Elasticity is defined as the property of a body due to which it regains its original shape and size when the deforming force is removed.

Stress: The restoring force per unit area developed inside the body is called stress.

Strain: The ratio of the change in dimensions to original dimensions is called strain.

① Tensile stress, longitudinal Stress, Linear stress
Tensile strain, longitudinal strain, Linear Strain
T Stress = F/a , T strain = x/L — original.

② Compressive stress. Volume stress – F/a.
Volume strain = v/V – original.

③ Shear stress, Tangential stress = F/a
Shear Strain = x/L — original.

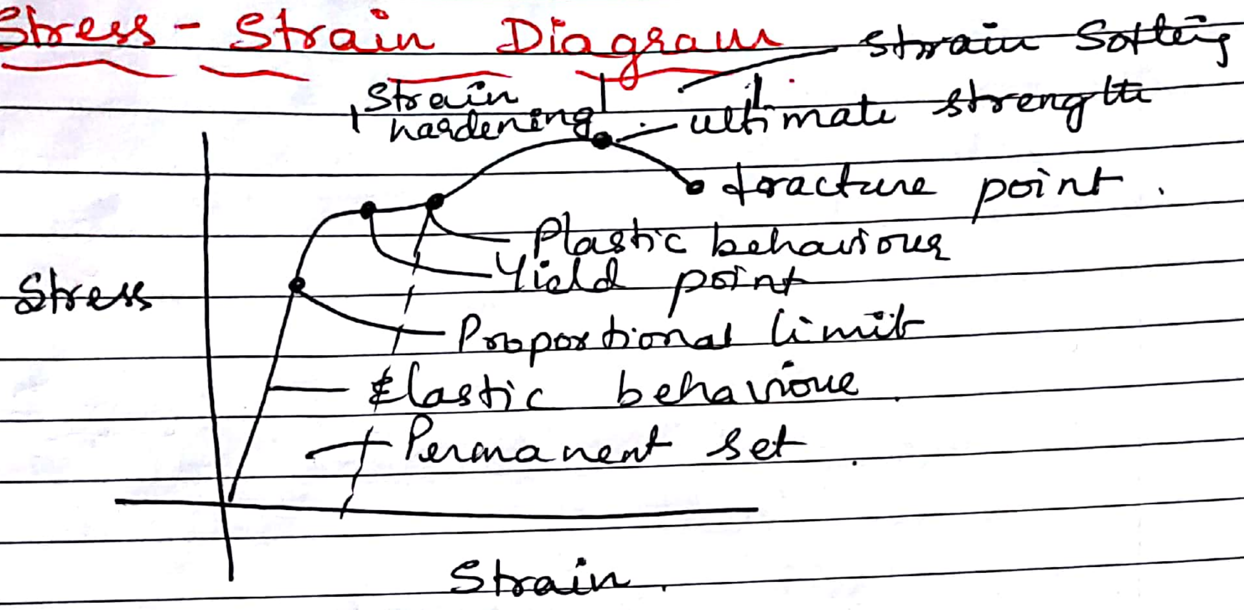
Hooke's law.

The stress is directly proportional to strain within elastic limits.

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant } E.$$

E → Elastic constant
Elastic module.

Stress - Strain Diagram



Elastic Moduli

1. Young's Modulus Y :-

The ratio of the longitudinal stress to linear strain within elastic limits

$$Y = \frac{\text{Longitudinal stress}}{\text{Linear strain}}$$

$$Y = \frac{F/a}{x/L}.$$

$$\boxed{Y = \frac{FL}{ax}} \quad N/m^2$$

2. Bulk Modulus K :-

The ratio of the compressive stress or pressure to the volume strain without change in shape of the body within elastic limits.

$$\text{Bulk modulus } K = \frac{\text{Compressive stress}}{\text{Volume strain}}$$

$$K = \frac{F/a}{v/V}$$

$$K = \frac{P}{v/V} \Rightarrow \boxed{K = \frac{PV}{v}} \quad N/m^2$$

Rigidity Modulus n :-

The ratio of the tangential stress to the shearing strain

$$n = \frac{\text{tangential stress}}{\text{shearing strain}}$$

$$n = \frac{F/a}{x/L} = \frac{FL}{ax}$$

$$\boxed{n = \frac{FL}{ax}} \quad N/m^2$$

Longitudinal Strain Coefficient α

$$\alpha = \frac{x}{TL} \qquad T\text{-applied stress}$$

Lateral strain Coefficient β

$$\beta = \frac{d}{TD} \qquad T\text{- applied stress}$$

Poisson's Ratio σ :

The ratio of lateral strain to longitudinal strain within elastic limits

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{d/D}{x/L}$$

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$$\sigma = \frac{Ld}{xD}$$

$$= \frac{d}{TD} \times \frac{F L}{x}$$

$$\therefore \quad \sigma = \frac{\beta}{\alpha}.$$

Ily

$$Y = \frac{1}{\alpha}$$

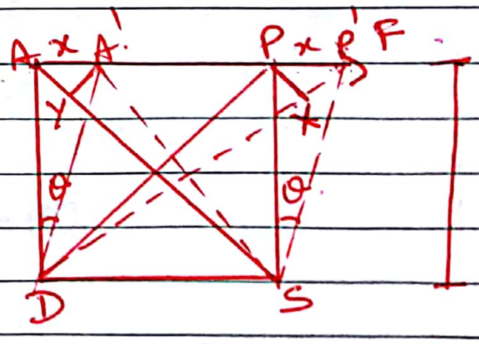
$$K = \frac{1}{3(\alpha + 2\beta)}$$

$$n = \frac{1}{2(\alpha + \beta)}.$$

Relation between Shearing strain Elongation strain & Compression strain.

