

Stanford CS224W: Advanced Topics in Graph Neural Networks

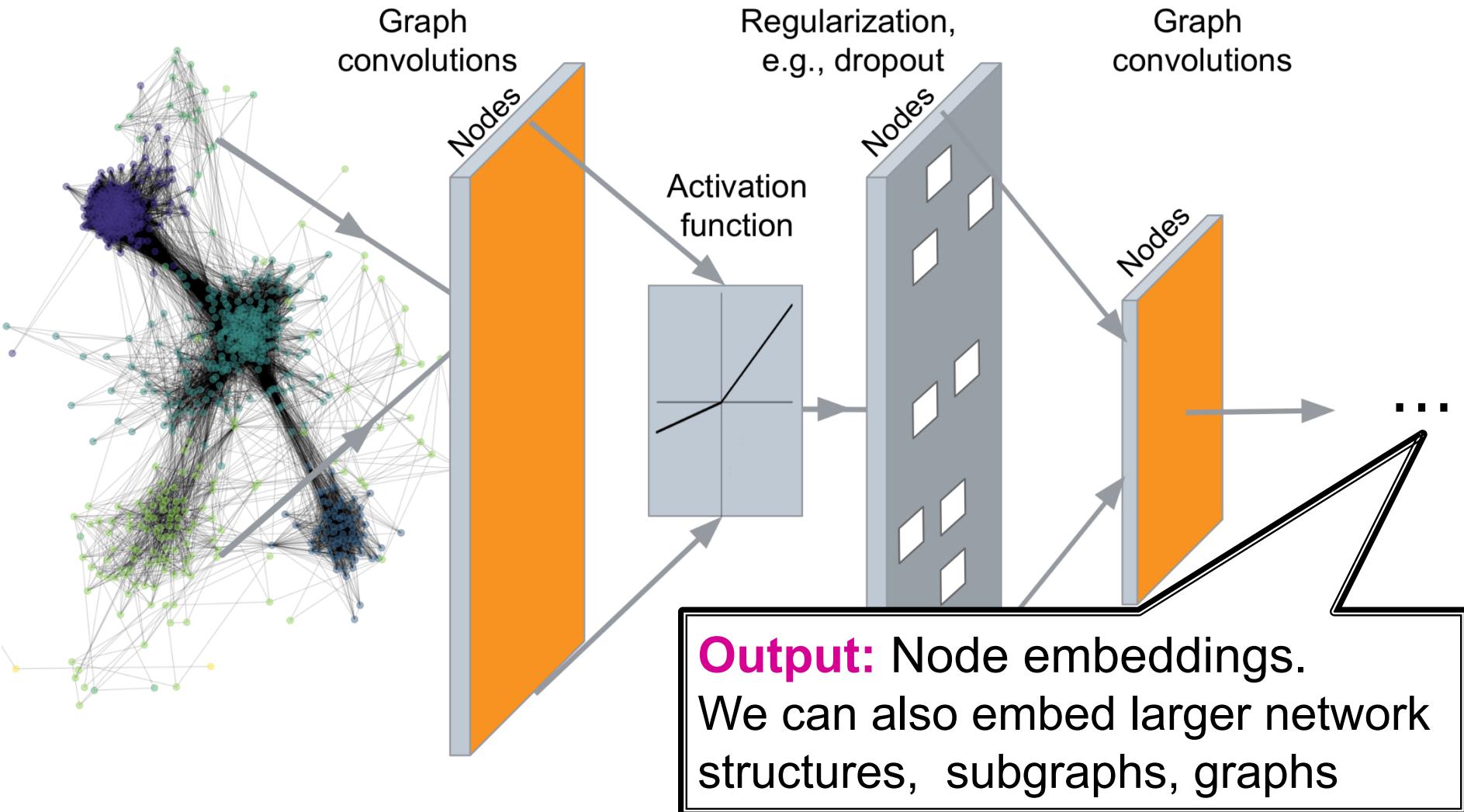
CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

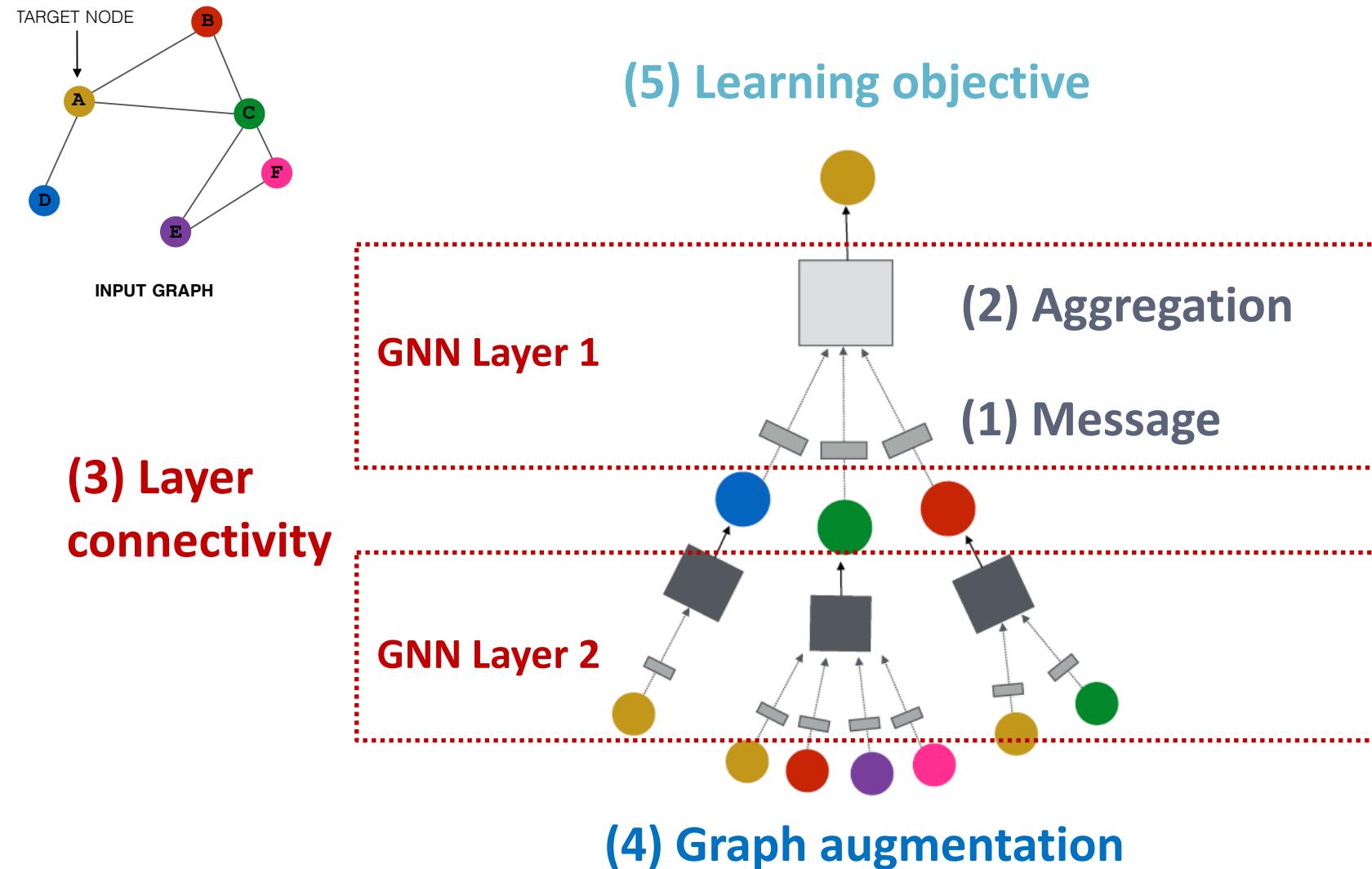
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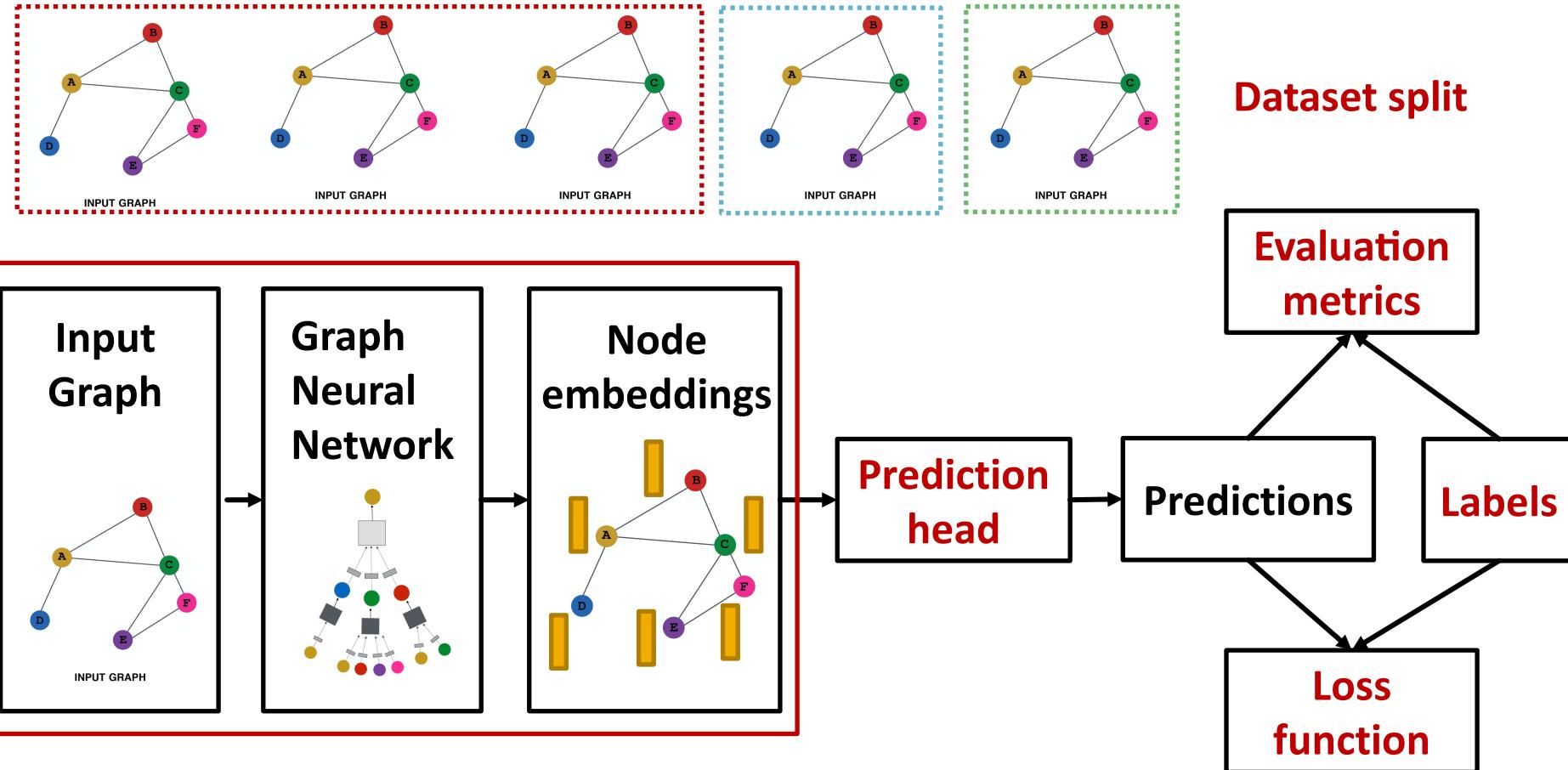
Recap: Graph Neural Networks



Recap: A General GNN Framework



Recap: GNN Training Pipeline



Today's lecture: Can we make GNN representation more expressive?

