

22417.202210 Homework 0 - Background

Pankaj Kumar Jatav

TOTAL POINTS

51.3 / 60

QUESTION 1

Course Policies 6 pts

1.1 Course Policies 1 1 / 1

✓ - 0 pts Correct

2.2 Q8 1 / 1

✓ - 0 pts Correct

1.2 Course Policies 2 1 / 1

✓ - 0 pts Correct

2.3 Q9 1 / 1

✓ - 0 pts Correct

1.3 Course Policies 3 1 / 1

✓ - 0 pts Correct

2.5 Q11 2 / 2

✓ - 0 pts Correct

1.4 Course Policies 4 1 / 1

✓ - 0 pts Correct

2.6 Q12 1 / 1

✓ - 0 pts Correct

1.5 Course Policies 5 1 / 1

✓ - 0 pts Correct

2.7 Q13 1 / 1

✓ - 0 pts Correct

1.6 Course Policies 6 1 / 1

✓ - 0 pts Correct

2.8 Q14 1 / 1

✓ - 0 pts Correct

QUESTION 2

Probability and Statistics 23 pts

2.1 Q7 0 / 1

✓ - 1 pts We need to take into account all 5 possible combinations that yield 4H-1T. So it's 5 times the fourth option (== the first option)

2.9 Q15 1 / 1

✓ - 0 pts Correct

2.10 Q16 1 / 1

✓ - 0 pts Correct

2.11 Q17 0.3 / 1

✓ - 0.7 pts Two correct answers missing. A-B, B-C,

and A-C are all not independent.

✓ - 0 pts Correct

2.12 Q18 2 / 2

✓ - 0 pts Correct

2.13 Q19 0 / 2

✓ - 2 pts This value **can** be computed.

The numerator if we try to compute this value will be $\$P(B=b_2 \cap A=a_3 \cap C=c_3)$ and we know from the table that $\$P(B=b_2 \cap C=c_e)=0$ which means that $B=b_2$ and $C=c_3$ cannot co-occur). So this value will be 0.

2.14 Q20 0 / 2

✓ - 2 pts Both sides of the equation some up to 1! So the equation is true.

2.15 Q21 2 / 2

✓ - 0 pts Correct

2.16 Q22 1 / 1

✓ - 0 pts Correct

2.17 Q23 2 / 2

✓ - 0 pts Correct

QUESTION 3

Calculus 8 pts

3.1 Q24 2 / 2

✓ - 0 pts Correct

3.2 Q25 2 / 2

✓ - 0 pts Correct

3.3 Q26 2 / 2

3.4 Q27 2 / 2

✓ - 0 pts Correct

QUESTION 4

Vectors and Matrices 13 pts

4.1 Q28 1 / 1

✓ - 0 pts Correct

4.2 Q29 1 / 1

✓ - 0 pts Correct

4.3 Q30 2 / 2

✓ - 0 pts Correct

4.4 Q31 2 / 2

✓ - 0 pts Correct

4.5 Q32 1 / 1

✓ - 0 pts Correct

4.6 Q33 2 / 2

✓ - 0 pts Correct

4.7 Q34 2 / 2

✓ - 0 pts Correct

4.8 Q35 1 / 2

✓ - 1 pts The solution could be correct. But eigenvectors do not need to be linearly dependent. The correct answer is none of the above.

QUESTION 5

CS Foundations 10 pts

5.1 Q36 1 / 1

✓ - 0 pts *Correct*

5.2 Q37 0 / 1

✓ - 1 pts *f(n) is polynomial. g(n) is exponential. So the correct answer is the first one.*

5.3 Q38 2 / 2

✓ - 0 pts *Correct*

5.4 Q39a 0 / 1

- 1 Point adjustment

5.5 Q39b 1 / 1

✓ - 0 pts *Correct*

5.6 Q40 2 / 2

✓ - 0 pts *Correct*

5.7 Q41 2 / 2

✓ - 0 pts *Correct*

QUESTION 6

6 Collaboration Questions 0 / 0

✓ - 0 pts *Correct*

HOMEWORK 0 BACKGROUND¹

CS 688 MACHINE LEARNING (SPRING 2022)

<https://nlp.cs.gmu.edu/course/cs688-spring22/>

OUT: Jan 27, 2022

DUE: Feb 2, 2022

Student Name: Pankaj Kumar Jatav

Student GID: G01338769

¹Compiled on Wednesday 2nd February, 2022 at 23:27

Instructions for Specific Problem Types

For “Select One” questions, please fill in the appropriate bubble completely:

Select One: Who is teaching this course?

