1. Introduction to linear programming

Panithan Sriboriboon

A diet problem

• Polly wonders how much money she must spend on food to get:

  1. Energy 2000 kcal
  2. Protein 55 g
  3. Calcium 800 mg

Food	Serving size	Energy (kcal)	Protein (g)	Calcium (mg)	Price per serving (cents)
Oatmeal	28 g	110	4	2	3
Chicken	100 g	205	32	12	24
Eggs	2 large	160	13	54	13
Whole milk	237 cc	160	8	285	9
Cherry pie	170 g	420	4	22	20
Pork with beans	260 g	260	14	80	19

Serving per day limits

- She would not be able to stomach more than serving per day limits

Food	Serving size	Serving per day limits	Energy (kcal)	Protein (g)	Calcium (mg)	Price per serving (cents)
Oatmeal	28 g	4	110	4	2	3
Chicken	100 g	3	205	32	12	24
Eggs	2 large	2	160	13	54	13
Whole milk	237 cc	8	160	8	285	9
Cherry pie	170 g	2	420	4	22	20
Pork with beans	260 g	2	260	14	80	19

- Trail and error is not particular helpful here

To be systematic

- The meal order must satisfy serving per day limits:

1. $0 \leq X_1 \leq 4$ ← Oatmeal, no more than 4 serving

2. $0 \leq X_2 \leq 3$

3. $0 \leq X_3 \leq 2$

       .
       .
       .

4. $0 \leq X_4 \leq 8$

5. $0 \leq X_5 \leq 2$

6. $0 \leq X_6 \leq 2$ ← Pork with beans, no more than 4 serving

	Food	Serving size	Serving per day limits	Energy (kcal)	Protein (g)	Calcium (mg)	Price per serving (cents)
X_1	Oatmeal	28 g	4	110	4	2	3
X_2	Chicken	100 g	3	205	32	12	24
X_3	Eggs	2 large	2	160	13	54	13
X_4	Whole milk	237 cc	8	160	8	285	9
X_5	Cherry pie	170 g	2	420	4	22	20
X_6	Pork with beans	260 g	2	260	14	80	19

Energy, protein, and calcium

- Requirements for energy, protein, and calcium

1. Energy 2000 kcal ⟶ $100X_1 + 205X_2 + 160X_3 + 160X_4 + 420X_5 + 260X_6 \geq 2000$

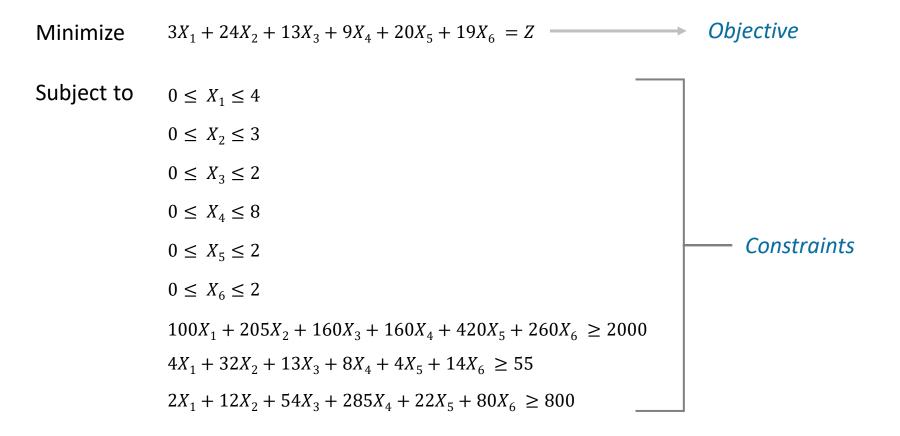2. Protein 55 g ⟶ $4X_1 + 32X_2 + 13X_3 + 8X_4 + 4X_5 + 14X_6 \geq 55$

3. Calcium 800 mg ⟶ $2X_1 + 12X_2 + 54X_3 + 285X_4 + 22X_5 + 80X_6 \geq 800$

4. Cost ⟶ $3X_1 + 24X_2 + 13X_3 + 9X_4 + 20X_5 + 19X_6 = Z$

	Food	Serving size	Serving per day limits	Energy (kcal)	Protein (g)	Calcium (mg)	Price per serving (cents)
X_1	Oatmeal	28 g	4	110	4	2	3
X_2	Chicken	100 g	3	205	32	12	24
X_3	Eggs	2 large	2	160	13	54	13
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X_5	Cherry pie	170 g	2	420	4	22	20
X_6	Pork with beans	260 g	2	260	14	80	19

Small cost as possible

- As data annalist, we consider that she wants to

Minimize $3X_1 + 24X_2 + 13X_3 + 9X_4 + 20X_5 + 19X_6 = Z$ ⟶   *Objective*

Subject to

$0 \leq X_1 \leq 4$

$0 \leq X_2 \leq 3$

$0 \leq X_3 \leq 2$

$0 \leq X_4 \leq 8$

$0 \leq X_5 \leq 2$ *Constraints*

$0 \leq X_6 \leq 2$

$100X_1 + 205X_2 + 160X_3 + 160X_4 + 420X_5 + 260X_6 \geq 2000$

$4X_1 + 32X_2 + 13X_3 + 8X_4 + 4X_5 + 14X_6 \geq 55$

$2X_1 + 12X_2 + 54X_3 + 285X_4 + 22X_5 + 80X_6 \geq 800$

- Her problem known as a *"diet problem"*

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Liner programing