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## Data Structure

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### 4. Stack & Queue

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# Stack & Queue

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## Stack Introduction & Implementation:

Stack is a linear data structure that follows LIFO (Last in First out) or FILO (First in Last out) order.

\* Operation:-

• **push**: Adds an item to the stack, if stack is full (overflow condition)

```

begin
    if stack is full
        return
    endif
    else
        increment top
        stack[top] assign value
    end else
end procedure.
```

• **pop**

Remove an item from the stack. The item are popped in reversed order in which they are pushed, if stack is empty [Underflow condition]

```

begin
    if stack is empty
        return
    endif
    else
        store value of stack[top]
        decrement top
        return value
    end else
end procedure.
```

• **Peek or top**: return top element

```

begin
    return stack[top]
end procedure
```

• **IsEmpty**: return true if stack is empty, else false

```

begin
    if top<1
        return true
    else
        return false
end procedure.
```

Note:- all push(), pop(), isEmpty(), peek() will take O(1) time.

### \* Applications of the Stack:

- Balancing of Symbols
- infix to post fix/prefix
- redo-undo features at many places like editors, photoshop
- forward and backward features in web browsers
- used in algorithm like Tower of Hanoi, tree traversals, stockspan problems, and histogram problems.
- Backtracking algorithm, recursion all uses stack features.
- In memory management. etc.

### \* Implementation:

- ① Using array ② Using linked list

Implementing stack using array.

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
#define MAX 1000
```

```
class Stack {
```

```
    int top;
```

```
public:
```

```
    int a[MAX];
```

```
    Stack() { top = -1; }
```

```
    bool push(int x);
```

```
    int pop();
```

```
    int peek();
```

```
    bool isEmpty();
```

```
{};
```

```
bool Stack::push(int x) {
```

```
    if (top >= (MAX - 1)) {
```

```
        cout << "Stack Overflow!";
```

```
        return false;
```

```
} else {
```

```
    a[++top] = x;
```

```
} return true;
```

```
} {
```

```

int Stack::pop() {
    if (top < 0) cout << "Stack Underflow";
    return 0;
}

int Stack::peek() {
    if (top < 0) cout << "Stack is Empty";
    return 0;
}

bool Stack::isEmpty() { return (top < 0); }

```

```

int main() {
    class Stack s;
    s.push(10);
    s.push(20);
    s.push(30);
    cout << s.pop() << endl;
    cout << "Elements in stack:" << endl;
    while (!s.isEmpty()) {
        cout << s.peek() << endl;
        s.pop();
    }
    return 0;
}

```

Op:- 30

Element in stack: 20 10

Note:- It is not dynamic, doesn't grow and shrink depending on needs at runtime.

## Implementing stack using linked list