Stanford CS224W: Limitations of Graph Neural Networks

CS224W: Machine Learning with Graphs

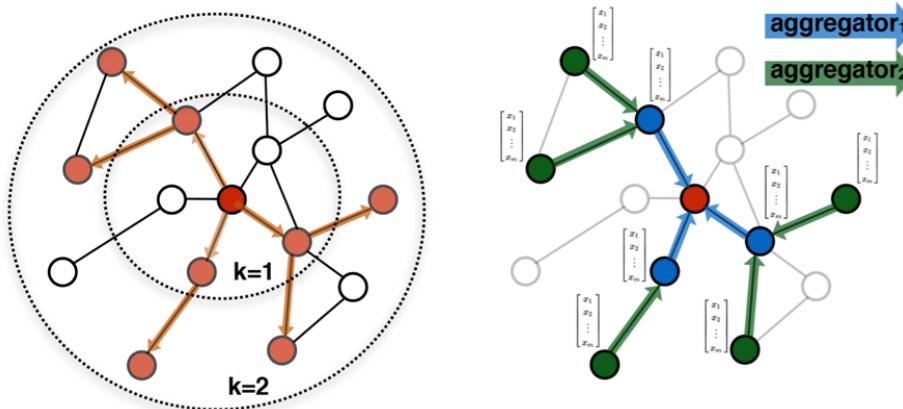
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A “Perfect” GNN Model

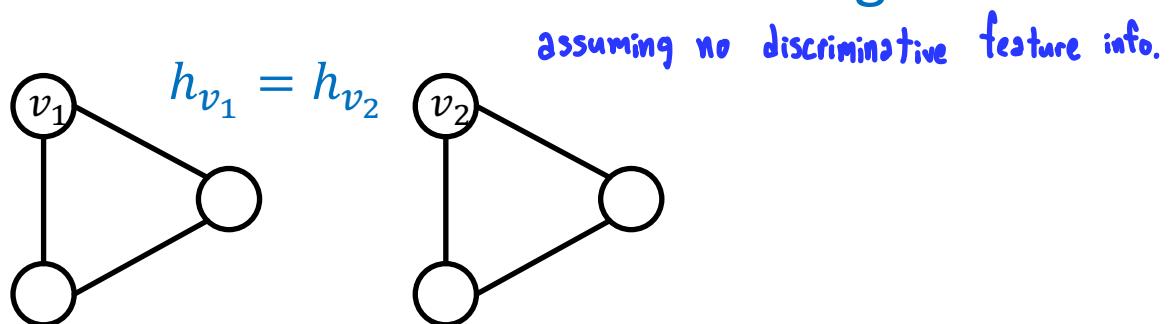
- A thought experiment: What should a perfect GNN do?
 - A k -layer GNN embeds a node based on the K -hop neighborhood structure



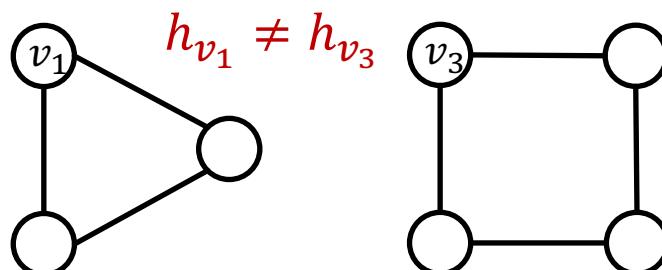
- A perfect GNN should build an **injective function** between **neighborhood structure** (regardless of hops) and **node embeddings**

A “Perfect” GNN Model

- Therefore, for a perfect GNN:
 - Observation 1:** If two nodes have the same neighborhood structure, they must have the same embedding

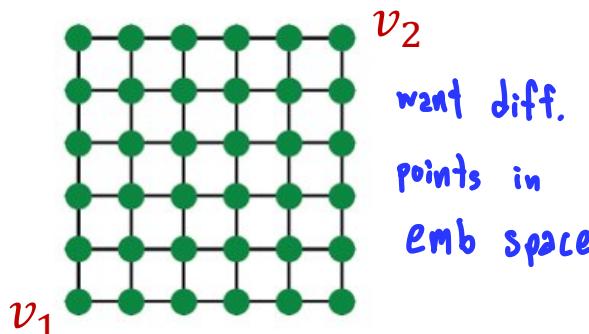


- Observation 2:** If two nodes have different neighborhood structure, they must have different embeddings



Imperfections of Existing GNNs

- However, Observations 1 & 2 are imperfect
- Observation 1 could have issues:
 - Even though two nodes may have the same neighborhood structure, we may want to assign different embeddings to them
 - Because these nodes appear in different positions in the graph
 - We call these tasks Position-aware tasks
 - Even a perfect GNN will fail for these tasks:



A grid graph

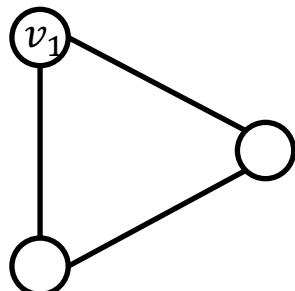


NYC road network

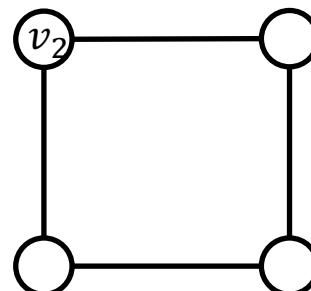
Imperfections of Existing GNNs

- **Observation 2 often cannot be satisfied:**
 - The GNNs we have introduced so far are not perfect
 - In Lecture 9, we discussed that their expressive power is **upper bounded by the WL test** graph isomorphism test
 - For example, message passing GNNs **cannot count cycle length**:

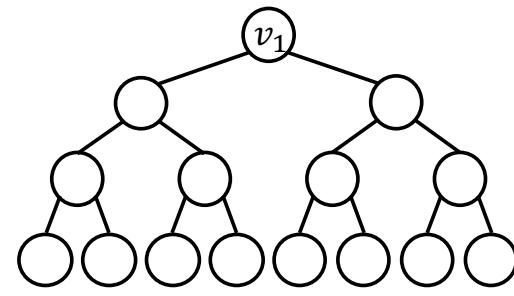
v_1 resides in a cycle with length 3



v_2 resides in a cycle with length 4



without discriminative node features
The computational graphs for nodes v_1 and v_2 are always the same



Plan for the Lecture

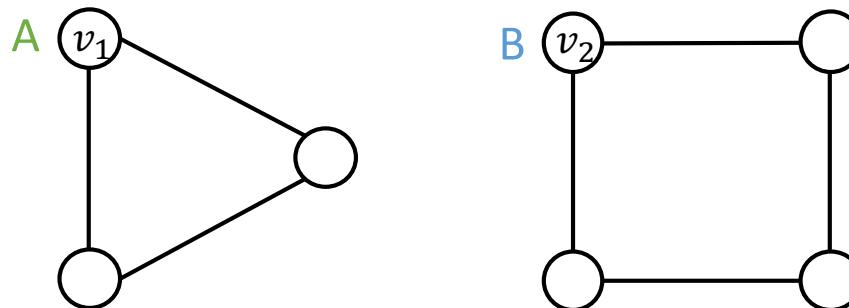
- We will resolve both issues by **building more expressive GNNs**
- **Fix issues in Observation 1:**
 - Create node embeddings based on their positions in the graph
 - Example method: Position-aware GNNs
- **Fix issues in Observation 2:**
 - Build message passing GNNs that are more expressive than WL test
 - Example method: Identity-aware GNNs

create reference points

Our Approach

- We use the following thinking:

- Two different inputs (nodes, edges, graphs) are labeled differently
- A “failed” model will always assign the same embedding to them
- A “successful” model will assign different embeddings to them
- Embeddings are determined by GNN computational graphs:



Two inputs: nodes v_1 and v_2

Different labels: A and B

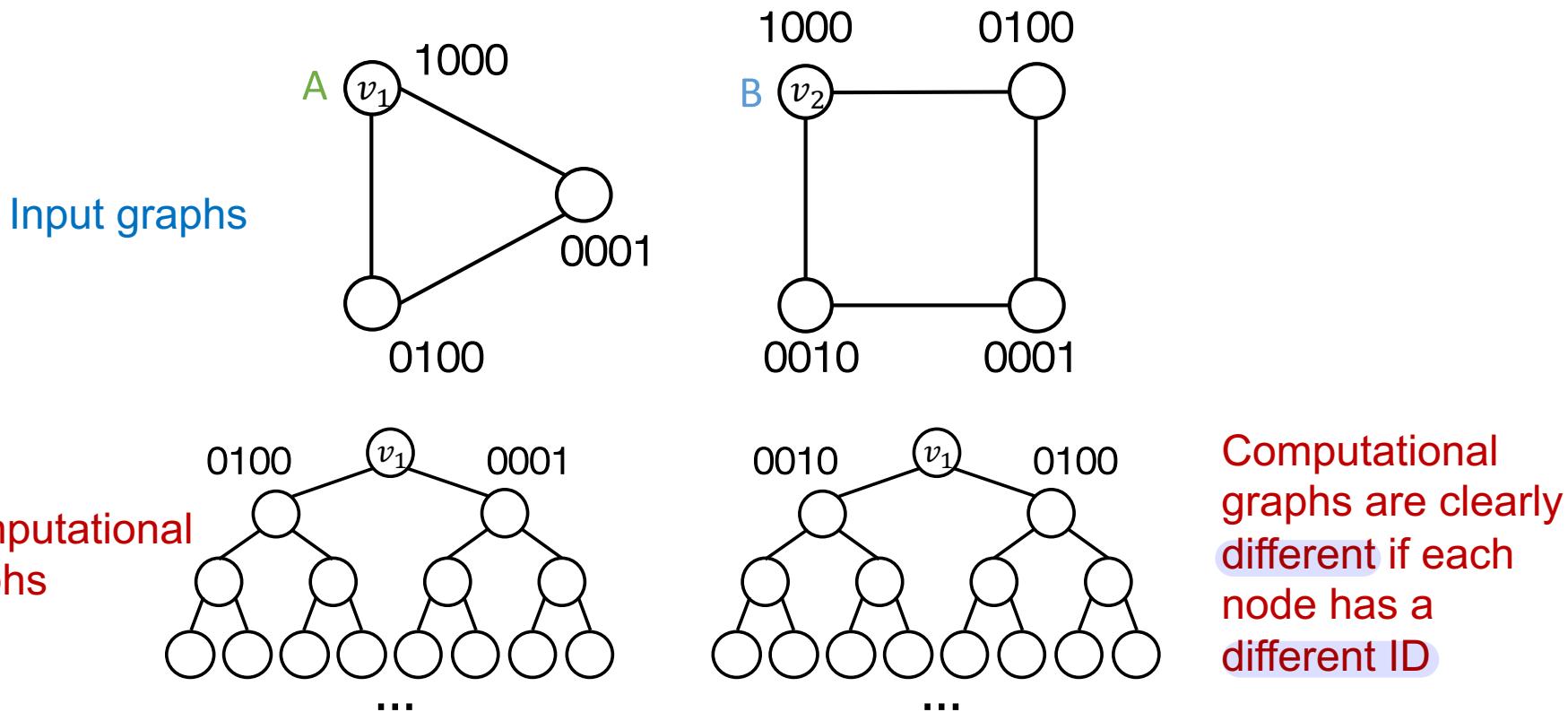
Goal: assign different embeddings to v_1 and v_2

Naïve Solution is not Desirable

Doesn't work

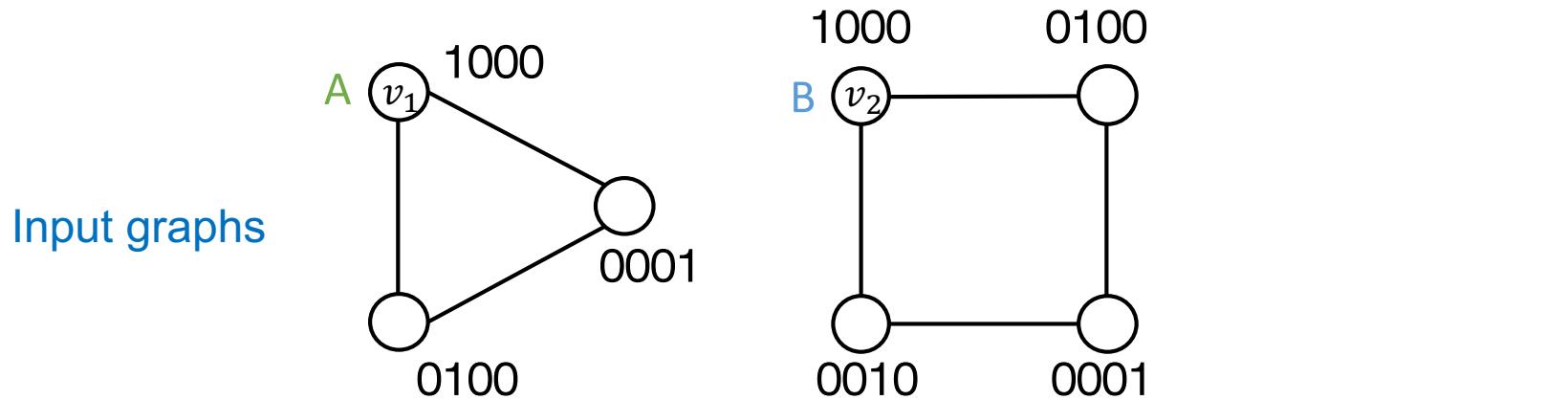
■ A naïve solution: One-hot encoding

- Encode each node with a different ID, then we can always differentiate different nodes/edges/graphs



Naïve Solution is not Desirable

- A naïve solution: One-hot encoding arbitrary
 - Encode each node with a different ID, then we can always differentiate different nodes/edges/graphs



