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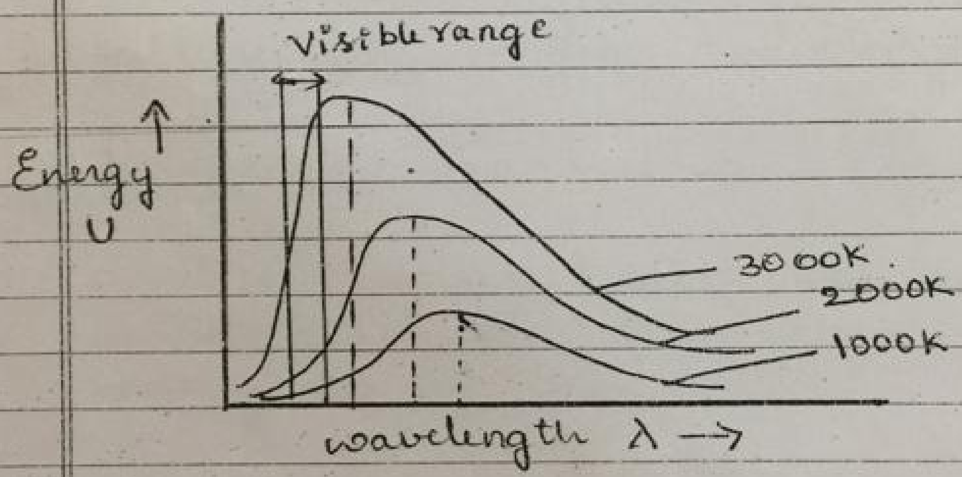
Radiation

The mode of transfer of energy in the form of electromagnetic waves is known as radiation.

Blackbody Radiation

Radiation emitted by black body is termed as Blackbody radiation.

Blackbody Radiation Spectrum



Observations

1. There are different curves for different ~~temps~~ temperatures
2. There is a peak for each of the curve
3. The peak shifts towards shorter wavelengths side as temperature increases

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Wien's law of Energy distribution:

The energy/unit volume for wavelengths in the range, λ and $\lambda+d\lambda$ is given by

$$U_\lambda d\lambda = C_1 \lambda^{-5} e^{-(C_2/\lambda T)} d\lambda$$

where C_1 and C_2 are constants.

Drawbacks of Wien's law:

Wien's law is applicable only for shorter wavelength region and is a failure for longer wavelength region.

Rayleigh - Jean's law:

The energy/unit volume for wavelengths in the range, λ and $\lambda+d\lambda$ is given by

$$U_\lambda d\lambda = 8\pi KT \lambda^{-4} d\lambda$$

where $K \rightarrow$ boltzmann constant

Drawbacks of wien's law:

Rayleigh - Jean's law is suitable only for longer wavelength region and could'nt explain the aspect of very little emission of radiation beyond violet region.

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Planck's law

Assumptions of Quantum theory of radiation.

① The walls of the experimental blackbody consists of a very large number of electrical oscillators vibrating with different frequencies.

② The energy possessed by the oscillators is an integral multiple of $h\nu$ where $h \rightarrow$ planck's constant and $\nu \rightarrow$ frequency of vibration

$$E = h\nu.$$

③ The oscillator may gain or lose energy by absorbing or emitting a radiation of frequency $\nu = \Delta E / h$

where $\Delta E \rightarrow$ energy diffrence between the two states, after & befor emission or absorption.

Based on these assumptions.

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{h\nu/kT}-1}\right] d\lambda$$

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Reduction of Planck's law to Wein's law.

For shorter wavelengths $\lambda \to$ small

$\therefore \nu = \frac{c}{\lambda} \to$ large.

$\nu \to$ large $e^{h\nu/kT}$ is very large.

$e^{h\nu/kT} \gg 1$.

$\therefore e^{h\nu/kT} - 1 \simeq e^{h\nu/kT} \simeq e^{hc/\lambda kT}$

$$\therefore U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT}} d\lambda.$$

$$\underline{U_\lambda d\lambda = C_1 \lambda^{-5} e^{-(C_2/\lambda T)} d\lambda}.$$

where $C_1 = 8\pi hc$.