```
class StackNode{
```

```
public:
```

```
int data;
```

```
StackNode* next;
```

```
{}
```

```
StackNode* newNode(int data){
```

```
StackNode* stackNode = new StackNode();
```

```
stackNode->data = data;
```

```
stackNode->next = NULL;
```

```
return stackNode;
```

```
}
```

```
int isEmpty(StackNode* root) { return !root; }
```

```
void push(StackNode** root, int data){
```

```
StackNode* stackNode = newNode(data);
```

```
stackNode->next = *root;
```

```
*root = stackNode;
```

```
}
```

```
int pop(StackNode** root){
```

```
if(isEmpty(*root)) return INT_MIN;
```

```
StackNode* temp = *root;
```

```
*root = (*root)->next;
```

```
int popped = temp->data;
```

```
free(temp);
```

```
return popped;
```

```
}
```

```
int main(){
```

```
StackNode* root = NULL;
```

```
push(&root, 10);
```

```
push(&root, 20);
```

```
push(&root, 30);
```

```
cout << pop(&root);
```

```
cout << "Element present in stack:";
```

```
while(!isEmpty(root)){
```

```
cout << peek(&root) << "
```

```
pop(&root);
```

```
int peek(StackNode* root)
```

```
{ if (isEmpty(root))
```

```
return INT_MIN;
```

```
return root->data;
```

```
}
```

```
cout << peek(&root);
```

```
cout << "
```

```
pop(&root);
```

```
cout << "
```

## Queue Introduction & Implementation

- Queue follows the First In First Out (FIFO) rule - the item that goes in first is the item that comes out first

### \* Basic Operations of Queue

A queue is an object (an abstract data structure - ADT) that allows the following operations.

- Enqueue: Add an element to the end of the queue.
- Dequeue: Remove an element from the front of the queue.
- isEmpty: Check if the queue is empty.
- isFull: Check if the queue is full.
- Peek: Get the value of the front of queue without removing it.

### \* Working

Queue works as follows:

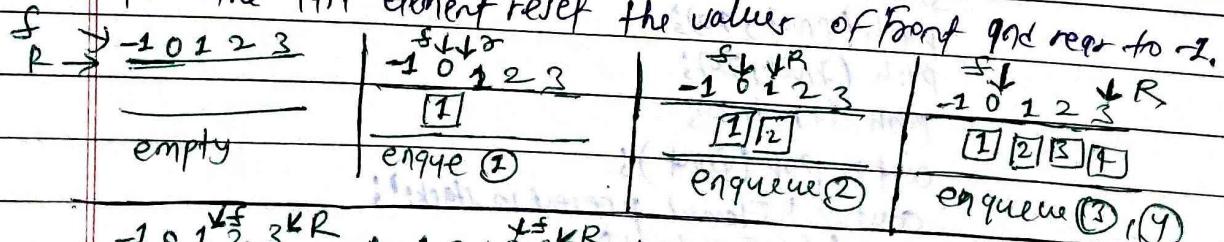
- two pointers FRONT & REAR
- FRONT track the first element of the queue
- REAR track the last element of the queue.
- initially, value of the FRONT and REAR is -1

#### Enqueue:

- Check if the queue is not full
- For first element set value of [Front] to 0.
- Increase [REAR] index by 1.
- Add the new element at position [REAR]

#### Dequeue:

- Check if the queue is not empty.
- return value pointed by FRONT
- increase the FRONT index by 1
- For the last element reset the values of Front and rear to -2.



### \* Implementation :-

```

#include <iostream>
#define SIZE 5
using namespace std;

class Queue {
private:
    int items[SIZE], front, rear;
public:
    Queue() { front = -1; rear = -1; }

    bool isFull() {
        if (front == 0 && rear == SIZE - 1) return true;
        return false;
    }

    bool isEmpty() {
        if (front == -1) return true;
        return false;
    }

    void enqueue(int ele) {
        if (isFull()) cout << "Queue is full";
        else {
            if (front == -1) front = 0;
            items[rear] = ele;
            cout << endl << "Inserted " << ele << endl;
        }
    }

    int dequeue() {
        int ele;
        if (isEmpty()) cout << "Queue is empty";
        else {
            ele = items[front];
            if (front == rear) front = -1, rear = -1;
            else front++;
            cout << endl << "Deleted " << ele << endl;
        }
        return ele;
    }
}

```