- Issues:
 - Not scalable: Need $O(N)$ feature dimensions (N is the number of nodes) need a separate feature for every individual node
 - Not inductive: Cannot generalize to new nodes/graphs
 - expand feature dimensionality
 - learn accord. to that node ordering

Stanford CS224W: Position-aware Graph Neural Networks

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

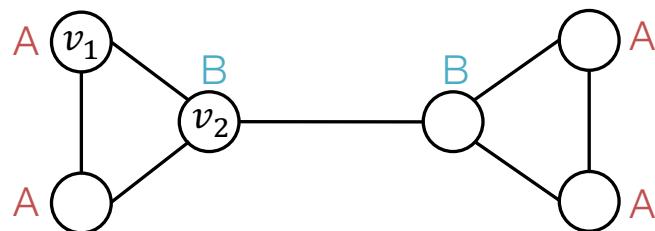
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Two Types of Tasks on Graphs

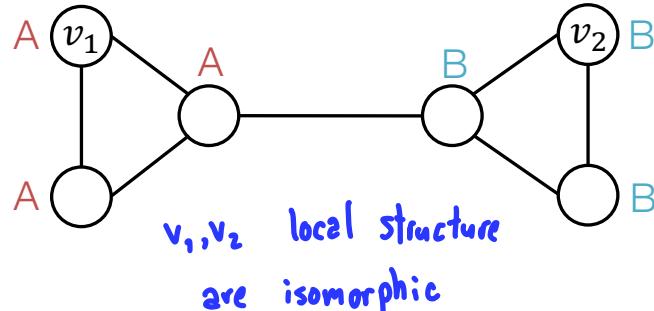
- There are two types of tasks on graphs

Structure-aware task



- Nodes are labeled by their **structural roles** in the graph

Position-aware task



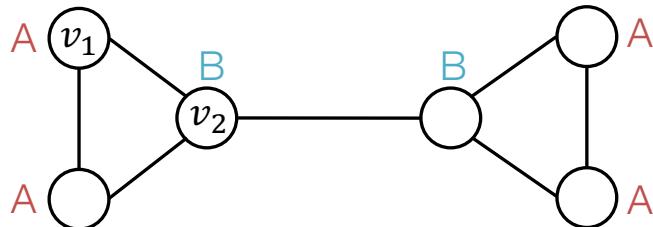
- Nodes are labeled by their **positions** in the graph

ex. community detection

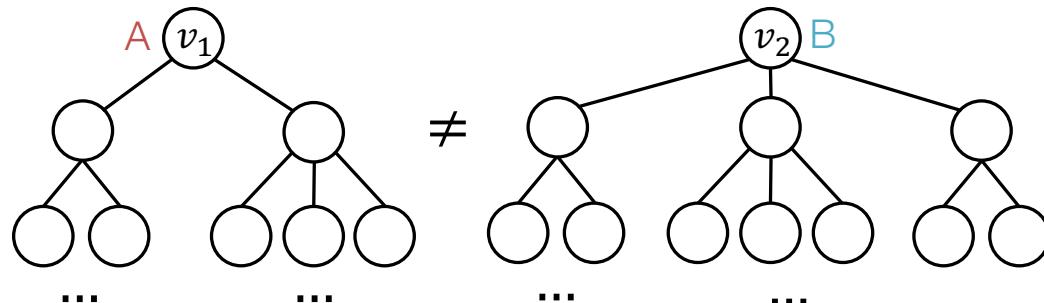
Structure-aware Tasks

- GNNs often work well for structure-aware tasks

Structure-aware task



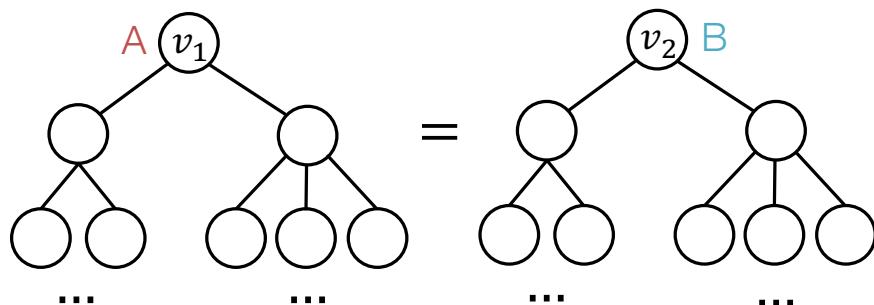
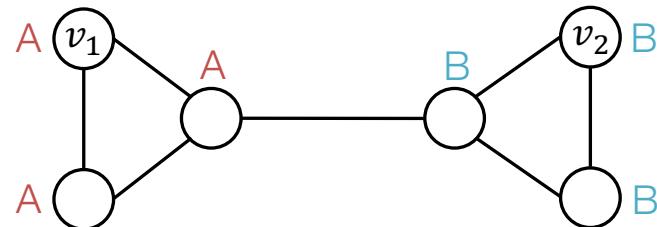
- GNNs work 😊
- Can differentiate v_1 and v_2 by using different computational graphs



Position-aware Tasks

- GNNs will always fail for position-aware tasks

Position-aware task

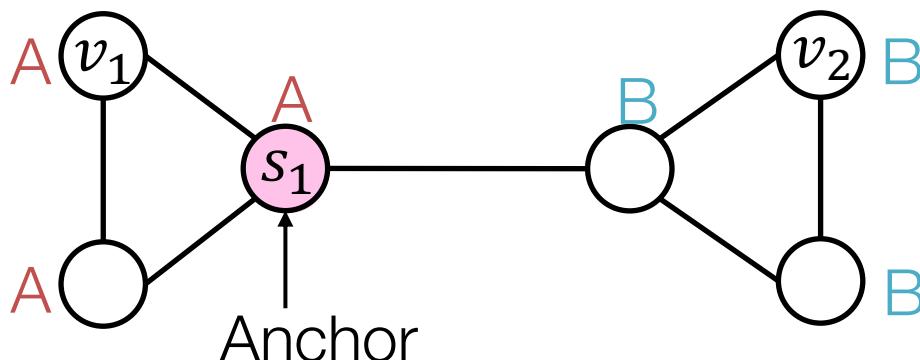


- GNNs fail 😞
- v_1 and v_2 will always have the same computational graph, due to structure symmetry
- Can we define deep learning methods that are position-aware?

Power of “Anchor”

reference point

- Randomly pick a node s_1 as an **anchor node**
- Represent v_1 and v_2 via their relative distances w.r.t. the anchor s_1 , **which are different**
- An anchor node serves as **a coordinate axis**
 - Which can be used to **locate nodes in the graph**



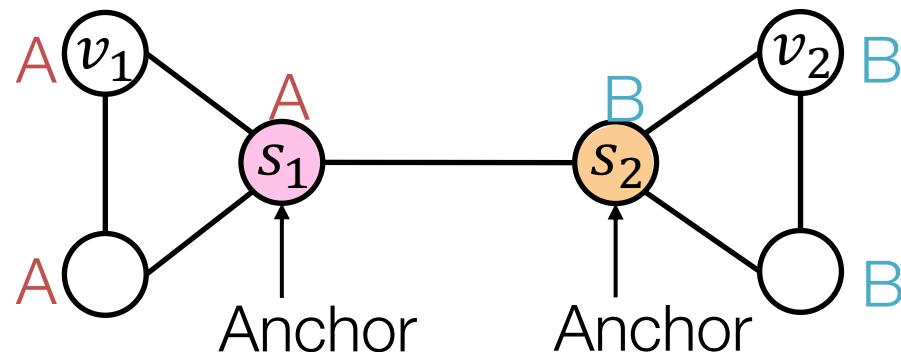
Relative
Distances

| | |
|-------|-------|
| | s_1 |
| v_1 | 1 |
| v_2 | 2 |

Power of “Anchors”

- Pick more nodes s_1, s_2 as **anchor nodes**
- **Observation:** More anchors can better characterize node position in different regions of the graph
- Many anchors → Many coordinate axes

too many → computationally hard



Relative
Distances

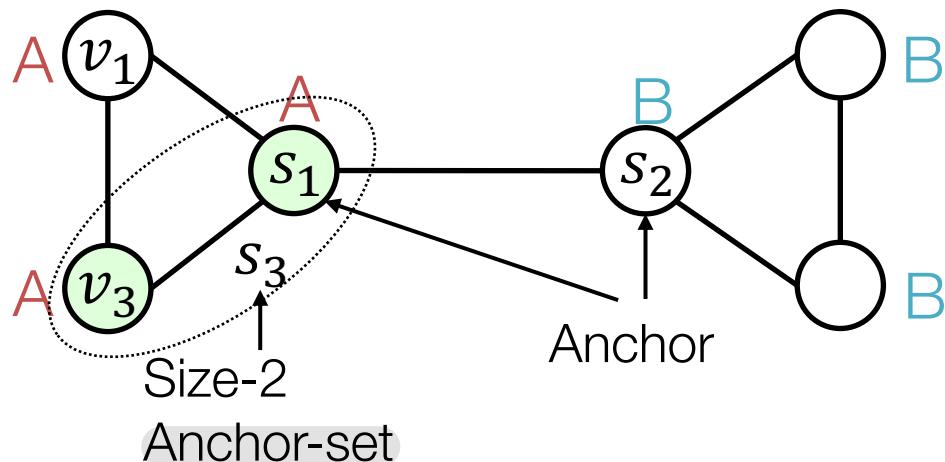
| | s_1 | s_2 |
|-------|-------|-------|
| v_1 | 1 | 2 |
| v_2 | 2 | 1 |

can distinguish
 v_1 from v_2

Power of “Anchor-sets”

more precise

- Generalize anchor from a single node to a set of nodes
 - We define distance to an anchor-set as the minimum distance to all the nodes in the anchor-set
- **Observation:** Large anchor-sets can sometimes provide more precise position estimate
 - We can save the total number of anchors



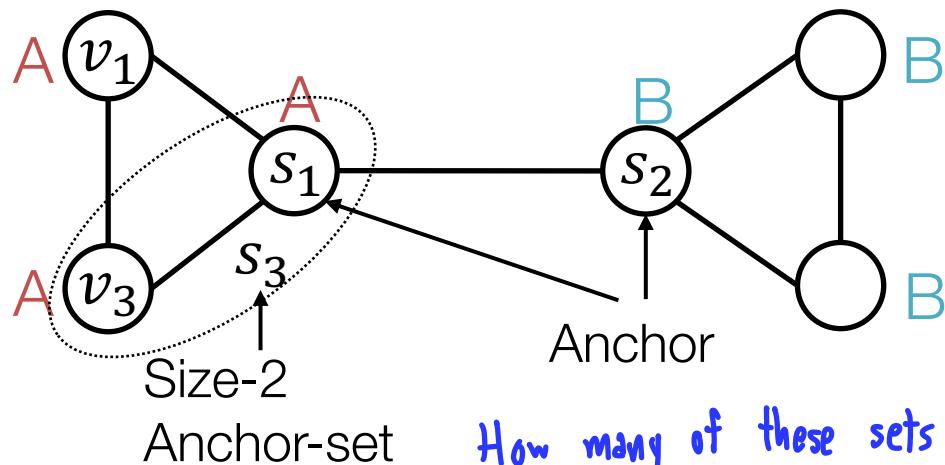
Relative Distances

| | s_1 | s_2 | s_3 |
|-------|-------|-------|-------|
| v_1 | 1 | 2 | 1 |
| v_3 | 1 | 2 | 0 |

Anchor s_1, s_2 cannot differentiate node v_1, v_3 , but anchor-set s_3 can

Position Information: Summary

- **Position encoding for graphs:** Represent a node's position by its distance to randomly selected anchor-sets
 - Each dimension of the position encoding is tied to an anchor-set
minimum distance to any node in the anchor-set



| | s_1 | s_2 | s_3 |
|-------|-------|-------|-------|
| v_1 | 1 | 2 | 1 |
| v_3 | 1 | 2 | 0 |

v_1 's Position
encoding

v_3 's Position
encoding

How many of these sets & how big they need to be?

