Three-phase Circuits

Any electrical apparatus such as generator, Motor transformer or rectifier having only one winding is called a single phase system. If there are two windings, connected in such a way that the Voltage generated by them or currents flowing through them have a phase difference of 90°, them they are called as two-phase systems. If there are three windings in them, connected in such a way that the Voltage generated by them or currents flowing through them have a phase difference of 120°. then they are called as three phase systems.

Advantages of three-phase Systems :-

① A three-phase apparatus is more efficient than a single phase apparatus.

② For the same capacity, a three phase apparatus costs less than a single phase apparatus.

③ The size of a three phase apparatus is smaller in size than the size of a single phase apparatus of the same capacity & hence, requires less material for construction.

④ For transmitting the same amount of power, over the same distance, under the same power loss, the amount of conductor material required is less in the case of a three phase system than in the case of $1-\phi$ system.

(5) Three-phase motors produce uniform torque whereas the torque produced by single phase motors is pulsating.

(6) Three phase motors are self starting whereas 1-ϕ motors are not self starting.

Generation of three-phase voltages

The electrical machine which generates three phase voltages is called an alternator. It consists of a stator & rotor. The stator is stationary & the rotor rotates. The stator is cylindrical in shape & has uniform slots on its inner periphery. The conductors which form the windings of the alternator are placed in these slots & connected together in such a way that the emf's induced in them are additive, forming one winding. The rotor which is the rotating part of the alternator is magnet of two poles N & S.

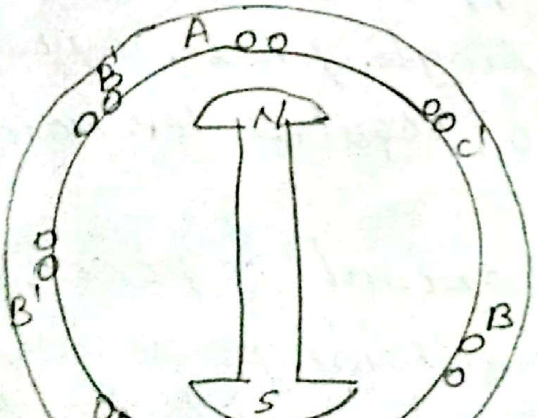


Fig 1

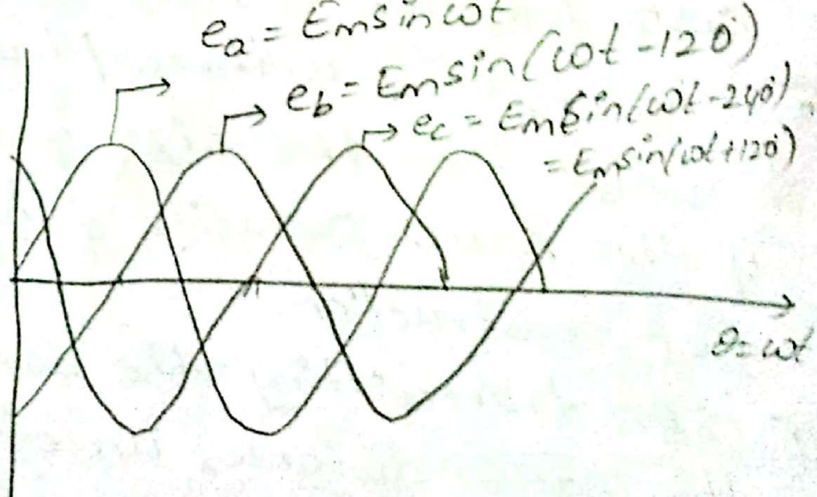
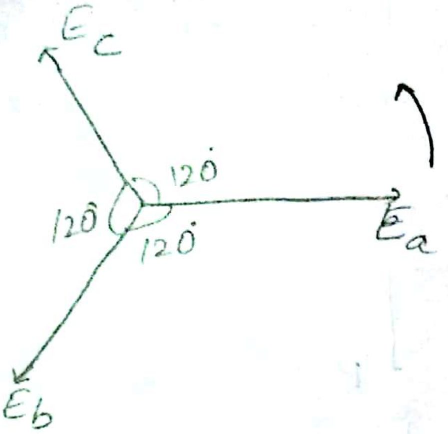