& $C_2 = hc/k$.

The equation reduces to Wein's law.

D D M M Y Y Y Y

Reduction of Planck's law to Rayleigh-Jeans law

For longer wavelengths. λ - large

$$\nu = \frac{c}{\lambda} \rightarrow \text{small}$$

$\nu \rightarrow$ small $\quad h\nu/kT \rightarrow$ small.

$e^{h\nu/kT}$ from Power series can be expanded as.

$$e^{h\nu/kT} = 1 + \frac{h\nu}{kT} + \left(\frac{h\nu}{kT}\right)^2 + \cdots$$

$$\simeq 1 + \frac{h\nu}{kT}$$

$$\therefore e^{h\nu/kT} - 1 \simeq \frac{h\nu}{kT}.$$

$$\therefore U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{\frac{h\nu}{kT}} d\lambda$$

$$= \frac{8\pi \cancel{hc}}{\lambda^{\cancel{5}4}} \frac{\cancel{\lambda} kT}{\cancel{hc}} d\lambda$$

$$\therefore U_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda.$$

The equation reduces to Rayleigh-Jean's law.

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Wave Particle dualism.

De Broglie suggested that waves sometimes behave as particle and conversely particles can have wave like characteristic properties.

$\therefore$ Particles are associated with matter waves or pilot waves or de Broglie waves.

de Broglie hypothesis.

Let us consider a photon ~~of~~ with frequency ν travelling in the velocity of light c. & wavelength λ.

According to Einstein energy E is given by

$$E = h\nu \qquad h \rightarrow \text{planck's Const.}$$

The momentum P is given by

$$p = \frac{E}{c}$$

$$P = \frac{h\nu}{\nu\lambda} \qquad c = \nu\lambda.$$

$$\Rightarrow \quad P = h/\lambda$$

$$\Rightarrow \boxed{\lambda = \frac{h}{P}} \Rightarrow \boxed{\lambda = \frac{h}{mv}}$$

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de Broglie wavelength in terms of K.E

$$E = \frac{1}{2} m v^2$$

$$= \frac{p^2}{2m}.$$

$$\therefore \quad p = \sqrt{2mE}$$

$$\therefore \quad \boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

de Broglie wavelength in terms of eV.

K.E = Loss in Potential energy

$$\frac{1}{2} m v^2 = eV.$$

$$\Rightarrow m^2 v^2 = 2meV.$$

$$\Rightarrow p^2 = 2meV.$$

$$\Rightarrow p = \sqrt{2meV}$$

$$\therefore \quad \boxed{\lambda = \frac{h}{\sqrt{2meV}}} \quad \Rightarrow \lambda = \frac{12.28}{\sqrt{V}} \text{ Å}$$

