Unit – 8

POWER TRANSMISSION

8.1 INTRODUCTION

Power is transmitted from the prime mover to machines like lathes, drilling machines, etc., by means of intermediate mechanisms called ***drives***. Refer figure 8.1. There are various drives, but the most commonly used among them are:

- Belt drives
- Rope drives
- Chain drives, and
- Gear drives

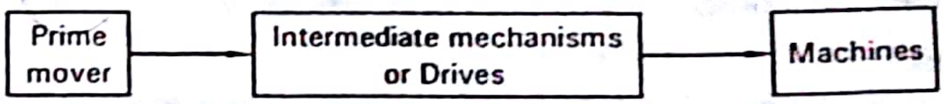


Figure 8.1 Line diagram of power transmission

The selection of a particular type of drive depends on the application, i.e., the amount of power transmitted, distance between the two shafts, etc. From the syllabus point of view, only *belt drives* and *gear drives* have been discussed in the present chapter.

8.2 BELT DRIVES

Belt drives are used to transmit power or motion from one shaft to the other by means of a thin inextensible belt running over two pulleys*. Figure 8.2 shows a simple arrangement of a belt drive. The arrangement consists of two pulleys mounted on two different shafts. One shaft called the *driving shaft* receives power from the mains and transmits it to another shaft called *driven shaft*. The pulley mounted on the driving shaft is called *driving pulley* or *driver*, while the other pulley mounted on a shaft to which the power is to be transmitted is called the *driven pulley* or *follower*. The belt passing over the two pulleys is kept in tension so as to avoid slip over the pulleys. This helps in transmitting power effectively from one shaft to another.

A pulley is a circular disc having a hole at the center so as to accommodate a shaft in it.

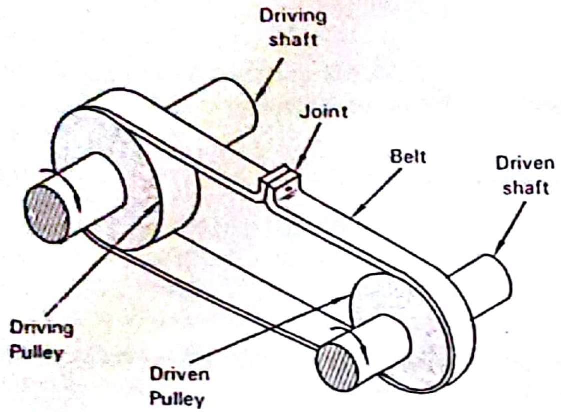


Figure 8.2 Belt drive arrangement

8.3 MATERIALS USED FOR BELTS

The materials used for manufacturing belts must be strong, flexible and durable. It must have a high coefficient of friction. The different materials used in manufacturing of belt includes

- Leather belts - cut from the back bone of steer hides.
- Cotton or fabric belts - made by folding canvass to three or more layers and stitching together.
- Rubber belt - layers of fabric impregnated with rubber.
- Balata belt - similar to rubber belts except that balata gum is used in place of rubber.

8.4 TYPES OF BELT DRIVES

There are two common types of belt drives by which power can be transmitted form one shaft to another. They are

1. Open belt drive and
2. Cross-belt drive

8.4.1 Open Belt Drive

Open belt drives are used to connect two shafts that are parallel and rotating in the same direction. Figure 8.3 shows an open belt drive.

The driver pulls the belt from the lower side *CD*, and delivers it to the upper side *AB*. Therefore, the tension in the lower side belt *CD*, will be more than the tension in the upper side belt *AB*. The lower side, because of more tension is known as the *tight side*, whereas the upper side belt, because of low tension is known as the *slack side*. Due to the lesser tension on the slack side, the belt sags due to its own weight.

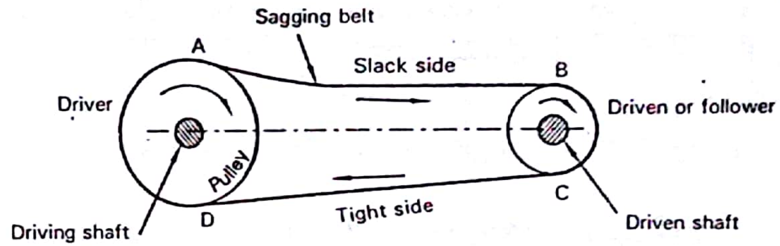


Figure 8.3 Open belt drive

8.4.2 Cross-Belt Drive

Cross-belt drives are used to connect two shafts that are parallel and rotating in opposite direction. Figure 8.4 shows the arrangement of cross belt drive.

In this type of drive, the driver pulls the belt from one side *BD*, and delivers it to the other side *AC*. Thus, the tension in the belt side *BD* will be more than the tension in the belt side *AC*. The belt side *BD*, because of more tension is known as the *tight side*, whereas, the belt side AC, because of less tension is known as the *slack side*.

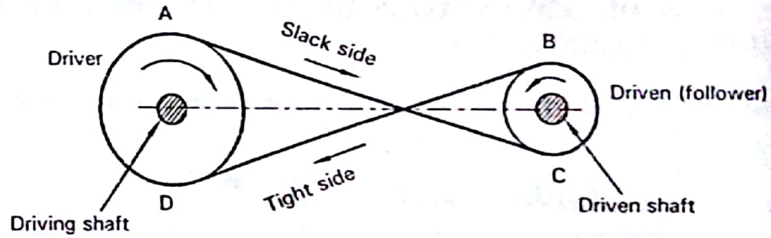


Figure 8.4 Cross belt drive

8.5 LENGTH OF BELT

The length of belt connecting the two pulleys will be different for open belt drive and cross belt drive. When the distance between the two pulleys varies, the length of the belt also varies. The equation for calculating the length of belt for both the types of drives have been derived in the following sections.

8.5.1 Length of Belt for Open Belt Drive

Figure 8.5 shows the arrangement of an open belt drive.

Let r_1 = radius of larger pulley.

r_2 = radius of smaller pulley.

C = center distance between the two pulleys.

L = total length of belt.

L_1 = Length of belt in contact with larger pulley

L_2 = Length of belt in contact with smaller pulley

L_3 = Length of belt, which is *not in contact* with either of the pulleys.

$$\therefore \quad L = L_1 + L_2 + L_3 \qquad \text{---- [1]}$$

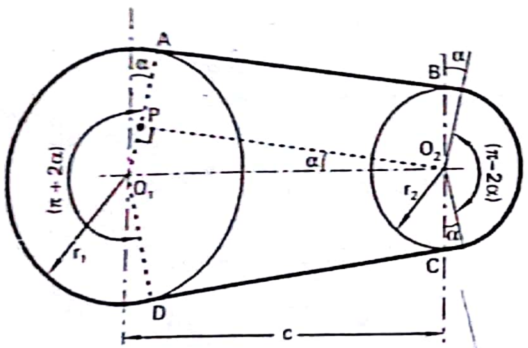


Figure 8.5 Belt length for open belt drive

To calculate L_1 and L_2

From the geometry of the figure 8.5, we have,

$$L_1 = (\pi + 2\alpha)\, r_1 \qquad \text{---- [2]}$$

$$L_2 = (\pi - 2\alpha)\, r_2 \qquad \text{---- [3]}$$

To calculate L_3

From O_2 draw a line O_2P parallel to the belt AB, which is not in contact with either of the pulleys.

From triangle O_1O_2P, $\quad O_2P = \sqrt{(O_1O_2)^2 - (O_1P)^2}$

$$= \sqrt{C^2 - (r_1 - r_2)^2}$$

multiplying numerator & denominator by C^2, we have, $O_2P = \sqrt{C^2\left[\dfrac{C^2 - (r_1 - r_2)^2}{C^2}\right]}$

$$\text{or} \quad O_2P = \sqrt{C^2\left[1 - \left(\frac{(r_1 - r_2)}{C}\right)^2\right]}$$

$$O_2P = C\left[1 - \left(\frac{r_1 - r_2}{C}\right)^2\right]^{\frac{1}{2}} \qquad \text{---- [4]}$$

Expanding the term in brackets using binomial theorm and neglecting higher powers, we have

$$\left[1 - \left(\frac{r_1 - r_2}{C}\right)^2\right]^{\frac{1}{2}} = 1 - \frac{1}{2}\left(\frac{r_1 - r_2}{C}\right)^2$$

$\therefore$ Equation (4) reduces to $\quad O_2P = C\left[1 - \dfrac{1}{2}\left(\dfrac{r_1 - r_2}{C}\right)^2\right]$

Line O_2P is parallel to the length of belt AB. Similarly by drawing a line from O_2 parallel to the belt CD, we obtain the equation similar to O_2P as given above.

$$\therefore \quad \text{Total length } L_3 = 2\, O_2P = 2C\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{C}\right)^2\right] \qquad \text{---- [5]}$$

Substituting equation (2), (3), and (5) in (1) we have,

$$L = [(\pi + 2\alpha)\, r_1] + [(\pi - 2\alpha)\, r_2] + 2C\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{C}\right)^2\right]$$

$$L = \pi r_1 + 2\alpha r_1 + \pi r_2 - 2\alpha r_2 + 2C\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{C}\right)^2\right]$$

$$L = \pi\,(r_1 + r_2) + 2\alpha\,(r_1 - r_2) + 2C - \frac{(r_1 - r_2)^2}{C} \qquad \text{---- [6]}$$

But α = ?

From triangle O_1O_2P $\sin\alpha = \dfrac{O_1P}{O_1O_2} = \dfrac{r_1 - r_2}{C}$ ---- [7]

For small values of α, $\sin\alpha = \alpha$

$\therefore$ equation (7) becomes $\alpha = \dfrac{r_1 - r_2}{C}$ ---- [8]

Substituting (8) in (6). $L = \pi(r_1 + r_2) + 2\dfrac{(r_1 - r_2)^2}{C} + 2C - \dfrac{(r_1 - r_2)^2}{C}$

$$L = \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{C} + 2C$$

Re-arranging $L = 2C + \pi(r_1 + r_2) + \dfrac{(r_1 - r_2)^2}{C}$

The above equation can be used to calculate the length of the belt for an open belt drive.

8.5.2 Length of Belt for Cross-Belt Drive

Robin KRh.

Figure 8.6 shows the arrangement of a cross belt drive.

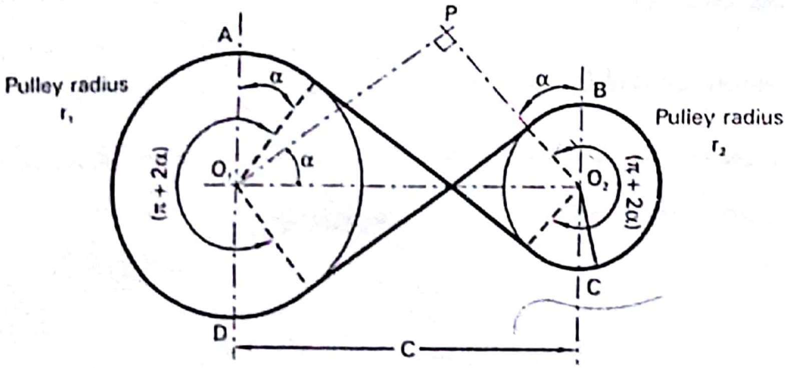


Figure 8.6 Belt length for cross belt drive

Let r_1 = radius of larger pulley.
r_2 = radius of smaller pulley.
C = center distance between the two pulleys.
L = total length of belt.

L_1 = Length of belt in contact with larger pulley
L_2 = Length of belt in contact with smaller pulley
L_3 = Length of belt, which is *not in contact* with either the pulleys.

$\therefore$ $L = L_1 + L_2 + L_3$

To calculate L_1 and L_2

From the geometry of the figure 8.6, we have,

$L_1 = (\pi + 2\alpha)\, r_1$
$L_2 = (\pi + 2\alpha)\, r_2$

To calculate L_3

From O_1 draw a line O_1P parallel to the belt BD as shown in the figure.

From triangle O_1O_2P, $O_1P = \sqrt{(O_1O_2)^2 - (O_2P)^2}$

$$= \sqrt{(C^2) - (r_1 + r_2)^2}$$

multiplying numerator & denominator by C^2, we have,

$$O_1P = \sqrt{C^2 \frac{\left[C^2 - (r_1 + r_2)^2\right]}{C^2}}$$

$$= \sqrt{C^2\left[1 - \left(\frac{r_1 + r_2}{C}\right)^2\right]}$$

or $O_1P = C\left[1 - \left(\dfrac{r_1 + r_2}{C}\right)^2\right]^{\frac{1}{2}}$

Expanding the term within the brackets by using binomial theorem and neglecting higher power

we have, $\left[1 - \left(\dfrac{r_1 + r_2}{C}\right)^2\right]^{\frac{1}{2}} = 1 - \dfrac{1}{2}\left(\dfrac{r_1 + r_2}{C}\right)^2$

$\therefore$ Equation (4) reduces to $O_1P = C\left[1 - \dfrac{1}{2}\left(\dfrac{r_1 + r_2}{C}\right)^2\right]$

$\therefore$ Total length $L_3 = 2O_1P = 2C\left[1-\frac{1}{2}\left(\frac{r_1+r_2}{C}\right)^2\right]$ ---- [5]

Substituting equation (2), (3), and (5) in (1), we have,

$$L = [(\pi + 2\alpha) r_1] + [(\pi + 2\alpha) r_2] + 2C\left[1-\frac{1}{2}\left(\frac{r_1+r_2}{C}\right)^2\right]$$

$$L = [(\pi + 2\alpha)r_1] + [(\pi + 2\alpha)r_2] + 2C - \frac{(r_1 + r_2)^2}{C}$$

$$L = \pi r_1 + 2\alpha r_1 + \pi r_2 + 2\alpha r_2 + 2C - \frac{(r_1 + r_2)^2}{C}$$

$$L = \pi (r_1 + r_2) + 2\alpha (r_1 + r_2) + 2C - \frac{(r_1 + r_2)^2}{C} \quad \text{---- [6]}$$

But $\alpha = ?$

From triangle O_1O_2P, $\sin\alpha = \frac{O_2P}{O_1O_2}$

$$\sin\alpha = \frac{r_1+r_2}{C} \quad \text{---- [7]}$$

For small values of α, $\sin\alpha = \alpha$

$\therefore$ equation (7) becomes, $\alpha = \frac{r_1+r_2}{C}$ ---- [8]

Substituting equation (8) in (6), we have,

$$L = \pi (r_1 + r_2) + \frac{2(r_1+r_2)^2}{C} + 2C - \frac{(r_1+r_2)^2}{C}$$

$$L = \pi (r_1 + r_2) + \frac{(r_1+r_2)^2}{C} + 2C$$

Re-arranging $L = 2C + \pi(r_1+r_2) + \frac{(r_1+r_2)^2}{C}$

The above equation can be used to calculate the length of the belt for a cross belt drive.

