Assignment 1

Pansha Gundelli

NIPER Hyderabad

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Question

cbse/math/10/2006/set2-Q17

Find the value of p for which the points
$A = \begin{pmatrix} -5 \\ 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ p \end{pmatrix}, C = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ are collinear.

Solution

Given:- $A = \begin{pmatrix} -5 \\ 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ p \end{pmatrix}, C = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

Given that the points are collinear, so we create a matrix

$$M = \begin{pmatrix} B - A & C - A \end{pmatrix}^{\top} \tag{1}$$

where $rank(M) = 1$.We have the matrix M as,

$$M = \begin{pmatrix} 1+5 & p-1 \\ 4+5 & -2-1 \end{pmatrix} \tag{2}$$

$$M = \begin{pmatrix} 6 & p-1 \\ 9 & -3 \end{pmatrix} \tag{3}$$

Solution

Now we row reduce the matrix M,

$$\begin{pmatrix} 6 & p-1 \\ 9 & -3 \end{pmatrix} \tag{4}$$

$$\overset{R_1 \leftrightarrow R_2}{\longleftrightarrow} \begin{pmatrix} 9 & -3 \\ 6 & p-1 \end{pmatrix} \tag{5}$$

$$\overset{R_1 \to \frac{R_1}{3}}{\longleftrightarrow} \begin{pmatrix} 3 & -1 \\ 6 & p-1 \end{pmatrix} \tag{6}$$

$$\overset{R_2 \to R_2 - 2R_1}{\longleftrightarrow} \begin{pmatrix} 3 & -1 \\ 0 & p+1 \end{pmatrix} \tag{7}$$

$$\tag{8}$$

Solution

$$\overset{R_1 \to \frac{R_1}{3}}{\longleftrightarrow} \begin{pmatrix} 1 & \frac{-1}{3} \\ 0 & p+1 \end{pmatrix} \tag{9}$$

Since $rank(\ \mathrm{M}) = 1$,we have

$$p + 1 = 0 \tag{10}$$

$$\implies p = -1 \tag{11}$$

Figure verifies that the points are indeed collinear for $p = -1$

Graphical solution