Dielectric Materials

Introduction:

The dielectric materials are the insulating materials used to store the electrical energy. In Insulators, all the states are completely filled by electrons. Where as a dielectric material have very few electrons for electrical conductivity and hence it has dipole.

Electric dipole:

Any two opposite charges separated by an arrow, which stars from the negative charge and ends at the positive charge. A dipole is as shown in figure

A dipole

Electric dipole moment, μ:

The product of the charge and distance of separation of a dipole is known as electric dipole moment. The electric dipole moment is given by

$$\mu = \mathbf{Q} \,.\, \mathbf{r} \quad \textbf{Coulomb meter}$$

Electric flux density, D :

Electric flux passing through the unit area of cross section is known as the electric flux density. It is represented by the letter D. It unit is Coulomb per square meter (Cm^2).

$$\mathbf{D} = \boldsymbol{\varepsilon_0}\, \mathbf{E} + \mathbf{P}$$

Where P is Polarization = $(\boldsymbol{\varepsilon_0}\ \boldsymbol{\varepsilon_r}\ \mathbf{E})$, E is electric field intensity, ε_0 is permittivity and ε_r is dielectric constant.

Electric field strength, E:

Electric field strength is the force experienced by a unit positive charge placed in the electric field region. It is represented by a letter, E. Its unit is Vm^2.

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{\boldsymbol{q}} \qquad \textbf{Where F} = \frac{q_1 q_2}{4\pi\varepsilon r^2}.$$

Polarization, P:

The electric dipole moment per unit volume is known as polarization. Its unit is Cm^{-2} and it is represented by the letter P.

$$\mathbf{P} = \frac{\textbf{Dipole moment}\ \ \boldsymbol{\mu}}{\textbf{Volume}\qquad\quad \mathbf{V}}$$

Dielectric Susceptibility, χ:

The polarization of dielectric material is found to be

P $\propto$ the applied electric field intensity
P $\propto$ the permittivity of free space

Therefore, P = $\boldsymbol{\varepsilon_0}\ \chi$ **E** **It has no unit.**
Dielectric susceptibility, χ **=** $(\boldsymbol{\varepsilon_r}$ **- 1)**

It is a measure of the extend up to which the material can be polarized by the application of electric field.

 # Dielectric Constant, ε_r:

The ratio of the permittivity of the medium to the permittivity of the free space is known as dielectric constant or relative permittivity of the medium. The dielectric constant is also defined as the ratio of the capacitance of a capacitor with the dielectric material as a dielectric (C) to the capacitance of the capacitor with vacuum as a dielectric (C_0).

$$\varepsilon_r = \frac{C}{C_0}$$

It is a measure of extent to material can be polarized by the application of the electric field.

#Dielectric Polarizability, α:

The induced dipole moment of an atom per unit electric field intensity is known as polarizability. It is also defined as 1/NE times of polarization. It is represented by the letter α and its unit is F m^2.

$$\alpha = \frac{\mu_{ind}}{E}$$

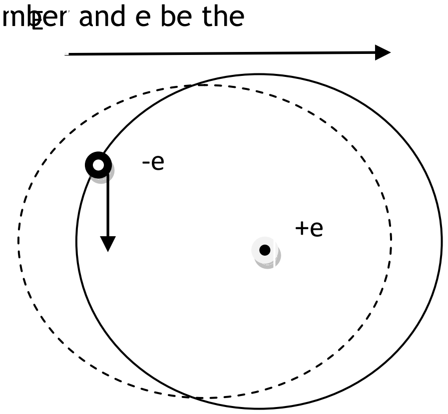
 # Polarization in a dielectric material:

Whenever an electric field is applied to a dielectric material, a displacement of electron or ions or rotation of dipoles takes place. Due to this phenomenon the dipoles are created and hence an induced dipole moment is produced. The induced dipole moment per unit volume is known as Polarization. There are four types of polarizations. They are as follows;

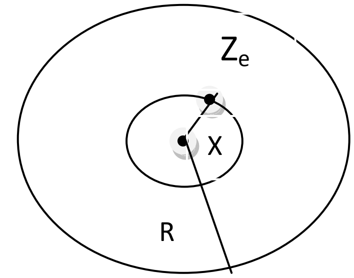
1. Electric Polarization
2. Ionic Polarization
3. Orientation Polarization
4. Space-charge Polarization

1. Electric Polarization:

Consider that the dielectric material is subjected to electric field intensity, E. Consider only one atom in a dielectric material as shown in figure. Let Z be the atomic number and e be the



Schematic illustration of the displacement of electron orbit relative to the nucleus under the influence of the field.

