Assignment No.6

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Download latex-tikz codes from

https://github.com/Panisha707/ASSIGNMENT07/blob/main/main.tex

Question taken from

Optimization , exercises 2.22

1 Question No 1

Find the shortest distance of the point $\begin{pmatrix}0\\c\end{pmatrix}$ from the parabola $y = x^2$, where $\frac{1}{2} \le c \le 5$

2 Solution

Let the points $\mathbf{P} = \begin{pmatrix}0\\c\end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix}x & y\end{pmatrix}$ on parabola $y = x^2$
Now the distance between the points can be calculated by

$$\mathbf{PQ} = \sqrt{\left(x-0\right)^2 + \left(y-c\right)^2} \tag{2.0.1}$$

As the equation of parabola is $y = x^2$, we can rewrite the above equation as

$$\mathbf{PQ} = \sqrt{\left(x-0\right)^2 + \left(x^2-c\right)^2} \tag{2.0.2}$$

$$\mathbf{PQ}^2 = \left(x\right)^2 + \left(x^2-c\right)^2 \tag{2.0.3}$$

Let $f(x) = x^2 + \left(x^2-c\right)^2$

$$f(x) = x^2 + \left(x^2-c\right)^2 \tag{2.0.4}$$

$$f'(x) = 2x + 2\left(x^2-c\right)2x \tag{2.0.5}$$

$$= 2x\left(1 + 2x^2 - 2c\right) \tag{2.0.6}$$

$$= 4x\left(x^2 - c + \tfrac{1}{2}\right) \tag{2.0.7}$$

$$= 4x\left(x - \sqrt{c-\tfrac{1}{2}}\right)\left(x + \sqrt{c-\tfrac{1}{2}}\right) \tag{2.0.8}$$

when $c > \frac{1}{2}$, For maxima, put $f'(x) = 0$

$$4x\left(x^2 - c + \tfrac{1}{2}\right) = 0 \tag{2.0.9}$$

$$\implies x = 0, x = \pm\sqrt{c-\frac{1}{2}} \tag{2.0.10}$$

Now,

$$f''(x) = 4\left(x^2 - c + \tfrac{1}{2}\right) + 4x\left(2x\right) \tag{2.0.11}$$

At,

$$x = \pm\sqrt{c-\frac{1}{2}}, f'' \ge 0 \tag{2.0.12}$$

$\therefore f''(x)$ is minimum
Hence, minimum value of $f(x) = \|\mathbf{PQ}\|$

$$= \sqrt{\left(\sqrt{c-\tfrac{1}{2}}\right)^2 + \left(\left(\sqrt{c-\tfrac{1}{2}}\right) - c\right)^2} \tag{2.0.13}$$

$$= \sqrt{c - \frac{1}{2} + \left(c - \tfrac{1}{2} - c\right)^2} \tag{2.0.14}$$

$$= \sqrt{c-\frac{1}{4}}, \because \frac{1}{2} \le c \le 5 \tag{2.0.15}$$