

Efficiently Inefficient Markets for Assets and Asset Management

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Abstract

We consider a model where investors can invest directly or search for an asset manager, information about assets is costly, and managers charge an endogenous fee. The efficiency of asset prices is linked to the efficiency of the asset management market: if investors can find managers more easily, more money is allocated to active management, fees are lower, and asset prices are more efficient. Informed managers outperform after fees, uninformed managers underperform, while the average manager's performance depends on the number of "noise allocators." Small investors should remain uninformed, but large and sophisticated investors benefit from searching for informed active managers since their search cost is low relative to capital. Hence, managers with larger and more sophisticated investors are expected to outperform.

Keywords: asset pricing, market efficiency, asset management, search, information

JEL Codes: D4, D53, D83, G02, G12, G14, G23, L10

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Asset managers play a central role in making financial markets efficient, as their size allows them to spend significant resources on acquiring and processing information. The asset management market is subject to its own frictions, however, since investors must search for informed asset managers. Indeed, institutional investors literally fly around the world to examine asset managers, assessing their investment process, trading infrastructure, risk management, and so on. Similarly, individual investors search for asset managers, some via local branches of financial institutions, others via the internet or otherwise.

How does this search for asset managers affect the efficiency of security markets? What type of manager is expected to outperform? Which type of investors should use active investing?

We seek to address these questions in a model with two levels of frictions: investors' costs of searching for informed asset managers and asset managers' cost of collecting information about assets. Despite this apparent complexity, the model is very tractable and delivers several new predictions that link the levels of inefficiency in the security market and the market for asset management: (1) If investors can find managers more easily, more money is allocated to active management, fees are lower, and security prices are more efficient; (2) as search costs diminish, asset prices become efficient in the limit, even if information-collection costs remain large; (3) managers of assets with higher information costs earn larger fees and are fewer, and such assets are less efficiently priced; (4) informed managers outperform after fees while uninformed managers underperform after fees; (5) the net performance of the average manager depends on the number of "noise allocators" (who allocate to randomly chosen managers) and, under certain conditions, is zero or negative; (6) searching for informed active managers is attractive for large or sophisticated investors, while small or unsophisticated investors should be uninformed; (7) managers with larger and more sophisticated investors are expected to outperform.

As a way of background, the key benchmark is that security markets are perfectly efficient (Fama (1970)), but this leads to two paradoxes: First, no one has an incentive to collect information in an efficient market, so how does the market become efficient (Grossman and

Stiglitz (1980))? Second, if asset markets are efficient, then positive fees to active managers implies inefficient markets for asset management (Pedersen (2015)).

Grossman and Stiglitz (1980) show that the first paradox can be addressed by considering informed investing in a model with noisy supply, but, when an agent has collected information about securities, she can invest on this information on behalf of others, so professional asset managers arise naturally (Admati and Pfleiderer (1988), Ross (2005), García and Vanden (2009)). Therefore, we introduce professional asset managers into the Grossman-Stiglitz model.

One benchmark for the efficiency of asset management is provided by Berk and Green (2004), who consider the implications of fully efficient asset-management markets (in the context of exogenous and inefficient asset prices). In contrast, we consider an imperfect market for asset management due to search frictions, consistent with the empirical evidence of Sirri and Tufano (1998), Jain and Wu (2000), Hortaçsu and Syverson (2004), and Choi et al. (2010). We focus on investors' incentive to search for informed managers and managers' incentives to acquire information about assets with endogenous prices, abstracting from how agency problems and imperfect contracting can distort asset prices (Shleifer and Vishny (1997), Stein (2005), Cuoco and Kaniel (2011), Buffa et al. (2014)).

We employ the term *efficiently inefficient* to refer to the equilibrium level of inefficiency given the two layers of frictions in the spirit of the Grossman-Stiglitz notion of “an equilibrium degree of disequilibrium.” Paraphrasing Grossman-Stiglitz, prices in efficiently inefficient markets reflect information, but only partially, so that some managers have an incentive to expend resources to obtain information, but only part of the managers, so investors have an incentive to expend resources to find informed managers.

Our equilibrium works as follows. Among the group of asset managers, an endogenous number decide to acquire information about a security. Investors must decide whether to expend search costs to find one of the informed asset managers. In an interior equilibrium, investors are indifferent between searching for an informed asset manager vs. uninformed investing (and both of these options dominate the investor collecting information herself).¹

¹Investors do not collect information on their own, since the costs of doing so are higher than the benefits

When an investor meets an asset manager, they negotiate a fee, and asset prices are set in a competitive noisy rational expectations market. The economy also features a group of “noise traders” (or “liquidity traders”) who take random security positions as in Grossman-Stiglitz. Likewise, we introduce a group of “noise allocators” who allocate capital to a random group of asset managers, e.g., because they place trust in these managers as modeled by Gennaioli et al. (2015).

We solve for the equilibrium number of investors who invest through managers, the equilibrium number of informed asset managers, the equilibrium management fee, and the equilibrium asset prices. The model features both search and information frictions, but the solution is surprisingly simple and yields a number of clear new results.

First, we show that informed managers outperform before and after fees, while uninformed managers naturally underperform after fees. Investors who search for asset managers must be compensated for their search and due diligence costs, and this compensation comes in the form of expected outperformance after fees. Investors are indifferent between active and uninformed investing in an interior equilibrium, so a larger search cost must be associated with a larger outperformance by active investors. Noise allocators invest partly with uninformed managers and therefore may experience underperformance after fees. The asset-weighted average manager (equivalently, their average investor) outperforms after fees if the number of noise allocators is small, and underperforms if many noise allocators exist. When the average manager outperforms, searching investors would have an incentive to “free ride” by choosing a random manager if this were free, but all manager allocations require a search cost in our baseline model. In a model extension with free search for a random manager, the equilibrium outperformance of the average manager is zero or negative.

The model consequently helps explain a number of empirical regularities on the performance of asset managers that are puzzling in light of the literature. Indeed, while the “old consensus” in the literature was that the average mutual fund has no skill (Fama (1970),

to an individual due to the relatively high equilibrium efficiency of the asset markets. This high equilibrium efficiency arises from investors’ ability to essentially “share” information collection costs by investing through an asset manager.

Carhart (1997)), a “new consensus” has emerged that the average hides significant cross-sectional variation in manager skill among mutual funds, hedge funds, private equity, and venture capital.² For instance, Kosowski et al. (2006) conclude that “*a sizable minority of managers pick stocks well enough to more than cover their costs.*” In our model, this outperformance after fees is expected as compensation for investors’ search costs, but it is puzzling in light of the prediction of Fama (1970) that all managers underperform after fees, and the prediction of Berk and Green (2004) that all managers deliver zero outperformance after fees. Further, the fact that top hedge funds and private equity managers deliver larger outperformance than top mutual funds is consistent with our model under the assumption that investors face larger search costs in these segments.

While the data support our novel prediction that some managers outperform others, we can test the model at a deeper level by examining whether it can also explain *who* outperforms. To do this, we extend the model by considering investors and asset managers who differ in their size or sophistication. We show that large and sophisticated investors benefit from searching for an informed manager, since their search cost is low relative to their capital. In contrast, small unsophisticated investors are better served by uninformed investing. As a result, active investors who are small must be noise allocators, while large active investors could be rational searching investors (or noise allocators). Hence, we predict that large investors perform better than small investors on average, because large investors are more likely to find informed managers. This prediction is consistent with the findings of Dyck and Pomorski (2016), who report that large institutional investors select managers who outperform those of small investors.

We also predict that asset managers who have larger and more sophisticated investors outperform those serving small unsophisticated investors. Consistent with this prediction, managers of institutional investors outperform those of retail investors (Evans and Fahlenbrach (2012), Dyck et al. (2013), Gerakos et al. (2016)).

²Evidence on mutual funds is provided by Grinblatt and Titman (1989), Wermers (2000), Kacperczyk et al. (2008), Fama and French (2010), Berk and Binsbergen (2015), and Kacperczyk et al. (2014)), on hedge funds by Kosowski et al. (2007), Fung et al. (2008), Jagannathan et al. (2010), and on private equity and venture capital by Kaplan and Schoar (2005).

The model also generates a number of implications of cross-sectional and time-series variation in search costs. The important observation is that, if search costs are lower such that investors more easily can identify informed managers, then more money is allocated to active management, fees are lower, and security markets are more efficient. If investors' search costs go to zero, then the asset market becomes efficient in the limit. Indeed, as search costs diminish, fewer and fewer asset managers with more and more asset under management collect smaller and smaller fees, and this evolution makes asset prices more and more efficient even though information-collection costs remain constant (and potentially large). It may appear surprising (and counter to the result of Grossman and Stiglitz (1980)) that markets can become close to efficient despite large information collection costs, but this result is driven by the fact that the costs are shared by investors through an increasingly consolidated group of asset managers.

These model-implied predictions are consistent with a number of empirical findings. For instance, if search costs have diminished over time as information technology has improved, markets should have become more efficient, consistent with the evidence of Wurgler (2000) and Bai et al. (2013), and linked to the amount of assets managed by professional traders (Rosch et al. (2015)).

In summary we complement the literature³ by introducing a new model of asset management and asset prices. The main innovation, search for managers, produces wide-ranging results in a surprisingly tractable manner. Thinking through the logic of search markets almost immediately yields new predictions on the performance of investors and managers, while other predictions require a deeper model analysis, such as the magnitude of market inefficiency (approximately 6%), fees, and the industrial organization of asset management.

The next section lays out the basic model, Section 2 provides the solution, and Section 3 derives the results. Section 4 extends the model to small and large investors and asset

³The related theoretical literature includes, beside the papers already cited, models of asset management (Pastor and Stambaugh (2012), Vayanos and Woolley (2013), Stambaugh (2014)), noisy rational expectations models (Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985)), other models of informed trading (Glosten and Milgrom (1985), Kyle (1985)), information acquisition (Van Nieuwerburgh and Veldkamp (2010), Kacperczyk et al. (2014)), and search models in finance (Duffie et al. (2005), Lagos (2010)); we discuss the related empirical literature in Section 5.

managers. Section 5 discusses our empirical predictions and Section 6 concludes. Appendix A contains further analysis and proofs and Appendix B describes the real-world issues related to search and due diligence of asset managers.

1 Model of Assets and Asset Managers

1.1 Investors and Asset Managers

The economy features several types of competitive agents trading in a financial market, as illustrated by Figure 1. Searching investors trade directly or through asset managers, asset managers trade on behalf of groups of investors, noise allocators make random allocations to asset managers, and noise traders make random trades in financial markets.

Specifically, the economy has \bar{A} searching investors (or “allocators”), each of whom can either (i) invest directly in asset markets after having acquired a signal s at cost k , (ii) invest directly in asset markets without the signal, or (iii) invest through an asset manager. Due to economies of scale, a natural equilibrium outcome is that investors do not acquire the signal, but, rather, invest as uninformed or through a manager. We highlight below (see the end of Section 2.3) some weak conditions under which all realistic equilibria take this form, and we therefore rule out that investors acquire the signal. Consequently, we focus on the number A of investors who make informed investments through a manager, inferring the number of uninformed investors as the residual, $\bar{A} - A$.

The economy has \bar{M} risk-neutral asset-management firms.⁴ Of these asset managers, only M elect to pay a cost k to acquire the signal s and thereby become informed asset managers. The remaining $\bar{M} - M$ managers seek to collect asset management fees and invest without information. The number of informed asset managers is determined as part

⁴The total number of asset managers \bar{M} can be endogenized based on an entry cost k^u for being an uninformed manager. Such an endogenous entry leaves the other equilibrium conditions unaffected when we interpret the information cost k as the additional cost that informed managers must incur, i.e., their total cost is $k^u + k$. Asset management firms are risk-neutral as they face only idiosyncratic risk that can be diversified away by their owners.

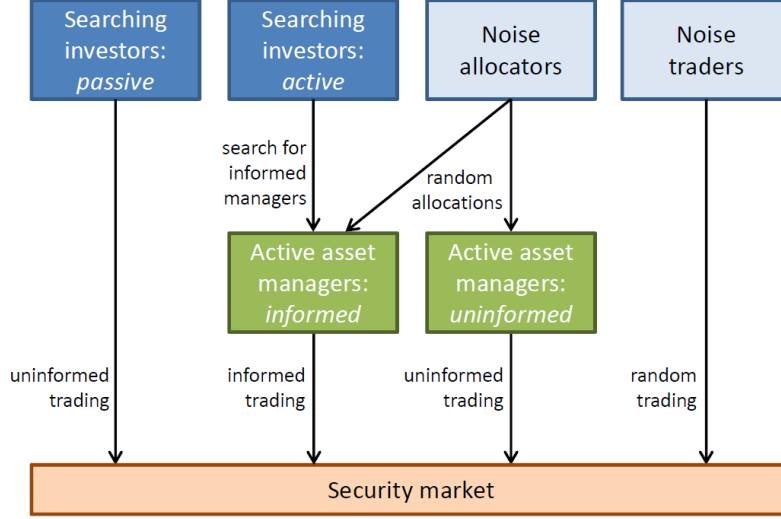


Figure 1: **Model Overview.**

of the equilibrium.⁵

To invest with an informed asset manager, investors must search for, and vet, managers, which is a costly activity. Specifically, the cost of finding an informed manager and confirming that she has the signal (i.e., performing due diligence) is given by a general continuous function $c(M, A)$, which depends on both the number of informed asset managers M and the number of their investors A .⁶ The search cost c captures the realistic feature that most investors spend significant resources finding an asset manager they trust with their money, as described in detail in Appendix B.

We assume that all investors have constant absolute risk aversion (CARA) utility over end-of-period consumption with risk-aversion parameter γ (following Grossman and Stiglitz (1980)). For convenience, we express the utility as certainty-equivalent wealth — hence, with end-date wealth \tilde{W} , an investor’s utility is $-\frac{1}{\gamma} \log(E(e^{-\gamma \tilde{W}}))$. Each investor is endowed with an initial wealth W .