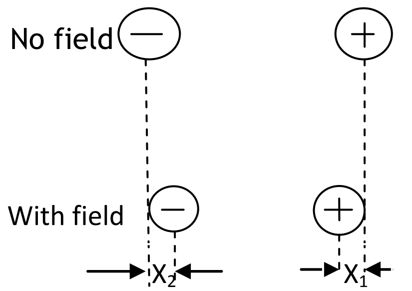


Induced dipole moment of an atom

charge of the electron. Ze is the charge of the nucleus. Due to the application of the electric field, the nucleus moves away from the field and electron moves towards the field. Due the moment of the electron and nucleus of the atom, there is a displacement and hence there is a dipole moment. This induced dipole moment per unit volume gives the induced polarization. This induced polarization is known as *Electric polarization*. Since this polarization is produced by the displacement of electron.

2. <u>Ionic Polarization:</u>

The Ionic polarization is produced in ionic substances such as NaCl, KBr, KCl etc. Conceder an electric field is applied to an ionic crystal. The positive ions move away from the applied electric field, where as the negative ions move towards the electric field. This moment of positive ion and negative ion produces displacement and hence an induced dipole moment is created. The induced dipole moment per unit volume is known as ionic polarization.



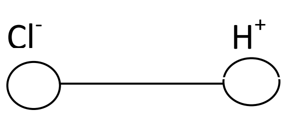
X_1 and X_2 be the distance moved by the positive ion and negative ion respectively.

Ionic Polarization

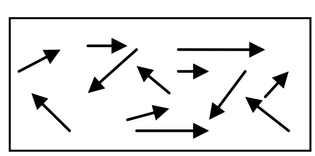
3. <u>Orientation Polarization</u>:

The orientation polarization occurs in polar molecules. The liquid dielectric materials those having dipole moment even in the absence of the field are known as *polar dielectrics*. Nitrobenzene, H_2O, HCl, etc. are the examples of polar dielectrics. The presence of dipole moment even in the absence of electric field is called *Permanent dipole moment*. In the case of water molecules, due to the arrangement of its molecular structure, it possesses dipole moment even in the absence of electric field. So it is a polar dielectric material. The molecules, those are not having any permanent dipole moment in the absence of electric field, are known as *non-polar dielectrics*. The example of non-polar dielectrics is CO_2 molecule.

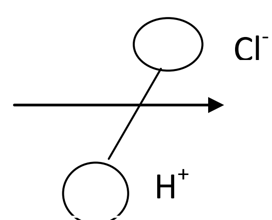
The polarization produced in the case of polar molecule due to the application of an electric field is known as *Orientation Polarization*.



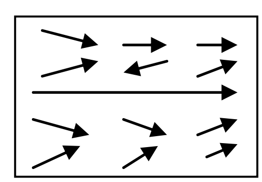
HCl molecule



Random orientation



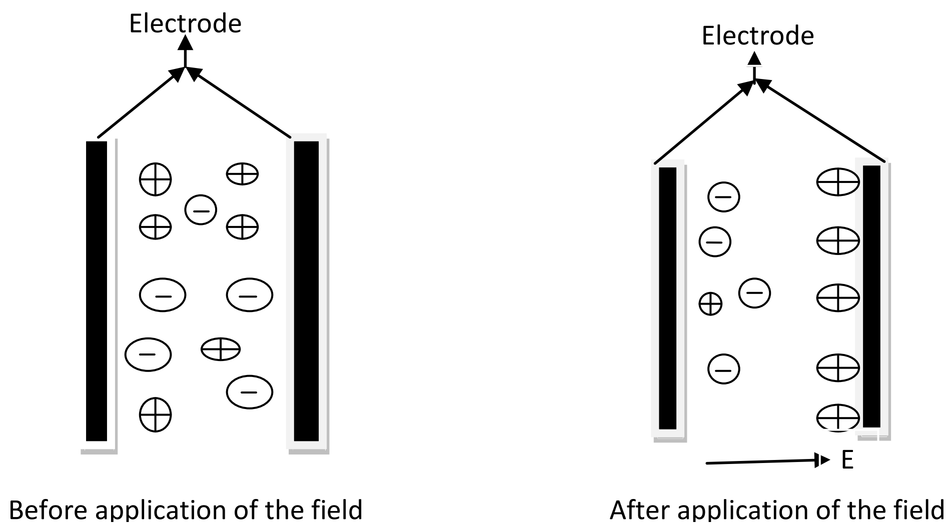
Rotation of dipoles due to the applied field



