



#### Introduction to NLP

Statistical Parsing



# **Need For Probabilistic Parsing**

- Time flies like an arrow
  - Many parses
  - Some (clearly) more likely than others
  - Need for a probabilistic ranking method



#### **Probabilistic Context-free Grammars**

- Just like (deterministic) CFG, a 4-tuple (N,Σ,R,S)
  - N: non-terminal symbols
  - $-\Sigma$ : terminal symbols (disjoint from N)
  - R: rules (A  $\rightarrow \beta$ ) [p]
    - $\beta$  is a string from  $(\Sigma \cup N)^*$
    - p is the probability  $P(\beta|A)$
  - S: start symbol (from N)



```
S -> NP VP
NP -> DT N | NP PP
PP -> PRP NP

VP -> V NP | VP PP
DT -> 'a' | 'the'
N -> 'child' | 'cake' | 'fork'
PRP -> 'with' | 'to'
V -> 'saw' | 'ate'
```



```
S -> NP VP
NP -> DT N
NP -> NP PP
PP -> PRP NP
VP -> V NP
VP -> VP PP
DT -> 'a'
DT -> 'the'
N -> 'child'
N -> 'cake'
N -> 'fork'
PRP -> 'with'
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V -> 'saw'
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S -> NP VP
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N -> 'child'
N -> 'cake'
N -> 'fork'
PRP -> 'with'
PRP -> 'to'
V -> 'saw'
V -> 'ate'
```



```
S -> NP VP
                [p0=1]
NP -> DT N
                [p1]
NP -> NP PP
                [p2]
                [p3=1]
PP -> PRP NP
VP -> V NP
                [p4]
VP -> VP PP
                [p5]
DT -> 'a'
                [p6]
DT -> 'the'
                [p7]
N -> 'child'
                [8q]
                [p9]
N -> 'cake'
N -> 'fork'
                [p10]
PRP -> 'with'
                [p11]
PRP -> 'to'
                [p12]
                [p13]
V -> 'saw'
V -> 'ate'
                [p14]
```



#### **Probability Of A Parse Tree**

The probability of a parse tree t given all n productions used to build it:

$$p(t) = \prod_{i=1}^{n} p(\alpha_i \to \beta_i)$$

The most likely parse is determined as follows:

$$\arg \max_{t \in T(s)} p(t)$$

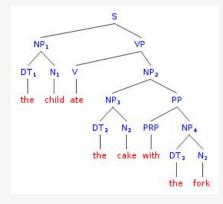
 The probability of a sentence is the sum of the probabilities of all of its parses



```
S -> NP VP
                [p0=1]
NP -> DT N
                [p1]
NP -> NP PP
                [p2]
                [p3=1]
PP -> PRP NP
VP -> V NP
                [p4]
VP -> VP PP
                [p5]
DT -> 'a'
                [p6]
DT -> 'the'
                [p7]
N -> 'child'
                [8q]
                [p9]
N -> 'cake'
N -> 'fork'
                [p10]
PRP -> 'with'
                [p11]
PRP -> 'to'
                [p12]
                [p13]
V -> 'saw'
V -> 'ate'
                [p14]
```

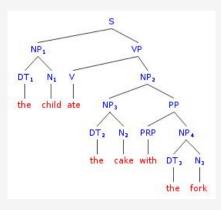


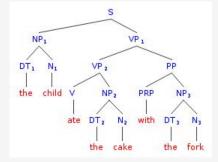
```
[p0=1]
S -> NP VP
NP -> DT N
                [p1]
NP -> NP PP
                [p2]
PP -> PRP NP
                [p3=1]
VP -> V NP
                [p4]
VP -> VP PP
                [p5]
DT -> 'a'
                [p6]
DT -> 'the'
                [p7]
N -> 'child'
                [8q]
N -> 'cake'
                [p9]
N -> 'fork'
                [p10]
PRP -> 'with'
                [p11]
PRP -> 'to'
                [p12]
                [p13]
V -> 'saw'
V -> 'ate'
                [p14]
```