⁵We note that we think of the sets of managers and investors as continua (e.g., M is the mass of informed managers), which keeps the exposition as simple as possible, but the model’s properties also obtain in a limit of a finite-investor model.

⁶We require continuity of c only on $[0, \infty)^2 \setminus \{(0, 0)\}$ as it is natural to assume that finding an informed manager is infinitely costly if none exists, $c(0, A) = \infty$ for all A .

When an investor has found an asset manager and confirmed that the manager has the technology to obtain the signal, they negotiate the asset management fee f . The fee is set through Nash bargaining and, at this bargaining stage, all costs are sunk — both the manager’s information acquisition cost and the investor’s search cost.⁷

We note that while the fee f is a total payment, which is the relevant quantity for the agents’ utilities, it can be achieved through an unlimited number of combinations of funds invested and percentage fees (as is typical in the literature). For instance, economic outcomes are unchanged if investors double their dollar investment in the fund and pay half the percentage fee, while the manager puts half of the portfolio in cash (or an index).

Lastly, the economy features a group of “noise traders” and one of “noise allocators.” As in Grossman and Stiglitz (1980), noise traders buy an exogenous number of shares of the security, $\bar{q} - q$, as described below. Noise traders create uncertainty about the supply of shares and are used in the literature to capture the fact that it can be difficult to infer fundamentals from prices. Noise traders are also called “liquidity traders” in some papers and their demand can be justified by a liquidity need, hedging demand, or behavioral reasons.

Following the tradition of noise traders, we introduce the concept of “noise allocators,” of total mass $N \geq 0$, who allocate their funds across randomly chosen asset managers. Noise allocators play a similar role in the market for asset management to noise traders in the market for assets — specifically, noise allocators can make it difficult for searching investors to determine whether a manager is informed by looking at whether she has other investors. Further, the existence of noise allocators changes the performance characteristics across managers and investors, giving rise to novel model predictions, particularly when we introduce agent heterogeneity in Section 4. Noise allocators pay the general fee f , which we can view as an assumption for simplicity. However, this fee and behavior of noise allocators are endogenized in Appendix A.4.

⁷Negotiation over terms is a common feature of the interaction between institutional investors and asset managers, but much less widespread for individual investors. For individual investors, our assumption can be interpreted as the result of other forms of (imperfect) competition among managers, for instance as in García and Vanden (2009). The main feature needed is that the fee provides incentives to search.

1.2 Assets and Information

We adopt the asset-market structure of Grossman and Stiglitz (1980), aiming to focus on the consequences of introducing asset managers into this framework. Specifically, there exists a risk-free asset normalized to deliver a zero net return, and a risky asset with payoff v distributed normally with mean \bar{v} and standard deviation σ_v . Agents can obtain a signal s of the payoff, where

$$s = v + \varepsilon. \quad (1)$$

The noise ε has mean zero and standard deviation σ_ε , is independent of v , and is normally distributed.

The risky asset is available in a stochastic supply given by q , which is jointly normally distributed with, and independent of, the other exogenous random variables. The mean supply is \bar{q} and the standard deviation of the supply is σ_q . We think of the noisy supply as the number of shares outstanding \bar{q} plus the supply $q - \bar{q}$ from the noise traders.

Given this asset market, uninformed investors buy a number of shares x_u as a function of the observed price p , to maximize their utility u_u (certainty-equivalent wealth), taking into account that the price p may reflect information about the value:

$$u_u(W) = -\frac{1}{\gamma} \log \left(\mathbb{E} \left[\max_{x_u} \mathbb{E} \left(e^{-\gamma(W+x_u(v-p))} | p \right) \right] \right) = W + u_u(0) \equiv W + u_u. \quad (2)$$

We see that, because of the CARA utility function, an investor's wealth level simply shifts his utility function and does not affect his optimal behavior. Therefore, we define the scalar u_u as the wealth-independent part of the utility function (a scalar that naturally depends on the asset-market equilibrium, in particular the price efficiency).

Asset managers observe the signal and invest in the best interest of their investors. This informed investing gives rise to the gross utility u_i of an active investor (i.e., not taking into account his search cost and the asset management fee — we study those, and specify their

impact on the ex-ante utility, later):

$$u_i(W) = -\frac{1}{\gamma} \log \left(\mathbb{E} \left[\max_{x_i} \mathbb{E} \left(e^{-\gamma(W+x_i(v-p))} | p, s \right) \right] \right) = W + u_i(0) \equiv W + u_i. \quad (3)$$

As above, we define the scalar u_i as the wealth-independent part of the utility function. The gross utility of an active investor differs from that of an uninformed via conditioning on the signal s .

We note that all investors with an informed manager want the same portfolio x_i since investors are homogeneous. Hence, we simply assume that the manager offers the portfolio x_i for anyone investing W . When we introduce small and large investors in Section 4, investors with smaller absolute risk aversions prefer larger multiples of the same x_i , which can naturally be achieved through a larger investment in the same fund.

1.3 Equilibrium Concept

We first consider the (partial) equilibrium in the asset market given the numbers of informed and uninformed investors. We denote the mass of informed investors by I and note that it is the sum of the number A of rational investors who decide to search for a manager and the number of the noise allocators who happen to find an informed manager, where the latter is the total number N of noise allocators times the fraction M/\bar{M} of informed managers:

$$I = A + N \frac{M}{\bar{M}}. \quad (4)$$

Clearly, the remaining investors, $\bar{A} + N - I$, invest as uninformed, either directly or via an uninformed manager. An *asset-market equilibrium* is an asset price p such that the asset market clears:

$$q = Ix_i + (\bar{A} + N - I)x_u, \quad (5)$$

where x_i is the demand that maximizes the utility of informed investors (3) given p and the signal s , and x_u is the demand of uninformed investors (2). The market clearing condition equates the noisy supply q with the total demand from all informed and uninformed investors.

Second, we define a *general equilibrium for assets and asset management* as a number of informed asset managers M , a number of active investors A , an asset price p , and asset management fees f such that (i) no manager would like to change her decision of whether to acquire information, (ii) no investor would like to switch status from active (with an associated utility of $W + u_i - c - f$) to uninformed (conferring utility $W + u_u$) or vice-versa, (iii) the price is an asset-market equilibrium, and (iv) the asset management fees are the outcome of Nash bargaining.

2 Solving the Model

2.1 Asset-Market Equilibrium

We first derive the asset-market equilibrium taking as given the number of informed investors I . We later solve for the equilibrium number of searching investors and managers, which yields I by (4). For a given I , the unique linear asset-market equilibrium is as in Grossman and Stiglitz (1980), but for completeness we record the main results here.⁸

In the linear equilibrium, an informed agent's demand for the asset is a linear function of prices and signals and the price is a linear function of the signal and the noisy supply:

$$p = \theta_0 + \theta_s ((s - \bar{v}) - \theta_q (q - \bar{q})), \quad (6)$$

where the coefficients θ are given in Appendix A.5. The key property of the price is its *efficiency* (or informativeness), which Grossman and Stiglitz (1980) define as $\frac{\text{var}(v|s)}{\text{var}(v|p)}$. For

⁸Our setup differs from the one of Grossman and Stiglitz (1980) by a change of variables, which leads to some superficial differences in the results. Palvolgyi and Venter (2014) derive interesting non-linear equilibria in the Grossman and Stiglitz (1980) model.

convenience, we concentrate on the quantity

$$\eta \equiv \log \left(\frac{\sigma_{v|p}}{\sigma_{v|s}} \right) = \frac{1}{2} \log \left(\frac{\text{var}(v|p)}{\text{var}(v|s)} \right), \quad (7)$$

which represents the price *inefficiency*. This quantity records the amount of uncertainty about the asset value for someone who only knows the price p , relative to the uncertainty remaining when one knows the signal s . The price inefficiency is a positive number, $\eta \geq 0$, since the price is a noisy version of the signal, $\text{var}(v|p) \geq \text{var}(v|p, s) = \text{var}(v|s)$. Naturally, a higher η corresponds to a more inefficient asset market and a zero inefficiency corresponds to a price that fully reveals the signal.

The price inefficiency η is linked to investors' value of information. Indeed, η gives the relative utility of investing based on the manager's information (u_i) vs. investing as uninformed (u_u):

$$\gamma(u_i - u_u) = \eta. \quad (8)$$

This is an important result because the relative utility $u_i - u_u$ plays a central role in the remainder of the paper, affecting investors' incentive to search, asset management fees, and managers' incentive to acquire information.

The inefficiency η can be written as an explicit function of the number of informed investors I :

$$\eta = -\frac{1}{2} \log \left(1 - \frac{\sigma_q^2 \sigma_\varepsilon^2}{I^2 / \gamma^2 + \sigma_q^2 \sigma_\varepsilon^2} \frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2} \right) \in (0, \infty). \quad (9)$$

We see that η is decreasing in I , which is natural since, when there are more informed investors, asset prices become less inefficient (lower η), implying that informed and uninformed investors receive more similar utilities (lower $u_i - u_u$).

We note that the price inefficiency does not depend directly on the the number of asset managers M . What determines the asset price efficiency is the risk-bearing capacity of agents investing based on the signal, and this risk-bearing capacity is ultimately determined

by the number of informed investors (not the number of managers they invest through). The number of asset managers does affect asset price efficiency *indirectly*, however, since M affects I as seen in (4), and, importantly, since the number of searching investors A and the number of asset managers are determined jointly in equilibrium, as we shall see.

2.2 Asset Management Fee

The asset-management fee is set through Nash bargaining between an investor and a manager. The bargaining outcome depends on each agent's utility in the events of agreement vs. no agreement (the latter is called the "outside option"). For the investor, the utility when agreeing on a fee f is $W - c - f + u_i$. If no agreement is reached, the investor's outside option is to invest as uninformed with his remaining wealth, yielding a utility of $W - c + u_u$ as the cost c is already sunk.⁹ Hence, the investor's gain from agreement is $u_i - u_u - f$.

Similarly, the asset manager's gain from agreement is the fee f . This is true because the manager's information cost k is sunk and there is no marginal cost to taking on the investor. The bargaining outcome maximizes the product of the utility gains from agreement:

$$\max_f (u_i - u_u - f) f. \tag{10}$$

The solution is the equilibrium asset management fee f given by

$$f = \frac{\eta}{2\gamma}, \tag{11}$$

[equilibrium asset management fee]

using $u_i - u_u = \eta/\gamma$ from equation (8). This equilibrium fee is simple and intuitive: The fee would naturally have to be zero if asset markets were perfectly efficient, so that no benefit of information existed ($\eta = 0$), and it increases in the size of the market inefficiency. Indeed, active asset management fees can be viewed as evidence that investors believe that security

⁹The investor's outside option is equal to the utility of searching again for another manager in an interior equilibrium. Hence, we can think of the investor's bargaining threat as walking away to invest on his own or to find another manager. Note also that we specify the bargaining objective in terms of certainty-equivalent wealth, which is natural and tractable.

markets are less than fully efficient.

We next derive the investors' and managers' decisions in an equally straightforward manner. Indeed, an attractive feature of this model is that it is very simple to solve, yet provides powerful results.

2.3 Investors' Decision to Search for Asset Managers

An investor optimally decides to look for an informed manager as long as

$$u_i - c - f \geq u_u. \tag{12}$$

Recalling the equality $\eta = \gamma(u_i - u_u)$, the investor's optimality condition can be written as $\eta \geq \gamma(c + f)$. This relation must hold with equality in an "interior" equilibrium (i.e., an equilibrium in which strictly positive amounts of investors decide to invest as uninformed and through asset managers — as opposed to all investors making the same decision). Inserting the equilibrium asset management fee (11), we have already derived the investor's indifference condition: $c = \frac{\eta}{2\gamma}$.

Using similar straightforward arguments, we see that an investor would prefer using an asset manager to acquiring the signal singlehandedly provided $k \geq c + f$. Using the equilibrium asset management fee derived in equation (11), the condition that asset management is preferred to buying the signal can be written as $k \geq 2c$. In other words, finding an asset manager should cost at most half as much as actually being one, which seems to be a condition that is clearly satisfied in the real world. We can also make use of (13) to express this condition equivalently as $A \geq 2M$, i.e., there must be at least two searching investors for every manager, another realistic implication.

Finally, we note that we have assumed that searching investors only allocate to an active manager when they have paid a search cost to ensure that the manager is informed. We could also allow investors to pick a random manager without paying a search cost, even pick a manager based on her assets under management (AUM). We consider such extensions in

Section 3.1.1 and Appendix A.2.

2.4 Entry of Informed Asset Managers

A prospective informed asset manager must pay the cost k to acquire information. Becoming an informed manager has the benefit that the manager can expect to have more investors. Specifically, each manager receives a noisy number of investors, but, since managers are risk neutral, they optimize the expected fee revenue net of information costs.

An uninformed manager expects N/\bar{M} investors, that is, the number of noise allocators divided by the total number of managers. An informed manager expects $A/M + N/\bar{M}$ investors since she expects a fraction of the searching investors in addition to the noise allocators. Therefore, she chooses to become informed provided that the expected *extra* fee revenue covers the cost of information:

$$f \frac{A}{M} \geq k. \quad (13)$$

This manager condition must hold with equality for an interior equilibrium, and we can easily insert the equilibrium fee (11) to get $M = \frac{\eta A}{2\gamma k}$.