- Antonios Anastopoulos
- Marie Curie
- Noam Chomsky

If you need to change your answer, you may cross out the previous answer and bubble in the new answer:

Select One: Who is teaching this course?

- Antonios Anastopoulos
- Marie Curie
- Noam Chomsky

For “Select all that apply” questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

- Stephen Hawking
- Albert Einstein
- Isaac Newton
- None of the above

Again, if you need to change your answer, you may cross out the previous answer(s) and bubble in the new answer(s):

Select all that apply: Which are scientists?

- Stephen Hawking
- Albert Einstein
- Isaac Newton
- I don't know

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

688

788 688

1 Written Questions [60 pts]

In this section, you will work through a number of problems covering prerequisite material: probability, statistics, calculus, linear algebra, and computer science. The first subsection covers common course policy questions

1.1 Course Policies

This section covers important course policies that every student should know and understand. These questions MUST be finished in order for the whole homework to be considered for grading.

1. (1 point) Assignments turned in late without prior approval will incur a daily penalty. How much is the penalty? Up to 1 day: ___ Up to 2 day: ___ Up to 3 day: ___ Up to 4 day: ___

Select one:

- 5%, 10%, 15%, 20%
- 10%, 20%, 30%, 40%
- 25%, 50%, 75%, 100%
- 20%, 40%, 60%, 80%

2. (1 point) How many grace days do you have in total for all homework? Can you combine grace days with late days to extend a homework submission deadline by more than 4 days?

Select one:

- As many as I want; Of course!
- 5; No
- 5; Yes
- 8; Yes

3. (1 point) Seeking help from other students in understanding course materials needed to solve homework problems is ALLOWED under which of the follow conditions?

Select all that apply:

- Any written notes are taken on an impermanent surface (e.g. whiteboard, chalkboard) and discarded before writing up one's solution alone.
- Learning is facilitated not circumvented; i.e., the purpose of seeking help is to learn and understand the problem instead of merely getting an answer
- Help both given and received is reported in collaboration questions in the homework
- The student updates their collaborative questions even if it is after submitting their own assignment
- None of the above

4. (1 point) Which of the following is (are) strictly forbidden in solving and submitting homework?

Select all that apply:

- Searching on the internet for solutions or sample codes
- Consulting people outside this class who have seen or solved the problem before
- Turning in someone else's homework
- Using anyone else's, or allowing other classmates to use your computer or Gradescope account in connection with this course
- None of the above

5. (1 point) If you solved your assignment completely on your own, you can skip the collaboration questions at the end of each homework.

Select one:

- True
- False

6. (1 point) Assume a difficult situation arises in the middle of the semester (e.g. medical, personal etc.) that might prevent you from submitting assignments on time or working as well as you would like. What should you do? Select all that apply

- Talk to the course staff early so they can point you to the available resources on campus and make necessary arrangements
- Do not speak to the course staff, try to finish the class, reach out to the course staff in the end of the semester explaining your special situation
- Reach out to your academic advisor so that they are aware of the situation
- None of the above

1.2 Probability and Statistics

Use the following data to answer questions 7-8. Consider data created by flipping a coin five times $S = [1, 1, 0, 1, 1]$, where 1 denotes that the coin turned up heads and 0 denotes that it turned up tails.

7. (1 point) What is the probability of observing any combination of this data (4 heads and 1 tails), assuming it was generated by flipping a coin X with an unequal probability of heads (1) and tails (0), where the distribution is $P(X = 1) = 0.75$, $P(X = 0) = 0.25$?

