Conductivity in semiconductors

Based on the electrical conductivity of the materials they can be classified into three categories.

Conductors- conductors are the materials that allow the electricity to pass through them.

Eg: aluminum, copper, silver, etc.

Insulators- insulators are the materials that do not allow the electricity to pass through them. Eg: paper, glass, etc.

Semiconductors- semiconductors are materials whose electrical conductivity lies between that of conductors and insulators. Eg: silicon and germanium.

Conductivity in semiconductors

Atoms of silicon and germanium have four electrons in their outer most shell. These electrons form covalent bond with the neighbouring atom and not free at low temperature. Hence they behave like insulators. However when a small amount of thermal energy is available from the surroundings a few covalent bonds are broken and few electrons are set free to move. Even at room temperature good number of electrons is dissociated from their atoms and this number increases with rise in temperature. This leads to conductivity. When an electron is detached from the covalent bond, it leaves a vacancy which behaves like a positive charge. An electron from a neighbouring atom can move onto this vacancy leaving a neighbor with a vacancy. Such a vacancy is called a hole. Hole acts as a positive charge.

Types of semiconductors

In a semiconductor there are two kinds of current carriers- Electrons and Hole. In a pure semiconductor electrons and holes are always present in equal numbers and it is called **Intrinsic semiconductor.**

Conductivity of the semiconductors can be changed by adding small amount of impurities (other elements) to it. These impurities are called dopants. Such semiconductors are impure or **Extrinsic semiconductors**. When a few atoms of trivalent or pentavalent element is added into pure crystals of Ge or Si an extrinsic semiconductors are produced. The process of adding impurity atoms is called **Doping**. When pentavalent impurity atoms like arsenic, antimony, phosphorous, etc are added to pure germanium or silicon crystal, we get an extrinsic semiconductor known as **n-type semiconductor**. When trivalent impurity atoms like indium, boron, gallium, aluminum, etc are added to pure germanium or silicon crystal, we get an extrinsic semiconductor known as **p-type semiconductor**.

Concentration of electrons and holes in intrinsic semiconductors

With an increase in temperature covalent bonds are broken in an intrinsic semiconductor and electron- hole pairs are generated. We expect that a large number of electrons can be found in the conduction band and similarly, a large number of holes in the valance bond. As electrons and holes are charged particles, they are together called **charge carriers**. Charge carriers are the number of electrons in the conduction band per unit volume (n) and number of holes in the valence band per unit volume (p) of the material. **Carrier concentration** is known as the density of charge carriers.

Expression for Concentration of electrons in intrinsic semiconductors

Let dn be the number of electrons whose energy lies in the energy interval E and $E+dE$ in the conduction band. Then,

$$dn = g(E)f(E)dE$$

where $g(E)dE$ is the density of states in the energy interval E and $E+dE$ and $f(E)$ probability that a state of energy is occupied by an electron.

The electron density in the conduction band is given by integrating the above equation between the limits E_C and ∞. $E_C\rightarrow$ energy corresponding to bottom edge of the conduction band and $\infty\rightarrow$ energy corresponding to the top edge of the conduction band. Since $f(E)$ of the electrons occupying the upper levels of conduction band rapidly approaches 0 for higher energies, the upper limit is taken as ∞. Then,

$$n = \int_{EC}^{\infty} g(E)f(E)dE$$

The density of states in the conduction band is given by,

$$g(E)dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} E^{1/2} dE \quad for\ E > E_c$$

E_C corresponds to the potential energy of the electron at rest. Thus $(E-E_C)$ will be the kinetic energy of the conduction electron at higher energy levels. Thus,

$$g(E)dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_c)^{1/2} dE$$

The probability of electron occupying an energy level is given by,

$$f(E) = \frac{1}{1 + exp\left[(E - E_f)/kT\right]}$$

When the number of particles is very small compared to the available energy levels, $f(E)$ occupied by more than one electron is very small thus it can be written as

$$f(E) = exp\left[-(E - E_f)/kT\right]$$

Using the value of $g(E)$ and $f(E)$ we can write

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{EC}^{\infty} (E - E_c)^{1/2} e^{(E_f - E_c)/kT} dE$$

or

$$n = \frac{4\pi}{h^2} (2m_e^*)^{3/2} e^{(E_f - E_c)/kT} \int_{E_C}^{\infty} (E - E_c)^{1/2} e^{-(E - E_c)/kT} dE$$

This integral is of the standard form which has a solution of the form

$$\int_0^\infty x^{1/2} e^{-ax}\, dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

where $a = 1/kT$ and $x = (E - E_c)$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2}\, e^{(E_f - E_c)/kT} \left[\frac{\sqrt{\pi}}{2} (kT)^{3/2} \right]$$