```
void display() {
    int i;
```

```
if (isEmpty()) cout << "Empty Queue" << endl;
else {
```

```
    for (i = Front; i <= rear; i++)
        cout << items[i] << " ";
```

```
    cout << endl;
```

{  
}

```
int main() {
```

```
    Queue q;
```

```
    q.dequeue(); // empty Queue
```

```
    q.enqueue(1);
```

```
    q.enqueue(2);
```

```
    q.enqueue(3);
```

```
    q.enqueue(4);
```

```
    q.enqueue(5);
```

```
    q.enqueue(6); // queue is Full
```

```
    q.display(); // 1 2 3 4 5
```

```
    q.dequeue(); // remove 1
```

```
    q.display(); // 2 3 4 5
```

```
    return 0;
```

{  
}

### \* Limitations of Queue

- after a bit of enqueing and dequeuing, the size of queue is reduced
- and we can only use index 0 and 1 only when queue is reset

not usable → 1 0 2 3 4 5

[ ] [ ] [ ]

- After REAR reaches the last index, if we can store extra ele in the empty spaces (0 and 1), we can make use of the empty spaces. This is done by using circular queue.

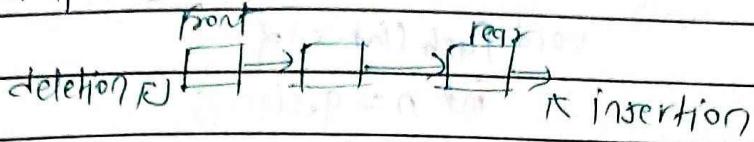
Note :- Dequeue & enqueue → O(1) time

## Applications

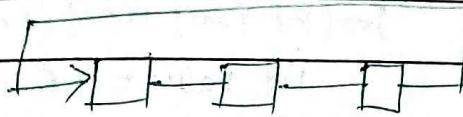
- CPU scheduling, Disk scheduling
- I/O Buffers, pipes, File I/O.
- Handling of interrupts in real-time systems.

## \* Types of Queues

### (1) Simple Queue



### (2) Circular Queue



• It is better memory utilization

### (3) Priority Queue

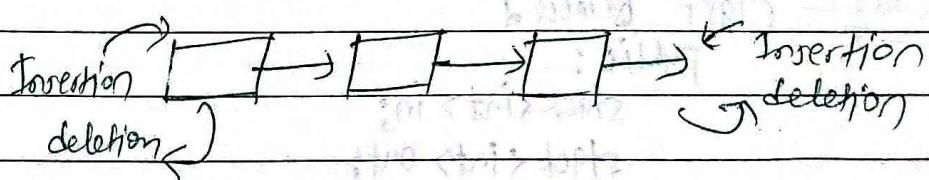
- a special type of queue in which each element is associated with a priority and is served according to its priority. If the element with same priority occurs, they are served according to their order in the queue.

Priority



### (4) Deque (Double Ended Queue)

- insertion and removal of elements can be performed from either from the front or rear. it doesn't follow FIFO



### (3) Implement stack using Queue:-

if push:- push into queue from rear and pop all the element,  
again push them into sequence.

if pop:- pop from queue from front.

Code:- class Stack {

queue<int> q;

public:

void Push(int x) {

int n = q.size();

q.push(x);

for (int i=0; i<n; i++) {

int value = q.front();

q.pop();

{ q.push(value);

}

int Pop() {

int value = q.front();

q.pop();

{ return value;

int Top() { return q.front(); }

int Size() { return q.size(); }

};

### (4) Implement a Queue using stack:-

→ use 2 stacks.

→ while pop(), shift all element in 2<sup>nd</sup> stack and pop from  
2<sup>nd</sup> stack and again shift all element <sup>to 2<sup>nd</sup></sup> when the 2<sup>nd</sup> got empty.

Code:- class Queue {

public:

stack<int> in;

stack<int> out;

void Push(int x) {

in.push(x);

};

int pop()

if (out.empty()) {

while (in.size()) {

out.push (in.top());

in.pop();

} int x = out.top();

out.pop();

return x;

int Top()

if (out.empty()) {