8.6 DEFINITIONS IN BELT DRIVES

8.6.1 Velocity Ratio

Velocity ratio of belt drive is defined as the ratio between the speed of the driven pulley (follower) and the speed of the driving pulley (driver).

Let d_1 = Diameter of driving pulley (driver).
d_2 = Diameter of driven pulley (follower / driven).
n_1 = Speed of the driving pulley and
n_2 = Speed of driven pulley

Assuming that there is no slip between the belt and the pulley rim, the linear speed at every point on the belt must be the same. Hence, $\pi d_1 n_1 = \pi d_2 n_2$

$$\text{or } d_1 n_1 = d_2 n_2$$

$$\text{or } \frac{n_2}{n_1} = \frac{d_1}{d_2}$$

$$\text{i.e., } \frac{\text{speed of driven}}{\text{speed of driver}} = \frac{\text{diameter of driver}}{\text{diameter of driven}}$$

The ratio $\frac{n_2}{n_1} = \frac{d_1}{d_2}$ is called *velocity ratio* or ***speed ratio*** or ***transmission ratio*** of belt drives.

Thus, in belt drives, the speeds are inversely proportional to the diameter of pulleys.

Note When thickness (t) of the belt is considered, then, velocity ratio is given by

$$\frac{n_2}{n_1} = \frac{d_1+t}{d_2+t}$$

8.6.2 Creep

In belt drives, the driver pulls the belt from the driven, and hence receives more length of the belt. In other words, the belt gets stretched as it comes out of the driven pulley. On the other hand, the driven pulley receives less length of the belt as it comes out from the driving pulley, and hence there is a contraction in the belt. The belt being an elastic material stretches out more compared to contraction and hence the increase in the length of the belt results in a *relative motion* of the belt on the pulley surface. This relative motion is called *creep* in the belt. Creep increases with load, since it is caused by the elasticity of the belt. It reduces the speed of the driven pulley, which in turn results in loss of power transmitted.

8.6.3 Slip

A common phenomenon encountered in belt drive is the *slipping* of belt. The power transmitted from one shaft to the other depends on the frictional grip between the belt and the pulley rim. There is always some amount of slip between the belt and the pulley rim that results in a slight reduction in the velocity ratio of the belt drive.

Slip may be defined as the relative motion between the pulley and the belt passing over it. It is generally expressed as a percentage.

$\therefore$ velocity ratio (when slip is considered) $\dfrac{n_2}{n_1} = \dfrac{d_1}{d_2}\left(\dfrac{100-S}{100}\right)$

where S = % slip.

Note When thickness of belt is considered, velocity ratio $\dfrac{n_2}{n_1} = \dfrac{d_1+t}{d_2+t}\left(\dfrac{100-S}{100}\right)$

8.7 IDLER PULLEY

In belt drives, when the enter distance between the two shafts is small, the *arc of contact* of the belt on the smaller pulley will be small. Due to this, the tensions in the belt are reduced and hence there will be a loss in power transmitted. The use of *idler pulley* or *jockey pulley* overcomes the problem, and the arrangement for the purpose is shown in figure 8.7.

The idler pulley, which is placed on the slack side, and near the smaller pulley exerts pressure on the belt thereby increasing the arc of contact and also tensions in the belt. Thus the power transmitted also increases.

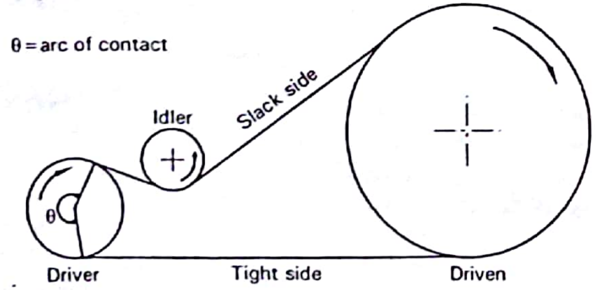
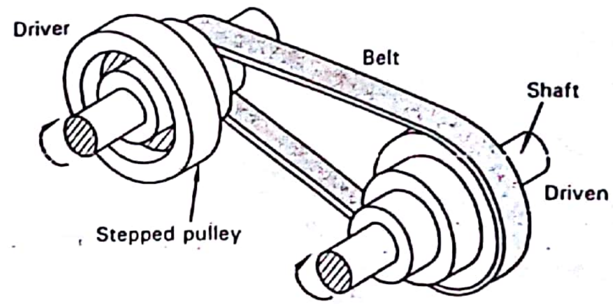


Figure 8.7 Idler pulley

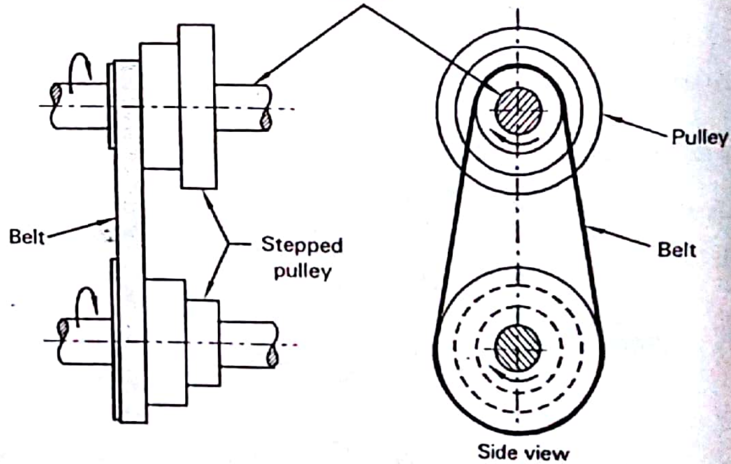
8.8 STEPPED PULLEY

A *stepped pulley* or a *cone pulley* is used for changing the speed of the driven shaft, while the driving shaft runs at constant speed. The arrangement of the belt on a stepped pulley is shown in figure 8.8.

Stepped pulleys are pulleys having several steps of varying diameters mounted on two parallel shafts, such that the smallest step of one pulley is opposite to the largest step of the other as shown in the figure. The velocity ratio of the belt drive can be varied by shifting the belt from one step of the pulley to the other. The diameters of the pulley are designed carefully so as to maintain sufficient grip of the belt for all positions on the pulley.



(a) Pictorial view of stepped pulley



(b) 2D view of stepped pulley

Figure 8.8 Stepped pulley

9 FAST & LOOSE PULLEY

n most belt drive applications, several machines obtain the drive from a single driving shaft. here arises a situation, where one of the shaft has to be stopped whenever desired without topping the main driving shaft. *A fast and loose pulley,* also called *tight and loose pulley* is sed in belt drives especially in situations when one of the driven shaft is to be started or stopped whenever desired without starting or stopping the driving shaft. Figure 8.9 shows the arrangement f fast and loose pulley. It consists of two pulleys: one called the *fast pulley* or *tight pulley* rmly keyed to the driven shaft, while the other pulley called *loose pulley* with a tight fitting rass bush is mounted on the same driven shaft but without rigidly securing to it. Hence the ose pulley rotates freely on the driven shaft.

ower is transmitted when the belt is running over the driving pulley and the tight pulley. However, hen the driven shaft is to be stopped, the driving shaft need not be disturbed, but the belt on the ght pulley is shifted on to the loose pulley by means of a belt shifter.

he shifting of the belt onto the loose pulley, causes the belt on the *driving pulley* to slip from and for this reason, the width of the driving pulley is made larger and is the sum of the width the tight and loose pulley. The driving pulley has a flat face which helps the belt to occupy fferent positions, whereas the tight and loose pulley have crowned faces, which helps the belt retain its position when shifted upon them.

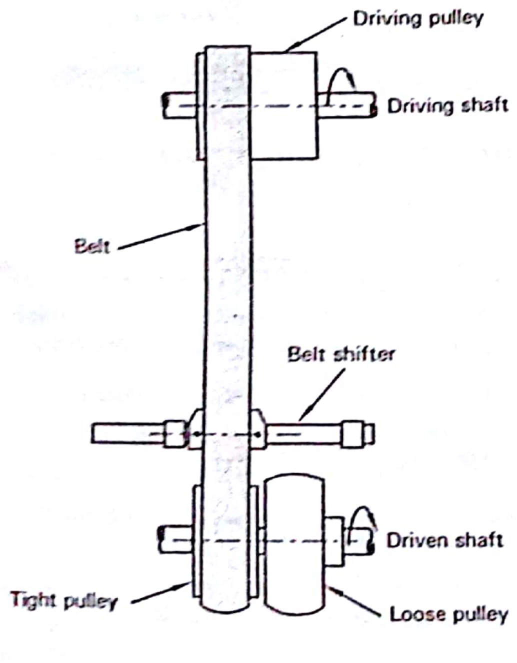


Figure 8.9 : Fast and loose pulley

8.10 POWER TRANSMITTED IN A BELT DRIVE

Let T_1 = tension in tight side of the belt (maximum tension) in Newton (N).
T_2 = tension in slack side of belt in N
d_1 = diameter of driver in m
d_2 = diameter of driven or follower in m

The effective turning force acting on the circumference of the follower is the difference in tensions on the tight side (T_1) and slack side (T_2) of the belt.

$\therefore$ Driving force is given by $F = (T_1 - T_2)$ Newton.

$$\text{or} \quad F = \frac{(T_1 - T_2)}{1000} \; kN \qquad \text{- - - - [1]}$$

we know that, Power (P) = F v, where v = velocity of the belt in m/sec.

$$\therefore \text{ equation (1) becomes } P = \frac{(T_1 - T_2)\cdot v}{1000} \text{ kW, where } v = \frac{\pi d_1 n_1}{60} \text{ or } \frac{\pi d_2 n_2}{60} \text{ m/sec}$$

8.11 RATIO OF BELT TENSIONS FOR FLAT BELT DRIVE

Consider a flat belt wound around a driven pulley as shown in Figure 8.10 (a). Let the driven pulley rotate in clockwise direction.

Let T_1 = tension on tight side of the belt and
T_2 = tension on slack side of the belt, & θ = arc of contact between the belt and pulley

Consider a small element PQ of the belt. Let $\delta\theta$ be the angle subtended by the element PQ. The element PQ is in equilibrium under the action of the following forces. Refer Figure 8.10 (b)

1. Slack side tension (T) acting at P
2. Tight side tension ($T + \delta T$) at Q
3. Normal reaction (R) exerted by the pulley on the belt element PQ.
4. Frictional force ($F = \mu R$) acting perpendicular to the normal reaction (R).

Resolving all forces horizontally, $T \sin\frac{\delta\theta}{2} + (T + \delta T)\sin\frac{\delta\theta}{2} = R$

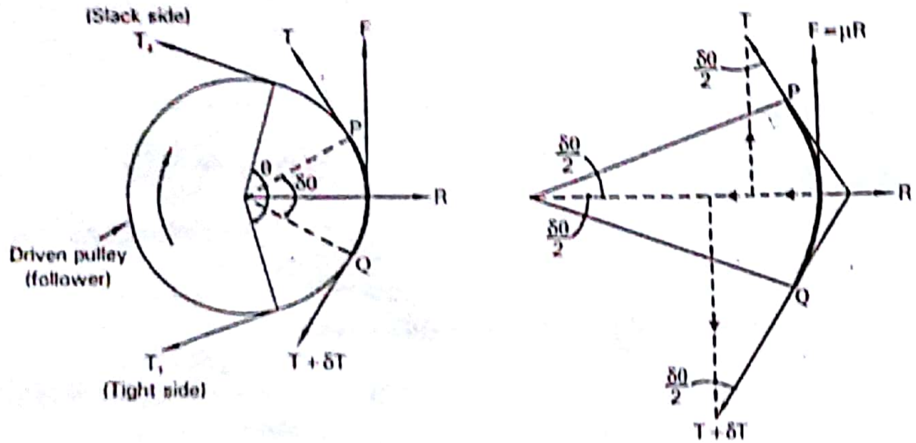
$$T\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} + \delta T\sin\frac{\delta\theta}{2} = R \qquad \text{- - - - [1]}$$

Since the angle $\frac{\delta\theta}{2}$ is very small, $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$\therefore$ equation (1) becomes, $T\dfrac{\delta\theta}{2} + T\dfrac{\delta\theta}{2} + \delta T\dfrac{\delta\theta}{2} = R$

$$R = 2T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2}$$

Neglecting $\delta T\cdot\dfrac{\delta\theta}{2}$ for small angles, we get $R = T\delta\theta$ ---- [2]



(a) Flat belt wound around driven pulley (b) Forces acting on belt element PQ

Figure 8.10 Ratio of belt tensions

Resolving all forces vertically, $T\cos\dfrac{\delta\theta}{2} + \mu R = (T+\delta T)\cos\dfrac{\delta\theta}{2}$

$$T\cos\frac{\delta\theta}{2} + \mu R = T\cos\frac{\delta\theta}{2} + \delta T\cos\frac{\delta\theta}{2}$$

$$\mu R = \delta T\cos\frac{\delta\theta}{2} \quad \text{---- [3]}$$

since $\dfrac{\delta\theta}{2}$ is small, $\cos\dfrac{\delta\theta}{2} = 1$

$\therefore$ Equation (3) becomes, $\mu R = \delta T$

$$\text{or } R = \frac{\delta T}{\mu}$$

Equating (2) and (4), we have, $T\delta\theta = \dfrac{\delta T}{\mu}$

$$\text{or } \frac{\delta T}{T} = \mu\delta\theta$$

Integrating between limits T_2 and T_1 and from 0 to θ respectively, we have,

$$\int_{T_2}^{T_1}\frac{\delta T}{T} = \int_0^{\theta}\mu\delta\theta$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\text{or } \frac{T_1}{T_2} = e^{\mu\theta}$$

The above equation is known as the ratio of belt tensions, which gives the relationship between the tensions T_1 and T_2, and the angle contact θ between the belt and the pulley.