Fig (2)



The equations for the voltages induced in the three windings are,

$e_a = E_m \sin \omega t$

$e_b = E_m \sin(\omega t - 120°)$

$e_c = E_m \sin(\omega t - 120°)$

$= E_m \sin(\omega t + 120°)$

From the wave diagram,

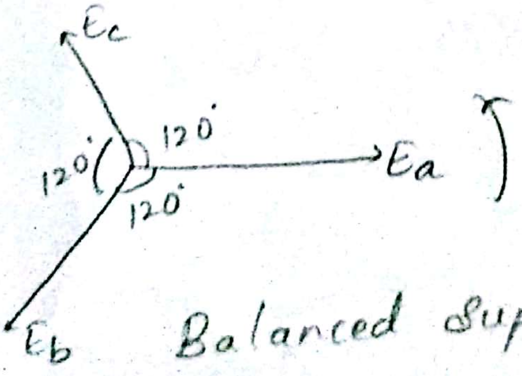
$$e_a + e_b + e_c = 0.$$

Phase Sequence :-

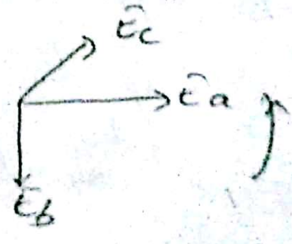
The phase sequence of the three supply is the order in which the Maximum Values of the three phase voltages occur. In Fig (2), the Maximum Values of the three phase Voltages occur in the order abc. Hence, the phase sequence of the supply is abc.

Balanced three-phase Supply :-

A three phase supply is said to be balanced, when all the three voltages have the same magnitude but differ in phase by 120° with respect to one another.



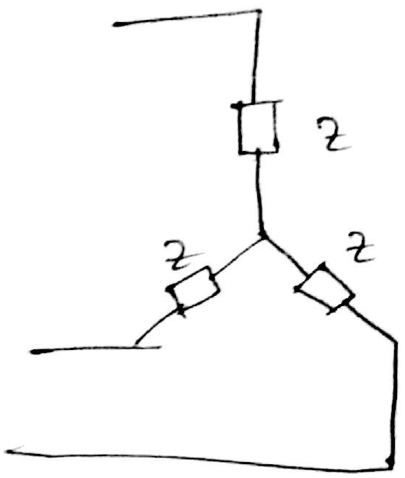
Balanced Supply



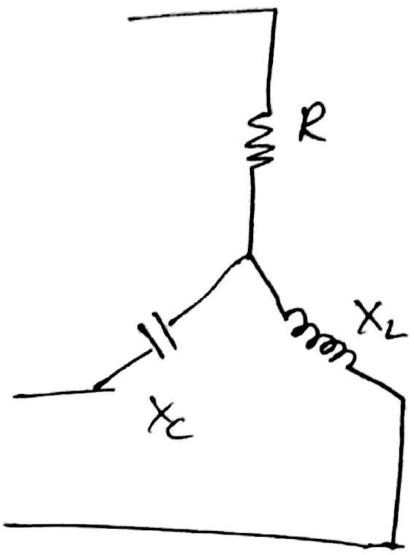
Unbalanced Supply

Balanced load :-

A three phase load is said to be balanced, when the impedances of all the three phases are exactly the same.



Balanced star connected load



Unbalanced star connected load.