while (in.size()) {

out.push (in.top());

in.pop();

} return out.top();

int size()

return in.size() + out.size();

### (5) Check for Balanced Parentheses

ex:- str = "[(())]" | str = "(()[]{}())"

false

true

Code:- bool isValid (string s) {

stack<char> st;

for (auto it : s) {

if (it == '(' || it == '[' || it == '{') st.push(it);

else if

if (st.size() == 0) return false;

char ch = st.top();

st.pop();

if ((it == ')' && ch == '(') || (it == ']' && ch == '[') || (it == '}' && ch == '{'))

|| (it == ')' && ch == '[') || (it == '}' && ch == '['))

```

        continue;
    else return false;
}
return st.empty(); // Time: O(n)
}
// Space: O(n)

```

### (6) Next Greater Element:-

- return the next greater element for every element in q1 array.

- next greater element is first greater element in q2 array while traversing.

<u>ex:- DT[4, 5, 2, 25]</u>	<u>q2[2, 4, 1, 3, 1, 6]</u>
$4 \rightarrow 5$	$2 \rightarrow 4$
$5 \rightarrow 25$	$4 \rightarrow 6$
$2 \rightarrow -1$	$1 \rightarrow 3$

res = [5, 25, 25, -1]

\* Approach :-

① Stack = [ ] ele = 25	NGE = -1	② Stack = [25, 2] ele = 5 $\because 2 < 5$ , pop(2) ∴ NGE = 25
③ Stack = [25] ele = 2	NGE = 25	④ Stack = [25, 5] ele = 4 NGE = 5

res = [5, 25, 25, -1]

\* Code :-

```

class Solution {
public:
    vector<int> nextGreaterElement(vector<int> q1, int n) {
        stack<int> st;
        vector<int> res(n);
        for(int i=n-1; i>=0; i--) {
            int currVal = q1[i];
            while(!st.empty() && st.top() < currVal)
                st.pop();
            res[i] = st.empty() ? -1 : st.top();
            st.push(currVal);
        }
        return res;
    }
}

```

};

// TC = O(n)  
SC = O(n)

### (7) Next Greater Element in Circular Array:

ex:-  $A[] = \{5, 1, 3, 7, 6, 0\}$   
 $O/P = [7, -1, 7, -1, 7, 5]$

Approach:- The only difference b/w q circular and non-circular array is that while searching for the next greater element in a non-circular array we don't consider the elements left to the concerned element. This can be easily done by inserting the elements of the array A at the end of A, thus making its size double. But we actually don't require any extra space, we can just traverse the array twice. i.e  $2^*N$  times.

Code:-

Class Solution {

public:

vector<int> nextGreaterElements(vector<int> &nums) {  
 int n = nums.size();

vector<int> nge(n, -1);

stack<int> st;

for (int i = 2 \* n - 1; i >= 0; i--) {

while (!st.empty() && st.top() <= nums[i + n])

st.pop();

if (i < n)

if (!st.empty()) nge[i] = st.top();

}

st.push(nums[i + n]);

return nge;

// TC: O(N)

SC: O(N)

};

### (8)

#### Next Smaller element:-

entire approach is similar to next greater element except for comparison.

ex:-  $A = [2, 4, 3]$ ,  $O/P = [-1, 3, -1]$

$A = [1, 3, 2]$ ,  $O/P = [-1, 2, 1]$