8.12 ANGLE OF CONTACT 'θ'

The angle of contact 'θ' will not be the same in case of open and cross-belt drive. Hence, it is important to know the value of 'θ' for both types of the belt drive.

a) Open belt drive

When power is transmitted between two pulleys of different diameters, then the angle of contact at the *smaller pulley* must be taken into consideration. This is because, the belt will slip first on the pulley having smaller angle of contact, i.e., on the smaller pulley. Hence, for an open belt drive, θ is given by,

$$\theta = \left(180 - 2\sin^{-1}\frac{(r_1 - r_2)}{C}\right)\frac{\pi}{180} \text{ radians.}$$

where r_1 = radius of larger pulley
r_2 = radius of smaller pulley
C = center distance between two pulleys.

b) Cross-belt drive

When two pulleys of different diameters are connected by means of a cross-belt drive, the angle of contact on both the pulleys will be same. Hence, for a cross-belt drive, θ is given by,

$$\theta = \left(180 + 2\sin^{-1}\frac{(r_1 + r_2)}{C}\right)\frac{\pi}{180} \text{ radians}$$

where r_1 = radius of larger pulley, r_2 = radius of smaller pulley
C = center distance between two pulleys.

8.13 INITIAL TENSION in BELT

When the belt is wound around the two pulleys, the two ends of the belt are joined together tightly & are fixed over the pulleys so as to maintain a tight grip between the belt and the pulley rim. Thus, even when the pulleys are stationary, the belt is subjected to some tension and this tension is called *Initial belt tension*. It is denoted by T_0 and is expressed as

$$T_0 = \frac{T_1 + T_2}{2}$$

where T_0 = Initial belt tension, T_1 = Tight side tension, and T_2 = Slack side tension

8.14 ADVANTAGES & DISADVANTAGES OF FLAT BELT DRIVES

Advantages

a) Belt drives can be used when the center distance between the two shafts is large.
b) The speeds can be varied by varying the diameters of the pulleys.
c) Simple in construction and operation.
d) Low operating costs.
e) Smoothness of operation and ability to absorb shocks due to elasticity of the belt.

Disadvantages

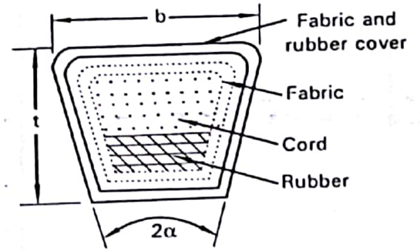
a) Belt drives are not efficient when the center distance between the two shafts is small.
b) Due to slip in belt drives, exact velocity ratio cannot be maintained.
c) Only moderate power can be transmitted.
d) The slip between the belt and the pulley causes the driven pulley to rotate at a lower speed. This reduces the power transmission.
e) Belt drives are used for transmitting power only between parallel shafts.

8.15 . V-BELT DRIVES

V-belts are used to transmit power between two shafts when the center distance between the

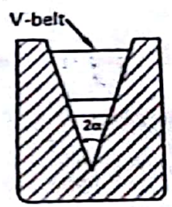
shafts is relatively small. V-belts are usually endless and trapezoidal in cross-section as shown in figure 8.11 (a). The included angle for the belt is usually 30°–40°. The belts are made from fabric and cords (that carry the load) moulded in rubber and covered with fabric and rubber as shown in the figure.

In case of flat belt drives, the belt runs over the pulley, whereas in V-belt drive, the rim of the pulley is grooved so as to accommodate the V-belt. Refer figure 8.11 (b). The effect of the groove is to increase the grip of the V-belt on the pulley, thereby reducing chances of slipping. For a V-belt drive, the ratio of tensions in the tight side and slack side of the belt is given by the equation; $\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\alpha}}$, where α = semi-groove angle. Remaining terms have their usual notations.



(a) Cross-section of v-belt

b = width of belt
t-thickness of belt
2α = grooved angle of pulley



(b) Cross-section of v-grooved pulley

Figure 8.11 V-Belt

8.15.1 Advantages and Disadvantages of V-Belt over Flat Belt

Advantages

1. V-belt transmits more power.
2. Slip between the belt and the pulley is negligible.
3. Can be used to transmit power for short center distances.
4. High velocity ratio can be obtained.
5. Operation is smooth and quiet.
6. Shaft axis may be horizontal, vertical or inclined.
7. Since V-belts are made endless, there is no joint trouble.

Disadvantages

1. Not suitable for large center distance.
2. V-belts are endless and also the pulley has to be provided with grooves. Hence, construction of belt and pulley are complicated.

8.16 PROBLEMS ON BELT DRIVES

Problem 1

In a belt drive, the velocity ratio is 3. The driving pulley runs at 400 rpm. The diameter of the driven pulley is 300 mm. Find the speed of the driven pulley and the diameter of the driving pulley.

Jan 06 - 06 m

Solution :

Step 1 Data collection

Drive - open belt type, velocity ratio = $\frac{n_2}{n_1} = 3$

Driving	Driven
$n_1 = 400$ rpm	$n_2 = ?$
$d_1 = ?$	$d_2 = 300$ mm $= 0.3$ m

Step 2 Speed of driven $n_2 = ?$

From data, $\frac{n_2}{n_1} = 3$

$\frac{n_2}{400} = 3$ $\quad\quad \therefore$ $\mathbf{n_2 = 1200}$ **rpm**

Step 3 Diameter of driving pulley $d_1 = ?$

w.k.t. velocity ratio $= \frac{n_2}{n_1} = \frac{d_1}{d_2}$

$\therefore d_1 = \frac{n_2 d_2}{n_1} = \frac{1200 \times 0.3}{400} = 0.9$ m $\quad\quad$ $\mathbf{d_1 = 0.9}$ **m or 900 mm**

Problem 2

It is required to drive a shaft A at 600 rpm by a belt using a pulley of 150 mm diameter on another parallel shaft B running at 240 rpm. What would be the diameter of the pulley on the shaft A. Find also the velocity ratio.

Jan 2008 - 08 m

Solution :

Students are advised to read the problem carefully to avoid confusion regarding driver and driven.

Step 1 Data collection

Drive - open belt type

Shaft B (driver)	Shaft A (driven)
$n_B = 240$ rpm	$n_A = 600$ rpm
$d_B = 150$ mm $= 0.15$m	$d_A = ?$

Step 2 diameter of pulley on shaft A = $d_A = ?$

w.k.t. velocity ratio= $\frac{n_A}{n_B} = \frac{d_B}{d_A}$

$\therefore d_A = \frac{n_B \cdot d_B}{n_A} = \frac{240 \times 0.15}{600}$ $\quad\quad$ $\mathbf{d_A = 0.06}$ **m or 60 mm**

Step 3 velocity ratio = ?

w.k.t. velocity ratio $= \frac{n_A}{n_B} = \frac{600}{240} = \frac{5}{2}$

velocity ratio $= \frac{n_A}{n_B} = \mathbf{5 : 2}$

Problem 3

Two pulleys are connected by a cross belt, the velocity ratio of the drive being 3. The driver runs at 1000 rpm and has a diameter of 120 cm. Find the speed and diameter of the driven pulley.

July 05 - 06 m

Solution :

Step 1 Data collection

Drive - cross belt type, velocity ratio = $\frac{n_2}{n_1} = 3$

Driver	Driven
$n_1 = 1000$ rpm	$n_2 = ?$
$d_1 = 120$ cm $= 1.2$ m	$d_2 = ?$

Step 2 To find $d_2 = ?$

w.k.t. velocity ratio $\frac{n_2}{n_1} = \frac{d_1}{d_2}$

$3 = \frac{1.2}{d_2}$

$\mathbf{d_2 = 0.4}$ **m = 400 mm**

Step 3 $n_2 = ?$

w.k.t. $\frac{n_2}{n_1} = 3$

$\frac{n_2}{1000} = 3$ $\therefore n_2 = 3000$ rpm

Problem 4

A shaft running at 100 rpm is to drive a parallel shaft at 150 rpm. The pulley on the driving shaft is 35 cm in diameter. Find the diameter of the driven pulley. Calculate the linear velocity of the belt and also the velocity ratio. *Jan 10 - 06 m*

Solution :

Step 1 Data collection

Driver	Driven
$n_1 = 100$ rpm $d_1 = 35$ cm $= 0.35$ m	$n_2 = 150$ rpm $d_2 = ?$

Step 2 $d_2 = ?$

w.k.t. velocity ratio $= \frac{n_2}{n_1} = \frac{d_1}{d_2}$

$\frac{150}{100} = \frac{0.35}{d_2}$

$\therefore d_2 = \frac{100 \times 0.35}{150} = 0.233$ m $\quad d_2 = 0.233$ m or 23.3 cm

Step 3 velocity ratio = ?

w.k.t. velocity ratio $= \frac{n_2}{n_1} = \frac{150}{100} = \frac{3}{2}$

velocity ratio $= \frac{n_2}{n_1} = 3 : 2$

Step 4 Linear velocity of belt = v = ?

w.k.t. Linear velocity at every point on the belt is same. Hence, $V = \pi d_1 n_1 = \pi d_2 n_2$

Taking $V = \pi d_1 n_1 = \pi(0.35)(100) = 109.95$

$V = 109.95$ m/min or $V = 1.83$ m/sec

Problem 5

An engine shaft running at 240 revolutions per minute is required to drive a generator by means of a flat belt drive. Pulley on an engine shaft has 160 cm diameter & on the generator shaft 60 cm diameter. Determine the speed of the generator shaft in the following cases.

(a) Neglecting thickness of belt, (b) When belt thickness is 6 mm

(c) Considering thickness 6 mm & a slip 3%. (d) Velocity of belt with thickness 6 mm.

Solution ;

Step 1 Data collection

Driver (Engine shaft)	Driven (Generator)
$n_1 = 240$ rpm	$n_2 = ?$
$d_1 = 160$ cm $= 1.6$ m	$d_2 = 60$ cm $= 0.6$ m

$t = 6$ mm $= 6 \times 10^{-3}$m, and slip = S = 3%

Step 2 To find n_2 by neglecting belt thickness

w.k.t. velocity ratio $= \frac{n_2}{n_1} = \frac{d_1}{d_2}$

$n_2 = \frac{d_1 \cdot n_1}{d_2} = \frac{1.6 \times 240}{0.6}$ $\quad n_2 = 640$ rpm

Step 3 To find n_2 by when belt thickness = t = 6 mm

In this case, velocity ratio $= \frac{n_2}{n_1} = \frac{(d_1 + t)}{(d_2 + t)}$

$\therefore n_2 = \frac{(d_1 + t) \cdot n_1}{(d_2 + t)} = \frac{\left[1.6 + (6 \times 10^{-3})\right] \times 240}{\left[0.6 + (6 \times 10^{-3})\right]}$ $\quad n_2 = 636.04$ rpm

Note When thickness of belt is considered, the speed of driven n_2 reduces from 640 rpm to 636.04 rpm.

Step 4 To find n_2 when t = 6 mm and S = 3%.

w.k.t. velocity ratio $= \frac{n_2}{n_1} = \frac{(d_1 + t)}{(d_2 + t)} \times \left[\frac{100-3}{100}\right]$

$\frac{n_2}{240} = \frac{\left[1.6 + (6 \times 10^{-3})\right]}{\left[0.6 + (6 \times 10^{-3})\right]} \times \left[\frac{100-3}{100}\right]$

$n_2 = 616.9$ rpm

Note When both thickness & slip are considered, the velocity of driven (n_2) reduces comparatively than that obtained in previous steps.