2.5 General Equilibrium for Assets and Asset Management

We focus on interior equilibria, but we provide a complete equilibrium characterization in Appendix A.1. We have arrived at the following two indifference conditions:

$$\frac{\eta(I)}{2\gamma} = c(M, A) \quad [\text{investors' indifference condition}] \quad (14)$$

$$\frac{\eta(I)}{2\gamma} = \frac{M}{A}k, \quad [\text{asset managers' indifference condition}] \quad (15)$$

where η is a function of $I = A + N\frac{M}{\bar{M}}$ given explicitly by (9). Hence, solving the general equilibrium comes down to solving these two explicit equations in two unknowns (A, M) . Recall that a general equilibrium for assets and asset management is a four-tuple (p, f, A, M) ,

but we have eliminated p by deriving the market efficiency η in a partial asset market equilibrium and we have eliminated f by expressing it in terms of η . We can solve equations (14)–(15) explicitly when the search-cost function c is specified appropriately as we show in the following example, but the remainder of the paper provides results for general search-cost functions.

Example: Closed-Form Solution. A cost specification motivated by the search literature is

$$c(M, A) = \bar{c} \left(\frac{A}{M} \right)^\alpha \text{ for } M > 0 \quad \text{and} \quad c(M, A) = \infty \text{ for } M = 0, \quad (16)$$

where the constants $\alpha > 0$ and $\bar{c} > 0$ control the nature and magnitude of search frictions. The idea is that informed asset managers are easier to find if a larger fraction of all asset managers are informed, while performing due diligence (which requires the asset manager's time and cooperation) is more difficult in a tighter market with a larger number of searching investors. With this search cost function, equations (14)–(15) can be combined to yield

$$\eta = 2\gamma (\bar{c}k^\alpha)^{\frac{1}{1+\alpha}}, \quad (17)$$

which shows how search costs and information costs determine market inefficiency η . We then derive the equilibrium number of informed investors I from (9):

$$I = \gamma \sigma_q \sigma_\varepsilon \sqrt{\frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2} \frac{1}{1 - e^{-2\eta}}} - 1 = \gamma \sigma_q \sigma_\varepsilon \sqrt{\frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2} \frac{1}{1 - e^{-4\gamma(\bar{c}k^\alpha)^{\frac{1}{1+\alpha}}}}} - 1. \quad (18)$$

The number of informed managers can be linearly related to the number of searching investors based on (15) and (17):

$$M = \frac{\eta}{2\gamma k} A = \left(\frac{\bar{c}}{k} \right)^{\frac{1}{1+\alpha}} A, \quad (19)$$

so the number of managers per investor M/A depends on the magnitude of the search cost \bar{c} relative to the information cost k . Combining (19) with the identity $I = A + M \frac{N}{M}$ yields the solution for A

$$A = I \left(1 + \frac{N}{\bar{M}} \left(\frac{\bar{c}}{k} \right)^{\frac{1}{1+\alpha}} \right)^{-1}, \quad (20)$$

concentrating on parameters for which $A < \bar{A}$.

When η is small — a reasonable value is $\eta = 6\%$, as we show in Section 3.3 — we can approximate the number of informed investors more simply as

$$I \cong \frac{\gamma}{(2\eta)^{1/2}} \frac{\sigma_q \sigma_\varepsilon \sigma_v}{(\sigma_\varepsilon^2 + \sigma_v^2)^{1/2}} = \frac{\gamma^{1/2}}{2(\bar{c}k^\alpha)^{\frac{1}{2(1+\alpha)}}} \frac{\sigma_q \sigma_\varepsilon \sigma_v}{(\sigma_\varepsilon^2 + \sigma_v^2)^{1/2}}, \quad (21)$$

illustrating more directly how search costs \bar{c} and information costs k lower the number of informed investors, while risk aversion γ and noise trading σ_q raise I .

Figure 2 provides a graphical illustration of the determination of equilibrium as the intersection of the managers' and investors' indifference curves. The figure is plotted based on the parametric example above,¹⁰ but it also illustrates the derivation of equilibrium for a general search function $c(M, A)$.

Specifically, Figure 2 shows various possible combinations of the numbers of active investors, A , and informed asset managers, M . The solid blue line indicates investors' indifference condition (14). When (A, M) is to the North-West of the solid blue line, investors prefer to search for asset managers because managers are easy to find and attractive to find due to the limited efficiency of the asset market. In contrast, when (A, M) is South-East of the blue line, investors prefer to be uninformed as the costs of finding a manager is not

¹⁰We use the following parameters. Starting with the investors, the total number of optimizing investors is $\bar{A} = 10^8$, the number of noise allocators is $N = 10^8$, the absolute risk aversion is $\gamma = 3 \times 10^{-5}$, corresponding to a relative risk aversion $\gamma^R = 3$ and an average invested wealth of $W = 10^5$. The total number of managers is $\bar{M} = 4,000$. Turning to asset markets, the number of shares outstanding is normalized to $\bar{q} = 1$, the expected final value of the asset equals total wealth $\bar{v} = (\bar{A} + N)W = 2 \times 10^{13}$, the asset volatility is 20% meaning that $\sigma_v = 0.2\bar{v}$, the signal about the asset has a 30% noise, $\sigma_\varepsilon = 0.3\bar{v}$, and the noise in the supply is 20% of shares outstanding, $\sigma_q = 0.2$. Lastly, the frictions are given by the cost of being an informed asset manager $k = 2 \times 10^7$ and the search cost parameters $\alpha = 0.8$ and $\bar{c} = 0.3$.

outweighed by the benefits. The indifference condition is naturally increasing as investors are more willing to be active when there are more asset managers.

Similarly, the dashed red line shows the managers' indifference condition (15). When (A, M) is above the red line, managers prefer not to incur the information cost k since too many managers are seeking to service the investors. Below the red line, managers want to become informed. Interestingly, the manager indifference condition is hump shaped for the following reason: When the number of active investors increases from zero, the number of informed managers also increases from zero, since the managers are encouraged to earn the fees paid by searching investors. However, the total fee revenue is the product of the number of active investors A and the fee f . The equilibrium fee f decreases with number of active investors because active investment increases the asset-market efficiency, thus reducing the value of the asset management service. Hence, when so many investors have become active that this fee-reduction dominates, additional active investment decreases the number of informed managers.

The economy in Figure 2 has two equilibria. In one equilibrium $(A, M) = (0, 0)$, meaning no investor searches for asset managers as there is no one to be found, and no asset manager sets up operation because there are no investors. We naturally focus on the more interesting equilibrium with $A > 0$ and $M > 0$.

Figure 2 also helps illustrate the set of equilibria more generally. First, if the search and information frictions c and k are strong enough, then the blue line is initially steeper than the red line and the two lines only cross at $(A, M) = (0, 0)$, meaning that this equilibrium is unique due to the severe frictions. Second, if frictions c and k are mild enough, then the blue line ends up below the red line at the right-hand side of the graph with $A = \bar{A}$. In this case, all investors being active is an equilibrium. Lastly, when frictions are intermediate — as in Figure 2 — the largest equilibrium is an interior equilibrium, i.e., $A < \bar{A}$ and $M < \bar{M}$. We focus on such interior equilibria since they are the most realistic and interesting ones. We note that, while Figure 2 has only a single interior equilibrium, more interior equilibria may exist for other specifications of the search cost function (e.g., because the investor indifference

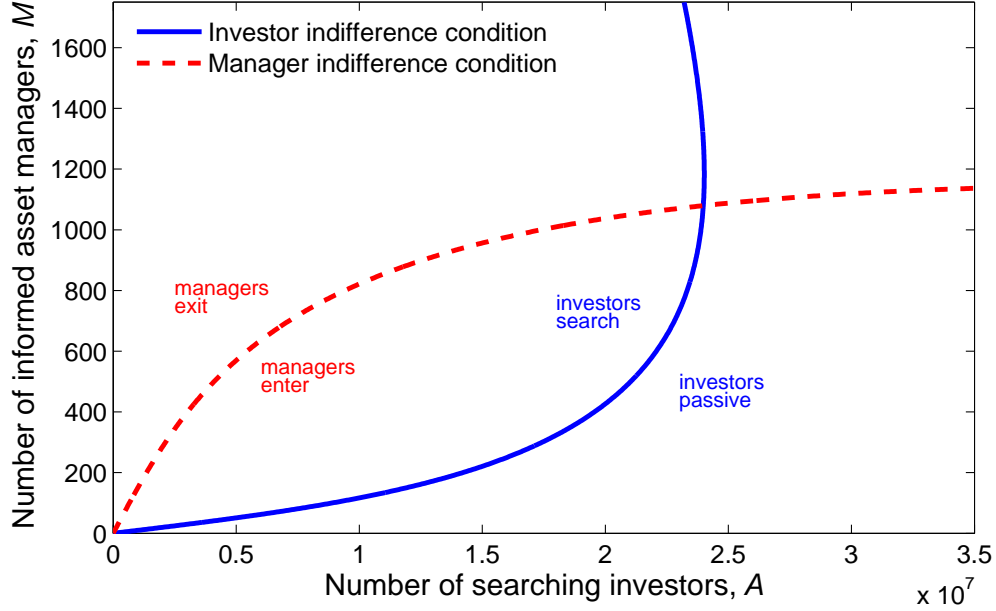


Figure 2: **Equilibrium for assets and asset management.** Illustration of the equilibrium determination of the number of searching investors A and the number of informed asset managers M . Each investor decides whether to search for an asset manager or invest uninformed depending on the actions (A, M) of everyone else, and, similarly, managers decide whether or not to acquire information. The right-most crossing of the indifference conditions is an interior equilibrium.

condition starts above the origin, or because it can in principle “wobble” enough to create additional crossings of the two lines).

3 Equilibrium Properties

3.1 Performance of Asset Managers and Investors

We start by considering some basic properties of performance in efficiently inefficient markets. We use the term outperformance to mean that an informed investor’s performance yields a higher expected utility than that of an uninformed, and vice versa for underperformance. We note that an investor’s expected utility is directly linked to his (squared) Sharpe ratio,

whose expectation is in turn proportional to the expected return.¹¹

Proposition 1 (Performance) *In a general equilibrium for assets and asset management:*

- (i) *Informed asset managers outperform uninformed investing before and after fees, $u_i - f > u_u$. Uninformed asset managers underperform after fees.*
- (ii) *Searching investors' outperformance net of fees just compensates their search costs in an interior equilibrium, $u_i - f - c = u_u$. Larger equilibrium search frictions means higher net outperformance for informed managers.*
- (iii) *The asset-weighted average manager (or, equivalently, the asset-weighted average investor) outperforms after fees if and only if the number N of noise allocators is small relatively to the number A of searching investors, $A \geq N \left(1 - 2\frac{M}{M}\right)$.*

These results follow from the fact that investors must have an incentive to incur search costs to find an asset manager and pay the asset-management fees. Investors who have incurred a search cost can effectively predict manager performance. Interestingly, this performance predictability is larger in an asset management market with larger search costs.

To the extent the search costs are larger for hedge funds than mutual funds, larger for international equity funds than domestic ones, larger for insurance products than mutual funds, and larger for private equity than public equity funds, this result can explain why the former asset management funds may deliver larger outperformance and why the markets they invest in are less efficient.

3.1.1 Searching for a Manager based on Assets Under Management

So far we have assumed that investors can only invest as uninformed or pay a search cost to find an informed manager. We illustrate here the implications of allowing investors, at a lower cost, to also draw a random manager according to some mechanism. This form of uninformed investment in the market for asset management parallels the uninformed

¹¹See Section 3.3 and the proof of Proposition 7 for these basic results of a mean-variance framework.

investment in the security market in Grossman and Stiglitz (1980), that is, investment based on freely available information. To make this alternative as attractive as possible, we take this search cost to be zero and, furthermore, make it more likely to draw a larger manager. To be precise, we assume that the investor obtains the industry-wide after-fee return. We analyze this extended model in Appendix A.2. As an alternative, we consider below an example in which investor are allowed to condition on all managers' AUM.

For some parameters, the equilibrium in the baseline model is the same as the equilibrium in the extended model. Indeed, if the asset-weighted net return is worse than from uninformed investing, which can be determined based on the condition in Proposition 1(iii), then the equilibrium does not change as this search for a random manager is not attractive.

If not, then the equilibrium in the extended model changes: some investors will switch from being uninformed or active to searching for a random manager, until the point at which the asset-weighted manager's net performance matches that of uninformed investing.

Proposition 1b. *In the equilibrium of the extended model, the asset-weighted manager's outperformance after fees is zero (interior equilibrium) or negative, $p_I u_i + (1 - p_I) u_u - f \leq u_i$, where p_I is the fraction of assets managed by informed managers.*

Hence, it may be no coincidence that the average manager in the data delivers a similar performance to index funds. Said differently, in an interior equilibrium of our extended model, the assumption of Berk and Green (2004) that asset managers deliver a zero outperformance after fees holds at the level of the overall asset management *industry*, but not at the level of each *individual* manager.

To understand the intuition for this result, start by recalling that an asset manager's AUM is noisy. Hence, while informed managers have higher AUM on average, any one informed manager could have lower AUM than any one uninformed manager by chance. Therefore, picking managers based on their AUM results in a mixture of informed and uninformed ones. Further, while informed managers are expected to outperform net of fees, uninformed

managers underperform after fees (because they charge a fee even though they don't add value). Therefore, a mixture of these can be (and, indeed, will be, in an interior equilibrium) just as good as direct uninformed investing and just as good as paying a search cost to find a manager who is surely informed.

Example: distribution of manager size and performance. The underperformance of the uninformed managers ($-f$) is as large in magnitude as the outperformance of informed managers ($u_i - f - u_u = f$). Therefore, picking a random manager is a good investment if the chance of getting an informed manager is at least 50%. In the numerical example of Section 2.5, there are more uninformed than informed managers in equilibrium, so picking a random manager would not be a good investment even if it were free. However, the informed managers have more investors on average, so investing with the “market portfolio” of managers would be better. Nevertheless, such an AUM-weighted manager investment is also dominated by investing directly as uninformed in the example.

