Assignment 1

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1 Question 1

Find the value of p for which the points $\mathbf{A} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ p \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$are collinear

2 Solution

Given:- Given:- $\mathbf{A} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ p \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$
Given that the points are collinear, so we create a matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^{\top} \tag{2.0.1}$$

where $rank(\mathbf{M}) = 1$.We have the matrix $\mathbf{M}$ as,

$$\mathbf{M} = \begin{pmatrix} 1+5 & p-1 \\ 4+5 & -2-1 \end{pmatrix} \tag{2.0.2}$$

$$\implies \mathbf{M} = \begin{pmatrix} 6 & p-1 \\ 9 & -3 \end{pmatrix} \tag{2.0.3}$$

Now we row reduce the matrix $\mathbf{M}$,

$$\begin{pmatrix} 6 & p-1 \\ 9 & -3 \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 9 & -3 \\ 6 & p-1 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_1 \to \frac{R_1}{3}} \begin{pmatrix} 3 & -1 \\ 6 & p-1 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 3 & -1 \\ 0 & p+1 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_1 \to \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{-1}{3} \\ 0 & p+1 \end{pmatrix} \tag{2.0.7}$$

Since $rank(\mathbf{M}) = 1$,we have

$$p + 1 = 0 \tag{2.0.8}$$

$$\implies p = -1 \tag{2.0.9}$$

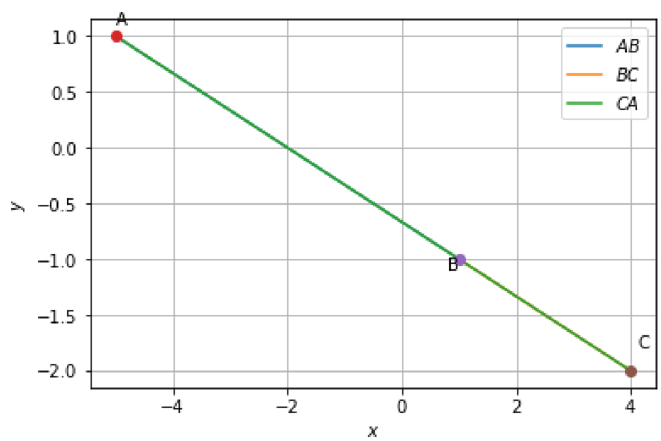

Fig. 2.1: Graphical solution