Step 5 To find velocity of belt with t = 6 mm

w.k.t. Linear velocity of belt at any point remains same

i.e., $V = \pi d_1 n_1 = \pi d_2 n_2$

When thickness is considered, $V = \pi(d_1+t)n_1 = \pi(d_2+t)n_2$

Taking $V = \pi(d_1+t)n_1$ we have,

$V = \pi[1.6+(6\times10^{-3})]240$

$V = 1210.89$ m/min   ***or* V = 20.18 m/sec**

Problem 6

The sum of diameter of two pulleys is 1000 mm and the pulleys are connected by a belt. If the pulleys rotate at 600 rpm and 1800 rpm, determine the diameter of each pulley. ***Aug 99 - 04 m***

Solution :

Step 1 Data collection

$$d_1\,(\text{driver}) + d_2\,(\text{driven}) = 1000 \text{ mm} \quad \text{or} \quad d_1 + d_2 = 1\text{m}$$

Note Since the speeds of driver & driven are not specifically mentioned in the given problem, one can assume speeds in 2 different combinations resulting in 2 different belt drive arrangements.

i.e., if $n_1 = 600$ rpm (driving), and $n_2 = 1800$ rpm (driven), then diameter of driving pulley (d_1) will be greater than that of driven pulley (d_2) resulting in speed reducing arrangement of belt drive (Speeds are inversely proportional to diameters according to velocity ratio).

If $n_1 = 1800$ rpm & $n_2 = 600$ rpm, then $d_1 < d_2$, resulting in speed increasing arrangement.

Case 1 Assuming $n_1 = 600$ rpm & $n_2 = 1800$ rpm

$$\text{w.k.t. velocity ratio} = \frac{n_2}{n_1} = \frac{d_1}{d_2}$$

$$\frac{1800}{600} = \frac{(1-d_2)}{d_2} \quad (\because d_1 + d_2 = 1 \text{ m})$$

$$1800\, d_2 = (1-d_2)\, 600$$

$$d_2 = 0.25 \text{ m}$$

$$\text{and,} \quad d_1 = 1 - d_2 = 1 - 0.25 = 0.75$$

$$d_1 = 0.75 \text{ m}$$

Thus when $n_1 = 600$ rpm & $n_2 = 1800$ rpm, we have,

d_1 = 0.75 m & d_2 = 0.25 m for speed increasing arrangement,

Case 2 Assuming $n_1 = 1800$ rpm & $n_2 = 600$ rpm

$$\text{w.k.t. velocity ratio} = \frac{n_2}{n_1} = \frac{d_1}{d_2}$$

$$\frac{600}{1800} = \frac{(1-d_2)}{d_2} \quad (\because d_1 + d_2 = 1 \text{ m})$$

$$600\, d_2 = 1800 - 1800\, d_2$$

$$\therefore d_2 = 0.75 \text{ m}$$

$$\text{and,} \quad d_1 = 1 - d_2 = 1 - 0.75 = 0.25$$

$$d_1 = 0.25 \text{ m}$$

Thus when $n_1 = 1800$ rpm & $n_2 = 600$ rpm, we have,

d_1 = 0.25 m & d_2 = 0.75 m for speed reducing arrangement.

Problem 7

An engine shaft running at 200 rpm is required to drive a generator at 300 rpm by means of a flat belt drive. Pulley on the driving shaft has 500 mm diameter. Determine the diameter of the pulley on the generator shaft if the belt thickness is 8 mm and slip is 4 percent. ***March 1999 - 10 m***

Solution :

Step 1 Data collection

Driver	Driven
$n_1 = 200$ rpm	$n_2 = 300$ rpm
$d_1 = 500$ mm = 0.5 m	d_2 = ?

Belt thickness = $t = 8$ mm $= 8\times10^{-3}$m, and slip = S = 4%

Step 2 To find d_2 when $t = 8\times10^{-3}$m and S = 4%

$$\text{w.k.t. velocity ratio} = \frac{n_2}{n_1} = \frac{(d_1+t)}{(d_2+t)} \times \left[\frac{100-S}{100}\right]$$

$$\frac{300}{200} = \frac{\left[0.5+\left(8\times10^{-3}\right)\right]}{\left[d_2+\left(8\times10^{-3}\right)\right]} \times \left[\frac{100-4}{100}\right]$$

$$d_2 + (8\times10^{-3}) = 0.325$$

$$d_2 = 0.317 \text{ m} \qquad \text{or} \quad \mathbf{d_2 = 317 \text{ mm}}$$

Problem 8

In a cross belt drive the difference in tension between tight and slack sides is 1200 N. If the angl of contact is 160° and the co-efficient of friction is 0.28, find the tension between slack and tigh sides. ***Jan 04 - 06 m***

Solution :

Step 1 Data collection

Tight side (T_1) – Slack side (T_2) = 1200 N ---- [1]

angle of contact = $\theta = 160^\circ = \frac{160\times\pi}{180} = 2.792$ radians ($180^\circ = \pi$ radians)

co-officient of friction = $\mu = 0.28$

Step 2 To find T_1 & T_2

w.k.t. $\frac{T_1}{T_2} = e^{\mu\theta}$

$\frac{T_1}{T_2} = e^{(0.28\times2.792)}$

$T_1 = 2.185\,T_2$ ---- [2]

From equation (1), we have, $T_1 = 1200 + T_2$ ---- [3]

Substituting equation (3) in (2), we have

$1200 + T_2 = 2.185\,T_2$

$T_2 = 1012.65$ **N slack side tension**

$\therefore T_1 = 1200 + 1012.65 = 2212.65$ N

$T_1 = 2212.65$**N tight side tension**

Problem 9

In a belt drive, the angle of lap on a driven pulley is 160° and the coefficient of friction is 0.3. If the maximum tension in the belt is 10000 N, find the initial tension in the belt drive. ***July 2001-10 m***

Solution :

Step 1 Data collection

angle of lap = $\theta = 160^\circ = \frac{160\times\pi}{180} = 2.792$ radians

$\mu = 0.3$, Maximum tension = $T_1 = 10000$ N

Step 2 To find initial tension (T_0)

w.k.t. $T_0 = \frac{T_1 + T_2}{2}$ N ---- [1]

but $T_2 = ?$

Using $\frac{T_1}{T_2} = e^{\mu\theta}$, we have $\frac{10000}{T_2} = e^{(0.3\times2.792)}$

$T_2 = 4327.47$ N

$\therefore T_0 = \frac{10000 + 4327.47}{2} = 7163.73$ N

$\therefore$ **Initial tension** = T_0 = **7163.73 N**

Problem 10

In a belt drive the ratio of tensions is 2, and the slack side tension is 500 N. If the speed and diameter of the driven pulley are 200 rpm and 120 cm respectively, find the power transmitted.

Jan 03 - 06 m

Solution :

Step 1 Data collection

$\frac{T_1}{T_2} = 2$ ---- [1]

$T_2 = 500$ N ---- [2]

$n_2 = 200$ rpm, and $d_2 = 120$ cm = 1.2 m

Step 2 To find power transmitted (P)

w.k.t. Power P = $\frac{(T_1 - T_2)\cdot v}{1000}$ kW

From equation (1) & (2), $T_1 = 1000$ N ; But V = ?

w.k.t. $V = \pi d_1 n_1 = \pi d_2 n_2$ m/min

Taking $V = \pi d_2 n_2$ we have, $V = \pi\times1.2\times200$

V = 753.98 m/min, **V = 12.56 m/sec**

Note Since power is expressed in kW (or kJ/sec), velocity (v) should be in m/sec.

$\therefore P = \frac{(1000-500)\times12.56}{1000} = 6.28$

Power transmitted = P = 6.28 kW

Problem 11

Two pulleys of 50 cm and 25 cm diameter are to be coupled by a belt drive. If the distance between their axis is 1.5 m, find the lengths of the belt for (a) open and (b) cross drives. ***July 2002-05 m***

Solution :

Step 1 Data collection

Assuming diameter of larger pulley $d_1 = 50$ cm = 0.5 m $\therefore$ radius $r_1 = 0.25$ m

$d_2 = 25$ cm = 0.25 m $\therefore$ radius $r_2 = 0.125$ m

center distance = c = 1.5 m

Step 2 To find L for open belt drive w.k.t. length of belt for open belt drive = $L = 2c + \pi(r_1+r_2) + \dfrac{(r_1-r_2)^2}{c}$

$$\therefore L = 2(1.5) + \pi(0.25+0.125) + \frac{(0.25-0.125)^2}{1.5} = 4.188$$

Length of open belt = L = 4.188 m

Step 3 To find L for cross belt drive

w.k.t. length of belt for cross-belt drive = $L = 2c + \pi(r_1+r_2) + \dfrac{(r_1+r_2)^2}{c}$

$$= 2(1.5) + \pi(0.25+0.125) + \frac{(0.25+0.125)^2}{1.5} = 4.271$$

Length of cross belt = 4.271 m

Problem 12

Two parallel shafts 6m apart are provided with 300 mm & 400 mm diameter pulleys & are connected by a cross-belt. The direction of rotation of the follower pulley is to be reversed by changing over to an open belt drive. How much length of the belt should be changed.

Feb 05 - 06 m

Solution :

Step 1 Data collection

Type of drive = cross belt type

c = 6 m

assuming larger diameter d_1 = 400 mm = 0.4 m ; $\therefore$ radius $r_1 = 0.2$ m

smaller diameter = d_2 = 300 mm = 0.3 m $\therefore$ radius $r_2 = 0.15$ m

Step 2 To find L for cross belt drive

$$\text{w.k.t. } L = 2c + \pi(r_1+r_2) + \frac{(r_1+r_2)^2}{c} \text{ for cross-belt drive}$$

$$L = 2(6) + \pi(0.2+0.15) + \frac{(0.2+0.15)^2}{6} = 13.12$$

L = 13.12 m for cross belt drive

Step 3 To find L for open belt drive

$$\text{w.k.t. } L = 2c + \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{c} \text{ for open-belt drive}$$

$$L = 2(6) + \pi(0.2+0.15) + \frac{(0.2-0.15)^2}{6} = 13.1$$

L = 13.1 m for open belt drive

Step 4 Change in length of belt (ΔL)

$\therefore$ change in length of belt = $\Delta L = 13.12 - 13.10 = 0.02 \text{ m} = 20$ mm

change in length of belt = $\Delta L = 20$ mm

Problem 13

It is desired to transmit a power of 20 kW between 2 parallel shafts by means of belt drive arrangement. The driving shaft rotates at 150 rpm, while the driven shaft at 250 rpm. The distance between the centers of the two shafts is 2.7 m. and the pulley mounted on the driven shaft has 60 cm diameter. Assuming the co-efficient of friction between the belt & the pulley rim as 0.25, determine the length of the belt and belt tensions for open belt drive & cross-belt drive.

Solution :

Step 1 Data collection

Power = P = 20 kW

Driving	Driven
$n_1 = 150$ rpm	$n_2 = 250$ rpm
$d_1 = ?$	$d_2 = 60$ cm $= 0.6$ m

c = 2.7 m

μ = 0.25

Step 2 Open belt drive

To find length of belt

$$\text{w.k.t. for open belt drive, } L = 2c + \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{c} \quad \text{----[1]}$$

$$r_2 = \frac{d_2}{2} = \frac{0.6}{2} = 0.3 \text{ m} \quad \text{But } r_1 = ?$$

$$\text{w.k.t. velocity ratio} = \frac{n_2}{n_1} = \frac{d_1}{d_2}$$

$$\therefore d_1 = \frac{n_2 \cdot d_2}{n_1} = \frac{250\times0.6}{150} = 1 \text{ m} \quad \text{or } r_1 = \frac{d_1}{2} = \frac{1}{2} = 0.5 \text{ m}$$

$$\therefore \text{ equation (1) becomes, } L = 2(2.7) + \pi(0.5+0.3) + \frac{(0.5-0.3)^2}{2.7}$$

L = 7.92 m for open belt drive

To find belt tensions, T_1 & T_2 = ?

$$\text{w.k.t.} \quad \frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{----[2]}$$

$$\text{for open belt drive,} \quad \theta = \left[180 - 2\sin^{-1}\left(\frac{r_1 - r_2}{c}\right)\right]\frac{\pi}{180} \text{ radians}$$

$$\theta = \left[180 - 2\sin^{-1}\left(\frac{0.5 - 0.3}{2.7}\right)\right]\frac{\pi}{180} = 2.993 \text{ radians}$$

Note Calculate θ up to 3 decimal places

$$\therefore \text{ equation (2) reduces to } \frac{T_1}{T_2} = e^{(0.25 \times 2.993)}$$

$$T_1 = 2.113\, T_2 \qquad \text{----[3]}$$

$$\text{w.k.t. Power} = P = \frac{(T_1 - T_2)\cdot v}{1000} \text{ kW} \qquad \text{----[4]}$$

$$V = \pi d_1 n_1 = \pi d_2 n_2 \quad \text{m/min}$$

$$\text{Taking} \quad V = \pi d_2 n_2 = \pi \times 0.6 \times 250 = 471.23 \text{ m/min or } V = 7.85 \text{ m/sec.}$$

$$\therefore \text{ equation (4) reduces to,} \quad 20 = \frac{(T_1 - T_2)\times 7.85}{1000}$$

$$\text{or} \quad 20 = \frac{(2.113\, T_2 - T_2)\times 7.85}{1000} \quad \text{from equation (3)}$$

$$1.113\, T_2 = 2547.77$$

$$\therefore T_2 = \mathbf{2289.1 \ N}$$

$$T_1 = 2.113\, T_2 = 2.113 \times 2289.1 = 4836.8 \text{ N}$$

$$T_1 = \mathbf{4836.8 \ N}$$

Step 3 Cross belt drive

To find length of belt

$$\text{w.k.t. for a cross belt drive, } L = 2c + \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{c}$$

$$L = 2(2.7) + \pi(0.5 + 0.3) + \frac{(0.5 + 0.3)^2}{2.7}$$

L = 8.15 m for cross belt drive.