We can further refine the example to explicitly consider the size distribution across asset managers. For instance, suppose that each manager receives a number of noise allocators that is exponentially distributed with mean N/\bar{M} (i.e., exponential parameter \bar{M}/N). This distribution can arise if noise allocators invest based on news stories, and news stories about each manager arrive at Poisson jumps such that each manager receives media attention for an exponentially-distributed time period. Each informed manager also receives A/M searching investors for sure (i.e., without randomness, for simplicity).

In this case, managers with fewer than A/M investors must be uninformed. Among managers with any number of investors greater than A/M , a constant proportion — 47%, given the parameters of our numerical example — are informed, and the remainder — 53% — are uninformed.¹² Hence, if we further extended the model to allow investors to pick a manager at any specific size (at some cost), then investors would not want to do so given that only 47% of managers are informed. Instead, investors would still prefer to either pay

¹²Specifically, this proportion equals $Me^{\frac{\bar{M}}{N} \frac{A}{M}} / (Me^{\frac{\bar{M}}{N} \frac{A}{M}} + \bar{M} - M)$.

a search cost to ensure finding an informed manager for sure or invest as uninformed.

3.1.2 On the Impossibility of Efficient Asset Management: A Paradox

The Grossman-Stiglitz paradox shows that security markets cannot be fully efficient since, if they were, no one would have an incentive to collect information. A similar paradox exists for asset management markets: public signals about asset managers such as their AUM cannot fully reveal which managers are informed since, if they did, no investor would have an incentive to search and do due diligence. This insight can be seen rigorously in the version of our model in which investors can invest based on AUM for free. If the number of noise allocators goes to zero, then AUM becomes very informative, leading fewer investors to pay for search, and, eventually, the only equilibrium is one in which no investor searches and no manager is informed. This equilibrium is fragile, however, as the market is so inefficient that investors have strong incentives to find an informed manager (should any exist) — but, as soon as someone succeeds in finding an informed manager, other investors can free ride. Thus, noise allocators are needed to resolve this paradox just as noise traders are needed for the Grossman-Stiglitz paradox.

3.1.3 Meaning of Efficiently Inefficient

We say that the asset price is *fully efficient* if $\eta = 0$, meaning that the price fully reflects the signal. In equilibrium, asset prices always involve some degree of inefficiency ($\eta > 0$), but efficiency can arise as a limit, as we shall see in the next section.

There can be several measures of the inefficiency of asset management markets. One measure of this inefficiency is the aggregate cost of locating asset managers plus their aggregate information cost, $cA + kM$. As we shall see next, this aggregate asset management inefficiency can be reduced towards zero if the search cost is reduced. Another measure of asset management efficiency could be the extent to which AUM reflects a manager's information as discussed in the paradox above.

We employ the term *efficiently inefficient* to refer to the equilibrium level of inefficiency

given the frictions (as discussed in the introduction). This definition applies both to markets for securities and asset managers.

3.2 Comparative Statics

We next consider how the economic outcomes depend on the exogenous parameters. To analyze such comparative statics in a model that could have multiple equilibria, we focus on the equilibrium with the largest value of I simply because we need to pick a given equilibrium. We start with the implications of changing the search cost:

Proposition 2 (Search for asset management)

- (i) *Consider two search cost functions, c_1 and c_2 , with $c_1 > c_2$ and the corresponding largest- I equilibria. In the equilibrium with the lower search costs c_2 , the numbers of active investors A and of informed investors I are larger, the number of managers M may be higher or lower, the asset price is more efficient, the asset management fee f is lower, and the total fee revenue $f(A + N)$ may be either higher or lower.*
- (ii) *If $\{c_j\}_{j=1,2,3,\dots}$ is a decreasing series of cost functions that converges to zero at every point, then $A = \bar{A}$ when the cost is sufficiently low, that is, all rational agents search for managers. If the number of investors $\{\bar{A}_j\}$ increases towards infinity as j goes to infinity, then η goes to zero (full price efficiency in the limit), the asset management fee f goes to zero, the number of asset managers M goes to zero, the number of investors per manager goes to infinity, and the total fee revenue of all asset managers $f(A + N)$ goes to zero.*

This proposition provides several intuitive results, which we illustrate in Figure 3. As seen in the figure, a lower search costs means that the investor indifference curve moves down, leading to a larger number of active investors in equilibrium. This result is natural, since investors have stronger incentives to enter when their cost of doing so is lower.

The number of asset managers can increase or decrease (as in the figure), depending on the location of the hump in the manager indifference curve. This ambiguous change in M

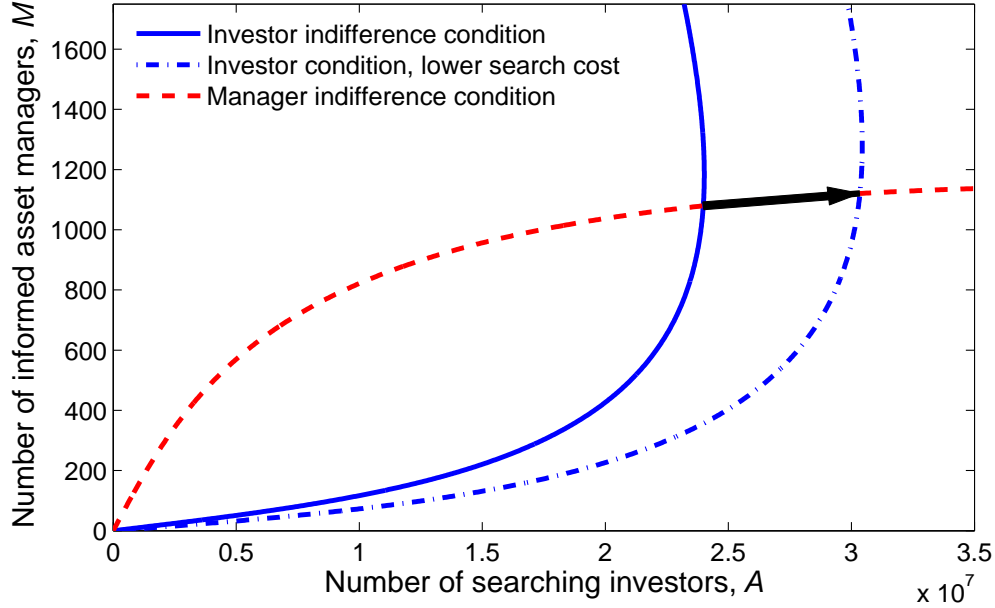


Figure 3: **Equilibrium effect of lower investor search costs.** The figure illustrates that lower costs of finding asset managers implies more active investors in equilibrium and, hence, increased asset-market efficiency.

is due to two countervailing effects. On the one hand, a larger number of active investors increases the total management revenue that can be earned given the fee. On the other hand, more active investors means more efficient asset markets, leading to lower asset management fees. When the search cost is low enough, the latter effect dominates and the number of managers starts falling as seen in part (ii) of Proposition 2.

As search costs continue to fall, the asset-management industry becomes increasingly concentrated, with fewer and fewer asset managers managing the money of more and more investors. This leads to an increasingly efficient asset market and market for asset management.

Perhaps surprisingly, the security market can become almost efficient despite a high Grossman-Stiglitz cost k . This finding is driven by the fact that, as search costs diminish, investors essentially share the information cost more and more efficiently. Indeed, the aggregate information cost incurred is kM , which decreases towards zero as the asset management industry consolidates.

We next consider the effect of changing the cost of acquiring information, which depends on some realistic properties of the search function.¹³

Proposition 3 (Information cost) *Suppose that c satisfies $\frac{\partial c}{\partial M} \leq 0$ and $\frac{\partial c}{\partial A} \geq 0$. As the cost of information k decreases, the largest equilibrium changes as follows: The number of informed investors I increases, the number of asset managers M increases, the asset-price efficiency increases, and the asset-management fee f goes down. The number of active investors A may increase or decrease.*

The results of this proposition are illustrated in Figure 4. As seen in the figure, a lower information cost for asset managers moves their indifference curve out. This leads to a higher number of asset managers and informed investors in equilibrium, which increases the asset-price efficiency. Finally, we consider the effect of risk.

Proposition 4 (Risk) *Suppose that $\frac{\partial c}{\partial M} \leq 0$ and $\frac{\partial c}{\partial A} \geq 0$. An increase in the fundamental volatility σ_v or in the noise-trading volatility σ_q leads to a larger number of active investors A , informed investors I , and informed asset managers M . The effect on the efficiency of asset prices and the asset-management fee f , as well as the total fee revenues $f(A + N)$, is ambiguous. The same results obtain with a proportional increase in $(\sigma_v, \sigma_\varepsilon)$ or in all risks $(\sigma_v, \sigma_\varepsilon, \sigma_q)$.*

3.3 Economic Magnitude of Market Inefficiency

The debate in financial economics is often centered around whether the market is inefficient or not. However, the Grossman-Stiglitz insight implies that we should really ask *how* inefficient. Our model can help provide an answer. As we show below, the answer is neither “yes” nor “no,” but “6%.”

¹³Proposition 3 relies on a regularity condition on the search cost function c . On one hand, finding an informed manager is easier if a larger fraction of all managers are informed: $\frac{\partial c}{\partial M} \leq 0$. On the other, it is more challenging if more other investors are competing for the asset manager’s time and attention: $\frac{\partial c}{\partial A} \geq 0$. This may be, for instance, why many managers have a minimum investment size. That said, there are potential channels, such as word-of-mouth communication, through which a larger number of searching investors may alleviate search costs. Our condition is satisfied for the search cost function considered in our example in equation (16).

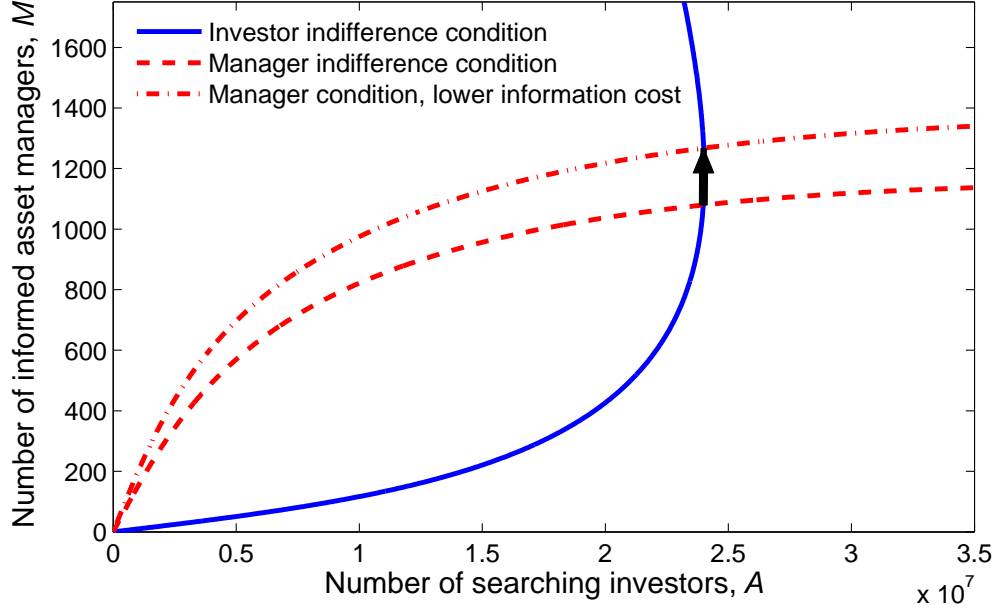


Figure 4: **Equilibrium effect of lower information acquisition costs.** The figure illustrates that lower costs of getting information about assets implies more active investors and more asset managers in equilibrium and, hence, increased asset-market efficiency.

To illustrate the economic magnitudes of some of the interesting properties of the model in a simple way, it is helpful to write our predictions in relative terms. Specifically, as seen in Section 4, investors' preferences can be written in terms of the relative risk aversion γ^R and wealth W such that $\gamma = \gamma^R/W$. Further, the asset management fee can be viewed as a fixed proportion of the investment size and we define the proportional fee as $f^\% = f/W$. To be specific, suppose that all investors have relative risk aversion of $\gamma^R = 3$ and that the equilibrium percentage asset management fee is $f^\% = 1\%$.

We can now link the market inefficiency η to the proportional asset management fee and relative risk aversion,

$$\eta = 2f\gamma = 2f^\% \gamma^R = 2 \cdot 1\% \cdot 3 = 6\%. \quad (22)$$

In other words, the standard deviation of the true asset value from the perspective of a trader who knows the signal is 6% smaller than that of a trader who only observes the price.

Further, we see that the inefficiency is greater in markets with higher percentage fees (e.g., private equity vs. public) and during times of high risk aversion (e.g., crisis periods).

We can also characterize the inefficiency by the difference in squared gross Sharpe ratios attainable by informed (SR_i) vs. uninformed (SR_u) investors using a log-linear approximation:¹⁴

$$E(SR_i^2) - E(SR_u^2) \cong 2\eta = 4f\% \gamma^R = 4 \cdot 1\% \cdot 3 = 0.12. \quad (23)$$

Hence, if uninformed investing yields an expected squared Sharpe ratio of 0.4^2 (similar to that of the market portfolio), informed investing must yield an expected squared Sharpe ratio around 0.53^2 (i.e., $0.53^2 - 0.4^2 = 0.12$). We see that, at this realistic fee level, the implied difference in Sharpe ratios between informed and uninformed managers is relatively small and hard to detect empirically. While the model-predicted magnitude of inefficiency appears reasonable, our model is of course quite stylized and needs to be supplemented with empirical analysis.

4 Small vs. Large Investors and Asset Managers

So far, we have considered an economy in which all investors and managers are identical ex ante, but, in the real world, investors differ in their wealth and financial sophistication and managers differ in their education and investment approach. Should large asset owners such as high-net-worth families, pension funds, or insurance companies invest differently than small retail investors and what type of asset managers are more likely to be informed?

