p8104_hw1_ps3194

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Problem 1

Prove the second half of the DeMorgan's Laws. i.e.
$(A \cap B)^c = A^c \cup B^c$

solution: we need to prove that $(A \cap B)^c \subset (A^c \cup B^c)$ and $A^c \cup B^c \subset (A \cap B)^c$

(i) :
$\forall x \in (A \cap B)^c$ definition of complement
$\Rightarrow x \notin (A \cap B)$ proof by contradiction
$\Rightarrow x \notin A$ or $x \notin B$ definition of complement
$\Rightarrow x \in A^c$ or $x \in B^c$ definition of union
$\Rightarrow x \in A^c$ or $x \in B^c$ we get $(A \cap B)^c \subset (A^c \cup B^c)$

(ii) :
$\forall x \in A^c \cup B^c$ definition of union
$\Rightarrow x \in A^c$ or $x \in B^c$ definition of complement
$\Rightarrow x \notin A$ or $x \notin B$ proof by contradiction
$\Rightarrow x \notin (A \cap B)$ definition of complement
$\Rightarrow x \in (A \cap B)^c$ we get $A^c \cup B^c \subset (A \cap B)^c$

$(i) + (ii)$ by definition of set equality
$\Rightarrow (A \cap B)^c = A^c \cup B^c$ complete the proof

Problem 2

Suppose that 3 events A, B, C are defined on sample space Ω. Use union, intersection, and complement operations to represent the following event:

(a) both A and B occur but not C

solution: $A \cap B \cap C^c$

(b) at least two events occur

solution: $(A \cap B) \cup (A \cap C) \cup (B \cap C)$

(c) at most one event occurs

solution: $(A \cap B)^c \cap (A \cap C)^c \cap (B \cap C)^c$

(d) exactly one event not occurs

solution: $[A \cap (B^c \cap C^c)] \cup [B \cap (A^c \cap C^c)] \cup [C \cap (A^c \cap B^c)]^c$

(e) at least one event not occurs

solution: $(A \cup B \cup C)^c$

Problem 3

A, B **are two events defined on sample space Ω. Prove that:**
(a) if $A \subset B$, then $P(A) \leq P(B)$

solution: Since $E \subset F$, it follows that we can express F as $F = E \cup E^c F$ Hence, because E and $E^c F$ are mutually exclusive, we obtain, from Axiom 3 of probability, $P(F) = P(E) + P(E^c F)$ which proves the result, since $P(E^c F) \geq 0$.

(b) $P(A \cap B^c) = P(A) - P(A \cap B)$

solution:
A can be written as the union of the two disjoint events $A \cap B^c$ and $A \cap B$. Thus, from Axiom 3 of probability, we obtain
$P(A) = P[(A \cap B^c) \cup (A \cap B)]$
$= P(A \cap B^c) + P(A \cap B)$. After manipulation we get $P(A \cap B^c) = P(A) - P(A \cap B)$. Thus complete the proof.

Problem 4

Susan took two tests. The probability of her passing at least one test is 0.9. Define events A and B as:
A : she passed the first test
B : she passed the second test
(a)suppose the probability of her only passing the second test is 0.4. What is $P(A)$?

solution:
$P(A \cup B) = P(A) + P(B \cap A^c) = 0.9$
$P(B \cap A^c) = 0.4$ Hence $P(A) = 0.9 - 0.4 = 0.5$

(b)suppose the probability of her passing the second test is 0.6. What is $P(A \cap B^c)$?

solution:
$P(A \cup B) = P(B) + P(A \cap B^c) = 0.9$
$P(B) = 0.6$ Hence $P(A \cap B^c) = P(A \cup B) - P(B) = 0.9 - 0.6 = 0.3$