Code : 326101

MCA 1st Semester Exam., 2024

COMPUTATIONAL MATHEMATICS

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **SEVEN** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question Nos. **1** and **2** are compulsory.*

SECTION—A

1. Choose the correct answer from the following : 2×10=20

(a) Let $\Sigma = \{a, b, \cdots, z\}$ and A = {Hello, World}, B = {Input, Output}, then

$$(A^* \cap B) \cup (B^* \cap A)$$

can be represented as

(i) {Hello, World, Input, Output, ε}

(ii) {Hello, World, ε}

(iii) {Input, Output, ε}

(iv) {}

(b) The maximum sum of in degree and out degree over a state in a DFA can be determined as :

$$\Sigma = \{a, b, c, d\}$$

(i) 4 + 4

(ii) 4 + 16

(iii) Depends on the language

(iv) 4 + 0

(c) Suppose a language $L1$ has 2 states and $L2$ has 2 states. After using the cross-product construction method, we have a machine M that accepts $L1 \cap L2$. The total number of states in M is

(i) 6

(ii) 4

(iii) 2

(iv) 8

(d) Which of the following statements is false?

(i) Context-free language is the subset of context sensitive language.

(ii) Regular language is the subset of context sensitive language.

(iii) Recursively enumerable language is the superset of regular language.

(iv) Context sensitive language is a subset of context-free language.

(e) Push-down automata accepts _____ languages.

(i) type 3

(ii) type 2

(iii) type 1

(iv) type 0

(f) Context-free languages are not closed under

(i) intersection

(ii) intersection with regular language

(iii) complement

(iv) All of the above

(g) Using the pumping constant n, if there is a string in the language of length between _____ and _____, then the language is infinite else not.

(i) n, $2n-1$

(ii) $2n$, n

(iii) $n+1$, $3n+6$

(iv) 0, $n+1$

(h) A language L is said to be _____ if there is a Turing machine M such that $L(M) = L$ and M halts at every point.

(i) Turing acceptable

(ii) decidable

(iii) undecidable

(iv) None of the above

(i) A problem is called _____ if it has an efficient algorithm for itself.

(i) tractable

(ii) intractable

(iii) computational

(iv) None of the above

(j) A recursively enumerable language L can be recursive if

(i) L' is recursively enumerable

(ii) every possible sequence of moves of T, the TM which accept L, causes it to halt

(iii) L' is recursively enumerable and every possible sequence of moves of T, the TM which accept L, causes it to halt

(iv) None of the above

SECTION—B

2. Answer any *four* of the following questions : 5×4=20

(a) Compare and contrast Dijkstra's algorithm with Bellman-Ford algorithm for finding the shortest path in a weighted graph. Highlight their strengths and weaknesses.

(b) Let L be the language, the set of strings over alphabet {0, 1, 2} that do not have two consecutive identical symbols. That is, strings of L are any string in {0, 1, 2} * such that there is no occurrence of 00, no occurrence of 11, and no occurrence of 22. Design a DFA for that.

(c) Convert the given context-free grammar into Chomsky normal form (CNF) :

$S \rightarrow AB$

$A \rightarrow aA|B$

$B \rightarrow bB|bC|d$

$C \rightarrow cC|Bc$

(d) Construct a Moore machine which determines the residue mod 3 for each binary string treated as binary integer and convert into the corresponding Mealy machine.

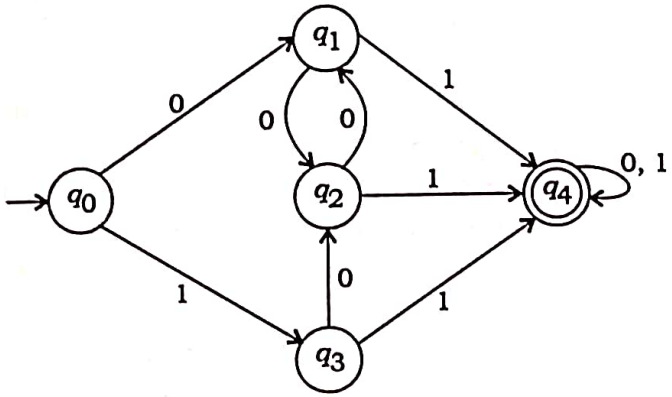
(e) Construct the context-free grammar for the given grammar :

$L = \{a^m b^n \mid m <= n\}$

SECTION—C

Answer any *three* of the following questions : 10×3=30

3. Consider the following DFA D over the alphabet $\Sigma_D = \{0, 1\}$:



Minimize the DFA D and show the major steps of your derivation. Also draw the transition diagram of the final, minimal DFA.

4. Design a push-down automata for accepting the string for the language

$$L = \{WW^R \mid W \in (a, b)^*\}$$

by the empty stack as well as final state.

5. Show that if G is a CFG in Chomsky normal form then for any string w in $L(G)$ of length $n \geq 1$, exactly $2n-1$ steps are required for any derivation of w.

6. Write short notes on the following :

(a) Universal Turing machine

(b) NP-hard problem

(c) Chomsky hierarchy

(d) Post-correspondence problem

7. Design a Turing machine that takes two unary-encoded positive integers as input and computes their product. Provide a step-by-step description of the machine's operations, transitions, and tape configurations, illustrating how the Turing machine can effectively perform the multiplication of the given integers. Additionally, discuss the time complexity of your Turing machine in terms of the input size and analyze its efficiency in solving the multiplication problem.

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