Parallel alignment of dipoles with field

4. Space-charge Polarization:

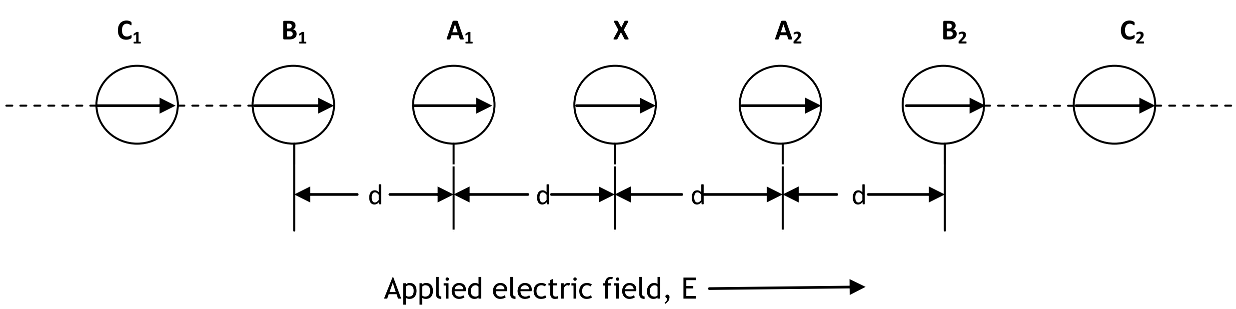
Consider that a container has an electrolyte and two electrodes are connected to a battery. The negative charge gets accumulated in positive electrode and positive charge gets accumulated in negative electrode. The separation of positive and negative charge produces an induced dipole moment and hence polarization takes places due to applied electric field are called *space-charge polarization*. The space charge polarization is very small and negligible.



Space- charge polarization

#Expression for Internal field:

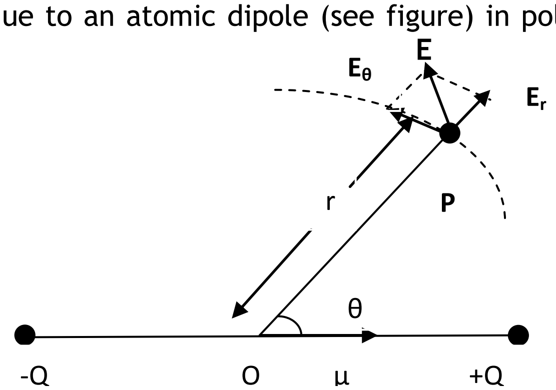
Consider a dielectric material, either solid or liquid, subjected to an external uniform electric field of intensity E. In a dielectric material, let us consider an infinite string of similar equidistant atomic dipoles, each of polarizability α_e, as shown in figure.



The components of the electric field at P due to an atomic dipole (see figure) in polar form are given by

$$E_r = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\mu\cos\theta}{r^3} \qquad \textbf{Eq----1}$$

$$E_\theta = \frac{1}{4\pi\epsilon_0} \cdot \frac{\mu\sin\theta}{r^3} \qquad \textbf{Eq----2}$$



Field components due to a dipole

Where r is the distance between the points O and P i.e, OP = r. and θ is the angle between r and μ. The electric field at X due to A can be found by putting r=d and θ=0 in the equation 1 and 2. Thus

$$E_r = \frac{\mu}{2\pi\varepsilon_0 d^3} \quad \textbf{Eq----3} \qquad E_\theta = 0 \quad \textbf{Eq---4}$$

Therefore, electric field at X due to A_1 is given by

$$\mathbf{E_r + E_\theta = \frac{\mu}{2\pi\varepsilon_0 d^3}} \quad \textbf{Eq----5}$$

Since the dipoles are situated symmetrically, the field seen by all atomic dipoles will also be the same. Therefore, electric field at X due to the dipole A_2 is also equal to

$$\mathbf{E_r + E_\theta = \frac{\mu}{2\pi\varepsilon_0 d^3}} \quad \textbf{Eq-----6}$$

Therefore, Electric field at X due to both the dipoles A_1 and A_2 is given by

$$E_1 = 2 \times \frac{\mu}{2\pi\varepsilon_0 d^3} = \frac{\mu}{\pi\varepsilon_0 d^3} \quad \textbf{Eq-----7}$$

Similarly, the field at X due to the dipole B_1 B_2 and C_1 C_2 situated at the distance 2d and 3d respectively are given by

$$E_2 = \frac{\mu}{\pi\varepsilon_0 (2d)^3} \quad \textbf{Eq-----8} \qquad E_3 = \frac{\mu}{\pi\varepsilon_0 (3d)^3} \quad \textbf{Eq------9}$$

