$\Rightarrow (A \cap B)^c = A^c \cup B^c$ complete the proof