Ans Exponentially increasing size, but use exponentially fewer of them

anchor size: 1 2 4 8 16
node No. : N $N/2$ $N/4$ $N/8$ $N/16$

How to Use Position Information

- **The simple way:** Use position encoding as an **augmented node feature** (works well in practice)
 - **Issue:** since each dimension of position encoding is tied to a **random anchor**, **dimensions of positional encoding can be randomly permuted, without changing its meaning**
 - Imagine you permute the input dimensions of a normal **NN**, the output will surely change

How to Use Position Information

- **The rigorous solution:** requires a special NN that can maintain the permutation invariant property of position encoding
 - Permuting the input feature dimension will only result in the permutation of the output dimension, the value in each dimension won't change
 - Refer to the Position-aware GNN paper for more details

Nodes won't only know their local neighborhood structure around them
but also their location/position in NN

Stanford CS224W: Identity-Aware Graph Neural Networks

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

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More Failure Cases for GNNs

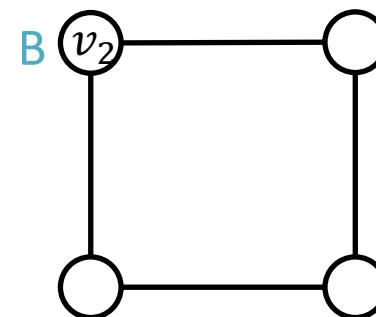
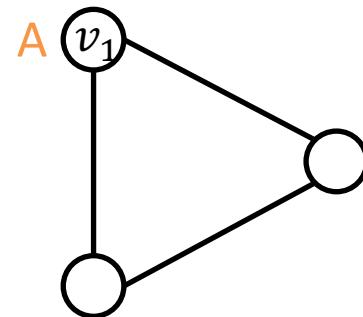
- We learned that **GNNs would fail for position-aware tasks**
- **But can GNN perform perfectly in structure-aware tasks?**
 - Unfortunately, **NO**
- GNNs exhibit three levels of failure cases in structure-aware tasks:
 - Node level
 - Edge level
 - Graph level

worse case

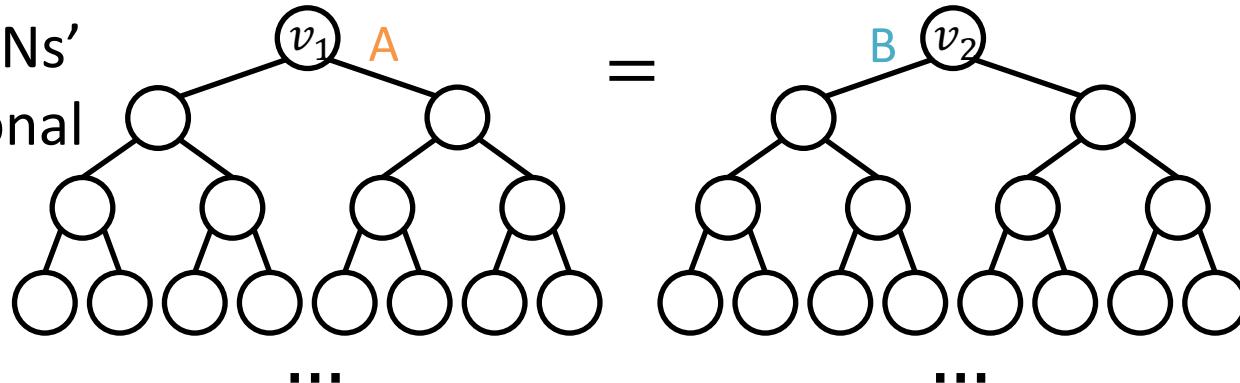
GNN Failure 1: Node-level Tasks

Different Inputs but the same computational graph → GNN fails

Example input
graphs



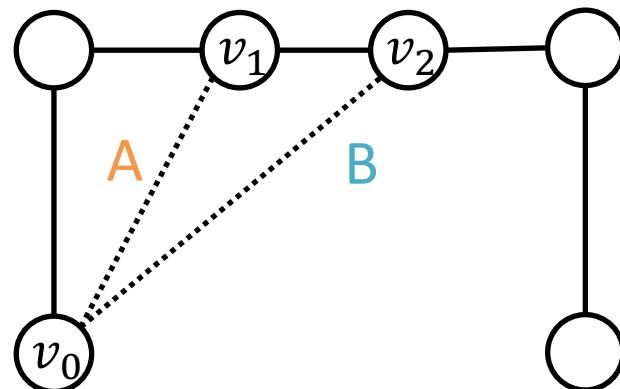
Existing GNNs'
computational
graphs



GNN Failure 2: Edge-level Tasks

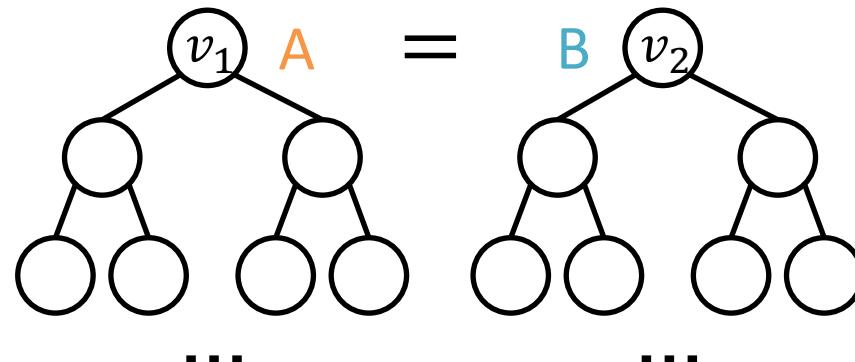
Different Inputs but the same computational graph → GNN fails

Example input graphs



Edge A and B share node v_0
We look at embeddings for v_1 and v_2

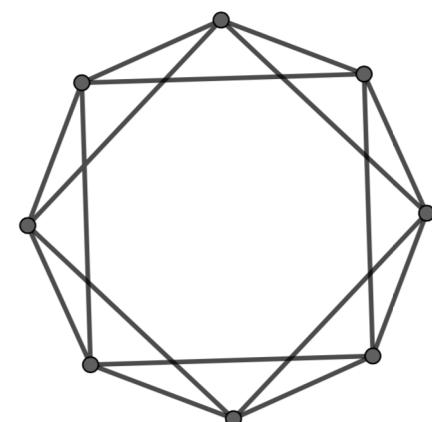
Existing GNNs' computational graphs



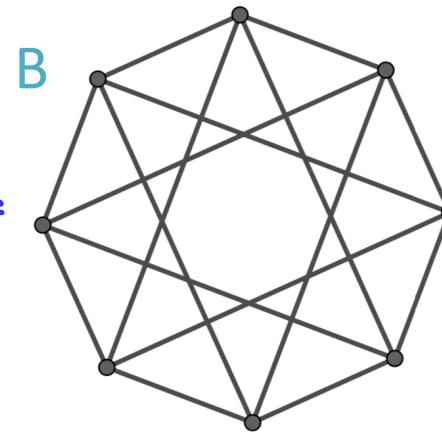
GNN Failure 3: Graph-level Tasks

Different Inputs but the same computational graph → GNN fails

Example input graphs



A

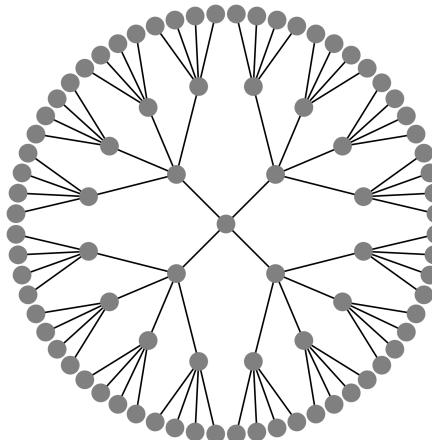


B

non-isomorphic

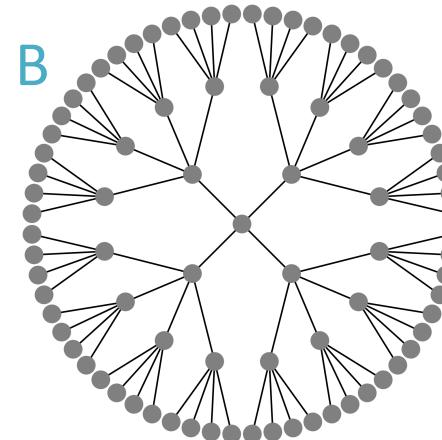
We look at embeddings for each node

For each node:



A

=



For each node:

Existing GNNs' computational graphs

Idea: Inductive Node Coloring

- Idea: We can assign a color to the node we want to embed

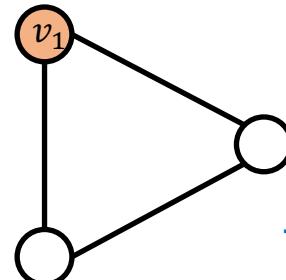


The node we want to embed



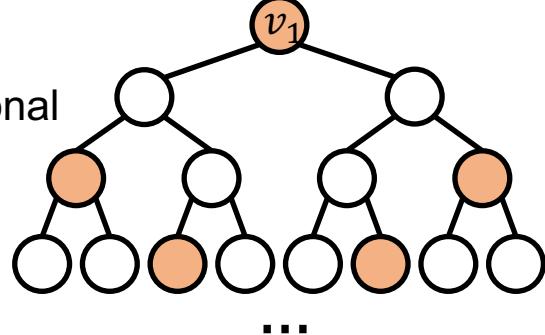
The rest of nodes

Input graph

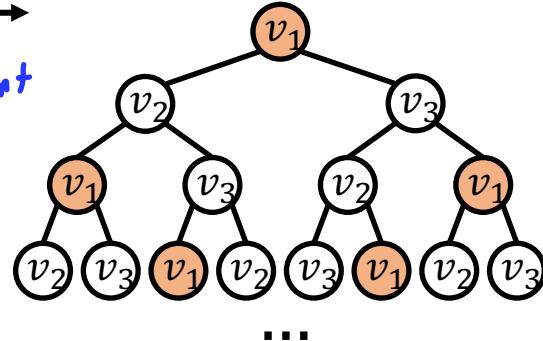
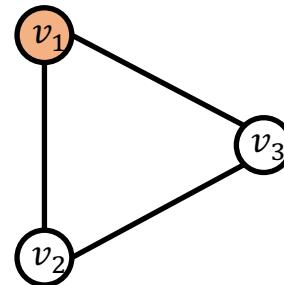


To assist understanding,
we label the nodes

Computational
graph



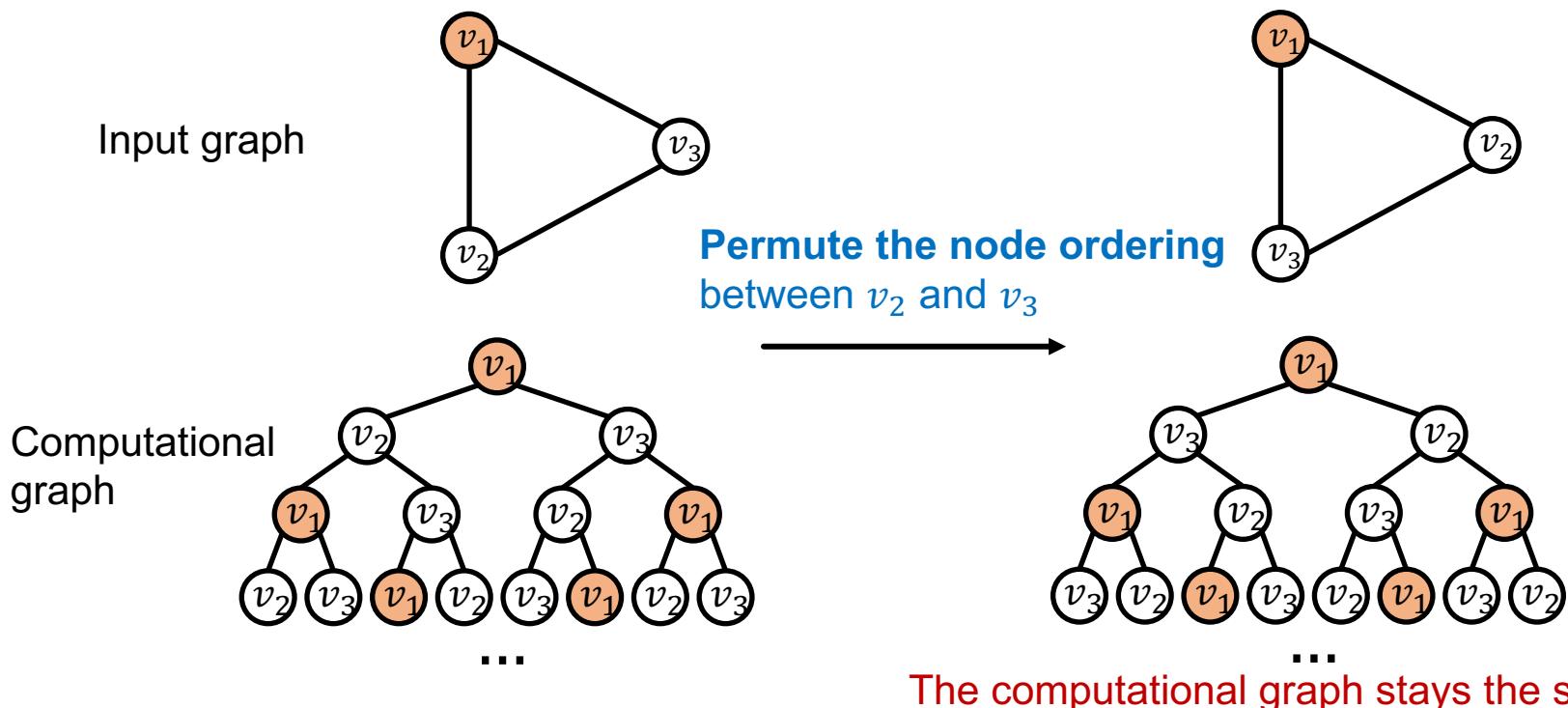
node order invariant



Idea: Inductive Node Coloring

- This coloring is **inductive**:
 - It is invariant to node ordering/identities

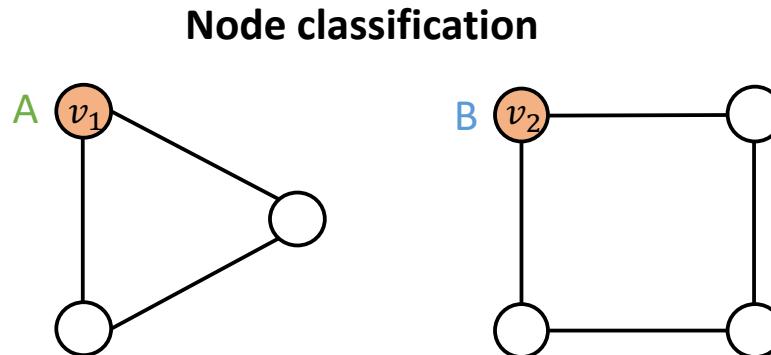
- The node we want to embed
- The rest of nodes