To address these issues, we extend the model to capture different types of investors and managers. Each investor $a \in [0, \bar{A}]$ has an investor-specific search cost c_a , where a smaller

¹⁴Since each type of investor $n = i, u$ chooses a position of $x = \frac{E_n(v) - p}{\gamma \text{Var}_n(v)}$, the investor's conditional Sharpe ratio is $SR_n = \frac{|E_n(v) - p|}{\sqrt{\text{Var}_n(v)}}$ (where E_n and Var_n are the mean and variance conditional on n 's information). We have $\eta = \log \left(E \left[e^{-\frac{1}{2}(v-p) \frac{E[v-p|p]}{\text{var}(v|p)}} \right] \right) - \log \left(E \left[e^{-\frac{1}{2}(v-p) \frac{E[v-p|s,p]}{\text{var}(v|s)}} \right] \right)$, which is approximated by $\frac{1}{2} \left(E \left[(v-p) \frac{E[v-p|s,p]}{\text{var}(v|s)} \right] - E \left[(v-p) \frac{E[v-p|p]}{\text{var}(v|p)} \right] \right)$, yielding (23) because the conditional variances are constant.

search cost corresponds to greater sophistication. Further, investors have different levels of absolute risk aversion, γ_a . We can interpret these as arising from different wealth W_a or different relative risk aversions γ_a^R , corresponding to a constant absolute risk aversion of $\gamma_a = \gamma_a^R/W_a$.¹⁵ The characteristics c_a , γ_a^R , and W_a are drawn randomly, independently of each other and across agents. Also, noise allocators $n \in [0, N]$ have (c_n, γ_n^R, W_n) drawn independently from the same distribution.¹⁶

To capture different types of asset managers, we assume that each manager $m \in [0, \bar{M}]$ has a manager-specific cost k_m of becoming informed — one can think of this feature as skill or education — and that they are ordered according to this cost. Hence, managers with lower index m have lower costs, that is, the function $k : [0, \bar{M}] \rightarrow \mathbb{R}$ is increasing.

We solve the model similarly to before, but we leave the details to Appendix A.3.

4.1 Who Should be Active?

We first study which types of investors should search for an active manager:

Proposition 5 (Which investors should be active?) *An investor a should invest with an active manager if he has a large wealth W_a , low relative risk aversion γ_a^R , or low search cost c_a , all relative to the asset-market inefficiency η , that is, if*

$$\frac{\gamma_a^R c_a}{W_a} = \gamma_a c_a \leq \frac{1}{2}\eta, \quad (24)$$

and otherwise should invest as uninformed.

This result is intuitive and consistent with the idea that the active investors should be those who have a comparative advantage in asset allocation, either large investors who can hire a serious manager-selection team or sophisticated investors with special insights on

¹⁵Wealth levels vary a lot more in the cross-section — easily by factors measured in thousands — than relative risk aversions, so that variation in γ_a is likely mostly driven by wealth differences in the real world.

¹⁶These independence assumptions only affect our performance results and we note that these results would only be strengthened under the realistic assumptions that high sophistication (low c) correlates with high wealth W , or if noise allocators are more likely to have low sophistication and wealth.

asset managers. Indeed, the cost of finding and vetting an informed asset manager is a smaller fraction of the investment for large investors as is captured by equation (24). In contrast, small retail investors are better served by low-cost uninformed investing. The next proposition states the corresponding result for asset managers.

Proposition 6 (Which managers should be informed?) *An asset manager m should acquire information if her information cost is low, $k_m \leq k_M$, and otherwise remain uninformed.*

Clearly, asset managers are more likely to have success in informed trading if they are well educated, experienced, and have access to an existing research infrastructure, while other managers who find it more difficult to collect useful information might prefer to limit their costs.

4.2 How Size and Sophistication Affect Performance

The model makes clear predictions about the expected performance differences across different types of investors and asset managers. Investors who are more wealthy (high W_a) and more sophisticated (low c_a) are more likely to search for an informed manager and, as a result, such investors allocate to better managers on average.

To state these performance predictions in terms of percentage returns, we suppose, without loss of generality, that a manager scales the portfolio such that any investor with a relative risk aversion of $\gamma_a^R = \bar{\gamma}^R$ optimally invest his entire wealth W_a with the manager. Then we can define the investor return with the manager as his dollar profit per capital committed W_a . An investor with relative risk aversion twice as high, $\gamma_a^R = 2\bar{\gamma}^R$, naturally invests only half his wealth with the manager and earns the same percentage return (before fees) on the committed capital.

Proposition 7 (Investor performance: size and sophistication) *Holding fixed other characteristics, larger investors (higher W_a) earn higher expected returns before and after fees and pay lower percentage fees, on average. Likewise, holding fixed other characteristics,*

more sophisticated investors (lower c_a) earn higher expected returns before and after fees and pay lower percentage fees.

These results are intuitive and give rise to several testable predictions that we confront with the existing evidence in Section 5. Since large and sophisticated investors can better afford to spend resources on finding an informed manager, they are more likely to find one and, as a result, they expect to earn higher returns.¹⁷ The higher returns are partly a compensation for the search costs that these agents incur, but they can even outperform after search costs when inequality (24) is strict.

Said differently, if a small investor with no special knowledge of asset managers (that is, an investor for whom (24) is not satisfied) invests with an active manager, then he must be a noise allocator. Since noise allocators pay fees even to uninformed managers, such investors are expected to earn lower returns.

On the other hand, noise allocators are under-represented among large sophisticated investors. We note that the model-implied effect is not linear in that, as investors become very large (or sophisticated), they search for a manager almost surely, and therefore an even larger size has a negligible effect on their expected performance. We next consider how performance varies across asset managers.

Proposition 8 (Asset manager performance)

- (i) Asset manager returns (before and after fees) and their average investor size covary positively. Similarly, returns and average sophistication covary positively.*
- (ii) Asset-manager size and expected returns (before and after fees) covary positively. Similarly, managers with a comparative advantage in collecting information ($k_m \leq k_M$) earn higher expected returns before and after fees.*

Part (i) shows that asset managers with larger and more sophisticated investors are more likely to have investors who have performed due diligence and confirmed that they

¹⁷The fact that investors with large absolute risk tolerance choose informed investing through managers (by paying search costs and fees) parallels the result of Verrecchia (1982) that more risk-tolerant investors purchase more precise (and expensive) signals.

are informed about security markets. These managers, being more likely to have passed a screening, should deliver higher expected return on average (even though some of them can still be uninformed as some large investors can also be noise allocators). Other measures proxying for the type of a manager’s clientele, such as the proportion of large investors (i.e., with wealth above a given threshold), would work as well.

Part (ii) shows that managers who find it easier to collect information are more likely to do it. Indeed, for the marginal manager, the cost of information equals the benefit, so anyone with higher costs will not acquire information. Hence, an asset manager may be more likely to be informed if she is well educated, experienced, and benefits from firm-wide investment research as part of an investment firm with multiple funds. Hence, investors’ search process might partly consist of examining whether an asset manager has such qualities, as discussed further in Appendix B.

5 Empirical Implications

Our model has implications for investors, asset managers, and financial markets — three layers that we represent schematically in Figure 5. We start by examining the predictions and empirical evidence at the middle level, i.e., concerning managers.

5.1 Performance of Asset Managers

The central prediction of asset market efficiency is that all managers underperform by an amount equal to their fees, and, indeed, the early empirical literature documented negative average after-fee returns for US mutual funds (Fama (1970)). More recent evidence suggests that the average alpha after fees is close to zero (Berk and Binsbergen (2015)). Further, the research over the past decades shows that the evidence for the average asset manager hides significant cross-sectional variation across managers. Indeed, the literature documents a significant difference between the net-of-fee performance of the best and worst managers of mutual funds (Kosowski et al. (2006), Kacperczyk et al. (2008), Fama and French (2010),

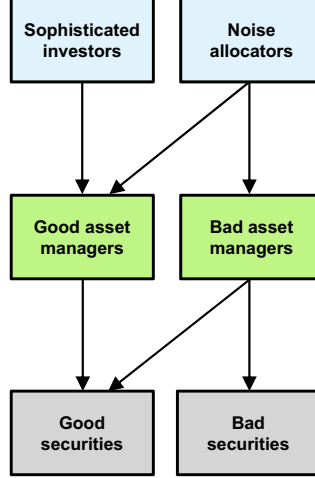


Figure 5: **Testing the model at three levels.** The figure illustrates stylistically the three layers for which our model has new cross-sectional implications: investors, asset managers, and financial markets. Further, the model also makes predictions on the interaction between these layers, that is, the interaction between investors, securities, and the industrial organization of asset management.

Keswani et al. (2016)), hedge funds (Kosowski et al. (2007), Fung et al. (2008), Jagannathan et al. (2010)), private equity, and venture capital funds (Kaplan and Schoar (2005)). For instance, Kosowski, Naik, and Teo (2007) report that *“top hedge fund performance cannot be explained by luck, and hedge fund performance persists at annual horizons... Our results are robust and relevant to investors as they are neither confined to small funds, nor driven by incubation bias, backfill bias, or serial correlation.”*

The strong performance of the best managers presents a rejection of Fama’s hypothesis that asset markets are fully efficient and all asset managers underperform by their fees. Further, the net-of-fee performance spread between the best and worst managers is a rejection of the hypothesis by Berk and Green (2004) that all managers deliver the same expected net-of-fee return. The existence of the performance spread is, however, consistent with our model’s predictions. In our model, top asset managers should be difficult to locate and their outperformance must compensate investors for their search costs.

We note the following subtlety concerning the relation between our model and empirical tests: Our model implies that investors should be able to find managers that outperform net

of fees only after incurring a search cost, but, then, should an empirical researcher be able to identify such managers? On one hand, researchers should not be able to locate informed managers based on public information that investors can easily process (as seen in the version of our model where investors can search for free based on AUM, Section 3.1.1). On the other hand, skilled researchers using large commercial databases and advanced statistical methods would be able to locate informed managers, to the extent that this research mimics in part investors’ costly search process. Of course, investors with access to more data (e.g., meetings with managers that reveal the trading infrastructure) should be able to do even better.

Again, there is a close parallel between the market for assets and the market for asset management: Just like finding good asset managers should be difficult, but not impossible, finding good securities should also be difficult, but not impossible. In both cases, researchers may identify good managers and good securities based on commercial data that are costly to process. For instance, asset pricing anomalies are typically based on such commercial data.

5.2 Manager Performance: Link to Our Search Mechanism

While the mere existence of a performance spread among the best and worst asset managers rejects existing theories and favors ours, this “victory” need not be that informative given that other theories might also predict such a performance spread. To test the model at a deeper level, we ask whether performance differences appear to be driven by our search mechanism, that is, are consistent with the predictions of Proposition 8.

Consistent with search costs being higher for alternative investments (hedge funds and private equity) than for mutual funds, we see larger performance spreads among alternative managers. However, comparisons across markets may be driven by multiple differences, so we need to dig deeper still, as we do in Table 1.

First, mutual funds that have an institutional share class outperform other mutual funds (Evans and Fahlenbrach (2012)), consistent with the idea that the institutional investors are more likely to have performed due diligence (Proposition 8(i)).

Second, the group of managers servicing all institutional investors outperform the mutual

funds servicing retail investors (Gerakos et al. (2016)). Indeed, Gerakos et al. (2016) find that the asset managers servicing institutions overall delivered outperformance after fees, in contrast to the evidence on the average retail mutual fund discussed above.

Third, Guercio and Reuter (2014) find that mutual funds sold directly to searching investors outperform those that are placed via brokers who earn commissions/loads (to noise allocators).

Fourth, consistent with Proposition 8(ii), Chevalier and Ellison (1999) find that *“managers who attended higher-SAT undergraduate institutions have systematically higher risk-adjusted excess returns”* and Chen et al. (2004) find that *“Controlling for fund size [...] the assets under management of the other funds in the family that the fund belongs to actually increase the fund’s performance.”*

Last, consistent with Proposition 1(ii), the outperformance of managers of searching investors is larger in less efficient markets. Dyck et al. (2013) find that *“active management in emerging market equity outperforms passive strategies by more than 180 bps per year, and that this outperformance generally remains significant when controlling for risk through a variety of mechanisms. In EAFE equities (developed markets of Europe, Australasia, and the Far East), active management also outperforms, but only by about 50 bps per year, consistent with these markets being relatively more competitive and efficient.”* Together, these findings provide significant and diverse evidence for the model’s performance predictions.

From investors’ perspective, the relevant measure of manager performance is average excess return (or alpha), but, when evaluating managers’ skill per se, Berk and Binsbergen (2015) argue that the manager’s “value added” is a better measure. They find that the *“cross-sectional distribution of managerial skill is predominantly reflected in the cross-sectional distribution of fund size,”* a result consistent with the prediction of our Proposition 8(ii).

5.3 Investor Performance: Link to Our Search Mechanism

We now turn to the predictions for the top layer of Figure 5, namely investors. We have already discussed that institutional investors outperform even after fees (Gerakos et al. (2016)),

Table 1: **Evidence on our predictions.** Panel A includes references on the performance differences between asset managers servicing investors who are more likely to be searching investors vs. those servicing noise allocators. Panel B includes quotes on the investors' performance. These references show that asset managers found by searching investors outperform those of noise allocators, consistent with our model's predictions.

