

Assignment No.7

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Download latex-tikz codes from

<https://github.com/Panisha707/ASSIGNMENT07/blob/main/main.tex>

Question taken from

Optimization , exercises 2.22

1 QUESTION No 1

Find the shortest distance of the point $\begin{pmatrix} 0 \\ c \end{pmatrix}$ from the parabola $y = x^2$, where $\frac{1}{2} \leq c \leq 5$

2 SOLUTION

Let the points $\mathbf{P} = \begin{pmatrix} 0 \\ c \end{pmatrix}$ and $\mathbf{Q} = (x \ y)$ on parabola $y = x^2$
Now the distance between the points can be calculated by

$$\mathbf{PQ} = \sqrt{(x-0)^2 + (y-c)^2} \quad (2.0.1)$$

As the equation of parabola is $y = x^2$, we can rewrite the above equation as

$$\mathbf{PQ} = \sqrt{(x-0)^2 + (x^2-c)^2} \quad (2.0.2)$$

$$\mathbf{PQ}^2 = (x)^2 + (x^2-c)^2 \quad (2.0.3)$$

Let $f(x) = x^2 + (x^2-c)^2$

$$f(x) = x^2 + (x^2-c)^2 \quad (2.0.4)$$

$$f'(x) = 2x + 2(x^2-c)2x \quad (2.0.5)$$

$$= 2x(1 + 2x^2 - 2c) \quad (2.0.6)$$

$$= 4x\left(x^2 - c + \frac{1}{2}\right) \quad (2.0.7)$$

$$= 4x\left(x - \sqrt{c - \frac{1}{2}}\right)\left(x + \sqrt{c - \frac{1}{2}}\right) \quad (2.0.8)$$

when $c > \frac{1}{2}$, For maxima, put $f'(x) = 0$

$$4x\left(x^2 - c + \frac{1}{2}\right) = 0 \quad (2.0.9)$$

$$\Rightarrow x = 0, x = \pm \sqrt{c - \frac{1}{2}} \quad (2.0.10)$$

Now,

$$f''(x) = 4\left(x^2 - c + \frac{1}{2}\right) + 4x(2x) \quad (2.0.11)$$

At,

$$x = \pm \sqrt{c - \frac{1}{2}}, f'' \geq 0 \quad (2.0.12)$$

$\therefore f''(x)$ is minimum

Hence, minimum value of $f(x) = \|\mathbf{PQ}\|$

$$= \sqrt{\left(\sqrt{c - \frac{1}{2}}\right)^2 + \left(\left(\sqrt{c - \frac{1}{2}}\right) - c\right)^2} \quad (2.0.13)$$

$$= \sqrt{c - \frac{1}{2} + \left(c - \frac{1}{2} - c\right)^2} \quad (2.0.14)$$

$$= \sqrt{c - \frac{1}{4}}, \because \frac{1}{2} \leq c \leq 5 \quad (2.0.15)$$