```

vector<int> nextSmallerElement(vector<int> arr, int n) {
    stack<int> st;
    vector<int> res(n);
    for (int i=n-1; i>=0; i--) {
        int currVal = arr[i];
        while (!st.empty() && st.top() >= currVal)
            st.pop();
        res[i] = st.empty() ? -1 : st.top();
        st.push(currVal);
    }
    return res;
}
    
```

$T \rightarrow O(n)$   
 $S \rightarrow O(n)$

9.

Stock Span problem: given price quotes of stock for  $n$  days.

We need to find span of stock on any particular day.

- The span of the stock's price today is defined as the max no. of consecutive days (starting from today and going back)

for which the stock price was less than or equal to today's price

Ex:-  $A = [100, 80, 60, 70, 60, 75, 85]$ ,  $o/p = [1, 2, 2, 2, 1, 4, 6]$

Approach:-  $stack = [\text{stores index}]$  |  $\text{while } (\text{currEle} > A[\text{stack.top}])$

$\text{span} = [0|0|0|0|0|0|0]$

$\text{pop stack}$

$\text{span}[i] = \text{currEle} - \text{stack.top}$

• 100:  $stack = [] \therefore \text{span}[0] = 1$   
 $\text{push}(0)$

• 80:  $stack = [0]$  |  $\therefore \text{span}[1] = 1 - 0 = 1$   
 $\because 80 < 100$   
 $\text{push}(1)$

• 60:  $stack = [0, 1]$

$\because 60 < 80 \therefore \text{span}[2] = 2 - 1 = 1$   
 $\text{push}(2)$

• 70:  $stack = [0, 1, 2]$

$\because 70 > 60 \therefore \text{pop}(2)$  |  $\downarrow$  idx of 80 top of stack  
 $\therefore 70 < 80 \therefore \text{span}[3] = 3 - 1 = 2$

$\text{push}(3) \rightarrow \text{stack} = [0, 1, 3]$

and 80.0n

Code:-

Class Solution of

public:

`vector<int> calculateSpan(int price[], int n)`

`vector<int> span;`

`Stack<int> st;`

`st.push(0);`

`span[0] = 1;`

`for (int i = 1; i < n; i++) {`

`int currPrice = price[i];`

`while (!st.empty() && currPrice >= price[st.top()])`

`st.pop();`

`if (st.empty())`

`span[i] = i + 1;`

`else`

`span[i] = i - st.top();`

`{ st.push(i);`

`} return span;`

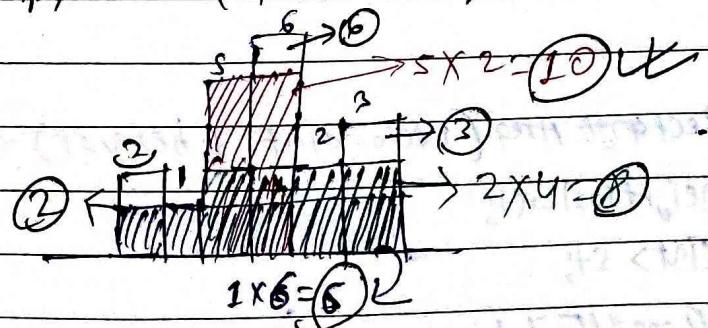
`}`

10.

Area of largest rectangle in Histogram:

Given an array of integers heights representing the histogram's bar height where the width of each bar is 1 return the area of largest rectangle in histogram.

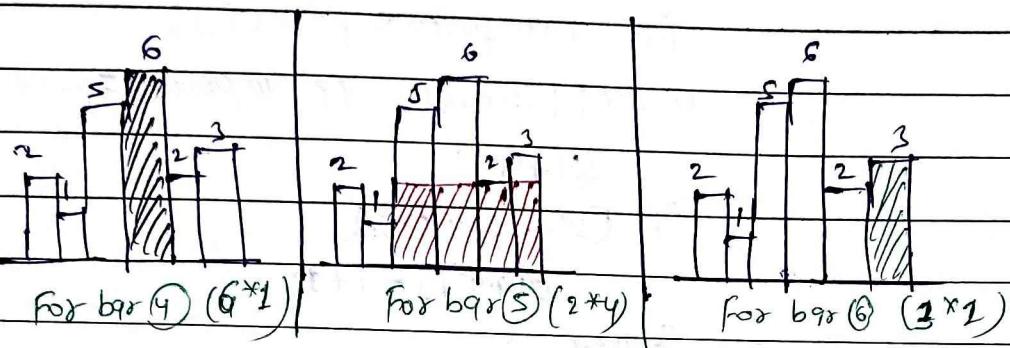
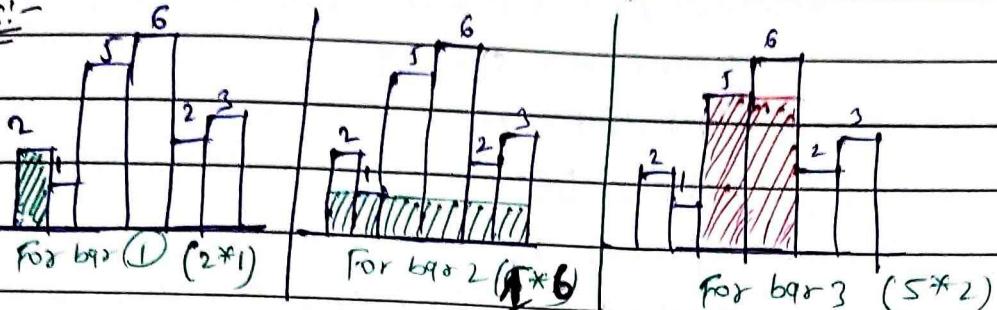
`int h[] = {2, 1, 5, 6, 2, 3}; Output: 10`



### Approach 1 :- Brute Force Approach :-

The intuition behind the approach is taking diff bars and finding the maxm width possible using the bars.

ex:-



- The approach is to find the right next smaller and left next smaller element and find the largest rectangle area in Histogram
- We can find this by using two loop. Time =  $O(N \times N)$   
Space =  $O(1)$

### Approach 2 :-

The intuition behind the approach is the same as finding the smaller element on both side but in an optimized way using the concept of next greater element and the next smaller element.

Code :-

class Solution {

public:

int largestRectangleArea(vector<int> &heights) {

int n = heights.size();

stack<int> st;

int leftsmall[n], rightsmall[n];