To find belt tensions T_1 & T_2 = ?

$$\text{w.k.t.} \quad \frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{----[5]}$$

$$\text{For cross belt drive,} \quad \theta = \left[180 + 2\sin^{-1}\left(\frac{r_1 + r_2}{c}\right)\right]\frac{\pi}{180} \text{ radians}$$

$$\theta = \left[180 + 2\sin^{-1}\left(\frac{0.5 + 0.3}{2.7}\right)\right]\frac{\pi}{180} = \mathbf{3.743 \ radians}$$

$$\therefore \text{ equation (5) reduces to,} \quad \frac{T_1}{T_2} = e^{(0.25 \times 3.743)}$$

$$T_1 = 2.549\, T_2 \qquad \text{----[6]}$$

$$\text{w.k.t. Power} \quad P = \frac{(T_1 - T_2)\cdot v}{1000} \text{ kW}$$

$$20 = \frac{(T_1 - T_2)\times 7.85}{1000} \quad \text{from equation (6)}$$

Note Velocity 7.85 m/s remains same for both open belt and cross **belt drive.**

$$20 = \left(\frac{2.549\ T_2 - T_2}{1000}\right)\times 7.85$$

$$1.549\, T_2 = 2547.77$$

$$T_2 = \mathbf{1644.7 \ N}$$

$$\text{and} \quad T_1 = 2.549\, T_2 = 2.549 \times 1644.7 = \mathbf{4192.3 \ N}$$

$$T_1 = \mathbf{4192.3 \ N}$$

Answers

Open Belt drive	Cross Belt drive
L = 7.92 m	L = 8.15 m
T_1 = 4836.8 N	T_1 = 4192.3 N
T_2 = 2289.1 N	T_2 = 1644.7 N

Problem 14

Two pulleys of diameter 300 mm and 750 mm mounted on two parallel shafts 1.5 mts apart are connected by leather belt 150 mm width. If maximum safe tension of belt is 14 N per mm width, determine maximum power transmitted in case of (i) open belt drive, and (ii) cross belt drive
Assume speed of the belt as 540 m/min & μ = 0.25 ***Jan 09 - 08 m***

Solution :

Step 1 Data collection

Assuming larger diameter = $d_1 = 750$ mm $= 0.75$ m $\therefore r_1 = 0.375$ m

$d_2 = 300$ mm $= 0.3$ m $\therefore r_2 = 0.15$ m

C = 1.5 m, width of belt = W = 150 mm

Maximum safe tension = $T_1 = 14$ N/mm width.

$\therefore$ For 150 mm width = $T_1 = 14 \times 150 = 2100$ N

$v = 540$ m/min & $\mu = 0.25$

Step 2 Power P for open belt-drive

w.k.t. Power = $P = \dfrac{(T_1 - T_2)\cdot v}{1000}$ kW

$$\dot{V} = 540 \text{ m/min} = \frac{540}{60} = 9 \text{ m/sec}$$

$$P = \frac{(2100 - T_2)\times 9}{1000} \text{ kW} \qquad \text{----[1]}$$

but $T_2 = ?$

$$\text{w.k.t. } \frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{----[2]}$$

But $\theta = ?$

w.k.t. for open belt drive, $\theta = \left[180 - 2\sin^{-1}\left(\dfrac{r_1 - r_2}{c}\right)\right]\dfrac{\pi}{180}$ radians

$$\theta = \left[180 - 2\sin^{-1}\left(\frac{0.375 - 0.15}{1.5}\right)\right]\frac{\pi}{180} = 2.84 \text{ radians}$$

$\therefore$ equation (2) reduces to, $\dfrac{2100}{T_2} = e^{(0.25\times 2.84)}$

$T_2 = 1032.4$ N

substituting $T_2 = 1032.4$ N in equation (1), we have, $P = \dfrac{(2100 - 1032.4)\times 9}{1000} = 9.6$ kW

P = 9.6 kW for open belt drive

Step 3 *Power P for cross belt drive*

Note Arc of contact will be different for open and cross belt drive.

$$\text{w.k.t. } \frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{----[2]}$$

For cross belt drive, $\theta = \left[180 + 2\sin^{-1}\left(\dfrac{r_1 + r_2}{c}\right)\right]\dfrac{\pi}{180}$ radians

$$\theta = \left[180 + 2\sin^{-1}\left(\frac{0.375 + 0.15}{1.5}\right)\right]\frac{\pi}{180} = 3.856 \text{ radians}$$

$\therefore$ equation (2) reduces to, $\dfrac{2100}{T_2} = e^{(0.25\times 3.856)}$

$T_2 = 800.86$ N

Substituting $T_2 = 800.86$ N in equation (1), we have $P = \dfrac{(2100 - 800.86)\times 9}{1000} = 11.69$ kW

$\therefore$ **P = 11.69 kW for cross-belt drive.**

Problem 15

Two parallel shafts 5 m apart are connected by an open flat belt drive. The diameter of the bigger pulley is 1.5m and that of the smaller pulley is 0.75m. The initial tension in the belt is 2.5 kN. The mass of the belt is 1.25 kg/m length, co-efficient of friction between the belt and pulley is 0.25 and angle of lap on the smaller pulley is 170°. Find the power transmitted in the following cases when the smaller pulley rotates at 450 rpm.

(i)Neglecting centrifugal tension, (ii) Considering centrifugal tension ***July 09 - 12***

Solution :

Step 1 Data collection

Type of drive - open belt

$c = 5$ m

$d_1 = 1.5$ m $\therefore$ radius $r_1 = 0.75$ m

$d_2 = 0.75$ m $\therefore r_2 = 0.375$ m

$T_0 = 2.5$ kN $= 2500$ N

mass of belt = m = 1.25 kg/m length, $\mu = 0.25$

$$\theta = 170^\circ = \frac{170\times\pi}{180} = 2.967 \text{ radians}$$

$n_2 = 450$ rpm

Step 2 To find power (P) neglecting centrifugal tension w.k.t. Power $P = \dfrac{(T_1 - T_2)\cdot v}{1000}$ kW ---- [1]

but V, T_1 & T_2 = ?

ing $V = \pi d_2 n_2 = \pi \times 0.75 \times 450 = 1060.28$ m/min or V = 17.67 m/sec

w.k.t. $\dfrac{T_1}{T_2} = e^{\mu\theta}$

$\dfrac{T_1}{T_2} = e^{(0.25\times 2.967)}$

$T_1 = 2.1\,T_2$ ---- [2]

Also, $T_0 = \dfrac{T_1 + T_2}{2}$

$2500 = \dfrac{2.1T_2 + T_2}{2}$

$T_2 = 1612.9$ N

$\therefore\ T_1 = 2.1\,T_2 = 2.1 \times 1612.9 = 3387.09$ N

$T_1 = 3387.09$ N

$\therefore$ equation (1) reduces to, $P = \dfrac{(3387.09 - 1612.9)\times 17.67}{1000}$

P = 31.35 kW neglecting centrifugal tension

ep 3 To find power (P) considering centrifugal tension

In this case, $T_0 = \dfrac{T_1 + T_2 + 2Tc}{2}$ ---- [3]

where T_c = centrifugal tension = mv^2 N

$T_c = 1.25 \times (17.67)^2 = 390.28$ N

$\therefore$ equation (3) reduces to, $2500 = \dfrac{T_1 + T_2 + 2(390.28)}{2}$ ---- [4]

$T_1 = 2.1\,T_2$ from equation (2)

$\therefore\ 2500 = \dfrac{2.1T_2 + T_2 + 780.56}{2}$

$T_2 = 1361.1$ N

$T_1 = 2.1\,T_2 = 2.1 \times 1361.1 = 2858.31$ N

$T_1 = 2858.31$ N

$\therefore$ Power = $P = \dfrac{(2858.31 - 1361.1)\times 17.67}{1000}$

P = 26.45 kW considering centrifugal tension

Problem 16

A v-belt drive transmits 10 kW power at 200 rpm. The grooved pulley has a mean diameter of 1.2m and the groove angle is 45°. Taking the co-efficient of friction as 0.25, and the angle of lap as 190°, determine the tension on each side of the belt.

Solution :

Step 1: Data collection

Type of drive - V-belt drive, Power = P = 10 kW speed n = 200 rpm

mean diameter of pulley = d = 1.2 m

groove angle = $2\alpha = 45°$ $\therefore$ semi-groove angle = $\alpha = 22.5°$

$\mu = 0.25,\ \theta = 190° = \dfrac{190\times\pi}{180} = 3.316$ radians

Step 2 To find belt tensions T_1 & T_2.

For a V-belt drive, belt tensions $= \dfrac{T_1}{T_2} = e^{(\mu\theta/\sin\alpha)}$ or $\dfrac{T_1}{T_2} = e^{\left(\frac{0.25\times 3.316}{\sin 22.5}\right)}$

$\dfrac{T_1}{T_2} = 2.166$ or $T_1 = 8.72\,T_2$ ---- [1]

But $T_1 = T_2 = ?$

w.k.t. Power = $P = \dfrac{(T_1 - T_2)\cdot v}{1000}$ kW ---- [2]

$V = \pi d n$ (general equation)

$V = \pi \times 1.2 \times 200 = 753.98$ m/min or **V = 12.56 m/sec.**

Equation (2) reduces to, $10 = \dfrac{(T_1 - T_2)\times 12.56}{1000}$

$10 = \dfrac{(8.72\,T_2 - T_2)\times 12.56}{1000}$ from equation (1)

$T_2 = 103.13$ N

$T_1 = 8.72\,T_2 = 8.72\times 103.13 = 899.31$

$T_1 = 899.31$ N

8.17 GEARS

A wheel provided with *teeth* is called a *gear*. In other words, gears are toothed wheels used to transmit power or motion from one shaft to another, where the distance between the two shafts is relatively small. Refer figure 8.12. Gears are generally used for one of four different reasons.

1) To reverse the direction of rotation.
2) To increase or decrease the speed of rotation.
3) To move rotational motion to a different axis.
4) To keep the rotation of two axes synchronized.

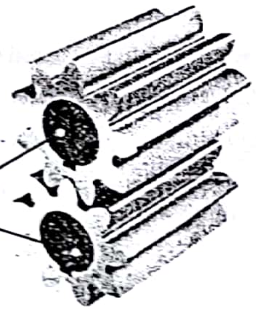


Figure 8.12 Spur gear drive

8.18 SUPER GEAR TERMINOLOGY

The terminology used in spur gear is shown in figure 8.13

Pitch circle It is a theoretical or an imaginary circle upon which all computations are made.

Pitch circle diameter (PCD) It is the diameter of the pitch circle, or it is also defined as the mean diameter of the gear wheel.

Pitch (circular pitch) It is the distance from a point on one tooth to the corresponding point on the next tooth measured along the pitch circle. It is denoted P_c and is given by the equation:

$$P_c = \frac{\pi d}{T}, \text{ where } d = \text{pitch circle diameter in } mm. \text{ and } T = \text{number of teeth.}$$

Module (m) It is the ratio of the pitch circle diameter of a gear in millimeter to the number to teeth. Module = $m = \frac{d}{T}$, where d = pitch circle diameter in *mm.* and T = number of teeth.

Module refers to the index of the tooth size.

Addendum It is the radial distance of a tooth from the pitch circle to the top of the tooth.

Dedendum It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

Face It is the surface of the gear tooth above the pitch circle. It is also defined as the action or working face of the addendum.

Flank It is the surface of the gear tooth below the pitch circle. It is also defined as the working face of the dedendum.

Top land It is the surface of the top of the gear tooth.

Bottom land It is the surface of the bottom of the tooth space.

Diametral pitch It is the number of teeth on a gear wheel per unit of its pitch diameter. It is denoted by P_d and is given by the equation $P_d = \frac{T}{d}$

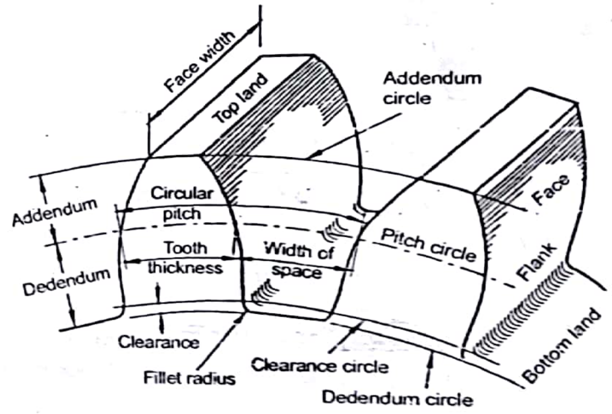


Figure 8.13 Spur gear terminology

8.19 TYPES OF GEARS

Gears are commonly classified based on the position of the axis of the shaft on which the gear is mounted. The most commonly used gears are:

1) Spur gear 2) Bevel gear 3) Helical gear 4) Worm gear, and 5) Rack and pinion

8.19.1 Spur gear

Spur gears are the simplest and the most commonly used gears designed to transmit motion between two parallel shafts. Refer Figure 8.14. The axis of the two shafts i.e., the driving shaft and the driven shaft are parallel to each other. The teeth are cut straight on the periphery (circumference) of the wheel and they are parallel to the axis of the wheel. Spur gears are used in machine tools, gearboxes, wind-up alarm clock and watches, precision measuring instruments, etc.

Spur gears are not widely used in automobiles, because each time a gear tooth engages a tooth on the other gear, the teeth collides and this impact makes a noise. It also increases the stress on the gear teeth.