References	More likely searching investors	More likely noise allocators	Finding of references
Panel A: Evidence on asset managers			
Evans and Fahlenbrach (2012)	Institutional investors	Retail investors	<i>"retail funds with an institutional twin outperform other retail funds by 1.5% per year"</i>
Gerakos, Linnainmaa, Morse (2015)	Institutional investors	Retail investors	<i>"institutional funds earned annual market-adjusted returns of 108 basis points before fees and 61 basis points after fees"</i>
Del Guercio and Reuter (2014)	Investors searching for direct-sold funds	Investors buying broker-sold funds	<i>"direct-sold actively managed funds outperform actively managed broker-sold funds"</i>
Dyck, Lins, and Pomorski (2013)	Institutional investors	Retail investors	<i>"the value of active management depends on the efficiency of the underlying market and the sophistication of the investor"</i>
Panel B: Evidence on investors			
Dyck and Pomorski (2015)	Larger institutional investors	Smaller institutional investors	<i>"A one standard deviation increase in PE holdings is associated with 4% greater returns per year"</i>
Sialm, Sun, and Zheng (2014)	Fund of funds investing locally	Fund of funds investing far away	<i>"funds of hedge funds overweight their investments in hedge funds located in the same geographical areas and that funds of hedge funds with a stronger local bias exhibit superior performance"</i>

hence performing better than the overall group of retail investors in their active mutual fund allocations. This mirror image of the result for asset managers is consistent with Proposition 7.

Further, as seen in Panel B of Table 1, Dyck and Pomorski (2016) find that large institutions outperform small ones in their private equity investments, further evidence for Proposition 7. Moreover, consistent with our model’s implication that size only matters up to a certain point (at which all investors decide to search), Dyck and Pomorski (2016) find a non-linear effect of size that eventually diminishes.

Likewise, funds of hedge funds perform better on their local investments where they likely have a search advantage (Sialm et al. (2014)), providing evidence on a different aspect of Proposition 7.

Lastly, Gerakos et al. (2016) find that larger investors pay lower percentage fees than small investors with the same asset manager, consistent with Proposition 7. This means that larger investors benefit more from active investing both because they pay lower fees and because their search costs are a smaller fraction of assets. We note that this empirical finding constitutes an even simpler rejection of Berk and Green (2004): if investors pay different fees in the *same* fund, then surely not all investors can earn a zero expected alpha after fees.

5.4 Implications for Asset Pricing and Market Efficiency

Turning to the bottom layer of Figure 5, let us consider the implications for capital markets. Our model is consistent with the existence of anomalies reflecting the types strategies that informed managers pursue.¹⁸ Given that other theories also may explain anomalies, we need to test the theory at a deeper level. Efficiently inefficient markets means that the marginal investor should be indifferent between uninformed investing and searching for asset managers, where the latter should deliver an expected outperformance balanced by asset management fees and search costs, consistent with the findings of Gerakos et al. (2016) for professional

¹⁸While the efficient market hypothesis is a powerful theory, it is nevertheless difficult to test because of the so-called “joint hypothesis” problem. However, the many documented violations of the Law of One Price (securities with the same cash flows that trade at different prices) constitute a clear rejection of fully efficient asset markets.

asset managers. Section 3.3 delivers further predictions on the magnitude of the inefficiency, which are yet to be tested.

Further, in an efficiently inefficient market, anomalies are more likely to arise the more resources a manager needs to trade against them (higher k) and the more difficult it is for investors to build trust in such managers (higher c). For instance, while convertible bond arbitrage is a relatively straightforward trade for an asset manager (low k), it might have performed well for a long time because it is difficult for investors to assess (high c).

5.5 Industrial Organization of Asset Management

Returns to scale. In our model, the overall asset management industry faces decreasing returns to scale since a larger amount of capital with informed managers (I) leads to more efficient markets (lower η), reducing manager performance, consistent with the evidence of Pastor et al. (2015).

In our model individual managers do not face decreasing returns to scale (controlling for the industry size) and larger managers are better on average (because searching investors look for informed managers). Consistently, Ferreira et al. (2013) find that larger asset managers perform better.¹⁹

In contrast, Berk and Green (2004) assume that individual managers face decreasing returns to scale (e.g., due to transaction costs). Pastor et al. (2015) study what happens when a given manager grows larger (seeking to eliminate the effect that larger managers may be different, cross-sectionally) and find that “all methods considered indicate decreasing returns, though estimates that avoid econometric biases are insignificant.”

Industry size. Our model has several implications for the size of the asset management industry. The asset management industry grows when investors’ search costs diminish or when asset managers’ information costs go down, leading to more efficient asset markets, consistent with the evidence of Pastor et al. (2015). Other important models that speak

¹⁹In the US, managers with larger fund family size perform better, but the size of the specific fund is a negative predictor. Outside the US, both the fund and family size predict returns positively. Since information costs may be more related to the size of the fund family (i.e., the overall asset management firm), it makes sense that family size appears to be the more robust predictor.

to the size of the asset management industry include Berk and Green (2004), García and Vanden (2009), and Pastor and Stambaugh (2012).

Concentration. When investors’ search costs go down, our model predicts that the number of managers will fall, but the remaining managers will be larger (in fact so much larger that the total size of the asset management industry grows as mentioned above). Such consolidation of the asset management industry is discussed in the press, but we are not aware of a direct test of this model prediction.

Fees. Finally, we have predictions for asset management fees. Asset-management fees should be larger for managers of more inefficient assets and in more inefficient asset-management markets. For instance, if search costs for managers are large, this leads to less active investing and higher management fees. Note that the higher management fee in this example is not driven by higher information costs for managers, but, rather, by the equilibrium dynamics between the markets for the asset and asset management. This may help explain why hedge funds have historically charged higher fees than mutual funds. Also, markets for assets that are costly to study should be more inefficient and have higher management fees. This can help explain why equity funds tend to have higher fees than bond funds and why global equity funds have higher fees than domestic ones. Lastly, in a cross-country study, Khorana et al. (2008) find that mutual fund *“fees are lower in wealthier countries with more educated populations,”* which may be related to lower search frictions for well educated investors.

6 Conclusion

We propose a theory of investors searching for informed asset managers — in short, Grossman-Stiglitz with asset management. The theory captures the time-consuming vetting process by which real-world investors examine an asset manager (portfolio construction, the number of employees, their professional pedigree, whether the manager operates a trading desk 24/7, co-location on major trading venues, costly information sources, risk management, valuation methods, financial auditors, and so on) and the costly process by which an asset manager examines a security. Our search-plus-information model turns out to be highly tractable and

it yields several novel results that help explain numerous empirical facts about asset prices and asset management that were puzzling in light of existing theories.

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Appendix

A Further Analysis and Proofs

A.1 Equilibrium Characterization: Interior and Corner Equilibria

Here we collect the conditions that define an equilibrium of the four endogenous variables (p, f, M, A) . First, the price p and the corresponding market inefficiency η are given by (9) as a function of the other endogenous variables. Second, the fee f depends on an investor's best outside option, which is the larger of the ex-ante utility of investing on one's own (given by u_u) and that from searching for another manager (given by $u_i - c - f$). It follows that

$$f = \min \left\{ \frac{\eta}{2\gamma}, c \right\}. \quad (\text{A.1})$$

We have the following types of equilibria (where p and f are given above):

(Interior equilibrium) Any pair $(M, A) \in [0, \bar{M}] \times [0, \bar{A}]$ satisfying equations (14)–(15) constitutes an interior equilibrium.

(Corner at zero) The trivial outcome $(M, A) = (0, 0)$ is always an equilibrium. If, for all $(M, A) > (0, 0)$, $f(M, A)A < kM$ or $c(M, A) > u_i - u_u = \frac{\eta}{\gamma}$, then $(M, A) = (0, 0)$ is the only equilibrium.

(Non-zero corner) The pair $(M, A) \in [0, \bar{M}] \times [0, \bar{A}]$ is an equilibrium if and only if

$$\frac{\eta(I)}{2\gamma} \geq c(M, A), \quad \left(\frac{\eta}{2\gamma} - c \right) (\bar{A} - A) = 0 \quad (\text{A.2})$$

$$c(M, A)A \geq kM, \quad (cA - kM)(\bar{M} - M) = 0. \quad (\text{A.3})$$

The conditions (A.2)–(A.3) encompass the conditions for interior equilibrium as well, but in addition allows for the corner solutions $A = \bar{A}$ and $M = \bar{M}$. Here, (A.2) states that investors must achieve at least as high utility from searching vs. not, and equality must hold unless everyone searches (“complementary slackness”). Likewise, (A.3) states that managers must expect a non-zero profit from becoming informed, and they must be indifferent unless all managers become informed.

A.2 Equilibrium when Investors can Search Based on Assets Under Management

We outline here the analysis of the model that incorporates the third investment option, namely making a random draw from the manager pool according to the AUM of each manager. We let $R > 0$ denote the mass of investors searching randomly and now an equilibrium consists of a collection of five endogenous variables (p, f, M, A, R) . As before, the price p and

the corresponding market inefficiency η are given by (9) as a function of the other endogenous variables, but now the number of informed investors I also depends on R :

$$I = A + Rp_I + N\frac{M}{M}, \quad (\text{A.4})$$

where p_I is the probability of finding an informed manager. The probability of randomly selecting an informed manager is the asset-weighted fraction of informed managers:

$$p_I = \frac{A + N\frac{M}{M}}{A + N} \quad (\text{A.5})$$

Next, the fee f is determined through bargaining as before. For this, it is helpful to introduce notation for the ex-ante utilities of the different type of investors. The ex ante utility of paying a search cost to be sure to find an informed manager is denoted $U_i = u_i - c - f$ and the ex ante utility of uninformed investors is $U_u = u_u$. Turning to the new part, the ex ante utility of investors searching randomly based on AUM is

$$U_r = p_I(u_i - f) + (1 - p_I)(u_u - f) = p_I u_i + (1 - p_I)u_u - f,$$

where p_I is the probability of finding an informed manager defined in (A.5) and we assume risk neutrality over certainty equivalents for simplicity.

To calculate the fee, the investor's best outside option now reflects his three options, that is, its value equals $\max\{U_u, U_i, U_r\}$. In any equilibrium, it cannot be the case that $U_r > U_i$, and therefore we can focus on $\max\{U_u, U_i\}$, which yields (A.1) just as before.

The following optimality conditions for investors determine A and R . First, for search to be optimal, we must have $U_i \geq U_u$, that is, as before,²⁰

$$\frac{\eta(I)}{2\gamma} \geq c(M, A), \quad (\text{A.6})$$

with equality if there are uninformed investors, i.e., $A + R < \bar{A}$. Likewise, for random search to be optimal, we must have $U_r = U_i$, that is,

$$p_I u_i + (1 - p_I)u_u - f = u_i - c - f, \quad (\text{A.7})$$

which can be shown to be equivalent to

$$(1 - p_I)\frac{\eta}{\gamma} = c \quad (\text{A.8})$$

²⁰Note that in any non-trivial equilibrium it cannot be the case that $U_u > U_i$, because then there would be no informed investors, and therefore no informed managers, meaning $p_I = 0$, and therefore $R = 0$.

Finally, the managers' optimally condition is now more complex. In particular, indifference between being informed and not means

$$\frac{1}{M} (A + Rp_I) c - k = \frac{1}{\bar{M} - M} R(1 - p_I)c, \quad (\text{A.9})$$

where the left-hand side represents the fees per informed manager, net of the cost of becoming informed, and the right-hand side the amount of fees for the uninformed manager. As before, the fees paid by noise allocators are not included, as they do not depend on manager type. Using the definition of p_I , the indifference condition simplifies to

$$cA \left(1 + \frac{R}{A + N} \right) = kM. \quad (\text{A.10})$$

To summarize, equilibrium with random search is characterized as follows (where, as before, p and f are given by the other endogenous variables).

(Interior equilibrium) The tuple $(M, A, R) \in \mathbb{R}_+^3$ is an interior equilibrium if $M \leq \bar{M}$, $A + R \leq \bar{A}$, and indifference conditions apply to investors ($U_u = U_i = U_r$) and managers:

$$\frac{\eta}{2\gamma} = c = (1 - p_I) \frac{\eta}{\gamma} \quad (\text{A.11})$$

$$cA \left(1 + \frac{R}{A + N} \right) = kM. \quad (\text{A.12})$$

(Corner at zero) The trivial outcome $A = R = M = 0$ is always an equilibrium. If, for all (M, A, R) with $M > 0$, $fA < Mk$ or $c > u_i - u_u = \frac{\eta}{\gamma}$, then $(M, A, R) = (0, 0, 0)$ is the only equilibrium.

(Non-zero corner) The tuple $(M, A, R) \in \mathbb{R}_+^3$ is an equilibrium if and only if $M \leq \bar{M}$, $A + R \leq \bar{A}$, and

$$\frac{\eta}{2\gamma} \geq c, \quad \left(\frac{\eta}{2\gamma} - c \right) (\bar{A} - A - R) = 0 \quad (\text{A.13})$$

$$cA \left(1 + \frac{R}{A + N} \right) \geq kM, \quad \left(cA \left(1 + \frac{R}{A + N} \right) - kM \right) (\bar{M} - M) = 0 \quad (\text{A.14})$$

$$(1 - p_I) \frac{\eta}{\gamma} \leq c, \quad \left((1 - p_I) \frac{\eta}{\gamma} - c \right) R = 0. \quad (\text{A.15})$$

As before, we see that there can be interior equilibria and corner equilibria. In an interior equilibrium, investors are indifferent between their three options (uninformed investing, paying for search, random search). In this case, agents who search randomly neither overperform, nor underperform, in the language of Proposition 1. We also note that (A.11) shows that $p_I = \frac{1}{2}$ in an interior equilibrium.

The set of corner equilibria is more complex now that there are more endogenous variables. In one set of such equilibria, random search does not occur. In particular, consider any equilibrium without the random-search option. Such an equilibrium continues to be an equilibrium the model that allows random search if and only if $U_r \leq \max(U_u, U_i)$. Another type of corner equilibrium involves positive numbers of investors searching at a cost and randomly, but no investor choosing uninformed investing, that is, $A > 0$, $R > 0$, and $\bar{A} - A - R = 0$.