Elongation strain

$$= \frac{P'X}{PD}$$



Compresion Strain $= \frac{AY}{AS}$.

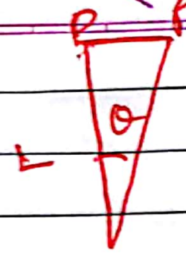
Pythog therm $= AS = PD = \sqrt{2}\,L$.

$$P'X = PP' \cos \angle PP'X.$$

$$= PP' \cos \angle AP'D.$$

Since $\angle APD = 45^\circ$ & θ small $\angle AP'D = 45^\circ$

$$\therefore P'X = PP' \cos 45^\circ$$
$$= \frac{PP'}{\sqrt{2}}.$$



$$\theta = \frac{PP'}{L}.$$

$\therefore$ Elongation strain $= \frac{PP'}{2L} = \frac{\theta}{2}$.

Illy Compression Strain $= \frac{\theta}{2}$

$\therefore$ Compression strain + Elong. Strain $= \frac{\theta}{2} + \frac{\theta}{2} = \theta$.

The shearing strain.

Relation bet^n Y n σ.

Total ext^n along DP = DP T $(\alpha + \beta)$.

$\therefore$ tensile stress $= T . DP . \alpha$.
Compression stress $= T . DP . \beta$.

$$P'X = DP\, T(\alpha + \beta).$$

$$P'X = (\sqrt{2} L)\, T\, (\alpha + \beta).$$

$$\boxed{P'X = \frac{PP'}{\sqrt{2}} = \frac{x}{\sqrt{2}}}$$

$$\therefore \frac{x}{\sqrt{2}} = (\sqrt{2} L)\, T\, (\alpha + \beta).$$

$$\Rightarrow 2(\alpha + \beta) = \frac{x}{TL}$$

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Inverting. $\frac{1}{2(\alpha+\beta)} = \frac{TL}{x} = \frac{T}{x/L} = \frac{T}{\theta} = n.$

$$\therefore \quad n = \frac{1}{2(\alpha+\beta)}$$

$$= \frac{1/\alpha}{2(1+\sigma)}.$$

Young's Mod $= \frac{1}{\text{Longitudinal } \frac{\text{strain}}{\text{stress}}}.$

$$Y = \frac{1}{\frac{\text{Strain along DP}}{\text{unit stress}}} = \frac{1}{\alpha}.$$

$$\Rightarrow \quad n = \frac{Y}{2(1+\sigma)}.$$

$$\Rightarrow \quad \boxed{Y = 2n(1+\sigma).}$$

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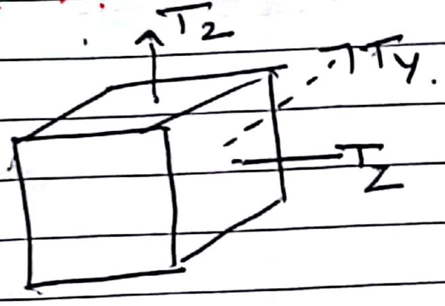
Relation between K, Y & σ :-

Let $\alpha \rightarrow$ elongation / unit length / unit stress.



$\beta \rightarrow$ Contraction / unit length / unit stress.

∴ length along X-direction.

$1 + \alpha T_x - \beta T_y - \beta T_z$

$1 + \alpha T_y - \beta T_z - \beta T_x$ Y-dirn

$1 + \alpha T_z - \beta T_y - \beta T_x$ Z-dirn.

∴ The new volume is

$= (1 + \alpha T_x - \beta T_y - \beta T_z)(1 + \alpha T_y - \beta T_z - \beta T_x)$
$(1 + \alpha T_z - \beta T_y - \beta T_x)$.

α & β are small. ∴ neglected the powers.

$V = 1 + \alpha(T_x + T_y + T_z) - 2\beta(T_x + T_y + T_z)$.

$= 1 + (\alpha - 2\beta)(T_x + T_y + T_z)$.

Since $T = T_x = T_y = T_z$.

Volume $= 1 + (\alpha - 2\beta) 3T$.

increase in volume = New volume − V_{orig} / V_0

$= [1 + 3T(\alpha - 2\beta)] - 1$.

$= 3T(\alpha - 2\beta)$.

T → Pressure.

Volume strain = $\dfrac{\text{change in vol.}}{\text{original vol}}$.