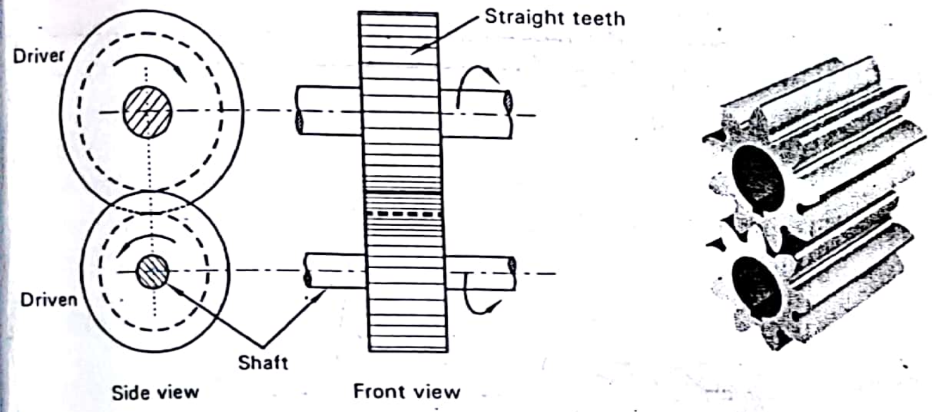


Figure 8.14 Spur gear

8.19.2 Bevel gear

Bevel gears are used for transmitting power between two intersecting shafts. Refer figure 8.15. They are usually mounted on shafts that are 90° apart, but can be designed to work at other angles as well. The teeth are cut on the outside of the conical surface and vary in cross-section throughout their length. Since the diameter of the cone is greatest at it base, the teeth will be thicker at the base as shown in the figure.

The teeth on bevel gears can be straight, spiral or hypoid. Straight bevel gear teeth actually have the same problem as straight spur gear teeth; as each tooth engages, it impacts the corresponding tooth all at once. The solution to this problem is to curve the gear teeth and such gears are called *spiral bevel* gears. Refer figure 8.16. In these gears, the contact starts at one end of the gear and progressively spreads across the whole tooth.

8.19.3 Helical Gears

Helical gears are used to transmit power or motion between two parallel or non-parallel, but non-intersecting shafts. Refer figure 8.16.

In helical gears, the teeth are curved, and helical in shape, hence the name *helical gears*. When two teeth on a helical gear engage, the contact starts at one end of the teeth and gradually spreads as the gears rotate, until the two teeth are in full engagement. This gradual engagement makes helical gears run much more smoothly and quietly than spur gears.

Helical gears are mostly used in automobile power transmission where smooth and quiet running is necessary at higher speeds.

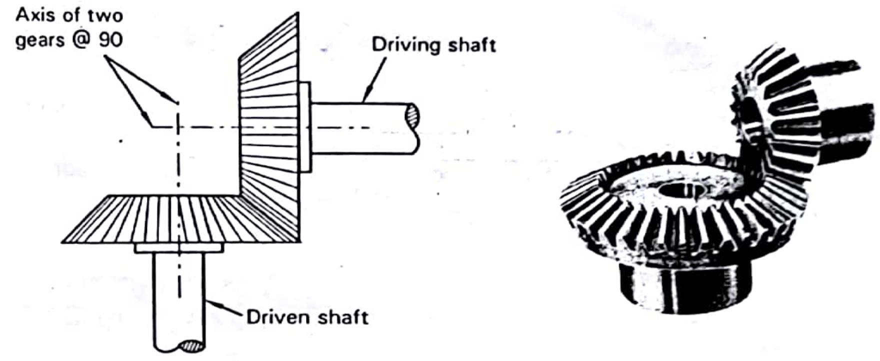


Figure 8.15 Bevel gear

Figure 8.16 Spiral Bevel gear

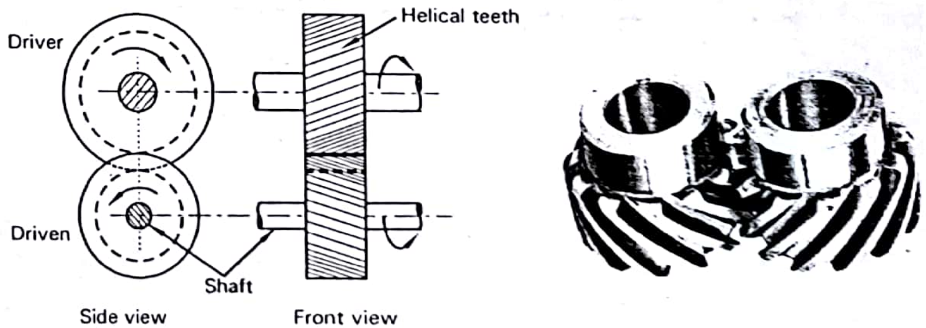
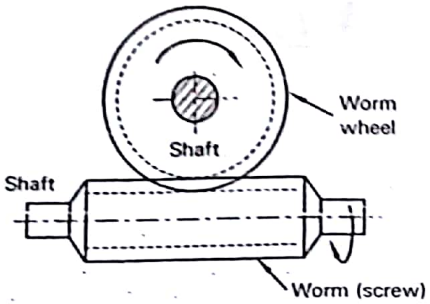


Figure 8.17 Helical gear

8.19.4 Worm Gears

Worm gears are used to transmit power or motion between two shafts having their axis at right angles and non-intersecting. Refer figure-8.18.

Worm gear is a type of screw gearing that consists of a screw meshing with a helical gear, or spur gear. The screw is called *worm* and the gear wheel meshing with the screw is called *worm gear or worm wheel.* Worm gears are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1 and even upto 300:1 or greater.



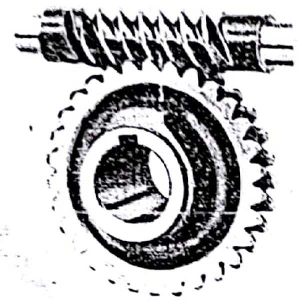
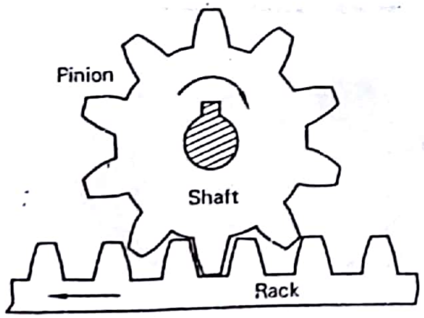


Figure 8.18 Worm gear

8.19.5 Rack and Pinion

A rack is a gear, having teeth cut along a straight line, while a pinion is a gear with teeth cut along its periphery. Figure 8.19 shows the rack and pinion. With the help of rack and pinion, rotary motion can be converted into linear motion.



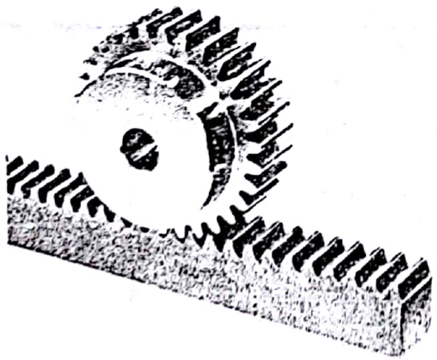
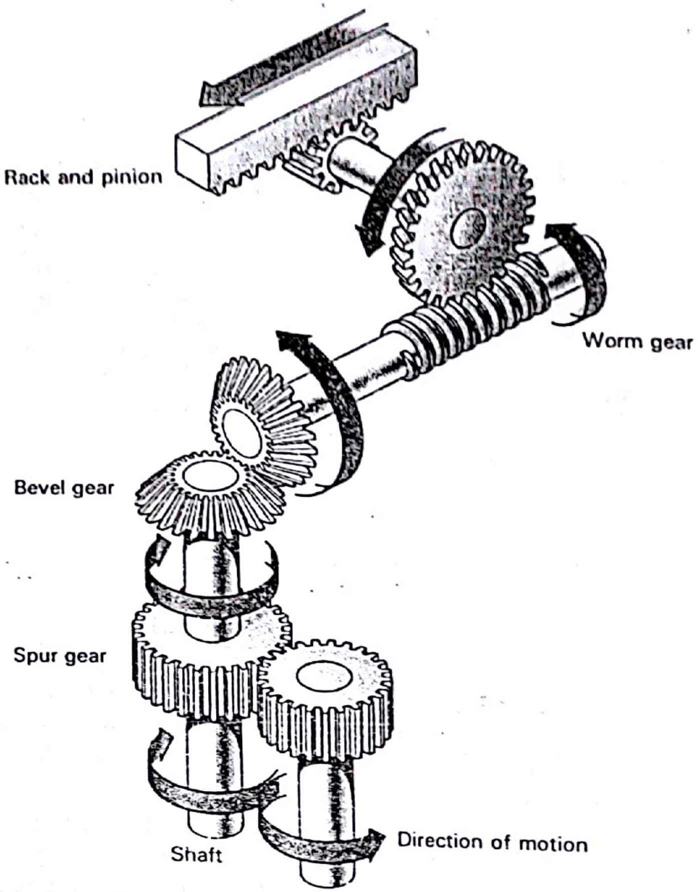


Figure 8.19 Rack and Pinion



Motion transmitted by different gears

8.20 GEAR DRIVES

Gear drives are used to transmit power or motion from one shaft to the other by means of gear Discussed below are few advantages and disadvantages of gear drives that helps one to understa their applications in power transmission.

Advantages of gear drives

a) Gear drives can be used to transmit power or motion between parallel, non-parallel, intersecti and non-intersecting shafts.

b) Gear drives are preferred to other drives (belt, rope, etc.), especially, when the cent distance between the two shafts is very small.

:) Power can be to transmitted with a constant speed ratio.
) Gear drives have higher power transmission efficiency.
:) They can be used for low, medium or high power transmission.

sadvantages of gear drives

) Gear drives are not suitable when the center distance between the two shafts is large.
) Since the teeth of driver and driven gears come in contact with each other, they always require some kind of lubrication.
) Cost for production of gears is high.
) Noise and vibration is a major problem when they rotate at higher speeds.
) Damage to single teeth affects the whole arrangement.

21 VELOCITY RATIO OF GEAR DRIVES

locity ratio of gear drive is defined as the ratio between the speed of the driven gear (follower) l the speed of the driving gear (driver). Consider the spur gear drive arrangement as shown in ıre 8.20.

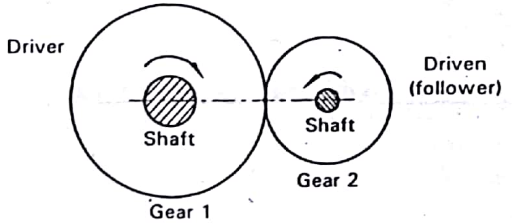


Figure 8.20 Spur gear

Let d_1 = pitch circle diameter of the driving gear. (driver)
d_2 = pitch circle diameter of the driven gear. (follower)
T_1 = number of teeth on driving gear.
T_2 = number of teeth on driven gear.
n_1 = speed of the driving gear.
n_2 = speed of the driven gear.

suming that there is no slip between the mating teeth, the linear speed of the driving gear must the same as that of the driven gear. Hence, $\pi d_1 n_1 = \pi d_2 n_2$

or $d_1 n_1 = d_2 n_2$

$$\frac{n_2}{n_1} = \frac{d_1}{d_2} \qquad \text{---- [1]}$$

i.e., $\frac{\text{speed of driven}}{\text{speed of driver}} = \frac{\text{diameter of driver}}{\text{diameter of driven}}$

The circular pitch for both the mating (meshing) gears remains same.
Hence, circular pitch (P_c) of driver = circular pitch of driven gear.

$$\text{i.e.,}\quad P_c = \frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2} \quad \text{or} \quad \frac{d_1}{d_2} = \frac{T_1}{T_2} \qquad \text{---- [2]}$$

Substituting equation (2) in (1), we have, $\frac{n_2}{n_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$

The ratio $\frac{n_2}{n_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$ is called the *velocity ratio* or *speed ratio* or *transmission ratio* of gear drives. Thus, in gear drives, the speeds are inversely proportional to the pitch circle diameter, or inversely proportional to the number of teeth.

8.22 BELT DRIVES v/s GEAR DRIVES

Table 8.1 gives a brief comparision between belt drives and gear drives.

Sl. No.	Belt Drives	Gear Drives
1.	They are non-positive drives, as there is a reduction in power transmission due to slip.	They are positive drives.
2.	Efficient when the center distance between the two shafts is greater.	Efficient when the center distance between the two shafts is very small.
3.	Used to transmit power between two parallel shafts.	Used to transmit power between parallel, non-parallel, intersecting and non-intersecting shafts.
4.	Due to slip, exact velocity ratio cannot be maintained.	Due to the absence of slip, constant velocity ratio can be maintained.
5.	Only moderate power can be transmitted.	Can be used for low, medium or high power transmission.
6.	Low power transmission efficiency.	High power transmission efficiency.
7.	Lubrication is not required	Requires some kind of lubrication

Table 8.1 Comparison between belt drives and gear drives

8.23 GEAR TRAINS

As discussed earlier, gears are toothed wheels used to transmit power from one shaft to another. When two or more gears are used to transmit power, the arrangement is then called *gear train*. The nature of *train* i.e., the number of gears used depends upon the desired velocity ratio and the relative position of the axis of shafts.

Depending on the arrangement of gears (wheels), gear trains are classified as below.

1) Simple gear train
2) Compound gear train
3) Reverted gear train and
4) Epicyclic gear train

Note From the syllabus point of view, only *simple* and *compound* gear trains have been discussed in the present chapter.

8.23.1 Simple Gear Train

A simple gear train consists of a number of gears mounted over the shaft, such that each shaft carries only one gear. Figure 8.21 shows a simple gear train in which gear-1, drives gear-2.