A.3 Equilibrium with Small and Large Investors and Asset Managers

Here we show how to derive an equilibrium with heterogeneous agents, but first we make a comment on the statistical structure of (c_a, γ_a^R, W_a) , beyond the independence already assumed. While γ_a^R and W_a are scalars, and therefore straightforward to specify, the costs c_a are less straightforward since they are functions. We could consider a number of choices, including (i) c_a scalars (constant functions), (ii) c_a proportional to c_0 for all a , or (iii) c_a in some general continuous-function space, possibly one in which all elements are ordered (thus the functions do not cross). The results in Section 4, though, hold for any general specification once an equilibrium is fixed.

An equilibrium with heterogenous agents consists of a price p , a fee f_a for each investor a , a set of active investors, and a set of informed managers. We show that the equilibrium has the following form. First, the set of managers is $\{m : k_m \leq k_M\} = [0, M]$, which is naturally characterized by the total number of informed manager M (as before).

Second, for any given investor, η is a given quantity. The same argument as above (subsection A.1), refined to take into account the agent's characteristics, leads to

$$f_a = \min \left\{ \frac{\eta}{2\gamma_a}, c_a \right\}. \quad (\text{A.16})$$

In equilibrium, agent a searches for a manager if and only if $u_{a,u} \leq u_{a,i} - c_a - f_a$, so that the only observed fee is $f_a = c_a$. The condition for searching, then, becomes $2c_a \leq u_{a,i} - u_{a,u}$, or $\gamma_a c_a \leq \frac{1}{2}\eta$. Hence, the set of active investors is $\{a : \gamma_a c_a \leq \frac{1}{2}\eta\}$, where the price inefficiency η is part of the equilibrium. As for the agent's risky investment, it is the same as that of an agent with unit risk tolerance, but multiplied by her own risk tolerance $1/\gamma_a$.

As before, the price can be characterized via the price efficiency η . The price efficiency depends on the aggregate risk tolerance of all investors with informed managers,

$$\tau = \bar{A} E \left(\frac{1}{\gamma_a} 1_{\{\gamma_a c_a \leq \frac{1}{2}\eta\}} \right) + N \frac{M}{M} E \left(\frac{1}{\gamma_n} \right). \quad (\text{A.17})$$

Here, the first part is the total risk tolerance of searching investors (those who decide to search based on (24)), and the second term is the total risk tolerance of all the noise allocators who happen to find an informed manager. Given the total risk-bearing capacity of investors with informed managers, equation (9), which determines price inefficiency η , is modified by

replacing I/γ with τ :

$$\eta(\tau) = -\frac{1}{2} \log \left(1 - \frac{\sigma_q^2 \sigma_\varepsilon^2}{\tau^2 + \sigma_q^2 \sigma_\varepsilon^2} \frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2} \right). \quad (\text{A.18})$$

Finally, the indifference condition for the marginal asset manager M states that the fee revenue from searching investors per manager covers her cost k_M :

$$\frac{\bar{A}}{M} E \left(c_a 1_{\{\gamma_a c_a \leq \frac{1}{2}\eta\}} \right) = k_M. \quad (\text{A.19})$$

Hence, a general equilibrium with many types of investors and managers is characterized by (η, τ, M) that satisfy (A.17)–(A.19).²¹

A.4 Endogenizing Noise Allocators

The behavior of, and fee paid by, noise allocators can be derived as an outcome by incorporating the following two features into the model. First, noise allocators use a poor search technology. They pay the same cost c to be matched with a manager, but they find a random manager, not necessarily an informed one (they draw the manager from the uniform distribution, not one given by AUM). Second, noise allocators face a high cost c^u of investing on their own even without information, a cost so high that they always search for a manager. Our results do not depend on whether noise allocators believe that the manager is informed or not, so noise allocators can be viewed as fully rational or biased. Our assumptions can be seen as capturing the idea that noise allocators invest based on trust as proposed by Gennaioli et al. (2015). We note that by taking a fixed number of noise allocators we rule out, for simplicity, that managers exploit behavioral biases to affect the number of noise allocators.

Since noise allocators face a high cost of investing on their own, they all search for an asset manager. A fraction M/\bar{M} randomly find an informed manager while the rest find uninformed managers. Since noise allocators cannot tell the difference between informed and uninformed managers, they pay the same fee either way. What specific fee they pay is not central to our results, but we can model the bargaining as before: Noise allocators receive a utility from investing with a random active manager that we denote by u_n . Given her unattractive option of investing on their own, noise allocators' alternative to investing with the current manager is paying the cost c again to find another manager and investing with him at an expected fee of \bar{f} . The gains from agreeing to pay the current manager a fee of f is therefore $W + u_n - c - f - (W + u_n - 2c - \bar{f}) = c + \bar{f} - f$.

The manager has a gain from agreement of f so the equilibrium fee maximizes $(c + \bar{f} - f)f$, which under $\bar{f} = f$ gives $f = c$. As seen from (11) and (14), the fee paid by noise allocators is the same as the fee paid by other investors in an interior equilibrium.

²¹While the indifference condition (A.19) applies with equality in an interior equilibrium, corner solutions are characterized by either $M = 0$ and the right-hand side being greater or $M = \bar{M}$ and the left-hand side being greater.

A.5 Partial Asset-Market Equilibrium

Here we outline the details of the asset-market equilibrium of Section 2.1 in the interest of being self-contained, although these results are effectively provided in Grossman and Stiglitz (1980). An agent having conditional expectation of the final value μ and variance V optimally demands a number of shares equal to

$$x = \frac{\mu - p}{\gamma V}. \quad (\text{A.20})$$

To compute the relevant expectations and variance, we conjecture the form (6) for the price and introduce a slightly simpler “auxiliary” price, $\hat{p} = v - \bar{v} + \varepsilon - \theta_q(q - \bar{q})$, with the same information content as p :

$$\mathbb{E}[v|p] = \mathbb{E}[v|\hat{p}] = \bar{v} + \beta_{v,\hat{p}}\hat{p} = \bar{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2}\hat{p} \quad (\text{A.21})$$

$$\mathbb{E}[v|s] = \mathbb{E}[v|v + \varepsilon] = \bar{v} + \beta_{v,s}(s - \bar{v}) = \bar{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}(s - \bar{v}) \quad (\text{A.22})$$

$$\text{var}(v|p) = \text{var}(v|\hat{p}) = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2} = \frac{\sigma_v^2(\sigma_\varepsilon^2 + \theta_q^2\sigma_q^2)}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2} \quad (\text{A.23})$$

$$\text{var}(v|s) = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\varepsilon^2} = \frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}. \quad (\text{A.24})$$

We can now insert these demands into the market-clearing condition (5), which is a linear equation in the random variables q and s . Given that this equation must hold for all values of q and s , the aggregate coefficients on these variables must zero, and similarly, the constant term must be zero. Solving these three equations leads to the coefficients in the price function (6) given by:

$$\theta_0 = \bar{v} - \frac{\gamma \bar{q} \text{var}(v|s)}{I + (\bar{A} + N - I) \frac{\text{var}(v|s)}{\text{var}(v|p)}} \quad (\text{A.25})$$

$$\theta_s = \frac{I \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} + (\bar{A} + N - I) \frac{\text{var}(v|s)}{\text{var}(v|p)} \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2}}{I + (\bar{A} + N - I) \frac{\text{var}(v|s)}{\text{var}(v|p)}} \quad (\text{A.26})$$

$$\theta_q = \gamma \frac{\sigma_\varepsilon^2}{I}. \quad (\text{A.27})$$

Hence, by construction, a linear equilibrium exists.

To compute the relative utility, we start by noting that, with $a \in \{u, i\}$,

$$e^{-\gamma u_a} = \mathbb{E} \left[e^{-\frac{1}{2} \frac{(\mu_a - p)^2}{V_a}} \right], \quad (\text{A.28})$$

where μ_a and V_a are the conditional mean and variance of v for an investor of type a . To complete the proof, one uses the fact that, for any normally distributed random variable $z \sim \mathcal{N}(\mu_z, V_z)$, it holds that (e.g., based on the moment-generating function of the noncentral chi-square)

$$\mathbb{E} \left[e^{-\frac{1}{2}z^2} \right] = (1 + V_z)^{-\frac{1}{2}} e^{-\frac{1}{2} \frac{\mu_z^2}{1+V_z}},$$

and performs the necessary calculations giving

$$u_u = \frac{1}{\gamma} \log \left(\frac{\sigma_{v-p}}{\sigma_{v|p}} \right) + \frac{1}{2\gamma} \frac{(\bar{v} - \theta_0)^2}{\sigma_{v-p}^2} \quad (\text{A.29})$$

$$u_i = \frac{1}{\gamma} \log \left(\frac{\sigma_{v-p}}{\sigma_{v|s}} \right) + \frac{1}{2\gamma} \frac{(\bar{v} - \theta_0)^2}{\sigma_{v-p}^2}. \quad (\text{A.30})$$

(We note that the last term, $\frac{1}{2\gamma} \frac{(\bar{v} - \theta_0)^2}{\sigma_{v-p}^2}$, represents the utility attainable by an agent who cannot condition on the price.)

By combining (A.29), (A.30), and the definition of η , we see that (8) holds. To see (9), we use (A.23), (A.24), and the expression (A.27) for θ_q .

A.6 Proofs

Before continuing with the proofs of the next propositions, we state an auxiliary result regarding the number of managers. First, we define the unique value of M that solves managers' indifference condition (15) for any I by

$$\mathcal{M}^I(I) = \max \left\{ \frac{\eta(I)I}{2\gamma k + \eta(I)\frac{N}{M}}, \bar{M} \right\}, \quad (\text{A.31})$$

where we have used that $I = A + N\frac{M}{M}$. Given this definition, the number of managers depends on I as follows.

Lemma 1 *The function of I given by $I\eta(I)$ increases up to a point \bar{I} and then decreases, converging to zero. Consequently, $\mathcal{M}^I(I)$ increases with I for I low enough, and decreases towards zero as I tends to infinity.*

Proof of Lemma 1. The function $x\eta(x)$ of interest is a constant multiple of

$$h(x) := x \log \left(\frac{a + x^2}{b + x^2} \right), \quad (\text{A.32})$$

with $a > b > 0$. Its derivative equals

$$\begin{aligned} h'(x) &= \log\left(\frac{a+x^2}{b+x^2}\right) + x \frac{b+x^2}{a+x^2} \frac{2x(b+x^2) - 2x(a+x^2)}{(b+x^2)^2} \\ &= \log\left(\frac{a+x^2}{b+x^2}\right) - \frac{2(a-b)x^2}{(a+x^2)(b+x^2)}. \end{aligned}$$

For $x = 0$, the first term is clearly higher: $h'(0) > 0$. For $x \rightarrow \infty$, the second is larger, so that $\lim h'(x) < 0$. Finally, letting $y = x^2$ and differentiating $h'(y)$ with respect to y one sees that $h''(y) = 0$ when y satisfies the quadratic

$$y^2 - (a+b)y - 3ab = 0, \quad (\text{A.33})$$

which clearly has a root of each sign. Thus, since $y = x^2$ is always positive, $h''(x)$ changes sign only once. Given that $h'(x)$ starts positive and ends negative and its derivative changes sign only once, we see that h' itself must change sign exactly once. This result means that h is hump-shaped. Finally, we can apply L'Hôpital's rule to $h(x) = \log\left(\frac{a+x^2}{b+x^2}\right)/(1/x)$ to conclude that $\lim_{x \rightarrow \infty} h(x) = 0$.

To make a statement about the number of informed managers M , we use (A.31) and the first result. ■

Proof of Propositions 1 and 1b. Part (i): The statement about informed managers follows from the fact that investors matched with good managers rationally choose to pay the fee and invest with the manager rather than invest as uninformed. The statement about uninformed managers follows from the facts that uninformed managers do not provide any investment value and that their fee is strictly positive. Part (ii) is the indifference condition for the active investors and we note that the outperformance $u_i - f - u_u = c$ is clearly larger if the equilibrium c is larger. Part (iii) follows from expressing the aggregate outperformance as

$$\left(A + N \frac{M}{M}\right) (u_i - f - u_u) + N \left(1 - \frac{M}{M}\right) (-f) = Af - N \left(1 - 2 \frac{M}{M}\right) f, \quad (\text{A.34})$$

using that $u_i - u_u = \frac{\eta}{\gamma} = 2f$. This outperformance is positive if and only if $N \left(1 - 2 \frac{M}{M}\right) \leq A$. Finally, Proposition 1b follows from (A.15) given that (A.7) and (A.8) are equivalent. ■

Proof of Proposition 2. (i) Consider the largest- I equilibrium under the search cost c_1 , denoted using the subscript 1. We show that, under c_2 , an equilibrium exists with larger I . To see this, note that since (14) holds with equality for c_1 , we have $\eta(I_1) \geq 2\gamma c_2(M_1, A_1)$. Consider now the set

$$\left\{ I \mid I \geq I_1, I - \mathcal{M}^I(I) \frac{N}{M} \leq \bar{A} \right\}, \quad (\text{A.35})$$

where $I - \mathcal{M}^I(I)\frac{N}{M}$ is the number of searching investors A corresponding to I . This set is not empty because it includes I_1 . Either $\eta(I) > 2\gamma c_2(\mathcal{M}^I(I), I - \mathcal{M}^I(I)\frac{N}{M})$ over the entire set, in which case $A = \bar{A}$ corresponds to an equilibrium for c_2 , or $\eta(I) = 2\gamma c_2(\mathcal{M}^I(I), I - \mathcal{M}^I(I)\frac{N}{M})$ for a value $I_2 \geq I_1$, which is the desired conclusion.

The asset-market efficiency and fee are determined monotonically by the level of I . The number M of managers can either increase or decrease given the result on the shape of \mathcal{M}^I .

Finally, if $\mathcal{M}^I(I_2) \leq \mathcal{M}^I(I_1)$, then $A_2 \geq A_1$ from $A = I - M\frac{N}{M}$. If $\mathcal{M}^I(I_2) \geq \mathcal{M}^I(I_1)$, then the same conclusion follows from (15).