Select one:

$\frac{405}{1024}$

$\frac{1}{32}$

$\frac{324}{1024}$

$\frac{81}{1024}$

8. (1 point) Note that the probability of this data sample would be greater if the value of $P(X = 1)$ was not 0.75, but instead some other value. What is the value of $P(X = 1)$ that maximizes the probability of the sample S ? Provide your answer as a fraction.

4/5

9. (1 point) State true or false. For events A and B, where $A \cap B$ indicates A AND B, and $A \cup B$ indicates A OR B,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Select one:

True

False

10. (1 point) State true or false. For events A and B,

$$P(A_1 \cap A_2 \cap A_3) = P(A_3|A_2 \cap A_1)P(A_2|A_1)P(A_1)$$

Select one:

True

False

11. (2 points) Whether your car is wet in the morning (W) is dependent on whether it rained last night (R) or not, however other factors may have lead to your car being wet. The following are probabilities of such events:

$$\begin{aligned}P(R) &= 0.4 \\P(W|R) &= 0.8 \\P(W|\neg R) &= 0.2\end{aligned}$$

What is the probability that your car is wet in the morning?

Select one:

- 0.64
- 0.56
- 0.44
- 0.4

Use the following information to answer questions 12-13. Consider the following joint probability table where both X and Y are binary variables:

X	Y	Probability
0	0	0.1
0	1	0.2
1	0	0.4
1	1	0.3

12. (1 point) What is $P(X = 1|Y = 1)$?

Select one:

- $\frac{2}{3}$
- $\frac{3}{7}$
- $\frac{4}{5}$
- $\frac{3}{5}$

13. (1 point) What is $P(Y = 0)$?

Select one:

- 0.2
- 0.6
- 0.5
- 0.3

Use the following information to answer questions 14-16. Let X be a random variable and the expected value of X is $E[X] = 1$ and the variance of X is $Var[X] = 1$.

14. (1 point) What is $E[6X]$?

Select one:

- 1
- 3
- 6
- 36

15. (1 point) What is $Var[3X]$?

Select one:

- 1
- 3
- 6
- 9

16. (1 point) What is $Var[2X + 3]$?

Select one:

- 3
- 4
- 5
- 7

Use the following information to answer questions 17-20: Let A, B, and C be random variables with discrete probability distributions. Consider the following two joint probability tables: one relating A and B, and the other relating B and C.

$A \setminus B$	b_1	b_2	b_3	$B \setminus C$	c_1	c_2	c_3	c_4
a_1	0.1	0.05	0.15	b_1	0.02	0.14	0.06	0.03
a_2	0.1	0.05	0.3	b_2	0.03	0.05	0	0.17
a_3	0.05	0.15	0.05	b_3	0.35	0.04	0	0.11

17. (1 point) Which of the following statements are necessarily **false**? Note $X \perp\!\!\!\perp Y$ indicates that random variable X is independent of random variable Y.

Select all that apply:

- A $\perp\!\!\!\perp$ B
- B $\perp\!\!\!\perp$ C
- A $\perp\!\!\!\perp$ C
- None of the above.

18. (2 points) What is $P(B = b_1 | A = a_2, C = c_4)$? If this value cannot be computed, write N/A.

19. (2 points) What is $P(B = b_2 | A = a_3, C = c_3)$? If this value cannot be computed, write N/A.

20. (2 points) True or False: $\sum_{i=1}^3 P(B = b_i | C = c_1) = \sum_{j=1}^4 P(C = c_j | B = b_1)$

Select one:

- True
- False

21. (2 points) Consider two random variables X, Y . Assume that we have $P(X = x) = \frac{1}{2^x}$ for $x \in \mathbb{Z}_{\geq 1}$ (integers greater than or equal to 1) and $P(Y = y | X = x) = \frac{1}{n}$ for $y \in \{1, 2, \dots, n\}$. Assume n is a fixed positive integer constant. What is $\mathbb{E}[Y]$?