Inductive Node Coloring – Node level

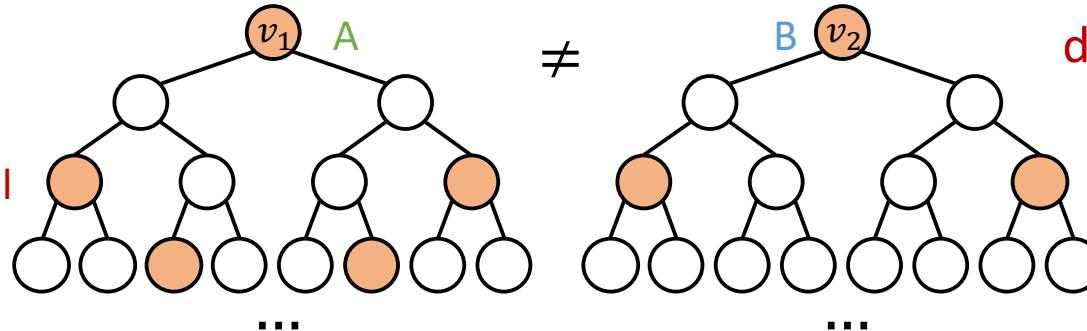
- Inductive node coloring can help **node classification**

Example input graphs



We color root nodes with identity

ID-GNNs' computational graphs



Different computational graphs
→ Successfully differentiate nodes

2 hops out

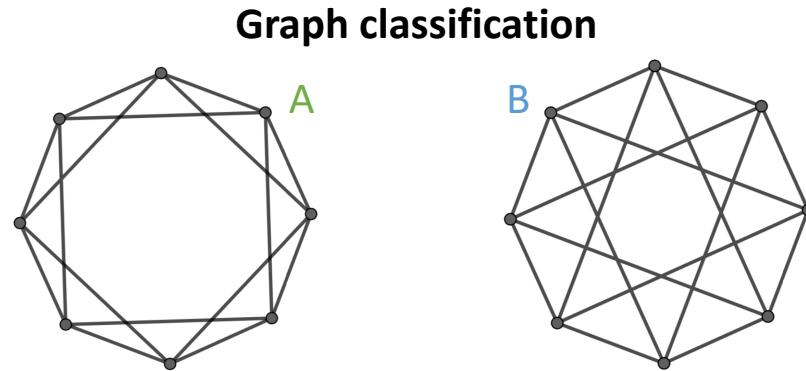
Two types of nodes:

- node with augmented identity (orange circle)
- node without augmented identity (white circle)

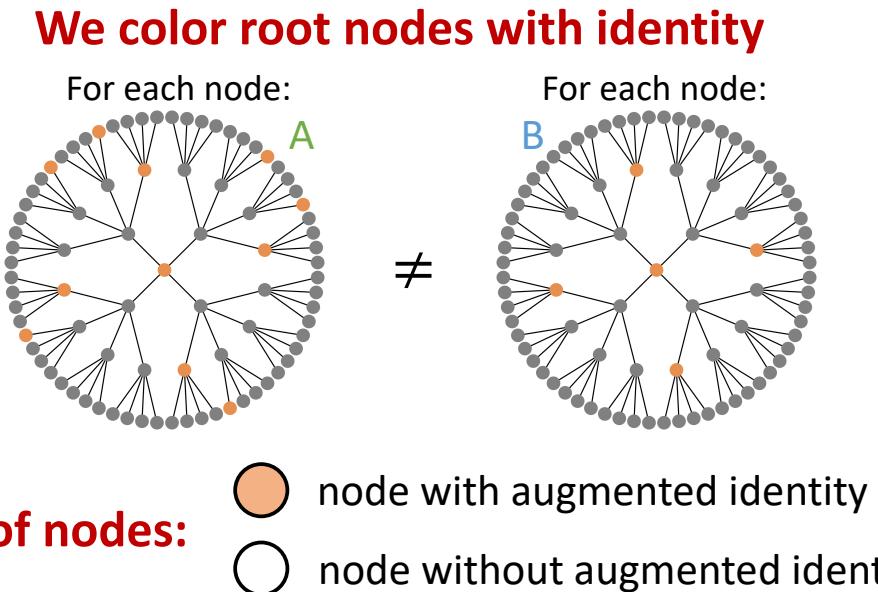
Inductive Node Coloring – Graph Level

- Inductive node coloring can help **graph classification**

Example input graphs



ID-GNNs' computational graphs

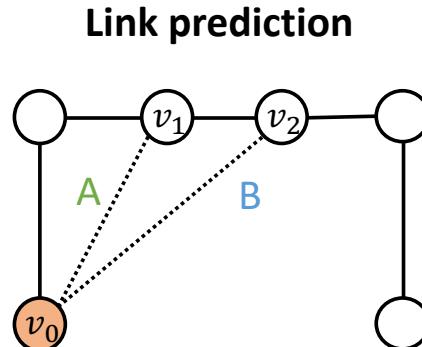


Different computational graphs
→ Successful differentiate graphs

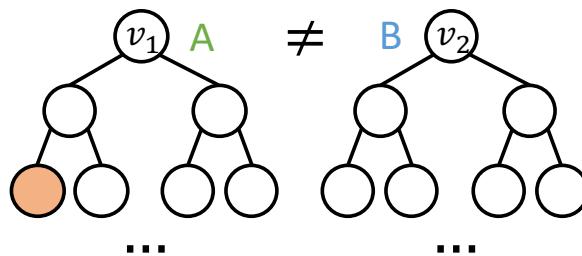
Inductive Node Coloring – Edge Level

- Inductive node coloring can help **link prediction**

Example input graphs



ID-GNNs'
computational
graphs



Two types of nodes:

- node with augmented identity (orange circle)
- node without augmented identity (white circle)

An edge-level task involves classifying a pair of nodes:

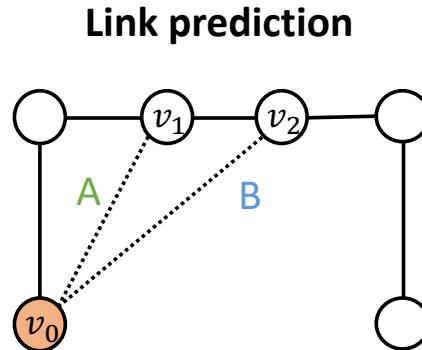
- We color one of the nodes (v_0)
- We then embed the other node in the node pair (v_1 or v_2)
- We use the node embedding for v_1 or v_2 conditioned on v_0 being colored or not to make edge-level prediction

Different
computational graphs
→ Successfully
differentiate edges

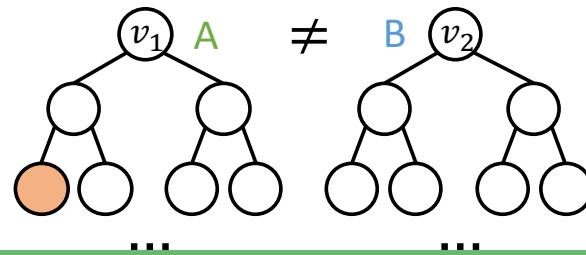
Inductive Node Coloring – Edge Level

- Inductive node coloring can help **link prediction**

Example input graphs



ID-GNNs'
computational
graphs



An edge-level task involves classifying a pair of nodes:

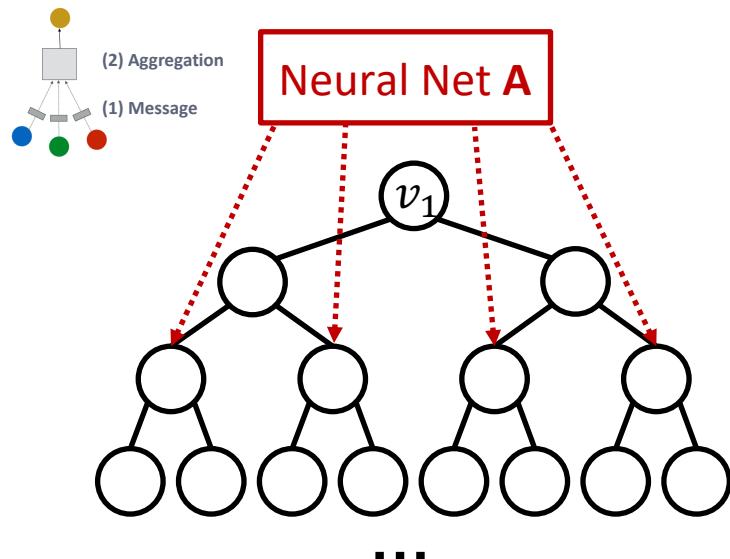
- We color one of the nodes (v_0)
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- We use the node embedding for v_1 or v_2 conditioned on v_0 being colored or not to make edge-level prediction

Different

Two How to build a GNN using node coloring?

Identity-aware GNN

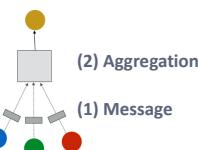
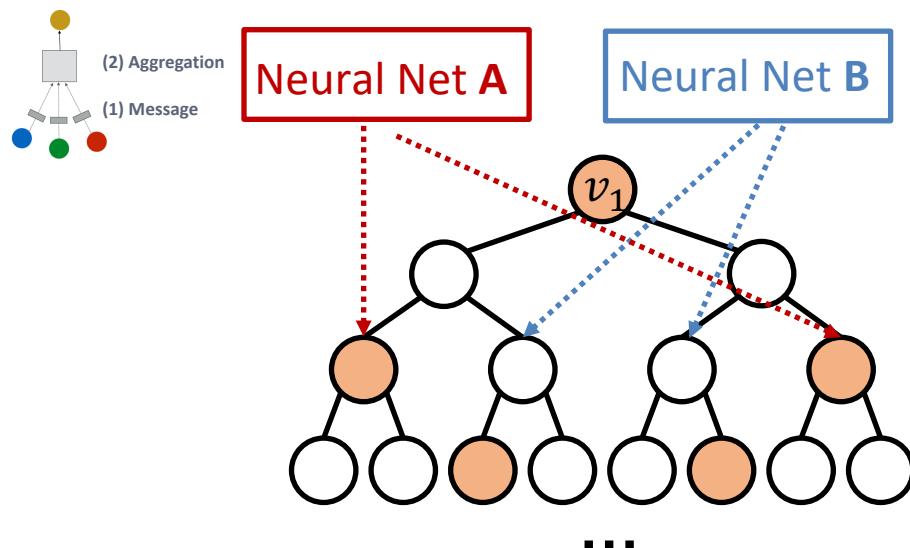
- Utilize **inductive node coloring** in embedding computation
 - Idea: **Heterogenous message passing**
 - Normally, a GNN applies **the same message/aggregation computation to all the nodes**