D D M M Y Y Y Y

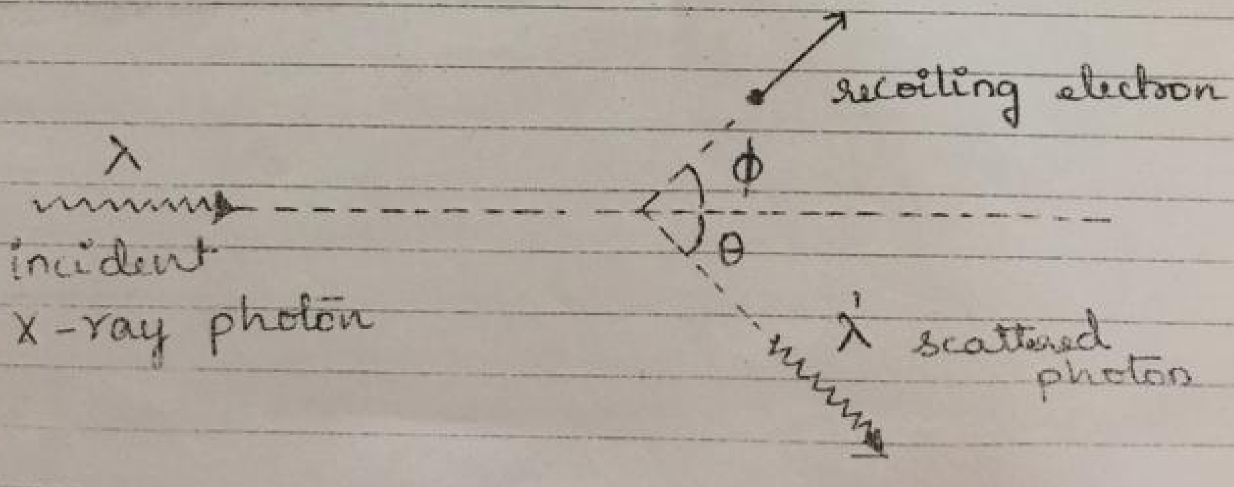
Compton Effect

The scattering of a photon by an electron is called Compton scattering

when a beam of x-rays (photon) of energy $E = hc/\lambda$ collides with an electron. the photon loses its energy and reduces to $E' = hc/\lambda'$ where λ & λ' are initial & final wavelengths. If the photon is scattered by an angle θ and the electron recoils by an angle ϕ, then the change in wavelength $\Delta\lambda = \lambda' - \lambda$ is given by the equation.

$$\Delta\lambda = (\lambda' - \lambda) = \frac{h}{m_0 c}(1 - \cos\theta)$$

where $m_0 \rightarrow$ rest mass of electron.



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The quantity (h/m_0c) is called Compton wavelength

The effect due to which there is an increase in wavelength accompanied by a change in the direction of the scattered X-rays compared to that of the incident x-rays. Consequent to the exchange of energy between the X-ray photons and the electrons in the target material, is called Compton effect

Physical significance of compton effect.

Compton effect demonstrates particle nature of x-rays.
In other words it signifies the particle nature of waves.

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Matter waves:

Waves associated with moving particles are called Matter waves.

Characteristic Properties of Matter waves

1) Matter waves cannot be observed. It is a wave model to describe and study matt
2) Matter waves travell even in vacuum, hence they are not mechanical wave.
3) Matter waves are ~~Pa~~ Probabilistic waves. as they represent the probability of finding a particle in space.
4) The phase velocity of matter waves can be greater than that of light.
5) Waves associated with macroparticles cannot be determined but with that of microparticles can be determined.
6) diffrent matter waves have diffrent phase velocities. Since mass and velocity are inversely proportional to wavelength.
7) Matter waves are not electromagnetic waves in nature
8) The Velocity of matter waves depends on the velocity of the material particle.

D D M M Y Y Y Y

Heisenberg's Uncertainty Principle.

1) In any simultaneous determination of the position (x) and momentum (p) of a particle, the product of the corresponding uncertainties ($\Delta x \, \Delta p_x$) inherently present in the measurement is equal to or greater than $h/4\pi$.

$$\Delta x \, \Delta p_x \geqslant \frac{h}{4\pi}.$$

2) In any simultaneous determination of energy (E) and time (t) in a physical process the product of the corresponding uncertainties ($\Delta E \, \Delta t$) inherently present in the measurement is equal to or greater than $\frac{h}{4\pi}$

$$\Delta E \, \Delta t \geqslant \frac{h}{4\pi}.$$

3) In any simultaneous determination of angular displacement (θ) and angular momentum (L) of a particle, the product of the corresponding uncertainties ($\Delta L \Delta \theta$) inherently present in the measurement is equal to or greater than $h/4\pi$

$$\Delta L \, \Delta \theta \geqslant \frac{h}{4\pi}.$$

Physical significance of HUP

1) It is impossible to determine precisely and simultaneously the value of both the position and momentum of a particl at the same time.

2) We can determine the probability of finding the particle at a certain position or probable value for the momentum.

3) The Probabilities can be determined by functions such as Probability density function in Quantum Mechanics.

Applications of Uncertainty Principle.

1) Non-existence of Electron in the nucleus.

2) Explanation for β-decay and Kinetic Energy of β-Particles.