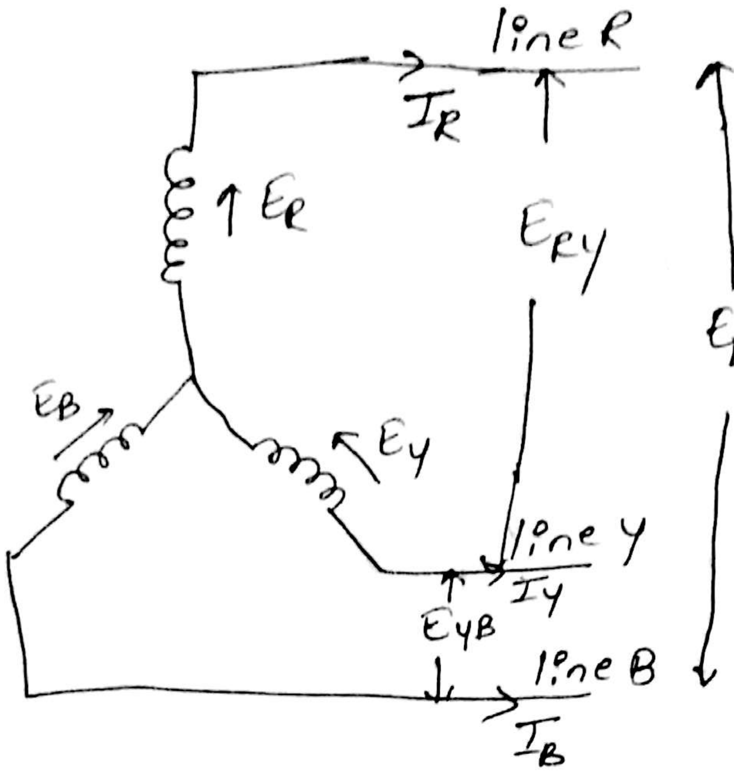
Three-phase Connections

(i) Star connection (Y) (ii) Delta connection (Δ)

Star connection :- A star connection is formed, when the ends of the three coils are joined together at point n, the other three ends being free as shown in Fig. The point n is known as neutral point. E_R, E_Y & E_B are the phase voltages & E_{RY}, E_{YB} & E_{BR} are the line voltages.