Rearranging

$$n = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2} e^{-(E_c - E_f)/kT}$$

This is the expression for the electron concentration in the conduction band of an intrinsic semiconductor. Designating N_c to be the effective density of states in the conduction band as

$$N_c = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2}$$

The electron concentration in the conduction band of an intrinsic semiconductor can be written as

$$n = N_c\, e^{-(E_c - E_f)/kT}$$

Expression for concentration of holes in intrinsic semiconductors

Let dp be the number of holes whose energy lies in the energy interval E and $E+dE$ in the valence band. Then,

$$dn = g(E)[1 - f(E)]dE$$

where $g(E)dE$ is the density of states in the energy interval E and $E+dE$ and $[1 - f(E)]$ probability that a state of energy is occupied not by an electron. Since $f(E)$ is the probability of occupancy of an electron, then the probability that the energy state is vacant is given by $[1 - f(E)]$.

Thus,

$$f(E) = 1 - \frac{1}{1 + e^{-(E_f - E)/kT}} \approx e^{-(E_f - E)/kT}$$

the density of states in the valence band is given by,

$$g(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2}\, E^{1/2}\, dE$$

The top edge of the valence band E_V corresponds to the potential energy of a hole at rest. Thus $(E_V - E)$ will be the kinetic energy of the hole at lower energy levels. Thus,

$$g(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{1/2} dE$$

Number of holes in the energy interval E and $E+dE$ is given by,

$$dp = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{1/2} e^{-(E_f - E)/kT} dE$$

To calculate the hole density this equation should be integrated between limits $-\infty$ and E_V. Thus the hole density is given by,

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_f - E)/kT} dE$$

$$= \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{-(E_f - E_V)/kT} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_V - E)/kT} dE$$

This integral is of the standard form which has a solution of the form

$$\int_0^\infty x^{1/2} e^{-ax} dx = \frac{\sqrt{x}}{2a\sqrt{a}}$$

where $a=1/kT$ and $x=(E_V - E)$, thus $dx=-dE$. The lower limit of the x becomes ∞ when $E=-\infty$. If we change the order of integration we introduce another minus sign.

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{-(E_f - E_V)/kT} \left[\frac{\sqrt{\pi}}{2} (kT)^{3/2} \right]$$

Rearranging

$$p = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2} e^{-(E_f - E_V)/kT}$$

This is the expression for the hole concentration in the valence band of an intrinsic semiconductor. Designating N_V to be the effective density of states in the valence band as

$$N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

The hole concentration in the conduction band of an intrinsic semiconductor can be written as

$$p = N_V \, e^{-(E_f - E_V)/kT}$$

Law of mass action

Statement: for a given semiconductor, the product of the electron and hole concentration is a constant at a given temperature and is equal to the square of the intrinsic carrier concentration.

Considering the electron concentration (n) and hole concentration (p) in a semiconductor,

$$n = N_c \; e^{-(E_c - E_f)/kT}$$

$$p = N_v \; e^{-(E_f - E_v)/kT}$$

Therefore

$$np = N_v \; e^{-(E_f - E_v)/kT} \; N_c \; e^{-(E_c - E_f)/kT}$$

$$np = N_c N_v \; e^{E_g/kT}$$

for an intrinsic semiconductor $n=p=n_i$,

$$np = n_i^2$$

Fermi level in intrinsic semiconductor

In a pure semiconductor, the electrons in the conduction band clusters very close to the bottom edge of the band, and the electrons are located at the bottom edge of the conduction band. Similarly the holes are at the top edge of the valence band. The electron concentration is given by,