GNN: At a given layer, we apply the same message/aggregation to each node

Identity-aware GNN

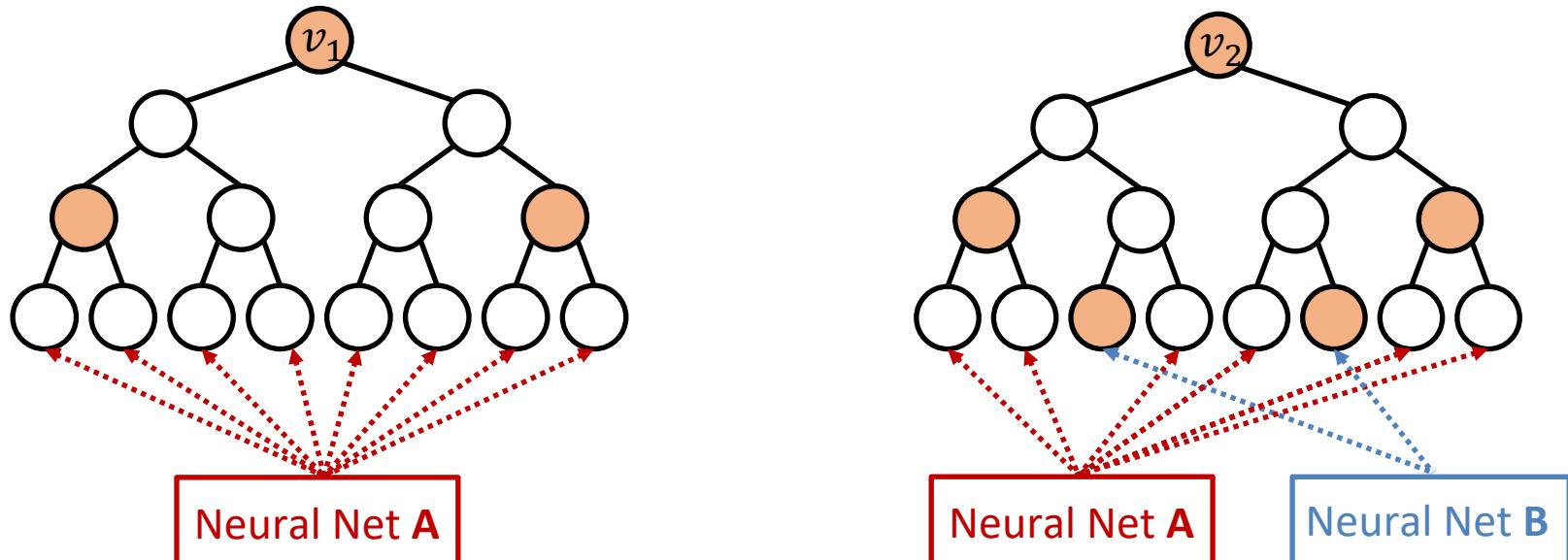
- Idea: **Heterogenous message passing**
 - **Heterogenous:** different types of message passing is applied to different nodes based on the color
 - An ID-GNN applies **different message/aggregation to nodes with different colorings**



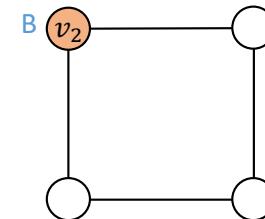
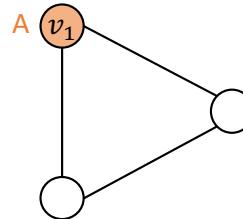
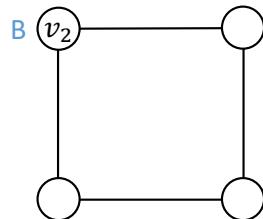
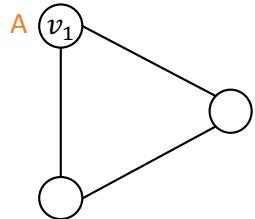
ID-GNN: At a given layer, different message/aggregation to nodes with different colorings

Identity-aware GNN

- Why does heterogenous message passing work:
 - Suppose two nodes v_1, v_2 have the same computational graph structure, but have different node colorings
 - Since we will apply different neural network for embedding computation, their embeddings will be different

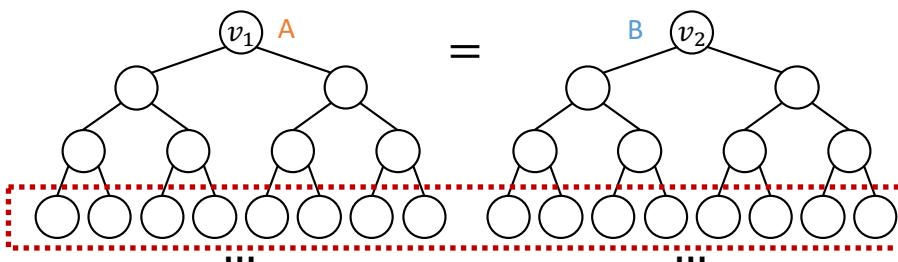


GNN vs ID-GNN

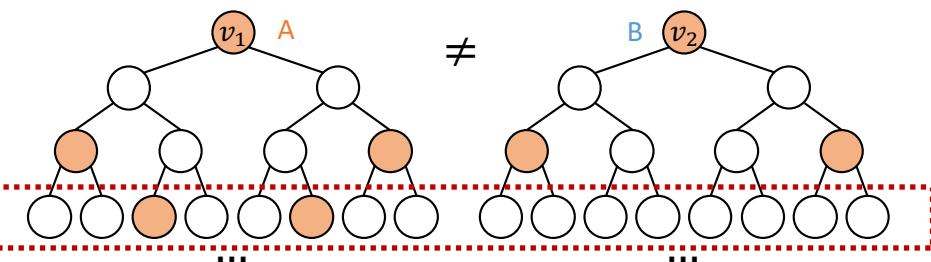


Goal: classify v_1 and v_2

GNN computational graph



ID-GNN rooted subtrees



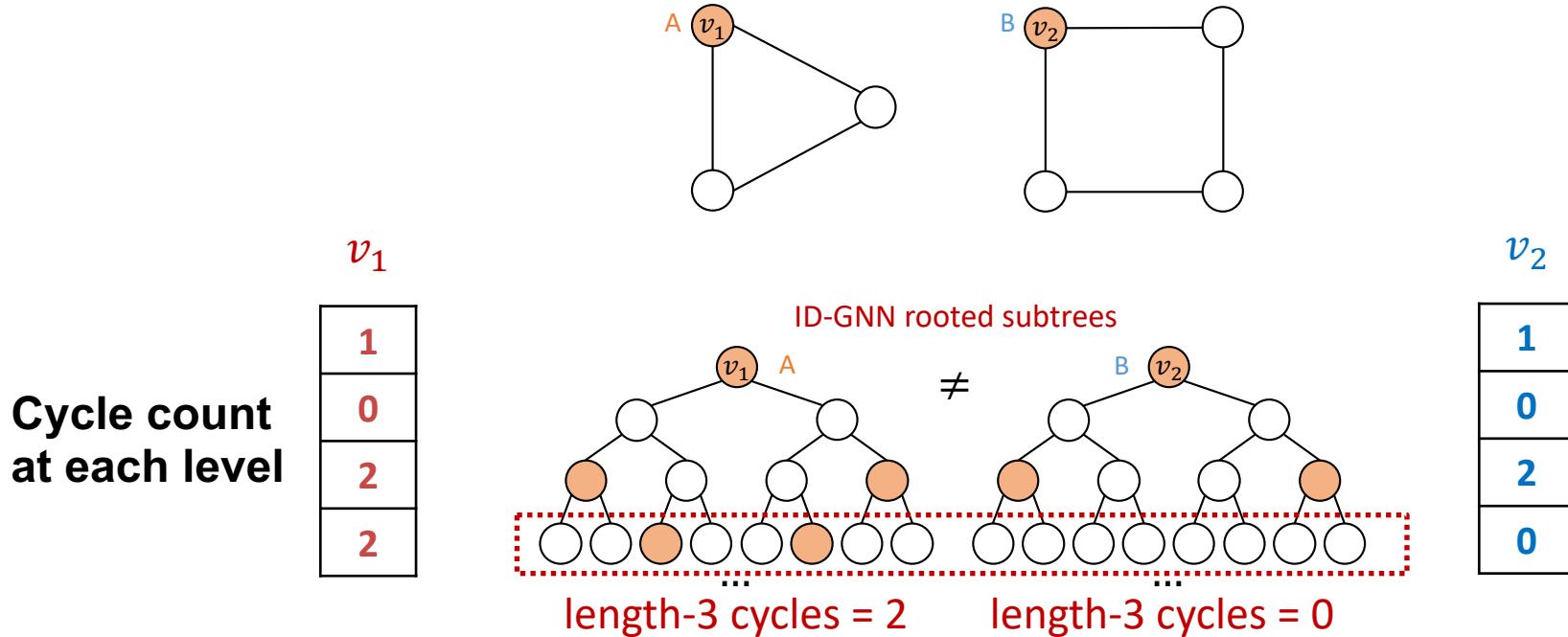
From the node coloring, we can tell that:

v_1 : length-3 cycles = 2

v_2 : length-3 cycles = 0

- Why does ID-GNN work better than GNN?
- Intuition: ID-GNN can count cycles originating from a given node, but GNN cannot

Simplified Version: ID-GNN-Fast



- Based on the intuition, we propose a simplified version **ID-GNN-Fast**
 - **Include identity information as an augmented node feature** (no need to do heterogenous message passing) *homogenous message-passing (drop the colors)*
 - **Use cycle counts in each layer as an augmented node feature.** Also can be used together with any GNN

Identity-aware GNN

- **Summary of ID-GNN: A general and powerful extension to GNN framework**
 - We can apply ID-GNN on **any message passing GNNs** (GCN, GraphSAGE, GIN, ...)
 - ID-GNN provides **consistent performance gain** in node/edge/graph level tasks
 - ID-GNN is **more expressive** than their GNN counterparts. ID-GNN is **the first message passing GNN that is more expressive than 1-WL test**
 - We can **easily implement** ID-GNN using popular GNN tools (PyG, DGL, ...) *color the root node*

Stanford CS224W: **Robustness of** **Graph Neural Networks**

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

<http://cs224w.stanford.edu>

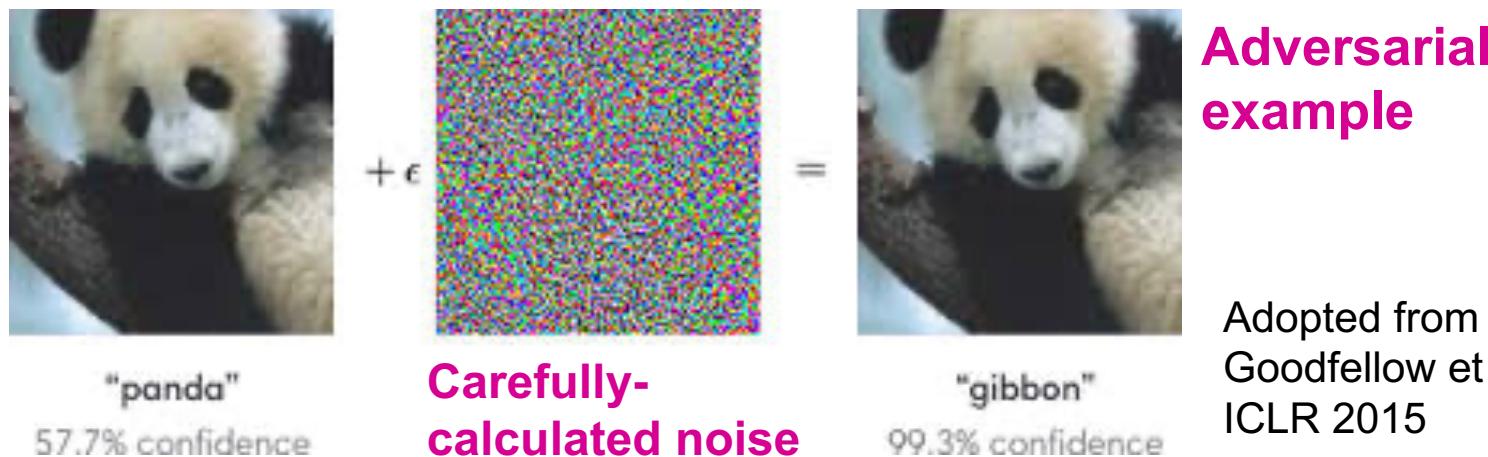


Deep Learning Performance

- Recent years have seen **impressive performance of deep learning models in a variety of applications.**
 - Ex) In computer vision, **deep convolutional networks** have achieved human-level performance on ImageNet (image category classification)
- **Are these models ready to be deployed in real world?**

Adversarial Examples

- Deep convolutional neural networks are vulnerable to **adversarial attacks**:
 - Imperceptible noise changes the prediction.



- Adversarial examples are also reported in natural language processing [Jia & Liang et al. EMNLP 2017] and audio processing [Carlini et al. 2018] domains.