$$\therefore \; E_R = E_Y = E_B = E_{ph}$$

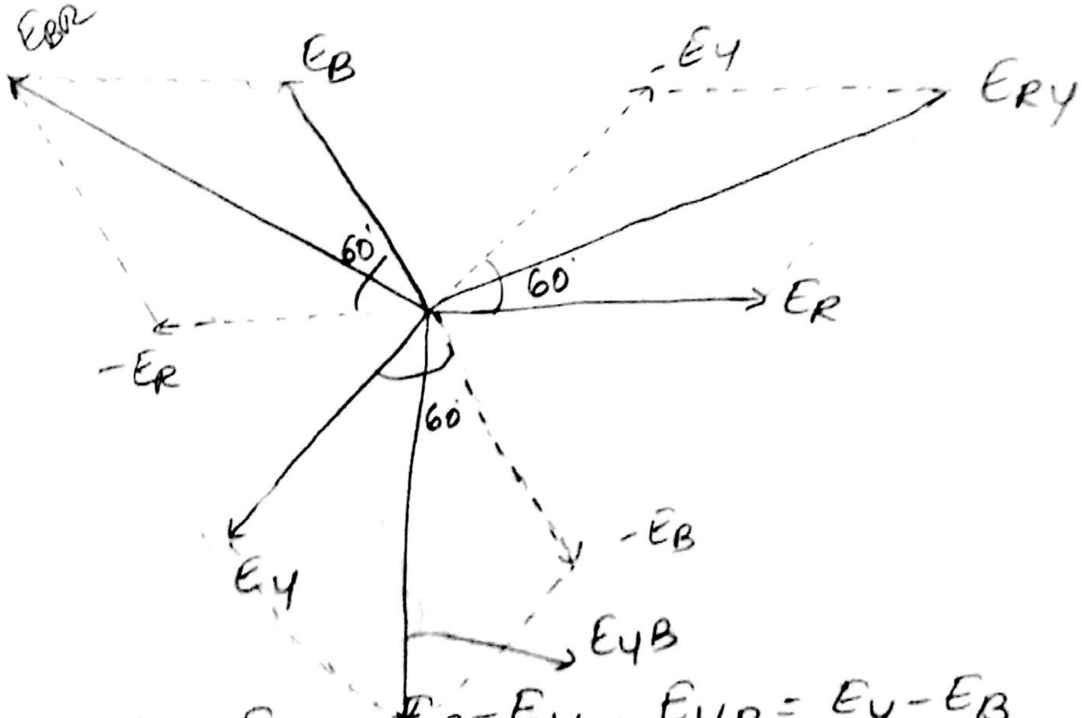
$$E_{RY} = E_{YB} = E_{BR} = E_L .$$



From the above diagram, we observe that, the currents flowing through the line are the same as currents flowing through the phases.

$\therefore I_L = I_{ph}$

The vector diagram of line voltages & phase voltages for the star connection is shown in fig



The line Vtg $E_{RY} = E_R - E_Y$, $E_{YB} = E_Y - E_B$

$E_{BR} = E_B - E_R$

From the diagram,

$$E_{RY} = \sqrt{(E_R)^2 + (E_Y)^2 + 2 E_R E_Y \cos 60^\circ}$$

$$E_L = \sqrt{(E_{ph})^2 + (E_{ph})^2 + 2 (E_{ph})^2 \times \tfrac{1}{2}} = \sqrt{3 E_{ph}^2}$$

$\therefore E_l = \sqrt{3} E_{ph}$

i.e., line Voltage = $\sqrt{3}$ phase Voltage.

The power consumed by the three phase ckt is given by.

$P = 3 \times$ power in each phase

$= 3 \times E_{ph} \cdot I_{ph} \cos\phi = 3 \frac{E_l}{\sqrt{3}} \cdot I_l \cos\phi$

$$\boxed{P = \sqrt{3} E_l I_l \cos\phi}$$

ϕ is the angle b/n E_{ph}, I_{ph} & not E_l & I_l.

Delta connection (Δ):-

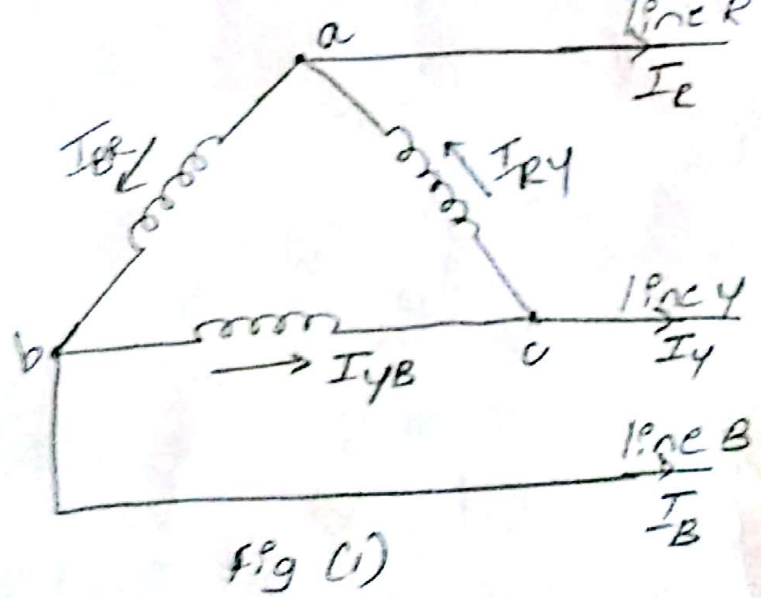


Fig (1)

I_R, I_Y, I_B are the line currents and each is equal to I_l. I_{RY}, I_{YB} & I_{BR} are the phase currents & each is equal to I_{ph}.