D D M M Y Y Y Y

Non-existence of electron in the nucleus

According to the theory of relativity the energy 'E' of a particle moving with the speed of light and having mass 'm' and moving with velocity v and having rest mass m_0 is given by

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}.$$

$$\Rightarrow E^2 = \frac{m_0^2 c^4}{1 - v^2/c^2}$$

$$\Rightarrow E^2 = \frac{m_0^2 c^6}{c^2 - v^2} \quad \text{-----} \quad (1)$$

The momentum of the particle is given by

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow p^2 = \frac{m_0^2 v^2}{(1 - v^2/c^2)} = \frac{m_0^2 v^2 c^2}{c^2 - v^2}$$

$$\Rightarrow p^2 c^2 = \frac{m_0^2 v^2 c^4}{c^2 - v^2} \quad \text{-----} \quad (2)$$

D D M M Y Y Y Y

Eq^n (1) − Eq^n (2)

$$E^2 - p^2c^2 = \frac{m_o c^4 (c^2 - v^2)}{(c^2 - v^2)}$$

$$\Rightarrow E^2 - p^2c^2 = m_o^2 c^4$$

$$\Rightarrow E^2 = p^2c^2 + m_o^2 c^4 \quad \text{----} \quad (3)$$

According to Heisenberg's Uncertainty principle.

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

Considering the electron to be in the nucleus. the maximum Δx should be the size of the nucleus.

$$\therefore \Delta x \leq 5 \times 10^{-15} m$$

$$\therefore \Delta P_x \geq \frac{h}{4\pi \times \Delta x}$$

$$\therefore \Delta P_x \geq 1.1 \times 10^{-20} \, kg\,m\,s^{-1}$$

taking the value of ΔP_x and value of m_o as 9.11×10^{-31} kg

we get the energy.

$$E \simeq 3.3 \times 10^{-12} J$$

D D M M Y Y Y Y

$\Rightarrow E \geq 20.6\ MeV$.

$\Rightarrow$ If an electron exists within the nucleus it must have a minimum energy of about 20 MeV.

But the maximum kinetic energy with which a β particle emitted from a radio active nuclei is of the order of 4 MeV.

$\therefore$ An electron cannot exist inside the nucleus.

Wave function:

A variable quantity that characterises the de Broglie wave is called the wave function. denoted by ψ

A wave function determines the entire space time behaviour of the system, so it it is also called the state function

$$\psi = A e^{i(kx - \omega t)}$$

where $A \rightarrow$ amplitude
$\omega \rightarrow$ angular frequency.
$k \rightarrow$ wave vector
$t \rightarrow$ time

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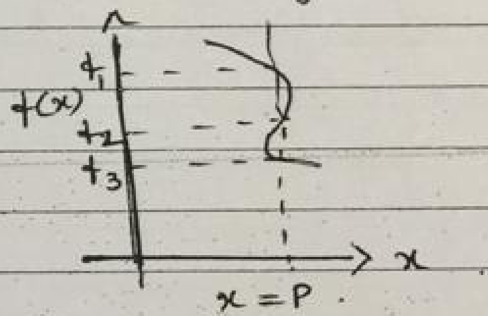
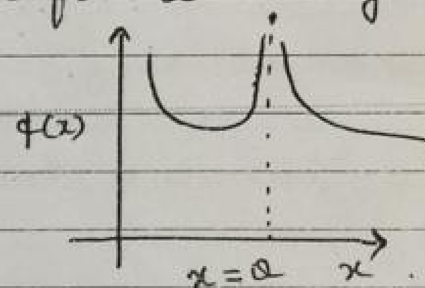
Properties of a wave function

1) Wave function ψ should be single valued everywhere.
If it is not single valued then the wave function is not acceptable.



multivalued function

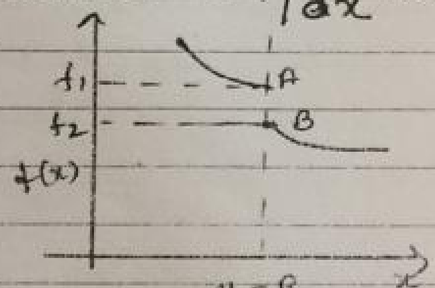
2) Wave function ψ should be finite everywhere.
the wave function is not acceptable if it reches infinite value at any point of time.



function not finite at a point.