```

for (int i=0; i<n; i++) {
    while (!st.empty() && height[st.top()] >= height[i]) {
        st.pop();
    }
    if (st.empty())
        leftsmall[i] = 0;
    else
        leftsmall[i] = st.top() + 1;
    st.push(i);
}

// Clear stack to reverse
while (!st.empty())
    st.pop();

for (int i=n-1; i>=0; i--) {
    while (!st.empty() && height[st.top()] >= height[i]) {
        st.pop();
    }
    if (st.empty())
        rightsmall[i] = n-1;
    else
        rightsmall[i] = st.top() - 1;
    st.push(i);
}

int maxA = 0;
for (int i=0; i<n; i++) {
    maxA = max(maxA, height[i] * (rightsmall[i] - leftsmall[i] + 1));
}

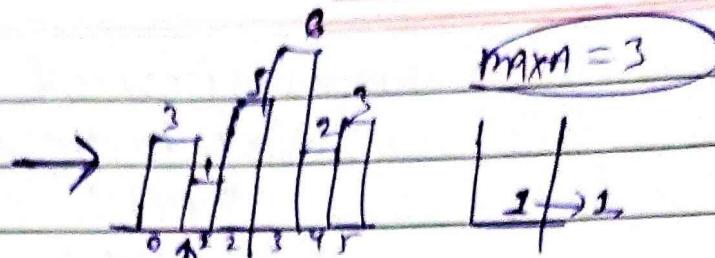
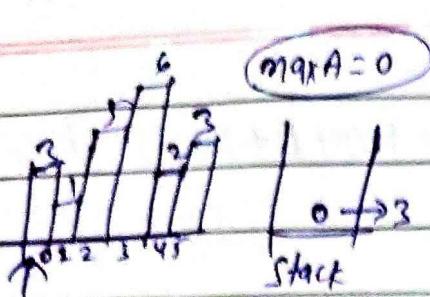
return maxA;
}

```

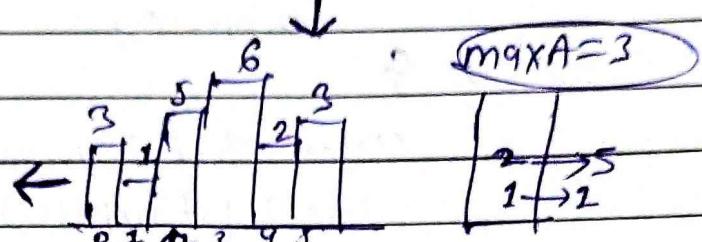
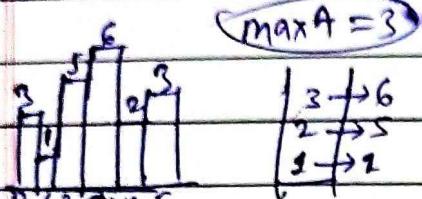
Time: O(n)  
Space: O(3N)

Approach 3: - This approach is a single pass approach. Instead of a two-pass approach, when we traverse the array by finding the next greater element, we found that some elements were inserted into the stack which signifies that after them the smallest element is themselves.

$$\text{So, Area} = arr[i] \times (\text{right smaller} - \text{left smaller} - 1)$$

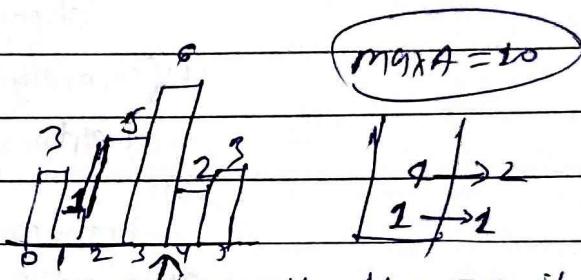
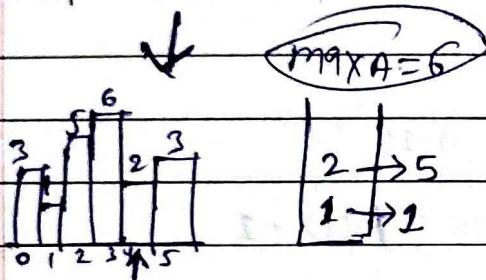


since 2 is smaller than 3 this means right smaller for 3 is 2 and left smaller is someone before 3 in stack.



since 5 is smaller than 6,  
but we don't know right smaller,  
so, push it directly.

since 2 is smaller than 5. but we don't  
know right smaller so, push it



$\therefore 2$  is smaller than 6, so it  
means it is right smaller, and  
popping out 6 from the stack, 5  
is left smaller for '6'

$\because 2$  is smaller than 5 so it  
means it is right smaller and  
popping out 5 from the stack, 2  
is left smaller for 5.

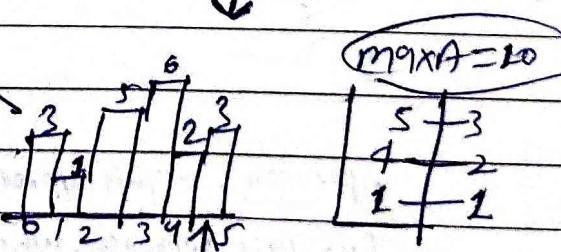
$$\therefore \text{maxA} = \text{max}(6 * (4 - 2 - 1), 3) = 6 \quad \therefore \text{maxA} = \text{max}(5 * (4 - 1 - 1), 6) = 10$$

now, 2 is smaller than 2 so it

① since now, there is no ele. to

consider 6 index as right smaller  
for '5', so pop out '3' and consider  
2 as left smaller for :

$$\text{maxA} = 3 * (6 - 4 - 2) = 3$$



② consider 6 index as right smaller  $\because 3$  is greater than 2 are will

for '4' index and 2 as left smaller for push directly '3'

'4' index and calculate maxA.

③ same for '1' index.

```

int largestRectangleArea (vector<int> &histo) {
    stack<int> st;
    int maxA = 0;
    int n = histo.size();
    for (int i = 0; i < n; i++) {
        while (!st.empty() && (i == n || histo[st.top()] >= histo[i])) {
            int height = histo[st.top()];
            st.pop();
            if (st.empty())
                width = i;
            else
                width = i - st.top() - 1;
            maxA = max(maxA, width * height);
        }
        st.push(i);
    }
    return maxA;
}
    