From the diagram, we observe that, the voltages b/n the lines are the same as the voltage b/n the phases

Hence, $\boxed{E_l = E_{ph}}$

phase currents :- I_{RY}, I_{YB}, I_{BR}

line currents :- I_R, I_Y, I_B

The Vector diagram of phase currents & line currents is as shown in Fig (2)

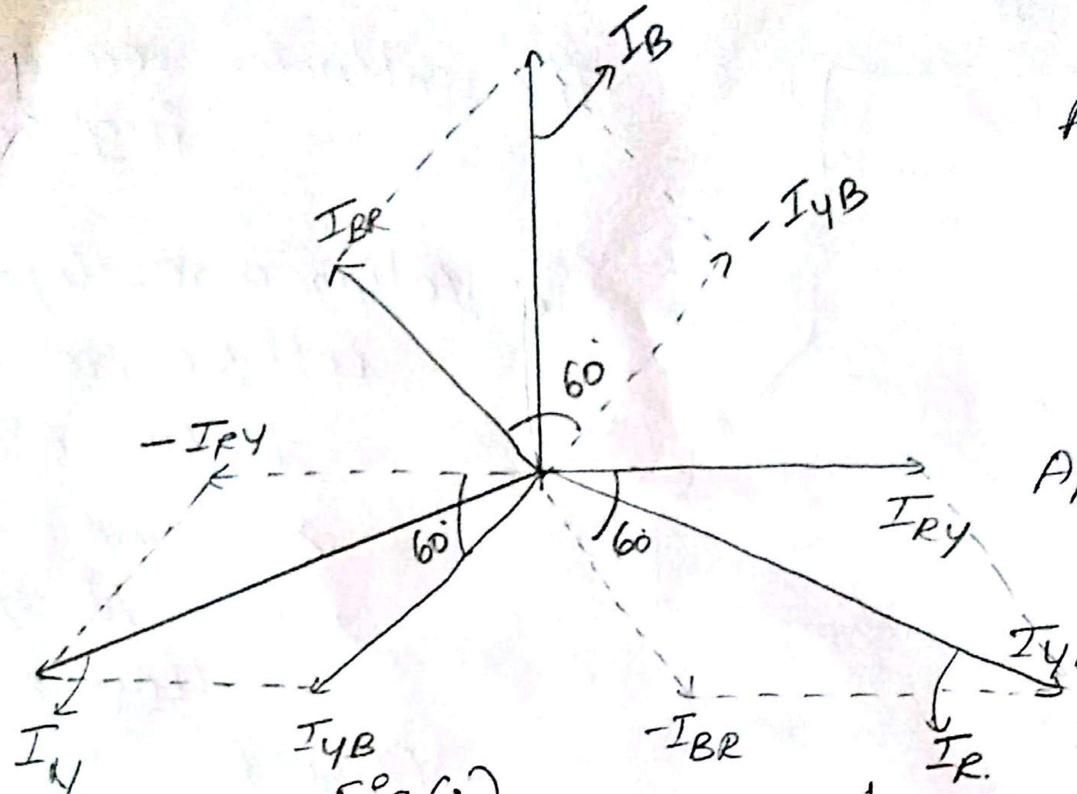


Fig (2)

Appling KCL at Node 'a'

$I_R + I_{BR} = I_{RY}$

$I_R = I_{RY} - I_{BR}$

Applying KCL at node 'b'

$I_{YB} + I_B = I_{BR}$

$I_B = I_{BR} - I_{YB}$

Applying KCL at node 'c'

$I_Y + I_{RY} = I_{YB}$

$I_Y = I_{YB} - I_{RY}$

From the diagram,

$$I_R = \sqrt{(I_{RY})^2 + (I_{BR})^2 + 2 I_{RY} I_{BR} \cos 60}$$

$$I_l = \sqrt{(I_{ph})^2 + (I_{ph})^2 + (I_{ph})^2}$$

$$\therefore \boxed{I_l = \sqrt{3}\, I_{ph}}$$

line current = $\sqrt{3}$ phase current.

