

# Assignment No.5

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Download latex-tikz codes from

<https://github.com/Panisha707/ASSIGNMENT05/blob/main/main.tex>

Download python codes from

<https://github.com/Panisha707/ASSIGNMENT05/blob/main/untitled31.py>

Question taken from

quad\_form, exercises 2.24

## 1 QUESTION No 1

Find the coordinates of the focus, axis, the equation of the directrix and latus rectum of the parabola  $y^2 = 8x$

## 2 SOLUTION

Given equation of the parabola is:

$$y^2 = 8x \quad (2.0.1)$$

$$y^2 - 8x = 0 \quad (2.0.2)$$

$$y^2 + 2(-4)x = 0 \quad (2.0.3)$$

comparing it with standard equation

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.4)$$

$\therefore a = b = e = 0, d = -4, c = 1, f = 0$

$$\mathbf{V} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.5)$$

$$\mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (2.0.6)$$

**Lemma 2.1.** The equation of a parabola is:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.7)$$

Then its vertex can be calculated as :

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -\mathbf{f} \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.8)$$

$$\text{where, } \eta = \mathbf{u}^T \mathbf{p}_1 \quad (2.0.9)$$

Equation of the parabola can be written as

$$\Rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.10)$$

We can find the eigen values corresponding to the  $\mathbf{V}$ ,

$$|\mathbf{V} - \lambda \mathbf{I}| = \left| \begin{pmatrix} 0 - \lambda & 0 \\ 0 & 1 - \lambda \end{pmatrix} \right| \quad (2.0.11)$$

$$(-\lambda)(1 - \lambda) = 0 \quad (2.0.12)$$

$\therefore$  Eigen values are  $\lambda_1 = 0, \lambda_2 = 1$

Calculating the eigen vectors corresponding to  $\lambda_1 = 0, \lambda_2 = 1$  respectively

$$\mathbf{V} \mathbf{x} = \lambda \mathbf{x} \quad (2.0.13)$$

$$\Rightarrow \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.15)$$

The vertex of the parabola can be given as

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -\mathbf{f} \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.16)$$

$$\text{where, } \eta = \mathbf{u}^T \mathbf{p}_1 = \begin{pmatrix} -4 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.17)$$

$$\begin{pmatrix} -8 & 1 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.18)$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.19)$$

Since  $\lambda_2 > \lambda_1$

Hence, the axis using  $\mathbf{p}_2$  is given by

$$\mathbf{p}_2^T (\mathbf{x} - \mathbf{c}) = 0 \quad (2.0.20)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad (2.0.21)$$

$$\Rightarrow y = 0 \quad (2.0.22)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.23)$$

**Theorem 2.1.** *The eccentricity, directrices and foci of parabola are given by*  
*Eccentricity,*

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \quad (2.0.24)$$

$$\mathbf{n} = \sqrt{\lambda_2} \mathbf{p}_1 \quad (2.0.25)$$

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2e^2 \mathbf{u}^T \mathbf{n}} \quad (2.0.26)$$

*Focus,*

$$\mathbf{F} = \frac{ce^2 \mathbf{n} - \mathbf{u}}{\lambda_2} \quad (2.0.27)$$

*Directrix,*

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.28)$$

From Equation 2.0.24, eccentricity can be calculated as

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \quad (2.0.29)$$

$$e = \sqrt{1 - \frac{0}{1}} = \sqrt{1} \quad (2.0.30)$$

$$\Rightarrow e = 1 \quad (2.0.31)$$

Substituting  $\lambda_2$  and  $\mathbf{p}_1$  values in Equation 2.0.25

$$\mathbf{n} = \sqrt{\lambda_2} \mathbf{p}_1 \quad (2.0.32)$$

$$\mathbf{n} = \sqrt{1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.33)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.34)$$

Equation 2.0.26 can be calculated as

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2e^2 \mathbf{u}^T \mathbf{n}} \quad (2.0.35)$$

$$c = \frac{16 - 1 \times 0}{2 \times 1^2 \times \begin{pmatrix} -4 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}} \quad (2.0.36)$$

$$= \frac{16}{-8} \quad (2.0.37)$$

$$\Rightarrow c = -2 \quad (2.0.38)$$

Focus of the parabolic equation can be calculated from the equation 2.0.27

$$\mathbf{F} = \frac{ce^2 \mathbf{n} - \mathbf{u}}{\lambda_2} \quad (2.0.39)$$

$$\mathbf{F} = \frac{-2 \times 1 \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ 0 \end{pmatrix}}{1} \quad (2.0.40)$$

$$\Rightarrow \mathbf{F} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (2.0.41)$$

Directrix of the parabolic equation can be calculated from equation 2.0.28

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.42)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2 \quad (2.0.43)$$

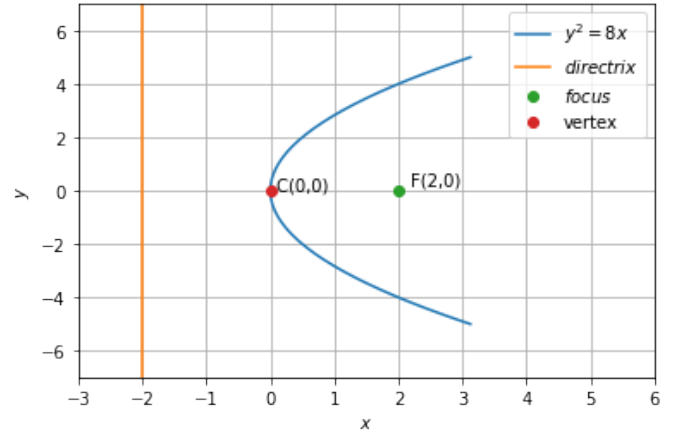


Fig. 2.1: Parabola