Select one:

- $\sum_{y=1}^n y \frac{1}{2^y}$
- $\sum_{y=1}^n y \frac{5}{3^y}$
- $\sum_{y=1}^n \frac{y}{n}$
- $\sum_{y=1}^n y$

22. (1 point) What is the mean, variance and entropy of a Bernoulli (p) random variable?

Select one:

- $p, p(1 - p), -(1 - p) \log(1 - p) - p \log(p)$
- $p(1 - p), p, -(1 - p) \log(1 - p) - p \log(p)$
- $p, p(1 - p), \log(1 - p) - p \log(p)$
- The entropy of a Bernoulli variable is not defined.

23. (2 points) Please match the probability density function of the random variable X to its corresponding distribution name.

1. $\text{prob}(X = x) = \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} \exp(-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu))$
2. $\text{prob}(X = x) = \lambda e^{-\lambda x}$ when $x \geq 0$; 0 otherwise
3. $\text{prob}(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$
4. $\text{prob}(X = x) = \frac{1}{b-a}$ when $a \leq x \leq b$; 0 otherwise
5. $\text{prob}(X = x) = p^x (1 - p)^{1-x}$

Multivariate Gaussian: 1

Exponential: 2

Uniform: 4

Bernoulli: 5

Binomial: 3

1.3 Calculus [8pts]

24. (2 points) Evaluate the derivative of y with respect to x , where $y = \ln(\frac{4}{x^2} - x^3)$ at $x = 1$.

-11/3

25. (2 points) Find the partial derivative of y with respect to x , where $y = 3x^2 \sin(z)e^{-x}$

Select one:

- $3x \sin(z)e^{-x}(2 + x)$
- $-6x \sin(z)e^{-x}$
- $3x \sin(z)e^{-x}(2 - x)$
- $6x \cos(z)e^{-x}$

26. (2 points) For the function $f(x) = 5x^3 + 2x^2 - 3x$ the value $x = \frac{1}{3}$ sets the derivative to be 0. Additionally, the second order derivative of $f(x)$ at $x = \frac{1}{3}$ is positive. What can you say about $f(x)$ at the point $\frac{1}{3}$:

Select one:

- a local minimum
- a local maximum
- a local minimum or a local maximum
- None of the above

27. (2 points) Suppose that $f(\mathbf{x}|\boldsymbol{\theta}) = \mathbf{x}^T \boldsymbol{\theta}$, where $\mathbf{x}, \boldsymbol{\theta} \in \mathcal{R}^n$. The function $g(\boldsymbol{\theta})$ is defined as $g(\boldsymbol{\theta}) = (f(\mathbf{x}^{(1)}|\boldsymbol{\theta}) - y^{(1)})^2$ for $\mathbf{x}^{(1)} \in \mathcal{R}^n$ and $y^{(1)} \in \mathcal{R}$. What is the function type of $g(\boldsymbol{\theta})$:

Select one:

- $g : \mathcal{R}^n \rightarrow \mathcal{R}$
- $g : \mathcal{R} \rightarrow \mathcal{R}$
- $g : \mathcal{R} \rightarrow \mathcal{R}^n$
- $g : (\mathcal{R}^n \times \mathcal{R}^n) \rightarrow \mathcal{R}$

1.4 Vectors and Matrices

28. (1 point) Consider the matrix \mathbf{X} and the vectors \mathbf{y} and \mathbf{z} below: $\mathbf{X} = \begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{z} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is the inner product of the vectors \mathbf{y} and \mathbf{z} ? (this is also sometimes called the dot product)

Select one:

$\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

9

$\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

7

29. (1 point) Using the same values for \mathbf{X} , \mathbf{y} , and \mathbf{z} as above, what is the product of $\mathbf{X}\mathbf{y}$?

Select one:

$\begin{bmatrix} 10 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 10 \end{bmatrix}$

$\begin{bmatrix} 7 \\ 11 \end{bmatrix}$

$\begin{bmatrix} 14 \\ 22 \end{bmatrix}$

30. (2 points) Consider $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{V} = \begin{bmatrix} 0 & 7 \\ 4 & 5 \\ -1 & 0 \end{bmatrix}$. Which of these are valid operations?