Three-phase power is given by,

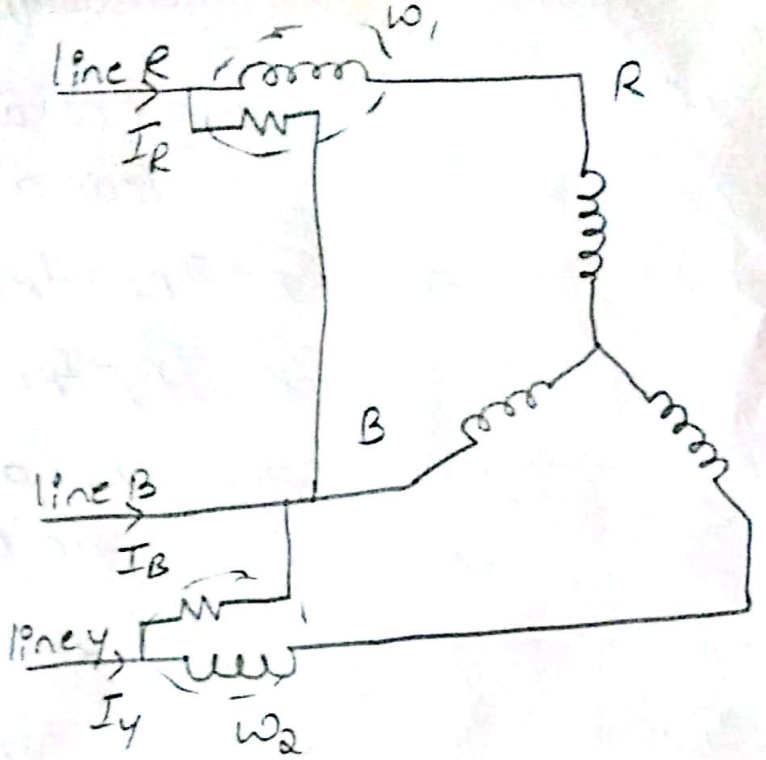
$$P = 3 E_{ph} I_{ph} \cos\phi = 3 E_l \cdot \frac{I_l}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3} E_l I_l \cos\phi$$

Measurement of power in a Three-phase circuit

Wattmeter is the instrument which is used to measure power in a Electrical circuit. It consist of (i) current coil (ii) a potential coil.

The wattmeter connections to measure power in a three phase balanced circuit is shown in Fig



The wattmeter reading W_0 is given by,

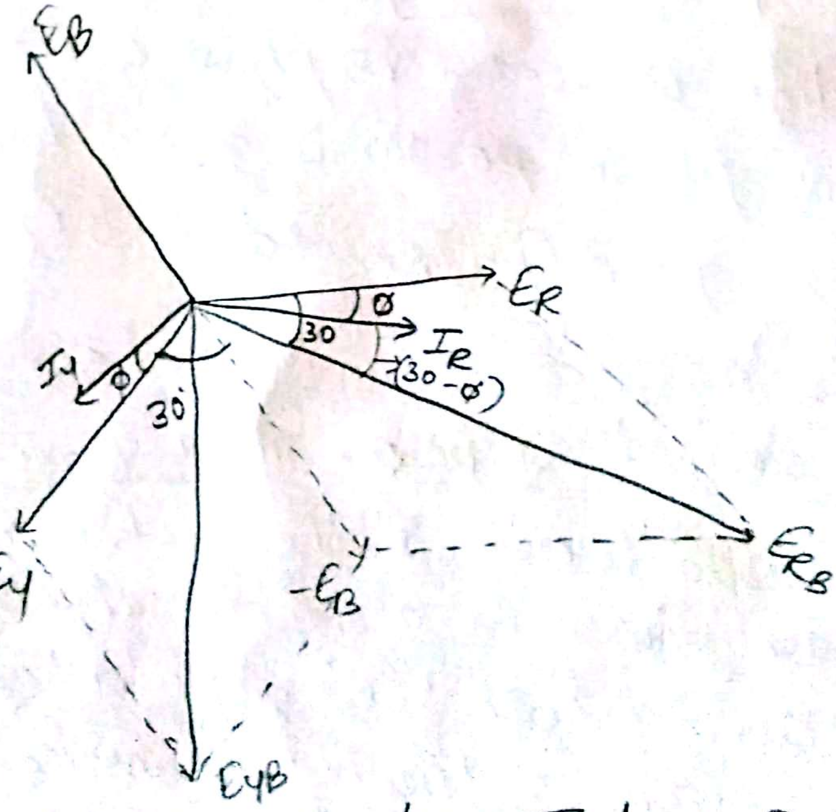
W_0 = Voltage across its potential coil × current through its current coil × cosine of the angle b/n the voltage & current