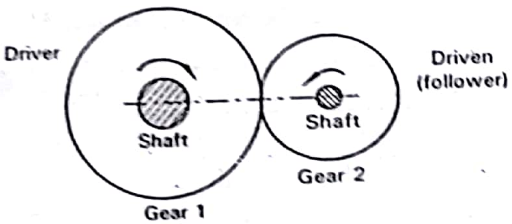


Figure 8.21 Simple gear drive

Let n_1 = speed of gear 1 or driver in rpm

n_2 = speed of gear 2 or follower/driven in rpm

T_1 = number of teeth on gear 1

T_2 = number of teeth on gear 2

We know that, in gear drives, speeds are inversely proportional to the number of teeth.

$$\therefore \quad \text{velocity ratio} = \frac{n_2}{n_1} = \frac{T_1}{T_2}$$

$$\text{or} \quad \frac{\text{speed of driven}}{\text{speed of driver}} = \frac{\text{number of teeth on driver}}{\text{number of teeth on driven}} \qquad \text{---- [1]}$$

There are certain cases, when power is to be transmitted between two shafts that are at large distances. This can be done by providing one or more intermediate gears as shown in figure. 8.22. These intermediate gears are called ***idler gears***. The idler gear does not affect the *velocity ratio* but only serves to fill up the space between the driver and the driven gears. They also help in achieving the required direction of rotation for the driven gear.

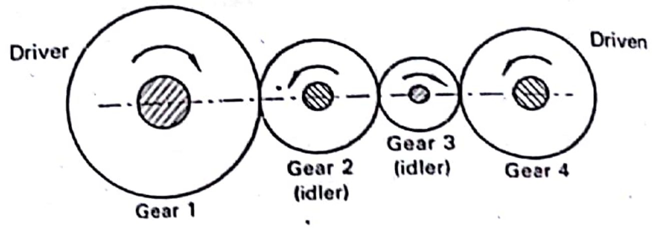


Figure 8.22 Simple gear train with idler gears

Let n_1, n_2, n_3 and n_4 be the speeds and T_1, T_2, T_3 and T_4 be the number of teeth on gear 1, 2, 3 and 4 respectively. Assume that gear-1 rotates in clockwise direction.

Now gear-1 drives gear-2. Hence gear-1 is driver and gear-2 is driven.

$$\therefore \quad \text{velocity ratio} = \frac{n_2}{n_1} = \frac{T_1}{T_2} \qquad \text{---- [2]}$$

Further, gear-2 drives gear-3. Now gear-2 is driver and gear-3 is driven.

$$\therefore \quad \text{velocity ratio} = \frac{n_3}{n_2} = \frac{T_2}{T_3} \qquad \text{---- [3]}$$

Similarly gear 3 drives gear 4

$$\therefore \quad \text{Velocity ratio} = \frac{n_4}{n_3} = \frac{T_3}{T_4} \qquad \text{---- [4]}$$

The velocity ratio of the gear train shown in figure 8.22 is obtained by multiplying equations (2), (3) and (4).

$$\text{i.e.,} \quad \text{Velocity ratio} = \frac{n_2 \times n_3 \times n_4}{n_1 \times n_2 \times n_3} = \frac{T_1 \times T_2 \times T_3}{T_2 \times T_3 \times T_4}$$

$$\text{Velocity ratio} = \frac{n_4}{n_1} = \frac{T_1}{T_4}$$

$$\text{Velocity ratio} = \frac{\text{speed of last gear(driven)}}{\text{speed of first gear(driver)}} = \frac{\text{number of teeth on first gear(driver)}}{\text{number of teeth on last gear(driven)}} \qquad \text{---- [5]}$$

Comparing equations (1) & (5), it is clear that, idler gears do not affect the velocity ratio. They only serve to fill the gap, and also help in achieving the required direction of rotation for the driven wheel.

Note

Case (1) Gear train with one idler

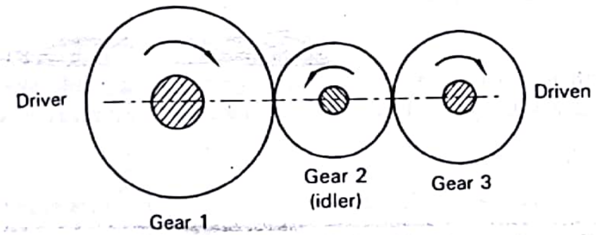
Consider a simple gear train with one idler gear as shown in figure 8.23(a). Let gear 1 rotate in clockwise direction. Therefore, gear 1 drives gear 2. Hence, gear 2 rotates in counter clockwise direction. Further, gear 2 drives gear 3. Hence, gear 3 rotates in opposite (clockwise) direction to that of gear 2 (anticlockwise direction).

Case (2) Gear train with two idlers

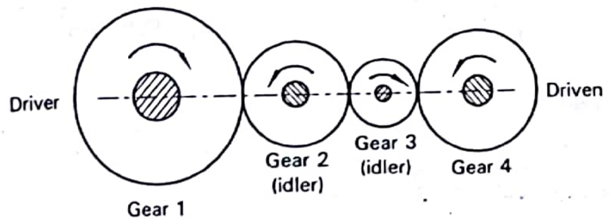
Consider a simple gear train with two idler gears as shown in figure 8.23(b). The rotation of each gear is shown in the figure.

From figure 8.23 (a) and (b), it can be concluded that,

- If **odd** number of idler gears are used, the first gear (driver) and the last gear (driven) will rotate in the same direction. Refer figure 8.23 (a).
- If **even** number of idler gears are used, the first gear (driver) and last gear (driven) will rotate in opposite directions. Refer figure 8.23 (b).



(a) Gear train with one idler



(b) Gear train with two idlers

Figure 8.23 Simple gear train with idlers

8.23.2 Compound Gear Train

A compound gear train is the one, in which ~~each~~ Intermediate shaft carries two or more gears. It was clear that, from discussions made in the previous section, intermediate gear does not affect the speed ratio. Thus, whenever the distance between the two shafts is large and at the same time, higher or much less speed is required, compound gears are provided with intermediate shafts. A compound gear train is shown in figure 8.24.

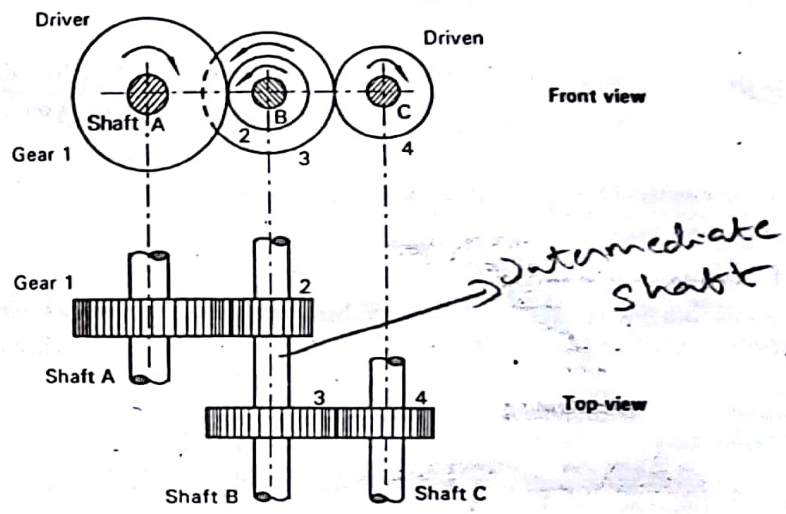


Figure 8.24 Compound gear train

In the gear train shown in figure 8.24, gear 1 and gear 4 are mounted on separate shafts (shaft A & C), but gear 2 and gear 3 are mounted (keyed) on a single shaft B. Hence, gear 2 and gear 3 are called *compound gears*. Since gear 2 and gear 3 are keyed to same spindle, they rotate at same speed.

Let n_1, n_2, n_3 and n_4 be the speeds, and T_1, T_2, T_3 and T_4 be the number of teeth on gear 1, 2, 3 and 4 respectively.

Gear-1 drives gear-2. $\therefore$ velocity ratio $= \dfrac{n_2}{n_1} = \dfrac{T_1}{T_2}$ ---- [1]

Note that, gear-2 and gear-3 are mounted on a common shaft B. Hence when gear-2 rotates, gear-3 also rotates in the same direction and at the same speed.

Now, gear-3 drives gear-4. $\therefore$ velocity ratio $= \dfrac{n_4}{n_3} = \dfrac{T_3}{T_4}$ ---- [2]

The velocity ratio of the gear train is obtained by multiplying equations (1) and (2).

i.e., Velocity ratio $= \dfrac{n_2 \times n_4}{n_1 \times n_3} = \dfrac{T_1 \times T_3}{T_2 \times T_4}$ - - - - [3]

but $n_2 = n_3$ since gear 2 and gear 3 are keyed to the same spindle.

$\therefore$ equation (3) becomes, velocity ratio $= \dfrac{n_4}{n_1} = \dfrac{T_1 \times T_3}{T_2 \times T_4}$

i.e., velocity ratio $= \dfrac{\text{speed of last gear(driven)}}{\text{speed of first gear(driver)}} = \dfrac{\text{product of number of teeth on driver}}{\text{product of number of teeth on driven}}$

8.24 PROBLEMS ON GEAR DRIVES

Problem 1

The velocity ratio of a gear drive is 2. Driving wheel has 16 teeth and turns at 120 r/min. Find the r/min and the number of teeth on the driven wheel. *July 08 - 05 m*

Solution :

Step 1 Data collection

$$\text{velocity ratio} = \frac{n_2}{n_1} = 2$$

Driver	Driven
$T_1 = 16$ $n_1 = 120$ rpm	$T_2 = ?$ $n_2 = ?$

Step 2 To find T_2

w.k.t. velocity ratio for simple gear train $= \dfrac{n_2}{n_1} = \dfrac{T_1}{T_2}$

$$2 = \frac{16}{T_2}, \qquad T_2 = 8 \text{ teeth}$$

Step 3 To find n_2

$$\text{using } \frac{n_2}{n_1} = \frac{T_1}{T_2} \text{ we have, } \frac{n_2}{120} = \frac{16}{8} \quad \text{or} \quad n_2 = 240 \text{ rpm}$$

Problem 2

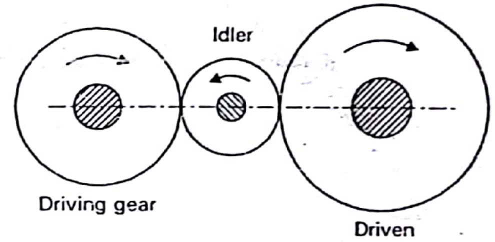
A simple gear train consists of 3 gears. The number of teeth on the driving gear is 60, on the idler 40, and on the driven gear 80. If the driving gear rotates at 1200 rpm, find the speed of the driven gear, and also the velocity ratio, sketch the arrangement of the gear drive.

Solution :

Step 1 Data collection

Driving gear	Idler gear	Driven gear
$T_1 = 60$ $n_1 = 1200$ rpm	$T_2 = 40$	$T_3 = 80$ $n_3 = ?$

Step 2 Sketching gear drive arrangement



Note Number of teeth is proportional to diameter of gear. Hence, while sketching the drive arrangement, the diameter of the gears should be proportionate to the numerical values given.

Step 3 To find n_3 & velocity ratio

Assuming driver to rotate in clockwise direction, we have, velocity ratio for simple gear train as:

$$\frac{n_3}{n_1} = \frac{T_1}{T_3}$$

$$\frac{n_3}{1200} = \frac{60}{80} \qquad n_3 = 900 \text{ rpm}$$

$$\text{velocity ratio} = \frac{n_3}{n_1} = \frac{900}{1200} = \frac{3}{4}$$

$$\textbf{velocity ratio} = \frac{n_3}{n_1} = \mathbf{3 : 4}$$

Problem 3

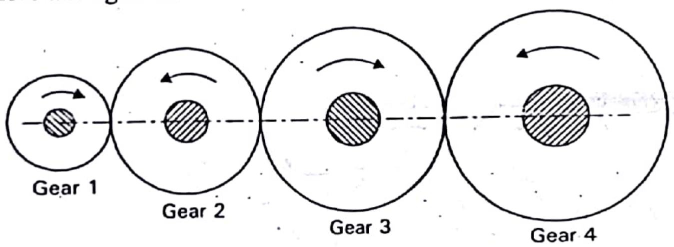
A simple gear train consists of four gears having 30, 40, 50 and 60 teeth respectively. Determine e speed and direction of rotation of the last gear if the first gear makes 600 rpm in clockwise rection. *MQP 2002 scheme - 06 m*

lution :

ep 1 Data collection

Gear 1	Gear 2	Gear 3	Gear 4
$T_1 = 30$ $n_1 = 600$ rpm	$T_2 = 40$	$T_3 = 50$	$T_4 = 60$ $n_4 = ?$

ep 2 Gear drive arrangement



tep 3 $n_4 = ?$

w.k.t. for a simple gear train, velocity ratio = $\frac{n_4}{n_1} = \frac{T_1}{T_4}$

$$\frac{n_4}{600} = \frac{30}{60} \qquad \boldsymbol{n_4 = 300 \text{ rpm in anticlockwise direction}}$$

roblem 4

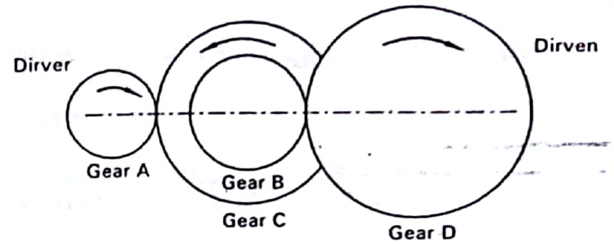
gear train consists of four gears A, B, C and D of 20, 25, 50 and 75 teeth respectively. A meshes ith C and B is a compound gear with C. B meshes with D. If A has a speed of 300 rpm, what is he speed of D. Sketch the gear train. *Jan 10 - 08 m*

olution :

tep 1 Data collection

Gear A	Gear B	Gear C	Gear D
$T_A = 20$ $n_A = 300$ rpm	$T_B = 25$	$T_C = 50$	$T_D = 75$ $n_D = ?$

Step 2 Gear drive arrangement



Step 3 $n_D = ?$

Assume gear A to rotate in clockwise direction. Hence, Gear A drives Gear C, and Gear B drives Gear D. Gear B and Gear C are compound gears, which rotate in same direction and speed.