$$n = N_c \; e^{-(E_c - E_f)/kT}$$

And the hole concentration is given by,

$$p = N_v \; e^{-(E_f - E_v)/kT}$$

for an intrinsic semiconductor $n=p$

therefore

$$N_c \; e^{-(E_c - E_f)/kT} = N_v \; e^{-(E_f - E_v)/kT}$$

taking log on both sides,

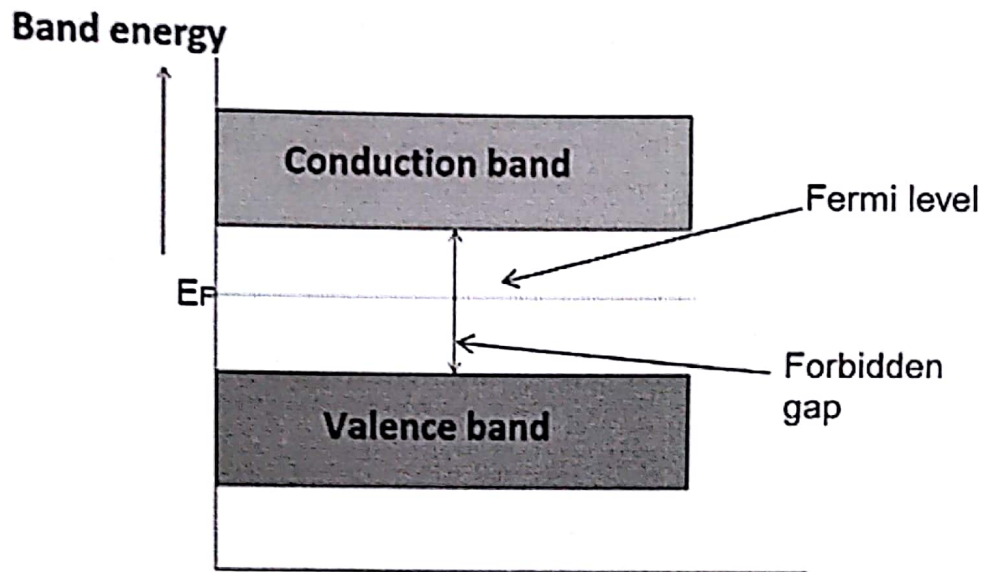$$-\frac{(E_c - E_f)}{kT} = \ln\frac{N_v}{N_c} - \frac{(E_f - E_v)}{kT}$$

considering the value for N_v and N_c and rearranging,

$$E_f = \frac{3}{4}kT \ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{(E_c + E_v)}{2}$$

If the effective mass of free electron is equal to the effective mass of free hole, $m_h^* = m_e^*$

We get,

$$E_f = \frac{(E_c + E_v)}{2} = \frac{E_g}{2}$$

Band energy

Hall Effect

When a conductor carrying a current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both the current and the magnetic field. This phenomenon is known as Hall effect and the generated voltage is called as Hall voltage.

Illustration of hall effect

Consider a thin rectungular semiconductor wafer mounted on an insulating strip and the two parts of electrical contacts are provided on the opposite sides of the wafer. A pair of contact is connected to a constant current source and the other pair is connected to a sensitive voltmeter. The whole arrangement is mounted between two pole pieces of an electromagnet so that the magnetic field acts perpendicular to the lateral faces of the semiconductor wafer.
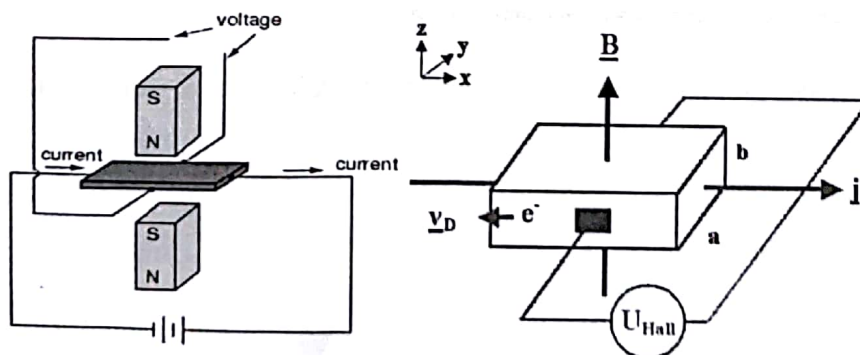
Hall coefficient

Hall coefficient 'R_H' is defined as Hall field per unit current density per unit magnetic induction. When a potential difference is applied across the ends of a p-type semiconductor, the current (I) is given by,

$I = peAv_d$

where p- concentration of holes, e- charge of a hole, A- area of cross-section of the end face, v_d average drift velocity

Current density J is

$$J = \frac{I}{A} = pev_d$$

When the force due to the magnetic field F_L is in equilibrium with the force due to the transverse electric field F_E

$$eE_H = ev_d B$$

i.e.

$$\frac{E_H}{B} = \frac{1}{v_d}$$

since $v_d = \dfrac{J}{pe}$, therefore, $\dfrac{E_H}{JB} = \dfrac{1}{pe}$

the ratio $\dfrac{E_H}{JB}$ is known as Hall co-efficient, given by

$$R_H = \frac{E_H}{JB}$$

For a p-type semiconductor the carriers are positively charged holes,

$$R_H = \frac{1}{pe}$$

For an n-type semiconductor the carriers are negatively charged electrons,

$$R_H = \frac{1}{ne}$$

Hall voltage (V_H)

If a rectangular slab of semiconductor of thickness t and width w is taken. A current I is passed through it with potential difference V. when the slab is placed between two pole pieces of an electromagnet, the Hall voltage V_H is given by,

$$V_H = E_H w = R_H JBw$$

Hall voltage is given by,

$$V_H = \frac{R_H I B w}{wt} = = \frac{R_H I B}{t}$$

Where J=I/A, A=wt.