

Continuity and Differentiability

1. Check the Continuity and Differentiability of function $y = |x| + |x + 1|$
2. Check the Continuity and Differentiability of function $y = [x] + |x|$, $-2 \leq x \leq 2$

3. Check the Continuity and Differentiability of function

$$f(x) = \begin{cases} -2x + 1, & x < 0 \\ x + 4, & x \geq 0 \end{cases}, \quad -1 \leq x \leq 2$$

4. Draw the graph of the function $y = |x| + |x + 1| + |x - 1|$
5. Draw the graph of the function $f(x) = \frac{x^2 - 4}{x - 2}$
6. Find the values of a and b so that $f(x) = \begin{cases} ax + b & \text{if } x < 0 \\ 2\sin x + 3\cos x & \text{if } x \geq 0 \end{cases}$
7. Find the number c that makes $f(x) = \begin{cases} \frac{x-c}{c+1} & \text{if } x \leq 0 \\ x^2 + c & \text{if } x > 0 \end{cases}$
8. Choose the correct option.
 - a. Let $f(x) = \begin{cases} 0 & : x > 0 \\ x^2 & : x \leq 0 \end{cases}$, then for all x
 - (a) f' is differentiable
 - (b) f' is continuous
 - (c) f is differentiable
 - (d) f is continuous

- b. If $x + |y| = 2y$, then y as a function of x is

(a) defined for all real x

(b) continuous at $x = 0$

(c) differentiable for all x

(d) such that $\frac{dy}{dx} = \frac{1}{3}$, for $x < 0$

9. The function $f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \leq x < \frac{\pi}{4} \\ 2x\cot x + 6, & \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \\ a\cos 2x - b\sin x, & \frac{\pi}{2} < x \leq \pi \end{cases}$ is

continuous for $[0, \pi]$ then find the value of a, b .

10. Let $f(x + y) = f(x)f(y)$ for all x and y. Suppose $f(5) = 2$, $f'(0) = 3$. find $f'(5)$.

11. Find the values of a, b so that the function $f(x) = \begin{cases} (x^2 + 3x + a) & \text{when } x \leq 1 \\ (bx + 2) & \text{when } x > 1 \end{cases}$ is differentiable at each $x \in \mathbb{R}$.

12. Find the values of A, B, C so that function $f(x) = \begin{cases} A \tan^{-1}\left(\frac{1}{x-4}\right) & \text{If } 0 \leq x \leq 4 \\ C & \text{If } x = 4 \\ B \tan^{-1}\left(\frac{2}{x-4}\right) & \text{If } 4 < x < 6 \\ \sin^{-1}(7-x) + A\frac{\pi}{4} & \text{If } 6 \leq x \leq 8 \end{cases}$ is continuous in the interval $[0, 8]$

13. Discuss the continuity of $f(x) = [\tan^{-1} x]$ $\{-\tan 1, 0, \tan 1\}$

14. Let $y = f(x)$ be defined parametrically as $x = t^2 + t|t|$, $y = 2t - |t|$, $t \in \mathbb{R}$. Then, at $x = 0$, Find $f(x)$ and discuss continuity.
Continuous in all \mathbb{R}

Differentiation

Differentiate the following function w.r.t x

1. $y = xe^x \cos x$
2. $y = \sin x \cos x$
3. $y = \sqrt{\sqrt{x}}$
4. $y = \sqrt{\tan \sqrt{x}}$
5. $y = \cos^2(3x + 5)$
6. $y = \sqrt{\sin x} + \sin \sqrt{x}$
7. If $y = \sin(\log x)$ find $\frac{dy}{dx}$
8. $y = \frac{\cos x}{e^x}$
9. $y = \frac{\sin x}{xe^x}$
10. If $(x) = (x + 1)(x + 2)(x + 3) \dots (x + n)$, then find then value of $f'(0)$
11. If $x^y + y^x = a^b$ find $\frac{dy}{dx}$
12. $y = x^{x^{x^{\dots \alpha}}}$
13. If $\phi = ax^2 + by^2 + 2gx + 2fy + 2hxy + c$, find $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$
14. If $\frac{x^2 y^2}{x+y}$, then Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u$
15. $u = x \sin(y - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{u}{x}$

16. If $f(x, y) = x \cos y + y \cos x$ (i) Find (i) f_{yy} (ii) Prove that $\frac{\delta^2 f}{\delta x \delta y} = \frac{\delta^2 f}{\delta y \delta x}$
17. If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$, find $\frac{dy}{dx}$ at $\theta = \pi/4$
18. $y = \sin^{-1} x + \cos^{-1} x$
19. $y = \sin 2x + 2 \sin x$
20. $y = \sin x^2 + \cos^2 x$
21. $y = x^2 + 2^x$
22. $y = e^x \sin x + \sin e^x$
23. $y = \log x + \log x^2$
24. $s = t^3 - 2t^2 - 5$, find $\frac{ds}{dt}$ at $t = 3$
25. $y = x^5 + 4x^4 + 3x^3 + 2x + 4$ find $\frac{d^3 y}{dx^3}$ at $x = 4$
26. If $e^y(x+1) = 1$, Show that $\frac{d^2 y}{dx^2} = \left(\frac{dy}{dx}\right)^2$
27. If $y = \cos^{-1} x$, Find $\frac{d^2 y}{dx^2}$ in terms of y alone
28. If $y = \tan(x+y)$, find $\frac{dy}{dx}$
29. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, Show that $x^2 y_2 + x y_1 + y = 0$
30. If $e^x + e^y = e^{x+y}$, then prove that $\frac{dy}{dx} = -e^{y-x}$
31. Verify $x + y = \tan^{-1} y$ is the solution of differential equation $y^2 y' + y^2 + 1$
32. Verify $y = \sqrt{a^2 - x^2}$ $x \in (-a, a)$, $x + y \frac{dy}{dx} = 0$ ($y \neq 0$)
33. Verify $xy = \log y + C$, $y' = \frac{y^2}{1-xy}$ ($xy \neq 1$)
34. Verify $y = x \sin x$, $xy' = y + x\sqrt{x^2 - y^2}$ ($x \neq 0$ and $x > y$ or $x < -y$)

Differentiate the following at $x=1$

35. $y = [|\sin x| + |\cos x|]$
36. $y = e^{-|x|}$
37. $y = x^2 - 3x + 2$
38. $y = \min \{|x|, |x+1|, |x-1|\}$