```

$T.C = O(N) + O(N)$   
 $S.C = O(N)$

## (11) Sliding Window Maximum :-

Given an array, and a window of size  $k$ , which is moving from the very left of the array to the very right. there can be only  $k$  numbers in window. Return maxm sliding window.

ex:- arr = [4, 0, -1, 3, 5, 3, 6, 8],  $k=3$ .

[4, 0, -1, 3, 5, 3, 6, 8]

Approach 1: (Brute force):- we take a window of size  $k$  and calculate the maximum element in it. then shift our window to next position and do the same process until reach end of array.

Time complexity:  $O(N^2)$  to iterate over each element

Space complexity:  $O(k)$  to store k elements in window

### Approach 2: (Using deque): -

- we address this problem with the help of a data structure that keeps checking whether the incoming element is larger than the already present elements. This could be implemented with the help of a deque. When shifting our window, we push the new element in from the rear of our deque.
- Every time before entering a new element, we first need to check whether the element present at the front is out of bounds of our present window's size. If no we need to pop out. Also we need to check from the rear that the element is smaller than the incoming ele. If yes, there is no point to store them so pop them out. Finally at the front be our largest element.

Code:-

```

vector<int> maxSlidingWindow(vector<int> &nums, int k) {
    deque<int> dq;
    vector<int> ans;
    for (int i = 0; i < nums.size(); i++) {
        if (!dq.empty() && dq.front() == i - k) dq.popFront();
        while (!dq.empty() && nums[dq.back()] < nums[i]) dq.popBack();
        dq.pushBack();
        if (i >= k - 1) ans.push_back(nums[dq.front()]);
    }
    return ans;
}
  
```

Time: O(N)

Space: O(k)

### (12): The Celebrity Problem

- A celebrity is a person who is known to all but does not know anyone at a party. If you go to a party of N people, find if there is a celebrity in the party or not.

Ex:-  $N=3 \quad 0 \ 1 \ 2$   
 $M = 0 [ [0, 1, 0],$   
 $1 [0, 0, 0],$   
 $2 [0, 1, 0] ]$

$$O/P = 1.$$

0th & 2nd person both know 1  
 $\therefore 1$  is the celebrity.

$N=2 \quad 0 \ 1$   
 $M = [ [0, 1],$   
 $1 [1, 0],$

$$O/P = 1,$$

The two people both know each other at party so, none of them is celebrity.

### ① Using stack:

```
int celebrity (vector<vector<int>> &M, int n) {
```

```
stack<int> s;
```

```
for (int i=0; i<n; i++) {
```

```
    s.push(i);
```

```
while (s.size() > 1) {
```

```
    int a = s.top();
```

```
s.pop();
```

```
int b = s.top();
```

```
s.pop();
```

```
if (M[a][b] M[a][b]) push(b);
```

```
else push(a);
```

```
}
```

```
int ans = s.top();
```

```
for (int i=0; i<n; i++) {
```

```
if (m[ans][i] != 0) return -1;
```

```
if (ans == i and m[i][ans] != 1)
```

```
return -1;
```

```
}
```

```
return ans;
```

Time: O(n)

Space: O(n)

### (2) Using Two pointer :-

```

int celebrity (vector<vector<int>>m, int n) {
    int i=0, j=n-1;
    while (i < j) {
        if (m[i][j] == 1)
            j--;
        else
            i++;
    }
    int ans = i;
    for (i=0; i < n; i++) {
        if (i != ans) {
            if (m[i][ans] == 0 || m[ans][i] == 1)
                return -1;
        }
    }
    return ans;
}
    