The total electric field at P due to all the atomic dipoles in linear array is given by

$$E' = E_1 + E_2 + E_3 + \text{-----------}$$

$$= \frac{\mu}{\pi\varepsilon_0 d^3} + \frac{\mu}{\pi\varepsilon_0 (2d)^3} + \frac{\mu}{\pi\varepsilon_0 (3d)^3} + \text{---------} \quad \textbf{Eq---------------10}$$

$$E' = \frac{\mu}{\pi\varepsilon_0 d^3}\left[1 + \frac{1}{2^3} + \frac{1}{3^3} + ----\right]$$

$$E' = \frac{\mu}{\pi\varepsilon_0 d^3}\sum_{n=1}^{\infty}\frac{1}{n^3} \quad \text{where } n = 1, 2, 3........\infty$$

$$E' = \frac{\mu}{\pi\varepsilon_0 d^3} \times 1.2 \qquad \text{since, } \sum_{n=1}^{\infty}\frac{1}{n^3} = 1.2$$

$$\boxed{\mathbf{E}' = \frac{1.2\mu}{\pi\varepsilon_0 d^3}}$$ **Eq--------11**

The total internal field $\mathbf{E_i}$ at X is sum of the applied field **E** and the field $\mathbf{E}'$ due to all the dipoles

Therefore,

Total Internal field, $\mathbf{E_i} = \mathbf{E} + \mathbf{E}'$

$$\boxed{E_i = E + \frac{1.2\mu}{\pi\varepsilon_0 d^3}}$$ Eq-----------12

The dipole moment induced in each of the atoms of the string is

$\boxed{\mu_{ind} = \alpha E}$ Where α is the polarizibility for the material.

The local field in a 3-dimentional solid is determined by the structure of the solid. We know that the number density N of an atom is equal to $1/d^3$ and also we know that $\mu_{ind} \times N = P$, Therefore, the expression for internal field may be written as

$$\mathbf{E_i} = \mathbf{E} + \left\{\frac{\gamma}{\epsilon_0}\right\} \boldsymbol{P}$$

Where γ is the proportionality constant known as internal field constant and P is polarization. The value of γ is dependent on the internal arrangement of atoms in dielectric. In general, it is in the order of unity. In case of cubic symmetry $\gamma = 1/3$ and the internal field is given by

$$\boxed{\mathbf{E_{Lorentz}} = \mathbf{E} + \left\{\frac{P}{3\epsilon_0}\right\}}$$

The field given by the above equation is called **Lorentz field**

Consider an elemental solid dielectric material which exhibits only electronic polarizibility. Let the dielectric constant of the material (such as Diamond, silicon and germanium) be ε_r. The dipole moment of a single atom is proportional to the local field i.e, dipole moment, $\mu = \alpha_e E_i$ where α_e is the polarizibility of the atom.

The value of polarization P as an electric moment per unit volume of a dielectric is given by the relation,

$$P = N\mu = N \alpha_e E_i \qquad \text{--------- Eq1}$$

Where N is the number of atoms per unit volume of the matter and E_i is local field.

We know that $P = \varepsilon_0 (\varepsilon_r - 1) E$ -------Eq2

Where E is the applied electric field $\mathbf{E} = \frac{P}{\epsilon_0(\epsilon_r - 1)}$

Internal electric field, $\mathbf{E_i} = \mathbf{E} + \frac{\gamma P}{\epsilon_0}$

Eq1 becomes,

$P = N\alpha_e\left\{E + \frac{\gamma P}{\epsilon_0}\right\}$ Eq--------3

If internal field is assumed to be Lorentz field, then, γ = 1/3, then

$P = N\alpha_e\left\{E + \frac{P}{3\epsilon_0}\right\}$ Eq---------4

Equating Eq2 and Eq4, we have

$N\alpha_e\left\{E + \frac{P}{3\epsilon_0}\right\} = \varepsilon_0 (\varepsilon_r - 1) E$ Eq---------5

$N\alpha_e E + \frac{N\alpha_e P}{3\epsilon_0} = \varepsilon_0 (\varepsilon_r - 1) E$ Eq. -------6

$N\alpha_e E + \frac{N\alpha_e \epsilon_0 E(\epsilon_r - 1)}{3\epsilon_0} = \varepsilon_0 (\varepsilon_r - 1) E$ Eq--------7

$N\alpha_e\left[1 + \frac{\varepsilon_{0(\epsilon_r - 1)}}{3\epsilon_0}\right] = \varepsilon_0 (\varepsilon_r - 1)$ Eq--------8

$N\alpha_e\left[\frac{3\epsilon_0 + \varepsilon_0\varepsilon_r - \epsilon_0}{3\epsilon_0}\right] = \varepsilon_0 (\varepsilon_r - 1)$ Eq--------9

$N\alpha_e \frac{(\varepsilon_r + 2)}{3} = \varepsilon_0 (\varepsilon_r - 1)$ Eq-------10

$$\boxed{\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}}$$ Eq--------11

The equation 11 is known as **Clausius-Mossotti equation**. This is valid for non-polar solids having cubic crystal structure.

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