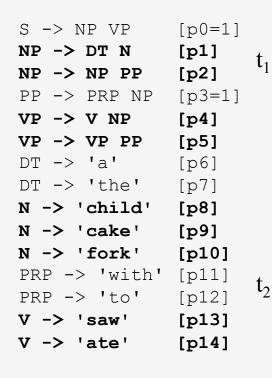


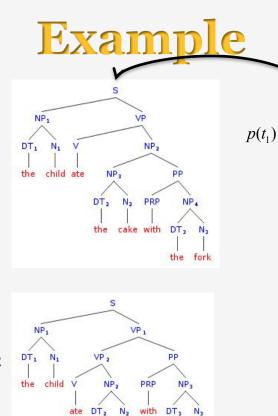
```
[p0=1]
S -> NP VP
                [p1]
NP -> DT N
NP -> NP PP
                [p2]
PP -> PRP NP
                [p3=1]
VP -> V NP
                [p4]
VP -> VP PP
                [p5]
DT -> 'a'
                [p6]
DT -> 'the'
                [p7]
N -> 'child'
                [8q]
N -> 'cake'
                [p9]
N -> 'fork'
                [p10]
PRP -> 'with'
                [p11]
                        t_2
PRP -> 'to'
                [p12]
                [p13]
V -> 'saw'
V -> 'ate'
                [p14]
```











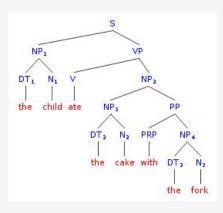
the cake

the fork

$$p(t_1) = p_0 p_1 p_4 p_7 p_8 p_{14} p_2 p_1 p_3 p_7 p_9 p_{11} p_1 p_7 p_{10}$$



S -> NP VP	[p0=1]
NP -> DT N	[p1]
NP -> NP PP	[p2]
PP -> PRP NP	[p3=1]
VP -> V NP	[p4]
VP -> VP PP	[p5]
DT -> 'a'	[p6]
DT -> 'the'	[p7]
N -> 'child'	[8q]
N -> 'cake'	[p9]
N -> 'fork'	[p10]
PRP -> 'with'	[p11]
PRP -> 'to'	[p12]
V -> 'saw'	[p13]
V -> 'ate'	[p14]



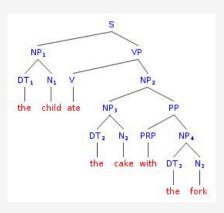
 $t_2$ 

$$p(t_1) = p_0 p_1 p_4 p_7 p_8 p_{14} p_2 p_1 p_3 p_7 p_9 p_{11} p_1 p_7 p_{10}$$

$$p(t_2) = p_0 p_1 p_5 p_7 p_8 p_4 p_3 p_{14} p_1 p_{11} p_1 p_7 p_9 p_7 p_{10}$$



S -> NP VP	[p0=1]
NP -> DT N	[p1]
NP -> NP PP	[p2]
PP -> PRP NP	[p3=1]
VP -> V NP	[p4]
VP -> VP PP	[p5]
DT -> 'a'	[p6]
DT -> 'the'	[p7]
N -> 'child'	[p8]
N -> 'cake'	[p9]
N -> 'fork'	[p10]
PRP -> 'with'	[p11]
PRP -> 'to'	[p12]
V -> 'saw'	[p13]
V -> 'ate'	[p14]

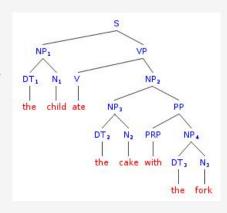


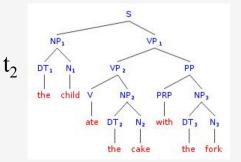
$$p(t_1) = p_0 p_1 p_4 p_7 p_8 p_{14} p_2 p_1 p_3 p_7 p_9 p_{11} p_1 p_7 p_{10}$$

$$p(t_2) = p_0 p_1 p_5 p_7 p_8 p_4 p_3 p_{14} p_1 p_{11} p_1 p_7 p_9 p_7 p_{10}$$



S -> NP VP NP -> DT N NP -> NP PP	[p0=1] [p1] [p2]	$t_1$
PP -> PRP NP	[p3=1]	
VP -> V NP	[p4]	
VP -> VP PP	[p5]	
DT -> 'a'	[p6]	
DT -> 'the'	[p7]	
N -> 'child'	[8q]	
N -> 'cake'	[p9]	
N -> 'fork'	[p10]	
PRP -> 'with'	[p11]	t.
PRP -> 'to'	[p12]	$\iota_2$
V -> 'saw'	[p13]	
V -> 'ate'	[p14]	











### Main Tasks With Pcfgs

- Given a grammar G and a sentence s, let T(s) be all parse trees that correspond to s
- Task 1
  - find which tree t among T(s) maximizes the probability p(t)
- Task 2
  - find the probability of the sentence p(s) as the sum of all possible tree probabilities p(t)



### **Probabilistic Parsing Methods**

- Probabilistic Earley algorithm
  - Top-down parser with a dynamic programming table
- Probabilistic Cocke-Kasami-Younger (CKY) algorithm
  - Bottom-up parser with a dynamic programming table



