



# Lambda Calculus

Ques: For the following forms apply  $\beta$ -reduction and  $\alpha$ -substitution to reduce to lowest form. Indicate at each step the rule you are applying.

$$(1) (\lambda x.x) (\lambda x.x) \\ \Rightarrow (\lambda x.x) (\lambda y.y) \\ \Rightarrow \boxed{\lambda y.y} \text{ Ans.}$$

[ $\alpha$ -substitution: Substituting  $x$  to  $y$ ]

[ $\beta$ -reduction:  $\{(\lambda y.y)/x\} x$ ]

$$(2) (\lambda x.xx) (\lambda x.\lambda y.xx) \\ \Rightarrow (\lambda x.xx) (\lambda z.\lambda y.zz)$$

[ $\alpha$ -substitution: Substituting  $x$  with  $z$ ]

$$\Rightarrow (\lambda z.\lambda y.zz) (\lambda z.\lambda y.zz)$$

[ $\beta$ -reduction:  $\{(\lambda z.\lambda y.zz)/x\} xx$ ]

$$\Rightarrow (\lambda z.(\lambda y.zz)) (\lambda p.\lambda y.pp)$$

[ $\alpha$ -substitution: Substituting  $z$  with  $p$ ]

$$\Rightarrow [\beta\text{-reduction: } \{(\lambda p.\lambda y.pp)/z\} \{(\lambda y.zz)\}]$$

$$\Rightarrow \lambda y (\lambda p.\lambda y.pp) (\lambda p.\lambda y.pp)$$

Since, this is a non-terminating expression, it can not be reduced to lowest form.





$$(3) ((\lambda x. (xy)) (\lambda z. z))$$

$$[\beta\text{-reduction: } \{\lambda z. z / x\} xy]$$

$$\Rightarrow (\lambda z. z)(y)$$

$$[\beta\text{-reduction: } \{y / z\} z]$$

$$\Rightarrow \boxed{y} \text{ Ans.}$$

$$(4) (\lambda z. z)(\lambda y. yy)(\lambda x. xa)$$

$$[\beta\text{-reduction: } \{(\lambda y. yy) / z\} z]$$

$$\Rightarrow (\lambda y. yy)(\lambda x. xa)$$

$$[\beta\text{-reduction: } \{(\lambda x. xa) / y\} yy]$$

$$\Rightarrow (\lambda x. xa)(\lambda x. xa)$$

$$[\beta\text{-reduction: } \{(\lambda x. xa) / x\} xa]$$

$$\Rightarrow (\lambda x. xa)a$$

$$[\beta\text{-reduction: } \{a / x\} xa]$$

$$\Rightarrow \boxed{aa} \text{ Ans.}$$

$$(5) (\lambda z. z) (\lambda z. zz) (\lambda z. zy)$$

$[\alpha\text{-Substitution: Substituting } z \text{ with } x]$

$$\Rightarrow (\lambda z. z) \cancel{(\lambda z. zz)} (\lambda z. zy)$$

$[\alpha\text{-Substitution: Substituting } z \text{ with } p]$

$$\Rightarrow (\lambda z. z) (\lambda x. xx) (\lambda p. py)$$

$$[\beta\text{-reduction: } \{(\lambda x. xx) / z\} z]$$

$$> (\lambda x. xx) (\lambda p. py)$$



$$[\beta\text{-reduction} : \{ \lambda p. py / x \} xx]$$

$$\Rightarrow (\lambda p. py)(\lambda p. py)$$

$$[\beta\text{-reduction} : \{ (\lambda p. py) / p \} py]$$

$$\Rightarrow (\lambda p. py)y$$

$$[\beta\text{-reduction} : \{ y / p \} py]$$

$$\Rightarrow \boxed{yy} \text{ Ans.}$$

$$(b) (\lambda x. \lambda y. xyy)(\lambda a. a)b$$

$$\Rightarrow (\lambda x. (\lambda y. xyy))(\lambda a. a)b$$

$$[\beta\text{-reduction} : \{ (\lambda a. a) / x \} (\lambda y. xyy)]$$

$$\Rightarrow (\lambda y. (\lambda a. a)yy)b$$

$$[\beta\text{-reduction} : \{ b / y \} (\lambda a. a)yy]$$

$$\Rightarrow (\lambda a. a)bb$$

$$[\beta\text{-reduction} : \{ bb / a \} a]$$

$$\Rightarrow \boxed{bb} \text{ Ans.}$$



(7)  $(\lambda x. x x) (\lambda y. y x) z$



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[ $\alpha$ -Substitution: Substituting  $x$  with  $p$ ]

$\Rightarrow (\lambda x. x x) (\lambda y. y p) z$

$\Rightarrow$  [ $\beta$ -reduction:  $\{\lambda y. y p / x\} x x$ ]

$\Rightarrow (\lambda y. y p) (\lambda y. y p) z$

[ $\beta$ -reduction:  $\{(\lambda y. p p) / y\} y p$ ]

$\Rightarrow (\lambda y. y p) p z$

$\Rightarrow$  [ $\beta$ -reduction:  $\{p / y\} y p$ ]

$\Rightarrow \boxed{p p z}$  Ans.

(8)  $(\lambda x. (\lambda y. (x y)) y) z$

$\Rightarrow (\lambda x. (\lambda p. (x y)) y) z$  [ $\alpha$ -substitution: Substituting  $y$  with  $p$ ]

$\Rightarrow$  [ $\beta$ -reduction:  $\{y / p\} x y$ ]

$\Rightarrow (\lambda x. (x y)) z$

[ $\beta$ -reduction:  $\{z / x\} x y$ ]

$\Rightarrow \boxed{z y}$  Ans

~~$\Rightarrow$~~   ~~$\lambda x. x x$~~   ~~$\lambda y. y$~~

(9)  $((\lambda x. x x) (\lambda y. y)) (\lambda y. y)$

[ $\beta$ -reduction:  $\{x y - y / x\} x x$ ]

$\Rightarrow (\lambda y. y) (\lambda y. y) (\lambda y. y)$



[ $\beta$ -reduction:  $\{\lambda y.y / y\} y]$

$$\Rightarrow (\lambda y.y) (\lambda y.y)$$

[ $\beta$ -reduction:  $\{\lambda y.y / y\} y]$

$$\Rightarrow \boxed{\lambda y.y} \quad \underline{\text{Ans.}}$$

$$(10) ((\lambda x. \lambda y. (xy)) (\lambda y.y)) \omega$$

[ $\alpha$ -Substitution: Substituting  $y$  with  $p$ ]

$$\Rightarrow ((\lambda x. \lambda y. (xy)) (\lambda p.p)) \omega$$

$$\Rightarrow ((\lambda x. (\lambda y. (xy))) (\lambda p.p)) \omega$$

[ $\beta$ -reduction:  $\{\lambda p.p / x\} \lambda y.(xy)$ ]

$$\Rightarrow (\lambda y. (\lambda p.p) y) \omega$$

$\neq$  [ $\beta$ -reduction:  $\{y / y\} \lambda p.p$ ]

$$\Rightarrow (\lambda p.p) \omega$$

[ $\beta$ -reduction:  $\{\omega / p\} p$ ]

$$\Rightarrow \boxed{\omega} \quad \underline{\text{Ans.}}$$