```

Time : O(n)  
Space : O(1)

### (3) LRU Cache :

- Design a data structure that follows the constraints of a Least Recently Used (LRU) cache.
- Implement the LRU Cache class:
  - LRUcache (int capacity) / initialize the LRU cache with fixed size  $\frac{\text{capacity}}{\text{size}}$
  - int get (int key) / Return the value of the key if key exists, otherwise return -1.
  - void put (int key, int value) / Update the value of the key if the key exists. Otherwise, add the key-value pair to cache. If the number of keys exceeds the capacity from this operation, evict the least recently used key.

Note:- The function get and put must each run in  $O(1)$  average time complexity.

Input:-

["LRUcache", "put", "put", "get", "put", "get", "put", "get", "put", "get", "put", "get", "get", "get"]  
 [[2], [1, 1], [2, 2], [1, 1], [3, 3], [2, 2], [4, 4], [1, 1], [3, 3], [4, 4]]

Output:-

[null, null, null, 1, null, -2, null, -2, 3, 4]

Prerequisite :- Hashmaps & DLL (Doubly linked list) :-

Dry run :- size = 3

put(1, 10)

put(3, 15)

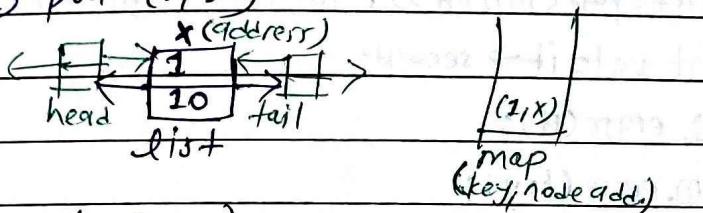
put(2, 12)

get(3)

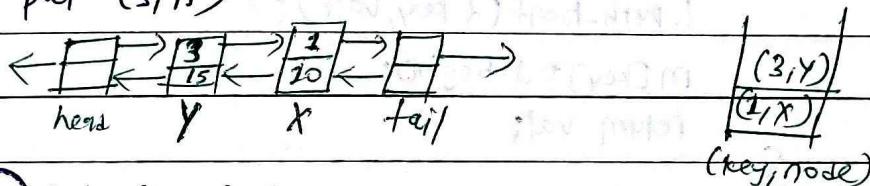
put(4, 25)

① Create a DLL & hashmap.

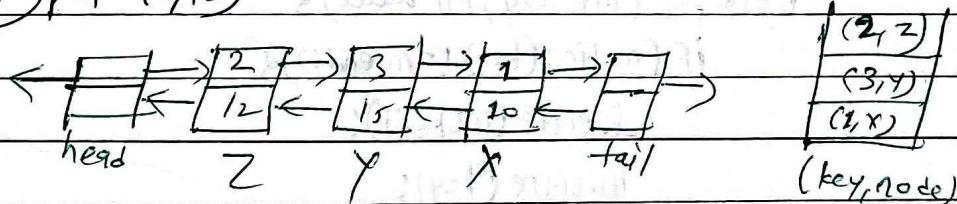
② put(1, 10)



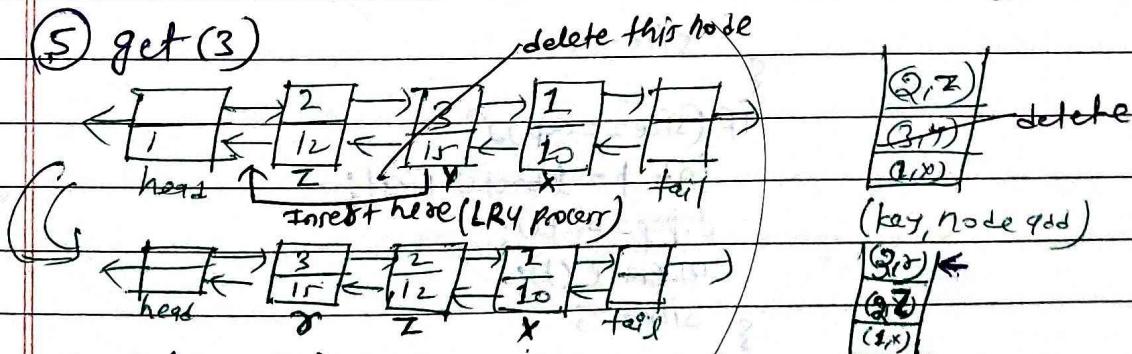
③ put(3, 15)



④ put(2, 12)

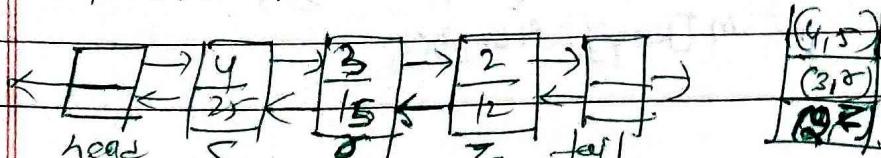


⑤ get(3)



⑥ put(4, 25)

∴ the size is full so delete the last (from back) and insert (4, 25) at front



code :-

```

class LRUCache {
    int size;
    int cap;
    unordered_map<int, list<pair<int, int>>::iterator> m;
    list<pair<int, int>> l;
public:
    LRUCache(int capacity) {
        cap = capacity;
        size = 0;
    }

    int get(int key) {
        if (m.find(key) == m.end()) return -1;
        list<pair<int, int>::iterator it = m[key];
        int val = it->second;
        l.erase(it);
        m.erase(key);
        l.push_front({key, val});
        m[key] = l.begin();
        return val;
    }

    void put(int key, int value) {
        if (m.find(key) != m.end()) {
            l.erase(m[key]);
            m.erase(key);
            size--;
        }
        if (size == cap) {
            int k = l.back().first;
            l.pop_back();
            m.erase(k);
            size--;
        }
        size++;
        l.push_front({key, value});
        m[key] = l.begin();
    }
}

```