Linear Econometrics for Finance - Fall II - Gianni De Nicolo'

1 The CAPM and Regression

The model:

$$E(R_{i,t} - R^f) = \beta_i E(R_{M,t} - R^f),$$

where

$$\beta_i = \frac{Cov(R_{i,t} - R^f, R_{M,t} - R^f)}{Var(R_{M,t} - R^f)}.$$

$R_{i,t}$ is the return on asset i at time t

R^f is the return on a risk-less asset

$R_{M,t}$ is the return on the market portfolio at time t

We run the regression:

$$R_{i,t} - R^f = \alpha_i + \beta_i(R_{M,t} - R^f) + \varepsilon_{i,t},$$

where we assume:

- $E(\varepsilon_{i,t}|R_{M,t}) = 0$
- $\varepsilon_{i,t}$ uncorrelated - $E(\varepsilon_t|R_{M,t}, R_{M,t-1}, ...\varepsilon_{t-1}, \varepsilon_{t-2}, ...) = 0$

Interpretation:

- The expected return that a generic risky stock i provides in excess of the risk-free return depends on a risk-premium.
- The risk premium comprises two components: a quantity of risk β_i and a price of risk $E(R_{M,t} - R^f)$
- There is only one source of risk: market fluctuations
- Exposure to market fluctuations are compensated
- The compensation is proportional to the risk exposure $\beta_i = \frac{Cov(R_{i,t}, R_{M,t})}{Var(R_{M,t})}$.

- Only market risk affects expected returns. The shock $\varepsilon_{i,t}$ is an idiosyncratic shock solely affecting actual returns.

- The *only* implication of the model is that $\alpha_i = 0$ for **all** stocks.

- β_i can be interpreted as the sensitivity of the return i to market risk. If $\beta_i = 0$, then asset i is not exposed to market risk. If $\beta_i > 0$, then asset i is exposed to market risk and $E(R_{i,t}) > R^f$, since $E(R_t^M - R^f) > 0$.

- The CAPM is a one factor model. The only factor is an excess return: the excess return on the market.

1.1 Estimation and testing

1. **Multifactor models: Econometrically, omitted variable bias.**

- What happens if you *assume* that market risk is the only risk you care about but this is not the case?

- Suppose the model *really* is:

$$R_{i,t} - R^f = \alpha_i + \beta_{1,i}(R_{M,t} - R^f) + \beta_{2,i} f_{2,t} + \varepsilon_{i,t},$$

  where $f_{2,t}$ is another factor (i.e., the excess return on another portfolio).

- However, you estimate $R_{i,t} - R^f = \widetilde{\alpha}_i + \widetilde{\beta}_{1,i}(R_{M,t} - R^f) + \widetilde{\varepsilon}_{i,t}$ where $\widetilde{\varepsilon}_{i,t} = \beta_{2,i} f_{2,t} + \varepsilon_{i,t}$

- We know that we get inconsistent estimates since the residuals have the potential to be correlated with the regressor.

- We would obtain consistent estimates only when $f_{2,t}$ and $R_{M,t} - R^f$ are uncorrelated.

2. **Roll's critique (Roll, 1977): Econometrically, errors in variables.**

- We do not observe the true market return $R_{M,t}$. We just have a proxy for it (say the S&P 500 return).

- Suppose the model is:

$$R_{i,t} - R^f = \alpha_i + \beta_i (R_{M,t} - R^f) + \varepsilon_{i,t}.$$

- However, we do not observe $R_{M,t}$. What we observe is a proxy:

$$R_{S\&P500,t} = R_{M,t} + v_t,$$

where $E(v_t) = 0$.

- We run the regression $R_{i,t} - R^f = \alpha_i + \beta_i (R_{S\&P500,t} - R^f) + \widetilde{\varepsilon}_{i,t}$, where $\widetilde{\varepsilon}_{i,t} = \varepsilon_{i,t} - \beta_{1,i} v_t$.
- Do we get good estimates of α_i and β_i? No. We get inconsistent estimates since the regressor $R_{S\&P500,t} - R^f$ is clearly correlated with v_t (and, hence, it is now correlated with the overall residuals $\widetilde{\varepsilon}_{i,t}$).

1.2 Testing the CAPM

- We can test if the α_is (the pricing errors) are small (individually).
- Testing whether each α_i is equal to zero, individually, is simple.
- Better: we *should* jointly test if $\alpha_1 = \alpha_2 = \alpha_3 = ... = \alpha_N = 0$. This is the main implication of the model, as the model applies to all assets. You will see this second method later.

2 Multifactor models (the other factors are also excess returns on specific portfolios)

- The regression is

$$R_{i,t} - R^f = \alpha_i + \beta_{1,i} f_{1,t} + \beta_{2,i} f_{2,t} + ... + \beta_{k,i} f_{k,t} + \varepsilon_{i,t}.$$

- Here, $f_{1,t} = R_{M,t} - R^f$, $f_{2,t}$ is the second factor, and so on.

- As earlier, all k factors are uncorrelated with the idiosyncratic shock $\varepsilon_{i,t}$.

The asset pricing model is:

$$E(R_{i,t} - R^f) = \beta_{1,i}E(f_{1,t}) + \beta_{2,i}E(f_{2,t}) + ... + \beta_{k,i}E(f_{k,t}).$$

- *Testing the model: are the cross-sectional pricing errors equal to zero*, $\alpha = 0$?
- What are the factors? Still an open question ...
- Economic arguments: business-cycle-variable, etc. (Chen, Roll, and Ross, 1986)
- Statistical arguments: principal components, etc.
- Other arguments: market (MKT), size (SMB), and book-to-market (HML) factors: a 3-factor model (Fama and French, 1993).

SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios,

SMB = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth).

HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios,

HML = 1/2 (Small Value + Big Value) - 1/2 (Small Growth + Big Growth).