Q How to actually solve to get complexity?

Ans few Few & ways :-

1) Plug & Chug

<div align="center">OR</div>

2) Master's theorem.

<div align="center">OR.</div>

3) Akra Bazzi { 1996 } :- ~~Solve any~~

** **Akra Bazzi** :-

formula :- $T(x) = \Theta\left(x^p + x^p \int_{1}^{x} \frac{g(u)}{u^{p+1}} \, du \right)$

Words :- Time complexity $= T(x)$

$g(u)$:- this g fun^n is basically a time complexity it-self. & we know that in time complexity like constants & more dominating terms are ignored.

we know that this going to be the simplest form of the function because all the less dominating terms and other things will be so removed.

P :

$$a_1 b_1^P + a_2 b_2^P + \cdots = 1$$

i.e $\sum\limits_{j=1}^{k} a_i + b_i^P = 1$ //

$$T(N) = T(N/2) + C$$

○ In constants why do we write $O(1)$?

Ans Because we don't really care about constants
for eg:- we can write $O(c)$ or $O(2k+c)$
all of these are constant, ∴ we can ignore
the constants

∴ Basically $O((2k+c) \times 1)$

∴ Anything multiplied by anything, you can
~~ing~~ ignore the ~~constant~~ constants.
So, Hence anything in constants can be written
as $O(1)$.

Eg: 1) $T(N) = 2T\left(\dfrac{N}{2}\right) + (N-1)$

$a_1 = 2$

$b_1 = \dfrac{1}{2}$

$g(x) = xN-1$

∴ $2 \times \left(\dfrac{1}{2}\right)^P = 1$

∴ $P = 1$.

Once you're found the P then substitute it into the Akra-Bazzi formula :-

$$T(n) = O\left(x^1 + x^1 \int_1^x \frac{u-1}{u^2} \, du\right)$$

$$= O\left(x + x \int_1^x \frac{1}{u} - \frac{1}{u^2} \, du\right)$$

$$= O\left(x + x\left[\int_1^x \frac{du}{u} - \int_1^x \frac{du}{u^2}\right]\right) \quad = u^{-2}$$
$$= \frac{u^{-1}}{-1}$$
$$= -\frac{1}{u}$$

$$= O\left(x + x\left[[\log u] + \left[\frac{1}{u}\right]\right]\right)$$
$$\underset{\text{put } \uparrow}{}$$

$$= O\left(x + x\left[\log x + \frac{1}{x} - 1\right]\right)$$

$$= O\left(x + x\log x + \frac{1}{\cancel{x}} - \cancel{x}\right)$$

$$= O\left(x\log x + 1\right)$$

$$\therefore O\left(\;\boxed{x \log x}\;\right). \;// \text{Time complexity.}$$

So, for array of size N :-
Merge sort complexity $= O(N \log N)$.

2) $T(N) = 2T\left(\dfrac{N}{2}\right) + \dfrac{8}{9}T\left(\dfrac{3}{4}N\right) + N^2$

$\therefore \quad 2 \times \left(\dfrac{1}{2}\right)^P + \dfrac{8^2}{9_3} \times \dfrac{3^1}{4_1}^P = 1$

$1 + \dfrac{\cancel{2}}{3} \qquad$ so, put $P = 21$

$\cancel{2} \times \dfrac{1}{4_2} + \dfrac{8}{9} \times \dfrac{69}{16_2} = 21$

$\therefore \quad \dfrac{1}{2} + \dfrac{1}{2} = 1$

$\therefore \quad P = 2$ //.

Subsitute.

$T(x) = O\left(x^2 + x^2 \displaystyle\int_{1}^{x} \dfrac{xy^2}{3y^3} \, dw \right.$

$= O\left(x^2 + x^2 \log x\right) \quad \begin{array}{l}\text{\{ ignore de} \\ \text{less dominati} \\ \text{term \}}\end{array}$

$= O\left(x^2 \log x\right)$

If you're unable to find value of P:

Q. $T(x) = 3T\left(\dfrac{x}{3}\right) + 4T\left(\dfrac{x}{4}\right) + x^2$

① P=1   Case 1

$$= 3 \times \left(\frac{1}{3}\right)^1 + 4 \times \left(\frac{1}{4}\right)^1 = 1$$

$$\therefore \quad 1 + 1 = 1$$

$$2 \neq 1 \qquad \qquad \therefore \ 2 > 1 \quad \text{This means}$$

what? 9 need to increase the denominator

② P=2   Case 2

$$3 \times \frac{1}{9}_3 + 4 \times \frac{1}{16}_4 = 1$$

$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12} < \not> 1 \qquad \textbf{✗}$$

Hence, P is less than 2.

So, P actually lies in range 1 & 2

Note:

When $P <$ power of $(g(x))$ than you your
ans $= g(x)$.

Here, $g(x) = x^2$

$P < 2 \ \{i.e \ power \ of \ g(x)\}$

Hence, ans $= O(g x)$.

$$T(x) = O\left(x^{p} + x^{p}\int_{1}^{x}\frac{u^{2}}{u^{p+1}}\,du\right)$$

$$= O\left(x^{p} + x^{p}\int_{1}^{x}u^{1-p}\,du\right)$$

$$= O\left(x^{p} + x^{2}\right)$$

$\because \; p < 2$

$\therefore \; x^{p}$ will become less dominating term x^{p} Hence, ignore.

$\therefore \; O(x^{2})$ //

Hence proved.