Select all that apply:

$\mathbf{u}^T\mathbf{V}$

$\mathbf{V}^T\mathbf{u}$

$\mathbf{u}\mathbf{V}$

$\mathbf{V}\mathbf{V}$

None of the above

31. (2 points) For the matrices $\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ -3 & 2 & 0 \\ 1 & 3 & -2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & 4 & 5 \\ 3 & -1 & 3 \\ 1 & 3 & -2 \end{bmatrix}$ What is the product \mathbf{AB} ?

Select one:

$\begin{bmatrix} 13 & 19 & 5 \\ -3 & -14 & -9 \\ 4 & -4 & 18 \end{bmatrix}$

$\begin{bmatrix} 13 & 19 & 28 \\ 19 & 9 & -7 \\ -10 & -2 & 13 \end{bmatrix}$

$\begin{bmatrix} 20 & -20 & -28 \\ 3 & -14 & 9 \\ 3 & 2 & 13 \end{bmatrix}$

$\begin{bmatrix} 13 & 19 & 5 \\ -3 & -14 & -9 \\ 10 & -5 & 18 \end{bmatrix}$

32. (1 point) True or False: The matrix \mathbf{A} from the previous question has an inverse?

Select one:

True

False

33. (2 points) Consider two vectors $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$, let $z = \mathbf{x}^T \mathbf{y}$. What is $\frac{\partial z}{\partial y_2}$?

Select one:

y_2

x_2

\mathbf{x}

\mathbf{y}

34. (2 points) Given matrix $\mathbf{X} = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix}$ and the column vector $\mathbf{y} = \begin{bmatrix} -6 \\ 1 \\ 1 \end{bmatrix}$, what is the eigenvalue of

\mathbf{X} associated with \mathbf{y} ? (Recall an eigenvector of a matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ is a nonzero vector $\mathbf{v} \in \mathbb{R}^n$ such that $\mathbf{Av} = \lambda\mathbf{v}$ where we call the scalar λ the associated eigenvalue for \mathbf{v} .)

Select one:

- y is not an eigenvector
- 3
- 2
- 1.5

35. (2 points) Preparing for his linear algebra final, Joe is finding eigenvectors and eigenvalues for different matrices. For one matrix (not given), he finds the following two **distinct** eigenvectors corresponding to

an eigenvalue of 4: $\begin{bmatrix} 3 \\ 117 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 39 \\ 3 \end{bmatrix}$. Which statement regarding his solution is true?

Select all that apply:

- The solution must be wrong because there cannot be multiple eigenvectors corresponding to a single eigenvalue.
- The solution must be wrong because the eigenvectors are linearly dependent.
- The solution is correct because eigenvectors should be linearly dependent and there may be multiple eigenvectors corresponding to an eigenvalue.
- None of the above.

1.5 CS Foundations

36. (1 point) If $f(n) = \ln(n)$ and $g(n) = \log_3(n)$ which of the following are true?

Select one:

- $f(n) \in O(g(n))$
- $g(n) \in O(f(n))$
- Both
- Neither

37. (1 point) If $f(n) = n^{10}$ and $g(n) = 10^n$ which of the following are true?

Select one:

- $f(n) \in O(g(n))$
- $g(n) \in O(f(n))$
- Both
- Neither

Britain's Royal Family
Review the royal family's line of succession to the throne.