$\therefore \quad = \dfrac{3P(\alpha - 2\beta)}{1}$.

$\therefore$ Bulk mod. $K = \dfrac{\text{Pressure}}{\text{Volu Strain}}$

$= \dfrac{P}{3P(\alpha - 2\beta)}$

$\therefore K = \dfrac{1}{3(\alpha - 2\beta)}$.

$\Rightarrow K = \dfrac{1/\alpha}{3(1 - 2\beta/\alpha)}$.

$\Rightarrow \boxed{K = \dfrac{Y}{3(1-2\sigma)}}$.

Relation bet K n & Y.

$Y = 2n(1+\sigma)$.

$K = \dfrac{Y}{3(1-2\sigma)}$

$\therefore \dfrac{Y}{n} = 2 + 2\sigma$.

$\dfrac{Y}{3K} = 1 - 2\sigma$.

$\therefore \dfrac{Y}{n} + \dfrac{Y}{3K} = 3$.

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$$\frac{Y(3k+\eta)}{3\eta k} = 3$$

$$\boxed{Y = \frac{9\eta k}{3k+\eta}}$$

Relation betⁿ k, η & σ:

$$Y = 2\eta(1+\sigma)$$
$$Y = 3k(1-2\sigma).$$

$$\therefore\ 2\eta + 2\sigma = 3k - 6k\sigma.$$

$$\sigma(2\eta + 6k) = 3k - 2\eta.$$

$$\therefore\ \boxed{\sigma = \frac{3k-2\eta}{2\eta+6k}}$$

Limiting Value of σ

WKT $Y = 2\eta(1+\sigma)$ & $Y = 3k(1-2\sigma)$.

$$\therefore\ 2\eta(1+\sigma) = 2k(1-2\sigma).$$

to make this +ve. $\sigma < \frac{1}{2}$.

or $\sigma < -1$

$\therefore$ σ is usually taken betⁿ 0 & 0.5.

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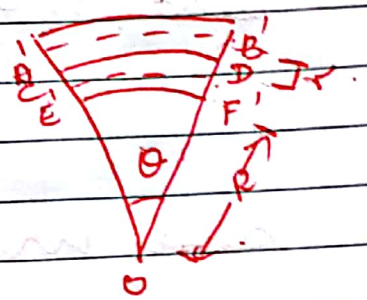
Change in length $= A'B' - AB$

$AB = CD = R\theta$



$A'B' = (R+r)\theta$

$\therefore$ change in length $= (R+r)\theta - R\theta$

$= r\theta$

Original length $= AB = R\theta$

$\therefore$ Linear Strain $= \dfrac{r\theta}{R\theta} = \dfrac{r}{R}$

$\therefore\ Y = \dfrac{\text{longi stress}}{\text{linear strain}}$

$\therefore$ Longitudinal stress $= Y \times$ Linear Str

$= Y\dfrac{r}{R}$

$\Rightarrow\ F/a = Y\dfrac{r}{R}$

$\Rightarrow\ F = \dfrac{Yar}{R}$

Moment of force $= F \times$ distance from neutral axis

$= F \times r$

$= \dfrac{Yar^2}{R}$

Moment of force on entire beam $= \sum \dfrac{Yar^2}{R}$

$= \dfrac{Y}{R}\sum ar^2$

$\boxed{\text{bending moment} = \dfrac{Y}{R} I_g}$ geometric moment of Inertia bending moment of beam of rectangular cross section

$$= \frac{Y}{R} \frac{bd^2}{12}$$

bending moment for beam of circular crossection

$$= \frac{Y}{R} \frac{\pi \rho^4}{R}$$

$\rho \rightarrow$ radius of beam

Single Cantilever

Absent :- 3, 7, 9, 12, 14, 15, 17, 29, 37, 40, 41, 54, 56, 72, 74, 78, classmate

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Work done per unit volume in elongation strain

Strain energy per unit volume.

Consider a wire of length L and area of cross section A be fixed at one end and streched by suspending a load M from other end. Let the interal elastic force be F and extension produced be x.

Then $Y = \frac{F/A}{x/L} = \frac{FL}{Ax}$.

$$\Rightarrow F = \frac{YAx}{L}.$$

$\therefore$ work done by external applied force producig dx is

$$dw = F\,dx.$$

$$\Rightarrow dw = \frac{YAx}{L}dx.$$

if l is the total extension produced

$$\therefore \int dw = \int_0^l \frac{YAx}{L}dx.$$

$$\Rightarrow w = \frac{YA}{L} \cdot \frac{l^2}{2}.$$

when wire is fully streched. $l = x$.

& $F = mg$.

$$\therefore w = \frac{YAx \times x}{L\,2}.$$

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$\therefore w = \frac{F \cdot l}{2}$.

strain energy $= \frac{1}{2}$ load × extension

$U = \frac{1}{2} F l$

$\frac{\cancel{U}}{AL} = \frac{1}{2} \frac{F}{A} \cdot \frac{l}{L}$.

$\Rightarrow$ strainenergy per unit volume.

$= \frac{1}{2}$ stress × strain