Implication of Adversarial Examples

- **The existence of adversarial examples prevents the reliable deployment of deep learning models to the real world.**
 - Adversaries may try to actively hack the deep learning models.
 - The model performance can become much worse than we expect.
- **Deep learning models are often not robust.**
 - In fact, it is an active area of research to make these models robust against adversarial examples

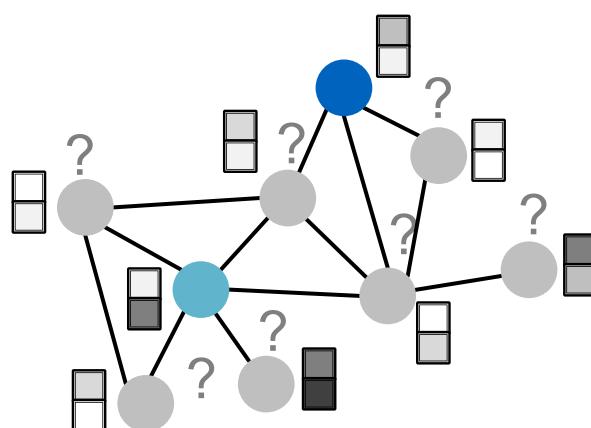
Robustness of GNNs

- **This lecture: How about GNNs? Are they robust to adversarial examples?**
- **Premise:** Common applications of GNNs involve **public platforms** and **monetary interests**.
 - Recommender systems
 - Social networks
 - Search engines
- Adversaries **have the incentive** to manipulate input graphs and hack GNNs' predictions.

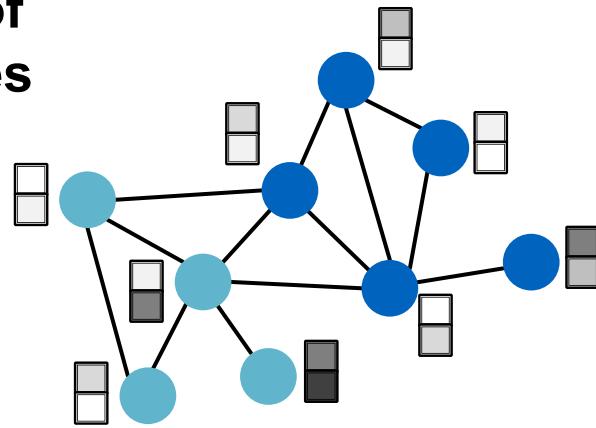
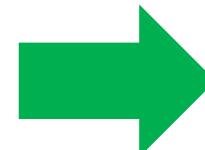
Setting to Study GNNs' Robustness

- To study the robustness of GNNs, we specifically consider the following setting:
 - Task:** Semi-supervised node classification
 - Model:** GCN [Kipf & Welling ICLR 2017]

?: Unlabeled



Predict labels of
unlabeled nodes

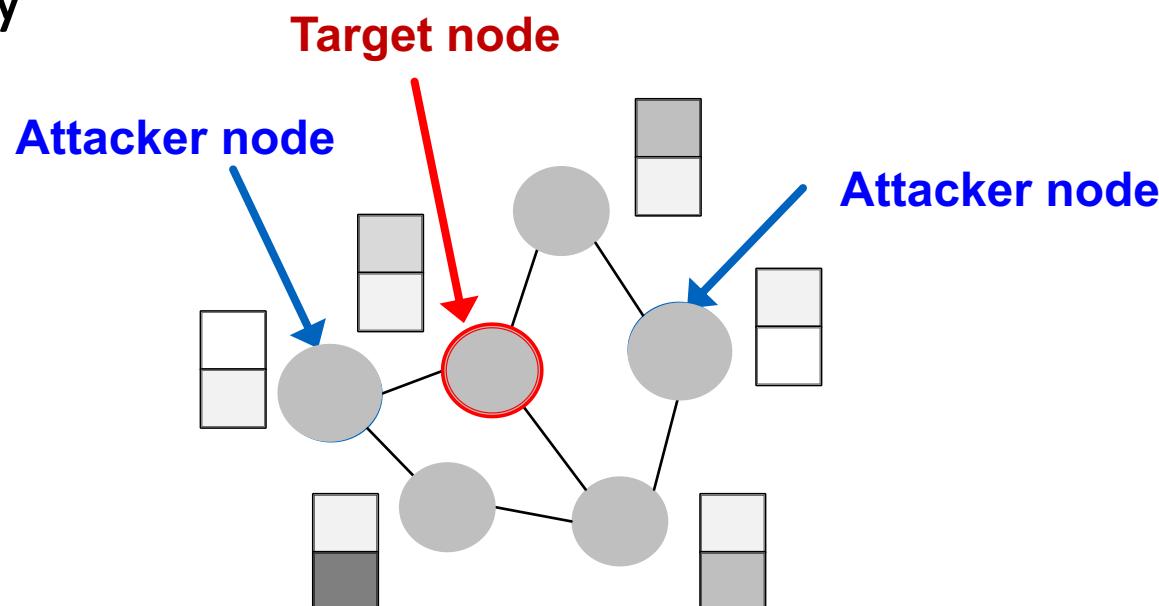


Roadmap

- We first describe several real-world **adversarial attack possibilities.**
- We then review the GCN model that we are going to attack (**knowing the opponent**).
- We mathematically **formalize the attack problem as an optimization problem.**
- **We empirically see how vulnerable GCN's prediction is to the adversarial attack.**

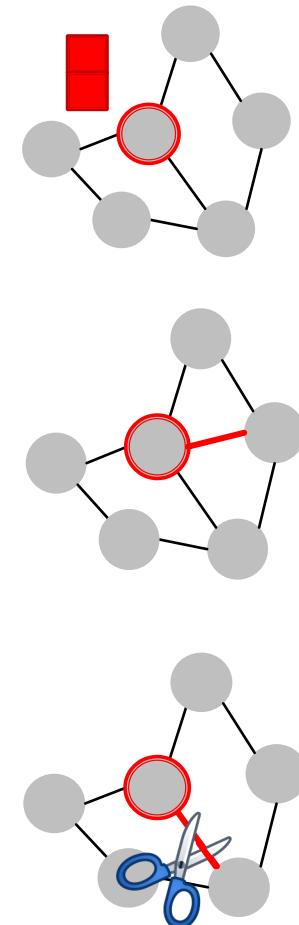
Attack Possibilities

- What are the attack possibilities in real world?
 - **Target node $t \in V$** : node whose label prediction we want to change
 - **Attacker nodes $S \subset V$** : nodes the attacker can modify



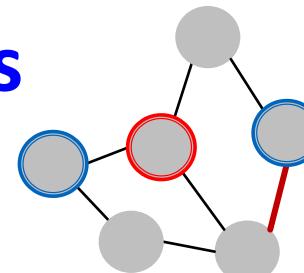
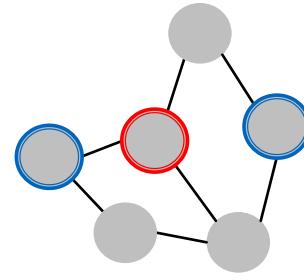
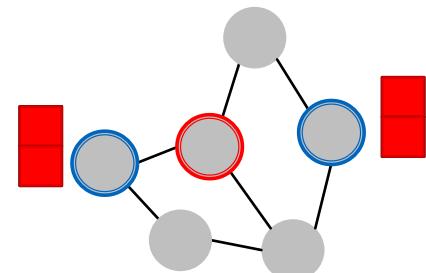
Attack Possibilities: Direct Attack

- **Direct Attack:** Attacker node is the target node: $S = \{t\}$
- Modify **target** node feature
 - Ex) Change website content
- Add connections to **target**
 - Ex) Buy likes/followers
- Remove connections from **target**
 - Ex) Unfollow users



Attack Possibilities: Indirect Attack

- **Indirect Attack:** The target node is **not** in the **attacker nodes**: $t \notin S$
- Modify **attacker** node features
 - Ex) Hijack friends of targets
- Add connections to **attackers**
 - Ex) Create a link, link farm
- Remove connections from **attackers**
 - Ex) Delete undesirable link



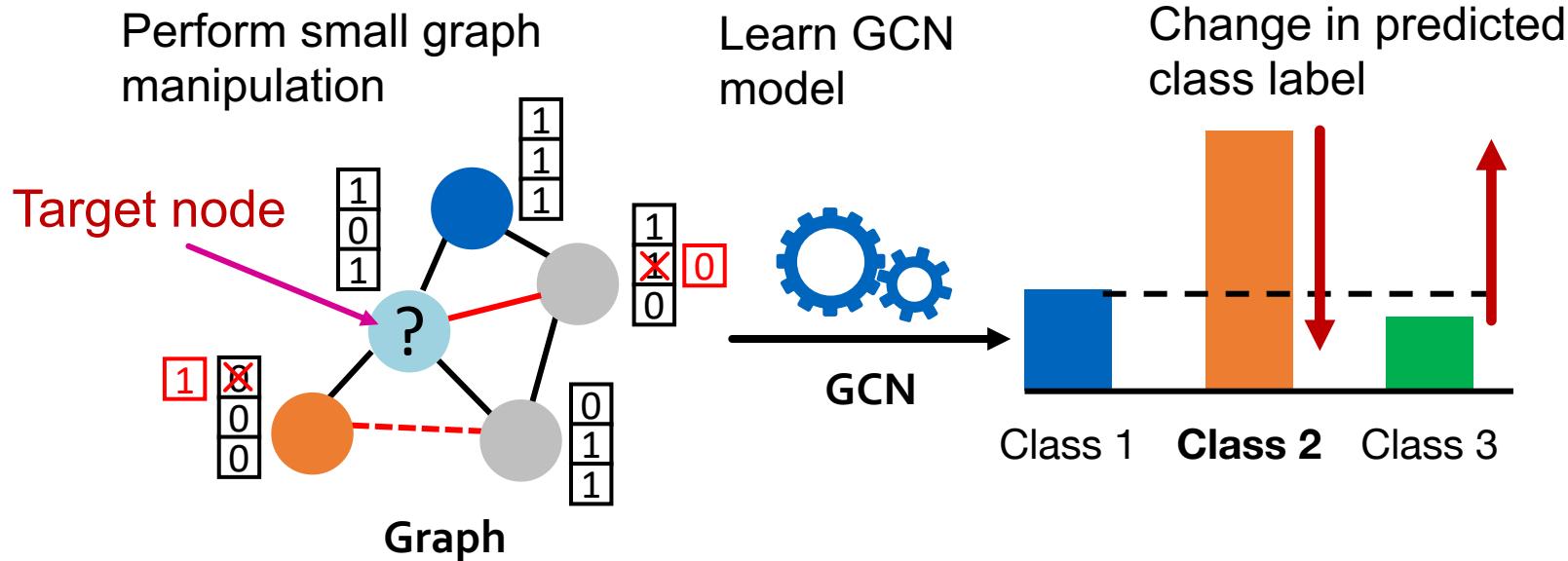
Formalizing Adversarial Attacks

■ Objective for the attacker:

Maximize (**change of target node label prediction**)

Subject to (**graph manipulation is small**) \rightarrow at least as possible

If graph manipulation is too large, it will easily be detected.
Successful attacks should change the target prediction
with “unnoticeably-small” graph manipulation.



Mathematical Formulation (1)

- **Original graph:**
 - A : adjacency matrix, X : feature matrix
- **Manipulated graph (after adding noise):**
 - A' : adjacency matrix, X' : feature matrix
- **Assumption:** $(A', X') \approx (A, X)$
 - Graph manipulation is **unnoticeably small**.
 - Preserving basic graph statistics (e.g., degree distribution) and feature statistics.
 - Graph manipulation is either **direct** (changing the feature/connection of target nodes) or **indirect**.

