

1 Solution

X1: 1.5, 1.2, 1.7, 1.8, 1.0, 1.1, 2.0

X2: 10.2, 10.4, 10.8, 11.5, 14.2, 9.4, 10.4

Weights: 0.5,1

Y: 5.6, 5.4, 5.7, 5.9, 4.9, 6.0, 5.5

$\lambda = 2$

$\alpha = 0.001$

- **Step1:** Compute

$$\begin{aligned}\sum_{j=1}^2 x_{ij} B_j &= \\ [(1.5*0.5+10.2*1), (1.2*0.5+10.4*1), (1.7*0.5+10.8*1), (1.8*0.5+11.5*1), (1*0.5+14.2*1), (1.1*0.5+9.4*1), (2*0.5+10.4*1)] \\ &= [10.95, 11, 11.65, 12.4, 14.7, 9.95, 11.4]\end{aligned}$$

- **Step2:** Compute

$$\begin{aligned}\sum_{i=1}^n (y_i - \sum_{j=1}^2 x_{ij} B_j)^2 &= \\ [5.35^2 + 5.6^2 + 5.95^2 + 6.5^2 + 9.8^2 + 3.95^2 + 5.9^2] &= 284.0875\end{aligned}$$

- **Step 3:** Compute

$$\begin{aligned}\lambda \sum_{j=1}^2 |\beta_j| &= \\ 2 * (0.5 + 1) &= 3\end{aligned}$$

- **Step 4:** Final value $\Rightarrow 284.0875 + 3 = 287.0875$

Let C represent the Cost. Now we compute the gradients to update the weights:

- Formula for gradient update for B_1 is:

$$\begin{aligned}\frac{\delta C}{\delta B_1} &= 2 * \sum_{i=1}^n (y_i - \sum_{j=1}^2 x_{ij} B_j) * (-x_{i1}) + \lambda = \\ 2 * [5.35 * 1.5 + 5.6 * 1.2 + 5.95 * 1.7 + 6.5 * 1.8 + 9.8 * 1 + 3.95 * 1.1 + 5.9 * 2] + 2 &= 127.01\end{aligned}$$

- Similarly for B_2 :

$$\begin{aligned}\frac{\delta C}{\delta B_2} &= 2 * \sum_{i=1}^n (y_i - \sum_{j=1}^2 x_{ij} B_j) * (-x_{i2}) + \lambda = \\ 2 * [5.35 * 10.2 + 5.6 * 10.4 + 5.95 * 10.8 + 6.5 * 11.5 + 9.8 * 14.2 + 3.95 * 9.4 + 5.9 * 10.4] + 2 &= 980.94\end{aligned}$$

- Hence updates for weights will be:

$$\begin{aligned}B_1^{new} &= B_1^{old} - \alpha * \frac{\delta C}{\delta B_1} \\ &= 0.5 - 0.001 * 127.01 = 0.37299 \\ B_2^{new} &= B_2^{old} - \alpha * \frac{\delta C}{\delta B_2} \\ &= 1 - 0.001 * 980.94 = 0.01906\end{aligned}$$