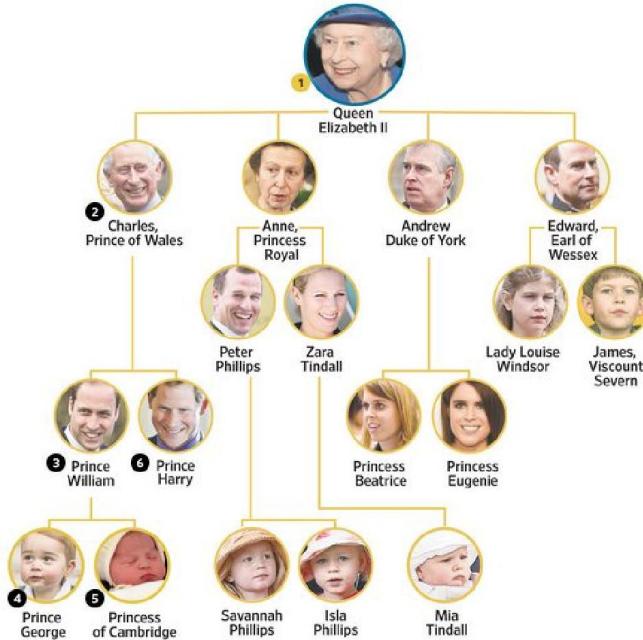


Figure 1.1: Britain's Royal Family

38. (2 points) Using the tree shown in Figure 1.1, how many nodes would depth-first-search visit in finding Mia Tindall (including her node)? Assuming we search left-to-right and top-down.

Select one:

- 3
- 12
- 15
- 18

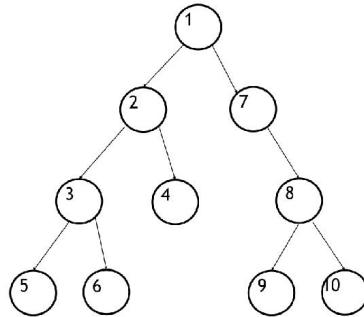


Figure 1.2: A Binary Tree with indexed nodes

39. (2 points) Figure 1.2 is a Binary Tree with indexed nodes. Assume root node is node 1. What is the node-visit order of **DFS** and **BFS** of the above Binary Tree?

A depth-first search (DFS) traversal of a binary tree starts with visiting the root node, and recursively searches down the left subtree (i.e., the tree rooted at the left node) before going to search the right subtree (i.e., the tree rooted at the right node) until the traversal is done.

Note: Alternatively, we can also look right subtree before left subtree too, for the question please consider left to right order!

A breadth-first search (BFS) traversal of a binary tree visits every node (assuming a left-to-right order) on a level (with the same distance to the root) before going to a lower level until the traversal is done.

The node-visit order of DFS is:

1 2 3 4 5 6 7 8 9 10

The node-visit order of BFS is:

1 2 7 3 4 8 5 6 9 10

40. (2 points) Fill in the blanks in the pseudo code for key search using recursive depth-first search (DFS) traversal.

```
class TreeNode:  
    def __init__(self, val):  
        self.val = val  
        self.leftNode = None  
        self.rightNode = None  
  
    # (a) the left/right node is denoted as  
    #      node.leftNode/node.rightNode  
    # (b) left/right node are of type TreeNode  
    # (c) the value of the node is denoted as node.val  
    # (d) recursive DFS to search for the node  
    #      with value key in a binary tree  
    # (e) the left node is assumed to be searched  
    #      before the right node  
  
def find_val(node, key):  
    if node is None:  
        return None  
  
    if (1) node.val == key :  
        return node  
  
    else:  
        result = find_val(node.leftNode, val)  
  
        if result is None:  
            result = find_val(node.rightNode, val)  
  
    return result
```

Consider writing a recursive program to solve the following question:

Lucas numbers are defined as:

$$L_n = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ L_{n-1} + L_{n-2} & \text{if } n > 1 \end{cases}$$

41. (2 points) Which of the following is the numerical value for L_{32} ?

Select one:

- 3010349
- 3524578
- 4870847
- 7881196

Collaboration Questions Please answer the following:

1. Did you receive any help whatsoever from anyone in solving this assignment? **No.**
 - If you answered ‘yes’, give full details: _____
 - (e.g. “Jane Doe explained to me what is asked in Question 3.4”)
2. Did you give any help whatsoever to anyone in solving this assignment? **No.**
 - If you answered ‘yes’, give full details: _____
 - (e.g. “I pointed Joe Smith to section 2.3 since he didn’t know how to proceed with Question 2”)
3. Did you find or come across code that implements any part of this assignment ? **No.** (See below policy on “found code”)
 - If you answered ‘yes’, give full details: _____
 - (book & page, URL & location within the page, etc.).