3) Wave function ψ and its derivative $\delta\psi/\delta x$ should be continuous everywhere.

discontinuous functions are not acceptable.



discontinuous function

4) For bound stages wavefunction ψ should vanish at infinity.

D D M M Y Y Y Y

Physical significance of wavefunction:

1) It gives a statistical relationship between the particle and wave nature

2) It is a complex quantity and hence one cannot measure it.

3) It is a function of wave and time coordinate. Hence it cannot locate the position of a particle.

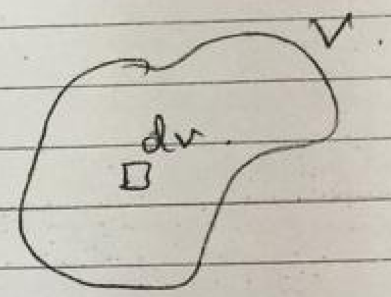
Physical significance of a wave function can be understood with Probability density and normalization

Probability density

The Probability of finding a particle in a small element of the given volume is termed as Probability density and is the square of the magnitude of the wave function ψ.

$$P\,D = |\psi|^2 = \psi^* \psi$$

if ψ is the wave function.
Let V be the given volume
Let dv be the small element in the given volume.



then the Probability of finding a particle in dv is given by

$$= |\psi|^2 dv$$

where $|\psi|^2 \rightarrow$ Probability density.

<u>Normalisation.</u>

when the particle is present in the given volume then the Probability density $\int_0 |\psi|^2 dv = 1$

If one wants to locate the particle anywhere in the given space then the limits will be $-\infty$ to ∞.

$$\int_{-\infty}^{\infty} |\psi|^2 dv = 1.$$

$\hat{O}\psi = \lambda\psi$

$\frac{d}{dx} e^{ax} = a e^{ax}$

This becomes the working equation for the wave function and the process is called normalisation.

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Schrodinger wave equation

Schrodinger developed a mathematical equation in two forms to rep. the dual nature of waves. These equation can be solved by knowing.

1. Potential energy of the particle
2. Initial conditions.
3. Boundary Conditions.

Form 1 - Time dependent SWE.

$$\frac{-h^2}{8\pi^2 m} \frac{d^2\psi}{dx} + V\psi = -\frac{ih}{2\pi} \frac{d\psi}{dt}$$

Form 2 :-

Time independent SWE

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h} (E-V)\psi = 0$$

Time independent SWE is applicable only to steady state Conditions. and wave function varies only with position but not with time

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Time independent Schrodinger wave equation

According to the de Broglie theory for a particle of mass m, moving with a velocity v the wavelength(λ) is given by.

$$\boxed{\lambda = h/p} \quad \text{——— ①}$$

where $p = mv$.

The wave function for a de Broglie wave travelling in +ve x-axis is given by

$$\psi = A e^{i(kx - \omega t)}.$$

taking the 2nd differentiation value we get:

$$\boxed{\frac{d^2\psi}{dt^2} = -\omega^2 \psi} \quad \text{——— ②}$$

For a travelling wave we have

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}.$$

Illy for de Broglie wave function we have.

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2} \frac{d^2\psi}{dt^2}.$$

from eq^n ②

$$\boxed{\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi} \quad \text{——— ③}$$

D D M M Y Y Y Y

we know $\omega = 2\pi\nu$ where ν - frequency
and
$v = \nu\lambda$ where λ - wavelength

$\therefore$ eq^n 3 becomes.

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2\cancel{\nu^2}}{\cancel{\nu^2}\lambda^2}\psi.$$

$$\Rightarrow \frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi.$$

re-arranging

$$\boxed{\frac{1}{\lambda^2} = \frac{-1}{4\pi^2\psi}\frac{d^2\psi}{dx^2}} \quad \text{——— (4)}.$$

The Kinetic energy of the particle of mass m and moving with a velocity v is given by

$$K.E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m}$$

$$\Rightarrow K.E = \frac{P^2}{2m}.$$

by de Broglie eq^n 1 we get

$$\boxed{K.E = \frac{h^2}{2m}\frac{1}{\lambda^2}} \quad \text{——— (5)}.$$