Mathematical Formulation (2)

standard

- **Target node:** $v \in V$
- GCN learned over the **original graph**

$$\theta^* = \operatorname{argmin}_{\theta} \mathcal{L}_{train}(\theta; A, X)$$

- GCN's original prediction on the **target node**:

$$c_v^* = \operatorname{argmax}_c f_{\theta^*}(A, X)_{v,c}$$

Predict the class c_v^* of vertex v that has the highest predicted probability

Mathematical Formulation (3)

Attacker comes into play → cannot change params of model (already trained)

- GCN learned over the **manipulated graph**

$$\boldsymbol{\theta}^{*''} = \operatorname{argmin}_{\boldsymbol{\theta}} \mathcal{L}_{train}(\boldsymbol{\theta}; \mathbf{A}', \mathbf{X}')$$

- GCN's prediction on the **target node v** :

$$c_v^{*''} = \operatorname{argmax}_c f_{\boldsymbol{\theta}^{**}}(\mathbf{A}', \mathbf{X}')_{v,c}$$

- We want the prediction to change after the graph is manipulated:

$$c_v^{*''} \neq c_v^*$$

want to flip the class
ex. spam/not spam

Mathematical Formulation (4)

- Change of prediction on target node v :

$$\Delta(v; A', X') =$$

$$\log f_{\theta^{*'}}(A', X')_{v, c_v^{*'}} - \log f_{\theta^{*'}}(A', X')_{v, c_v^*}$$

Predicted (log)
probability of the
newly-predicted
class $c_v^{*'}$

Predicted (log)
probability of the
originally-predicted
class c_v^*



Want to increase
this term



Want to decrease
this term

maximize $\Delta(v; A', X')$

Mathematical Formulation (5)

- **Final optimization objective:**

$$\operatorname{argmax}_{A', X'} \Delta(v; A', X')$$

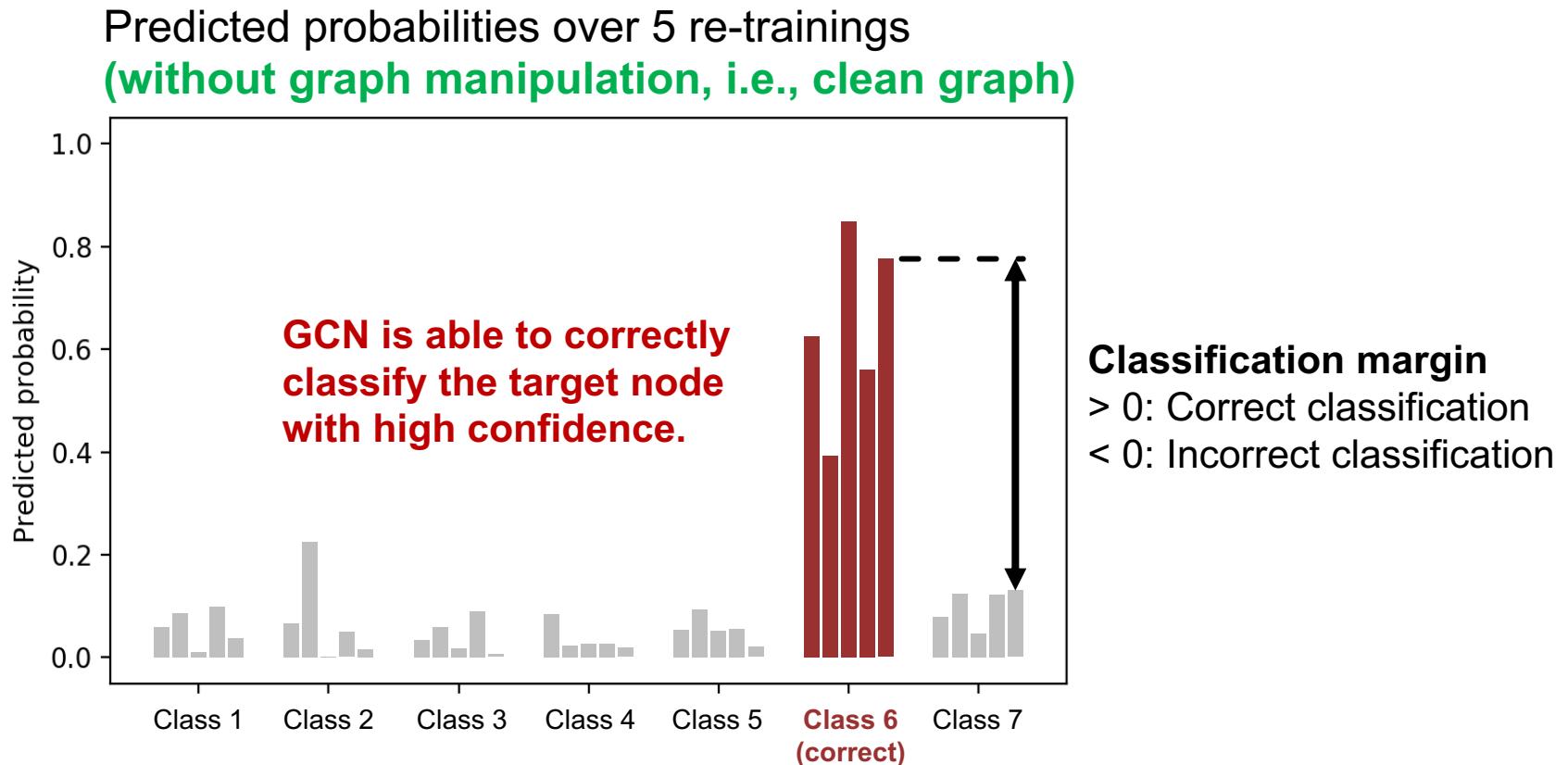
subject to $(A', X') \approx (A, X)$

- **Challenges in optimizing the objective**

- Adjacency matrix A' is a discrete object: gradient-based optimization cannot be used.
- For every modified graph A' and X' , GCN needs to be re-trained (this is computationally expensive):
 - $\theta^{*'} = \operatorname{argmin}_{\theta} \mathcal{L}_{train}(\theta; A', X')$
- Several approximations are proposed to make the optimization tractable [Zügner et al. KDD2018].

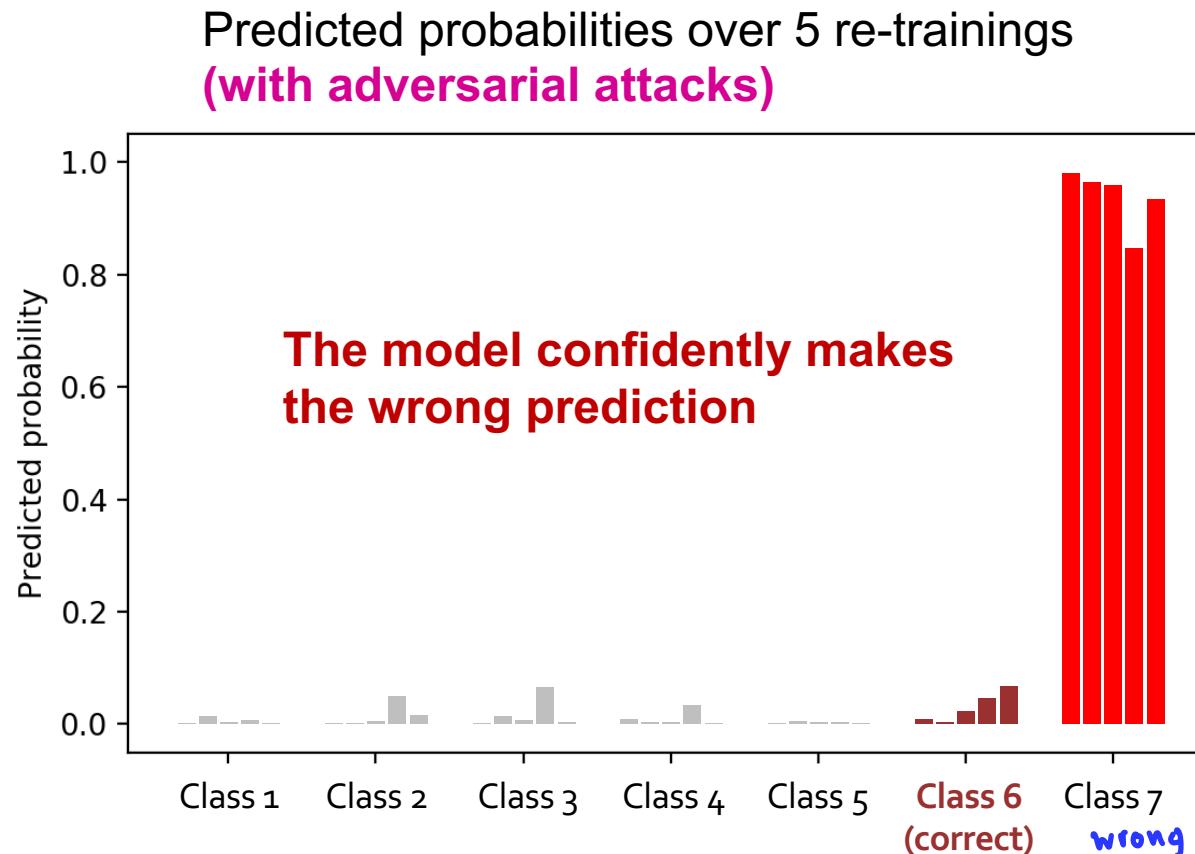
Experiments: Adversarial Attack

Semi-supervised node classification with GCN on a paper citation network (2,800 nodes, 8,000 edges).



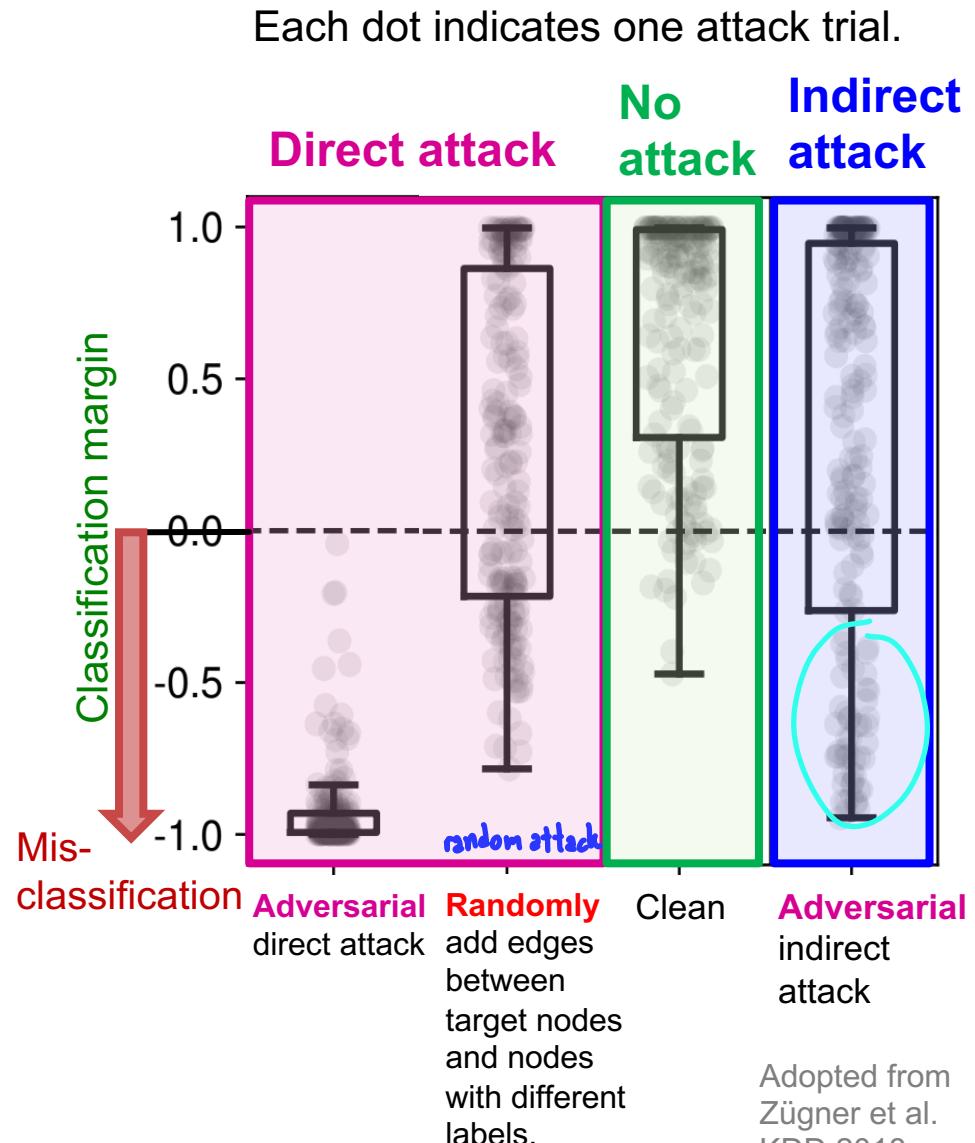
Experiments: Adversarial Attack

GCN's prediction after **carefully modifying just 5 edges attached to the target node (direct adversarial attack)**.



Experiments: Attack Comparison

- **Adversarial direct attack** is the strongest attack, significantly worsening GCN's performance (compared to **no attack**).
- **Random** attack is much weaker than **adversarial** attack.
- **Indirect attack** is more challenging than direct attack.



Summary

- We study the adversarial robustness of GCN applied to semi-supervised node classification.
- We consider different **attack possibilities on graph-structured data.** direct/indirect
- We mathematically **formulate the adversarial attack as an optimization problem.**
- We empirically demonstrate that GCN's prediction performance can be significantly harmed by adversarial attacks.
- **GCN is *not* robust to adversarial attacks but it is somewhat robust to indirect attacks and random noise.**