#### **Probabilistic Grammars**

- Probabilities can be learned from a training corpus (Treebank)
- Intuitive meaning
  - Parse #1 is twice as probable as parse #2
- Possible to do reranking
- Possible to combine with other stages
  - E.g., speech recognition, translation



#### **Maximum Likelihood Estimates**

Use the parsed training set for getting the counts

$$-P_{MI}(\alpha \rightarrow \beta) = Count(\alpha \rightarrow \beta)/Count(\alpha)$$

- Example:
  - $-P_{MI}(S \rightarrow NP VP) = Count(S \rightarrow NP VP)/Count(S)$



Grammar		Lexicon
$S \rightarrow NP VP$	[.80]	$Det \rightarrow that [.10] \mid a [.30] \mid the [.60]$
$S \rightarrow Aux NP VP$	[.15]	$Noun \rightarrow book [.10] \mid flight [.30]$
$S \rightarrow VP$	[.05]	meal [.15]   money [.05]
$NP \rightarrow Pronoun$	[.35]	flights [.40]   dinner [.10]
$NP \rightarrow Proper-Noun$	[.30]	$Verb \rightarrow book [.30] \mid include [.30]$
$NP \rightarrow Det Nominal$	[.20]	<i>prefer</i> ;[.40]
$NP \rightarrow Nominal$	[.15]	$Pronoun \rightarrow I[.40] \mid she[.05]$
$Nominal \rightarrow Noun$	[.75]	me[.15] you[.40]
$Nominal \rightarrow Nominal Noun$	[.20]	$Proper-Noun \rightarrow Houston [.60]$
$Nominal \rightarrow Nominal PP$	[.05]	<i>NWA</i> [.40]
$VP \rightarrow Verb$	[.35]	$Aux \rightarrow does [.60] \mid can [40]$
$VP \rightarrow Verb NP$	[.20]	$Preposition \rightarrow from [.30] \mid to [.30]$
$VP \rightarrow Verb NP PP$	[.10]	on [.20]   near [.15]
$VP \rightarrow Verb PP$	[.15]	through [.05]
$VP \rightarrow Verb NP NP$	[.05]	
$VP \rightarrow VP PP$	[.15]	
$PP \rightarrow Preposition NP$	[1.0]	



```
S -> NP VP
               [p0=1]
               [p1=.8]
NP -> DT N
               [p2=.2]
NP -> NP PP
PP -> PRP NP
                [p3=1]
               [p4=.7]
VP -> V NP
VP -> VP PP
               [p5=.3]
DT -> 'a'
               [p6=.25]
DT -> 'the'
               [p7=.75]
N -> 'child'
               [p8=.5]
               [p9=.3]
N -> 'cake'
N -> 'fork'
               [p10=.2]
PRP -> 'with'
               [p11=.1]
PRP -> 'to'
               [p12=.9]
               [p13=.4]
V -> 'saw'
               [p14=.6]
V -> 'ate'
```



the								
	child							
		ate						
			the					
				cake				
					with			
						the		
							fork	



the	.75							
	child							
		ate						
			the					
				cake				
					with			
						the		
							fork	

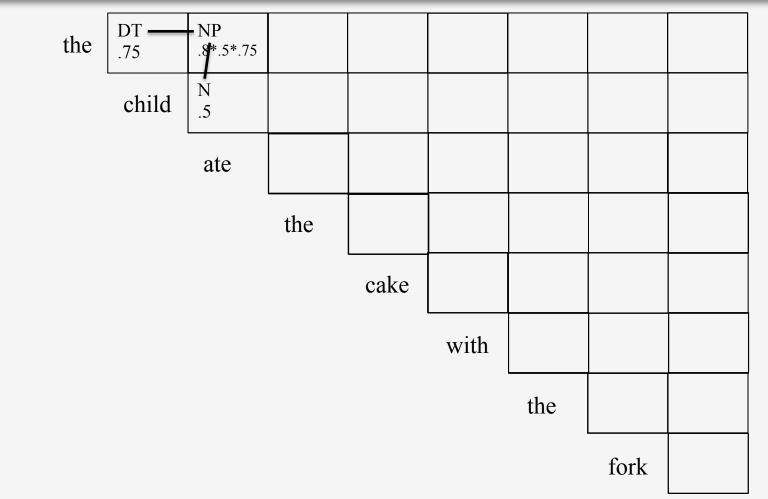


the	.75							
	child	N .5						
		ate						
			the					
				cake				
					with			
						the		
							fork	



the	DT .75	NP .8						
	child	N .5						
		ate						
			the					
				cake				
					with			
						the		
							fork	









# Question

- How, on your own, could you compute the probability of the entire sentence using Probabilistic CKY?
- Don't forget that there may be multiple parses, so you will need to add the corresponding probabilities.