$= VI\cos\phi$.

$$W_1 = E_{RB} \cdot I_R \cos(E_{RB} \hat{} I_R)$$

$$W_2 = E_{YB} \cdot I_Y \cos(E_{YB} \hat{} I_Y)$$

The angles b/n E_{RB} & I_R, E_{YB} & I_Y are found by the vector diagram as shown in fig,



Assuming the load to be inductive, I_R lags E_R by an angle ϕ & I_Y lags E_Y by an ϕ. The angle b/n E_{RB} & I_R

is $(30-\phi)$. & the angle b/n E_{YB} & I_Y is $(30+\phi)$

$$W_1 = E_{RB} \cdot I_R \cos(E_{RB} \hat{\ } I_R)$$
$$= E_l I_l \cos(30-\phi) \quad -①$$
$$W_2 = E_{YB} \cdot I_Y \cos(E_{YB} \hat{\ } I_Y)$$
$$= E_l \cdot I_l \cos(30+\phi) \quad -②$$

Adding equations ① & ②, we get

$$W_1 + W_2 = E_l I_l \cos(30-\phi) + E_l I_l \cos(30+\phi)$$
$$= \sqrt{3} E_l I_l \cos\phi = \text{Three-phase power.}$$

Thus, it is shown that, two wattmeters are sufficient to measure power in a three phase circuit.

Expression for P.f:

From ① & ②

$$W_1 - W_2 = E_l I_l \cos(30-\phi) - E_l I_l \cos(30+\phi)$$
$$= E_l I_l \sin\phi$$

$$W_1 + W_2 = \sqrt{3} E_l I_l \cos\phi \quad -③ \qquad W_1 - W_2 = E_l I_l \sin\phi \quad -④$$

Eq ④ / Eq ③

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\tan\phi}{\sqrt{3}} \quad \text{or} \quad \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right\}$$

$$\text{P.f} \rightarrow \cos\phi = \cos\left[\tan^{-1}\left\{\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right\}\right]$$

Effect of pf on W_1 & W_2 :-

(i) when pf = 1 , $\phi = 0$

$W_1 = E_l I_l \cos(30 - \phi) \Rightarrow E_l I_l \cos 30^\circ$

$W_2 = E_l I_l \cos(30 + \phi) \Rightarrow E_l I_l \cos 30^\circ$

The two wattmeter readings are positive and equal

(ii) when pf = 0.5, $\phi = 60^\circ$

$W_1 = E_l I_l \cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2} E_l \cdot I_l$

$W_2 = E_l I_l \cos(30^\circ + 60^\circ) = 0$

One of the wattmeter reads zero.

iii) when pf = 0, $\phi = 90^\circ$

$W_1 = E_l I_l \cos(30 - 90^\circ) = \frac{1}{2} E_l I_l$

$W_2 = E_l I_l \cos(30 + 90^\circ) = -\frac{1}{2} E_l \cdot I_l$