∴ Velocity ratio for Compound gear train =

$$= \frac{\textit{Speed of last gear (driven)}}{\textit{Speed of first gear (driver)}} = \frac{\textit{Product of teeth on driver}}{\textit{Product of teeth on driven}}$$

$$\text{i.e.,} \quad \frac{n_D}{n_A} = \frac{T_A \times T_B}{T_C \times T_D} \quad \text{or} \quad \frac{n_D}{n_A} = \frac{20 \times 25}{50 \times 75} \qquad \boldsymbol{n_D = 40 \text{ rpm}}$$

Problem 5

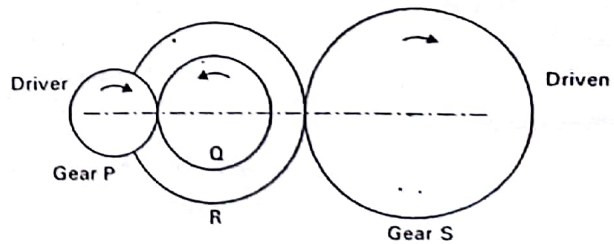
A compound gear train consists of 4 gears P, Q, R, S having 20, 40, 60 and 80 teeth respectively. The gear P is keyed to driving shaft; gear S to driven shaft, Q and R are compound gears, Q meshing with P and R meshes S. If P rotates at 150 rpm what is the rpm of gear S? Show gear arrangement. *Jan 04 - 06 m*

Solution :

Step 1 Data collection

Gear P (Driver)	Gear Q	Gear R	Gear S (Driven)
$T_P = 20$ $n_P = 150$ rpm	$T_Q = 40$	$T_R = 60$	$T_S = 80$ $n_S = ?$

Step 2 Gear drive arrangement



Step 3 To find n_S = ?

Assume Gear P to rotate in clockwise direction. Hence, Gear P drives Gear Q, and Gear R drives Gear S. Gear Q & R are compound gears.

∴ Velocity ratio for Compound gear train =

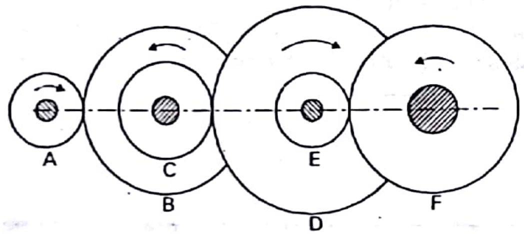
$$= \frac{\textit{Speed of last gear (driven)}}{\textit{Speed of first gear (driver)}} = \frac{\textit{Product of teeth on driver}}{\textit{Product of teeth on driven}}$$

i.e., $\frac{n_S}{n_P} = \frac{T_P \times T_R}{T_Q \times T_S}$ or $\frac{n_S}{150} = \frac{20 \times 60}{40 \times 80}$ $\quad n_S = 56.25$ **rpm**

Problem 6

Illustrated in figure below is the gear drive arrangement used in a machine tool. A motor drives gear A at 950 rpm. Gears B, C, D and E are fixed to parallel shafts rotating together. The number of teeth on each gear is given below. Determine the speed of gear F which is mounted on the output shaft of the machine tool.

Gear	A	B	C	D	E	F
No. of Teeth	20	50	25	75	25	65



Solution:

Step 1 Data collection

Assume Gear A as driver & Gear F as driven. Hence,

Gear A drives Gear B

Gear C drives Gear D

Gear E drives Gear F.

n_A = 950 rpm

∴ Velocity ratio for Compound gear train =

$$= \frac{\textit{Speed of last gear (driven)}}{\textit{Speed of first gear (driver)}} = \frac{\textit{Product of teeth on driver}}{\textit{Product of teeth on driven}}$$

i.e., $\frac{n_F}{n_A} = \frac{T_A \times T_C \times T_E}{T_B \times T_D \times T_F}$ or $\frac{n_F}{950} = \frac{20 \times 25 \times 25}{50 \times 75 \times 65}$

∴ n_F = **48.7 rpm**

Problem 7

A simple gear train of wheels consists of 4 gears A, B, C and D having 20, 40, 60 and 70 teeth respectively. Gear A is mounted on the driving shaft, while gear D on the driven shaft. Gears B and C are idler gears. If gear A rotates at 500 rpm in clockwise direction, calculate:

a) Speed and direction of gear D. b) Train value of the gear train.

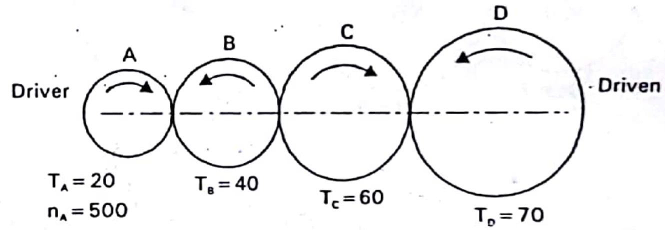
c) Speed and direction of intermediate gears.

Sketch the arrangement of the gear train using simple circles.

Solution :

Step 1 Data collection

The details of the gear train are shown in figure below:



Step 2 Consider gears A & B

Gear A drives gear B

w.k.t. velocity ratio: $\frac{n_B}{n_A} = \frac{T_A}{T_B}$ or $\frac{n_B}{500} = \frac{20}{40}$

∴ **speed of first idler** n_B = **250** ***rpm*** **in anticlockwise direction**

Now Gear B drives gear C

∴ velocity ratio = $\frac{n_C}{n_B} = \frac{T_B}{T_C}$ or $\frac{n_C}{250} = \frac{40}{60}$

∴ **speed of second idler** n_C = **166.67** ***rpm*** **in clockwise direction**

Now gear C drives gear D

∴ velocity ratio = $\frac{n_D}{n_C} = \frac{T_C}{T_D}$ or $\frac{n_D}{166.67} = \frac{60}{70}$

n_D = **142.86** ***rpm*** **in anticlockwise direction**

3 Train value of gear train

$$\text{w.k.t. Train value} = \frac{1}{speed\ (velocity)\ ratio}$$

$$\text{peed ratio of simple gear train} = \frac{n_D}{n_A} = \frac{T_A}{T_D} \text{ or } \frac{n_D}{n_A} = \frac{20}{70}$$

$$\frac{n_D}{n_A} = \frac{1}{3.5} = 0.285 = \text{Speed ratio of given drive}$$

$$\therefore \textbf{Train value} = \text{Inverse of speed ratio} = \frac{1}{0.285} = \mathbf{3.5}$$

lem 8

the pitch diameter, diametral pitch and module of a toothed gear having 36 teeth and circular of 13 mm.

tion :

1 Data collection

Number of teeth = T = 36, Circular pitch = P_C = 13 mm

2 To find pitch diameter or pitch circle diameter (d)

$$\text{w.k.t. circular pitch} = P_C = \frac{\pi \cdot d}{T} \text{ where d = pitch diameter \& T = number of teeth}$$

$$13 = \frac{\pi \times d}{36} \quad \therefore d = 148.96 = 149 \text{ mm}$$

$$\therefore \textbf{pitch diameter} = d \approx \mathbf{149\ mm}$$

p 3 To find diametral pitch (P_d)

$$\text{w.k.t. } P_d = \frac{T}{d} = \frac{36}{149}$$

$$P_d = \mathbf{0.241\ teeth/mm}$$

p 4 To find module (m)

$$\text{w.k.t. module} = m = \frac{d}{T} = \frac{149}{36} = 4.13 \text{ mm/tooth}$$

$$\text{or} \quad m = \mathbf{4.13 \quad mm/tooth}$$

blem 9

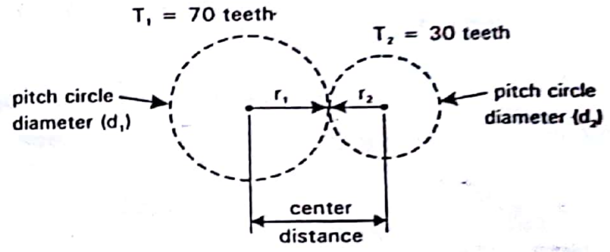
e common module of two mating gears having 70 and 30 teeth respectively is 5. Find the center-center distance between gear axis.

Solution :

Step 1 Data collection

$$T_1 = 70 \text{ teeth}, \ T_2 = 30 \text{ teeth}$$

w.k.t. Teeth are directly proportional to their diameters. The gear arrangement are shown with simple circles in figure below:



Step 2 To find center distance between 2 gear axis

w.k.t. module (m) = $\frac{d}{T}$. The module remains same for **2 meshing gears.**

$$\therefore m = \frac{d_1}{T_1} \text{ for gear 1 and } m = \frac{d_2}{T_2} \text{ for gear 2}$$

$$\therefore \text{pitch circle diameter } d_1 = m \cdot T_1 = 5 \times 70 = 350 \text{ mm}$$

$$d_1 = 350 \text{ mm}$$

$$\text{Similarly } d_2 = m\,T_2 = 5 \times 30 = 150 \text{ mm}$$

$$d_2 = 150 \text{ mm}$$

$$\therefore \text{center-to-center distance} = r_1 + r_2 = \frac{350}{2} + \frac{150}{2} = 175 + 75 = \mathbf{250\ mm}$$

$\therefore$ **center-to-center distance between gear axis = 250 mm**

8.25 EXERCISES

(1) Two pulleys on parallel shafts are connected by a cross-belt. The diameters of the pulleys are 450 mm and 200 mm. The center distance between the shafts is **1.95 m.** Calculate
   a) the length of the belt required. b) the angle of contact between the belt and each pulley.
   c) Power transmitted when the larger pulley rotates at 100 rpm. If the maximum permissible tension in the belt is 1000 N, and $\mu = 0.25$.

Answers: **L = 4.97 m; θ_S = 199.2°; P = 1.37 kW**

(2) Two parallel shafts are 6 m apart and are to be connected by a belt by means of pulleys having diameters 600 mm and 400 mm. Calculate the length of the belt required for

a) open belt drive arrangement b) cross-belt drive arrangement

Answers: **$L_{open\ belt}$ = 13.57 m, $L_{cross\ belt}$ = 13.61 m**

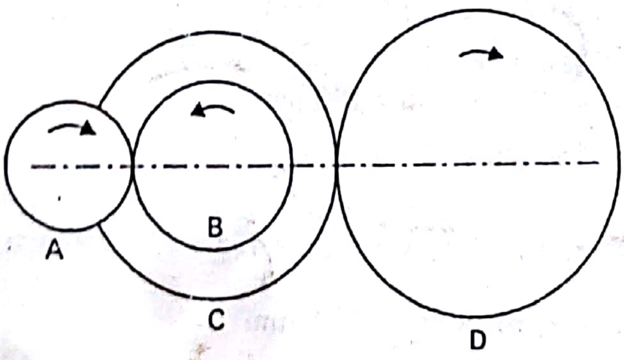
(3) The maximum permissable tension in a flat belt is 1500 N. The co-officient of friction between the belt and the pulley is 0.27. Neglecting the effect of centrifugal tension, calculate the net driving tension and power transmitted if the belt speed is 2 m/s.

Answers: **Tension = 826.7 N, Power = 1.65 kW**

4) A pulley is driven by a flat belt running at a speed of 500 m/min. The co-efficient of friction between the pulley and the belt is 0.3, and the angle of lap is 160°. If the maximum tension in the belt is 700 N, find the power transmitted by the belt. *Jan 07 - 06 m*
Answers: **Power = 3.3 kW**

5) Two pulleys are connected by a belt-drive. The tensions in the slack side and tight side are 800 N and 1200 N respectively. The diameter of the driven pulley is 1 meter, and its speed is 240 rpm. Determine the power transmitted. *Jan 07 - 10 m*
Answers: **Power = P = 5.026 kW**

6) A compound gear consists of 4 gears A, B, C and D and they have 20, 30, 40 and 60 teeth respectively. A is keyed to the driving shaft and D is keyed to the driven shaft. B and C are compound gears, B meshes with A and C meshes with D. Show the arrangements if A rotates at 180 rpm, find the rpm of D. *July 06, July 03 - 05m, 06 m*
Answers: **n_D = 80 rpm**



(7) The sum of diameters of two pulleys is 1000 mm and the pulleys are connected by a belt. If the pulleys rotate at 600 rpm & 1800 rpm, determine the diameter of each pulley and length of the belt in case of a belt drive system. Take center distance between the two pulleys c = 5 mts. *July 10 - 07 m*

(8) Two parallel shafts are 1m apart and are connected by a V-belt arrangement. The driving pulley is supplied with 30 kW and has an effective diameter of 0.4 m. It runs at an angular velocity of 100 rad/sec. The semi-groove angle of the pulley is 20° and the co-efficient of friction between the belt and the pulley rim is 0.28. Determine the tensions in the belt, if the velocity ratio is 3.