(ii) Since the functions c_j are continuous on $[0, \bar{A}] \times [M_0, \bar{M}]$ for any $M_0 > 0$, they converge to zero uniformly on this compact set. Pick M_0 low enough so that $\mathcal{M}^I(I) > M_0$ for any $I \in [\bar{A}, \bar{A} + N]$.

Since η is bounded away from zero on the set of interest, for high enough j there is an equilibrium with $A = \bar{A}$. By letting $\bar{A}_j \rightarrow \infty$, the equilibrium value A_j goes to ∞ . Hence, the market converges toward full efficiency in the limit. ■

Proof of Proposition 3. We note that equation (15) also defines a function $\mathcal{M}(A)$ associating a value A with a unique value M (because the left-hand side decreases in M and the right-hand side increases in M). Similarly, $c_A \geq 0$ ensures that (14) defines a function $\mathcal{A}(M)$ associating each value M with a unique value of A . Further, adding the condition $c_M \leq 0$ implies that $\mathcal{I}(M) \equiv \mathcal{A}(M) + M\frac{N}{M}$ increases with M .

Another helpful observation is that, from (14) and (15), A and M are positively related across equilibria, just as I and M are. The highest- I equilibrium is therefore also the highest- A equilibrium.

One can describe the effect of k using the language of graphs. (A more rigorous argument can be made following a similar logic to that in the proof of Proposition 2.) At the highest I , the increasing function \mathcal{I}^{-1} crosses \mathcal{M} from below; since a lower value of k translates into an upward shift of the function \mathcal{M} , there exists at least one equilibrium at the new k with a higher value of I than before. Since \mathcal{I} does not vary with k and it is increasing, M also increases. The inefficiency η decreases as I increases.

The level of A , on the other hand, can either increase or decrease. To see the latter fact, imagine a function c that increases abruptly in A around the original equilibrium, while being flat with respect to M . Since η decreases, A has to decrease from (14). Formally, make use of

$$\frac{d\eta}{dk} = c_M \frac{dM}{dk} + c_A \frac{dA}{dk}. \quad (\text{A.36})$$

Proof of Proposition 4. Letting x denote either σ_v^2 or σ_q^2 , we note that the partial derivatives are positive, $\frac{\partial \eta}{\partial x} > 0$ (i.e., keeping I constant). To derive the equilibrium effects

of a change in risk, we rewrite (14)–(15) abstractly as

$$0 = -\frac{1}{2}\eta + \gamma c(M, A) \equiv g^I(I, M) = g^I(\mathcal{I}(M), M) \quad (\text{A.37})$$

$$0 = -\frac{1}{2}\eta + \gamma k \frac{M}{A} \equiv g^M(I, M) = g^M(I, \mathcal{M}(I)), \quad (\text{A.38})$$

and note that I being maximal implies that the difference $\mathcal{I}^{-1}(I) - \mathcal{M}^I(I)$ increases in a neighborhood of the equilibrium I , or $\mathcal{M}'^I(I) < (\mathcal{I}^{-1})'(I)$. Using subscripts to indicate partial derivatives, this translates into²²

$$-\frac{g_I^M}{g_M^M} < -\frac{g_I^I}{g_M^I}, \quad (\text{A.39})$$

which is equivalent to

$$g_M^I g_I^M < g_I^I g_M^M \quad (\text{A.40})$$

because $g_M^I < 0$ and $g_M^M > 0$. The dependence of I and M on x is given as a solution to

$$\begin{pmatrix} g_I^M & g_M^M \\ g_I^I & g_M^I \end{pmatrix} \begin{pmatrix} I_x \\ M_x \end{pmatrix} = \begin{pmatrix} 1 & \partial \eta \\ 2 & \partial x \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (\text{A.41})$$

therefore by

$$\begin{pmatrix} I_x \\ M_x \end{pmatrix} = \frac{1}{g_M^I g_I^M - g_I^I g_M^M} \begin{pmatrix} g_M^I - g_M^M \\ g_I^M - g_I^I \end{pmatrix} \begin{pmatrix} 1 & \partial \eta \\ 2 & \partial x \end{pmatrix}. \quad (\text{A.42})$$

We note that $g_M^I - g_M^M < 0$ and $g_I^M - g_I^I < 0$, while the determinant $g_M^I g_I^M - g_I^I g_M^M$ is negative from (A.40). Thus, both I and M increase as σ_v^2 or σ_q^2 increases.

By dividing equation (14) by (15), A is seen to increase with M .

The effect on the efficiency of the asset market, on the other hand, is not determined. To see this clearly, differentiate (14) to get

$$\frac{1}{2} \frac{d\eta}{dx} = \gamma (c_M M_x + c_A A_x), \quad (\text{A.43})$$

and remember that $c_M \leq 0$ and $c_A \geq 0$. Since $M_x > 0$ and $A_x > 0$, by setting one of the partial derivatives c_M and c_A to zero and keeping the other non-zero, the sign of $\frac{d\eta}{dx}$ can be made either positive or negative. Consequently, the efficiency may increase as well as decrease, a conclusion that translates to the fee f .

Exactly the same argument works when increasing $(\sigma_v, \sigma_\varepsilon)$ or $(\sigma_v, \sigma_\varepsilon, \sigma_q)$ proportionally. ■

²²Note that $g^I(\mathcal{I}(M), M)$ can be written as $g^I(I, \mathcal{I}^{-1}(I))$ for $I = \mathcal{I}(M)$.

Proof of Proposition 5. This proposition follows from the discussion in Section A.3. ■

Proof of Proposition 6. The manager's utility decreases strictly with k . ■

Proof of Proposition 7. We compute the expected return on the wealth invested with a manager, working under the assumptions that all managers choose positions targeting investors with relative risk aversion $\bar{\gamma}_R$. Given the total wealth under management \bar{W} , the manager invests as an agent with absolute risk aversion $\bar{\gamma} = \bar{\gamma}_R/\bar{W}$. It is clear that all investors with an informed manager achieve the same gross excess return. The expected gross return is computed as the total dollar profit per capital invested \bar{W} , using the fact that the aggregate position is $(\bar{\gamma}\text{Var}(v|s))^{-1}(\mathbb{E}[v|s] - p)$, that is,

$$\bar{R}_i \equiv \mathbb{E} \left[\frac{1}{\bar{W}} (\bar{\gamma}\text{Var}(v|s))^{-1} (\mathbb{E}[v|s] - p) (v - p) \right] = \frac{1}{\bar{\gamma}_R} \mathbb{E} [SR_i^2]. \quad (\text{A.44})$$

Similarly, the expected gross return to an investor with an uninformed manager is

$$\bar{R}_u \equiv \mathbb{E} \left[\frac{1}{\bar{W}} (\bar{\gamma}\text{Var}(v|p))^{-1} (\mathbb{E}[v|p] - p) (v - p) \right] = \frac{1}{\bar{\gamma}_R} \mathbb{E} [SR_u^2]. \quad (\text{A.45})$$

There are two reasons why $\mathbb{E}[SR_i^2] > \mathbb{E}[SR_u^2]$: better information, and lower risk (which translates into higher leverage, in absolute value). We note that the second effect is not necessary for the result. As for the first effect, namely the fact that

$$\mathbb{E}[(\mathbb{E}[v - p|s, p])^2] > \mathbb{E}[(\mathbb{E}[v - p|p])^2], \quad (\text{A.46})$$

it follows immediately from Jensen's inequality (conditional on p).

Consider now the expected return of an investor in a fund conditional on the investor's characteristics:

$$\mathbb{E}[R|W_a, \gamma_a^R, c_a] = \text{Pr}(i|W_a, \gamma_a^R, c_a)\bar{R}_i + (1 - \text{Pr}(i|W_a, \gamma_a^R, c_a))\bar{R}_u, \quad (\text{A.47})$$

where $\text{Pr}(i|W_a, \gamma_a^R, c_a) = 1_{(2\gamma_a^R c_a < \eta W_a)} \frac{\bar{A} + \frac{M}{M}N}{\bar{A} + N}$ increases with W_a . Since $\bar{R}_i > \bar{R}_u$, we see that $\mathbb{E}[R|W_a, \gamma_a^R, c_a]$ increases with W_a .

Percentage fees for a given investor are a fixed multiple of $\frac{\gamma_a^R c_a}{W_a}$, a term that clearly decreases with W_a . Consequently, the conclusion holds for after-fee returns, as well.

Precisely the same argument works for the level of sophistication (c_a), albeit with reversed signs. ■

Proof of Proposition 8. Let $R^{(m)}$ denote the return of manager m and $\bar{W}^{(m)}$ the average wealth of across his investors. These two quantities are independent conditional on the manager's type (informed or uninformed). Since there are two manager types, $t = i$ and $t = u$, the covariance $\text{Cov}(R^{(m)}, \bar{W}^{(m)})$ is positive if and only if the conditional expectations

$E[R^{(m)}|t]$ and $E[\bar{W}^{(m)}|t]$ are ranked the same as a function of the type t of the manager.

In the present case, it is easy to see that the average investor of an informed manager has higher wealth. Specifically,

$$E[W_a | t = i] = \frac{A}{A + \frac{M}{M}N} E \left[W_a \mid \frac{\gamma_a^R c_a}{W_a} < \frac{\eta}{2} \right] + \frac{\frac{M}{M}N}{A + \frac{M}{M}N} E[W_a] > E[W_a], \quad (\text{A.48})$$

since $E \left[W_a \mid \frac{\gamma_a^R c_a}{W_a} < \frac{\eta}{2} \right] > E[W_a]$. We already saw that $\bar{R}_i > \bar{R}_u$.

The conclusion extends to after-fee returns, since the average percentage fees an informed manager perceives from searching investors is smaller than those paid by noise allocators:

$$E \left[\frac{\gamma_a^R c_a}{W_a} \mid \frac{\gamma_a^R c_a}{W_a} < \frac{\eta}{2} \right] < E \left[\frac{\gamma_n^R c_n}{W_n} \right]. \quad (\text{A.49})$$

The same argument works for any decreasing function of c_a , thus sophistication.

Part (ii) follows along the same lines, noting that the average size of an informed manager's AUM is higher than that of an uninformed manager. The statement about manager cost k is immediate. ■

B Real-World Search and Due Diligence of Asset Managers

Here we briefly summarize some of the main real-world issues related to finding and vetting an asset manager. While the search process involves a lot of details, the main point that we model theoretically is that the process is time consuming and costly. For instance, there exist more mutual funds than stocks in the U.S. Many of these mutual funds might be charging high fees while investing with little or no real information, just like the uninformed funds in our model (e.g., high-fee index funds, or so-called “closet indexers” who claim to be active, but in fact track the benchmark, or funds investing more in marketing than their investment process). Therefore, finding a suitable mutual fund is not easy for investors (just like finding a cheap stock is not easy for asset managers).

We first consider the search and due diligence process of institutional investors such as pension funds, insurance companies, endowments, foundations, funds of funds, family offices, and banks. Such institutional investors invite certain specific asset managers to visit their offices and also travel to meet asset managers at their premises. If the institutional investor is sufficiently interested in investing with the manager, the investor often asks the manager to fill out a so-called due diligence questionnaire (DDQ), which provides a starting point for the due diligence process. Here we provide a schematic overview of the process to illustrate the significant time and cost related to the search process of finding an asset manager and doing due diligence, but a detailed description of each of these items is beyond the scope of

the paper.²³

- **Finding the asset manager: the initial meeting.**

- **Search.** Institutional investors often have employees in charge of external managers. These employees search for asset managers and often build up knowledge of a large network of asset managers whom they can contact. Similarly, asset managers employ business development staff who maintain relationships with investors they know and try to connect with other asset owners, although hedge funds are subject to non-solicitation regulation preventing them from randomly contacting potential investors and advertising. This two-way search process involves a significant amount of phone calls, emails, and repeated personal meetings, often starting with meetings between the staff members dedicated to this search process and later with meetings between the asset manager’s high-level portfolio managers and the asset owner’s chief investment officer and board.
- **Request for Proposal.** Another way for an institutional investor to find an asset manager is to issue a request for proposal (RFP), which is a document that invites asset managers to “bid” for an asset management mandate. The RFP may describe the mandate in question (e.g., \$100 million of long-only U.S. large-cap equities) and all the information about the asset manager that is required.
- **Capital introduction.** Investment banks sometimes have capital introduction (“cap intro”) teams as part of their prime brokerage. A cap intro team introduces institutional investors to asset managers (e.g., hedge funds) that use the bank’s prime brokerage.
- **Consultants, investment advisors, and placement agents.** Institutional investors often use consultants and investment advisors to find and vet investment managers that meet their needs. On the flip side, asset managers (e.g., private equity funds) sometimes use placement agents to find investors.
- **Databases.** Institutional investors also get ideas for which asset managers to meet by looking at databases that may contain performance numbers and overall characteristics of the covered asset managers.

- **Evaluating the asset management firm.**

²³Standard DDQs are available online, e.g. from the Managed Funds Association (<http://www.managedfunds.org/wp-content/uploads/2011/06/Due-Diligence-Questionnaire.pdf>) or the Institutional Limited Partner Association (http://ilpa.org/wp-content/publicmedia/ILPA_Due_Diligence_Questionnaire_Tool.docx). See also “Best Practices in Alternative Investments: Due Diligence,” Greenwich Roundtable, 2010 (www.greenwichroundtable.org/system/files/BP-2010.pdf), the CFA Institute’s “Model RFP: A standardized process for selecting money managers” (http://www.cfainstitute.org/ethics/topics/Pages/model_rfp.aspx), and “Best Practices for the Hedge Fund Industry,” Report of the Asset Managers’ Committee to the President’s working group on financial markets, 2009 (<http://www.cftc.gov/ucm/groups/public/swaps/documents/file/bestpractices.pdf>). We are grateful for helpful discussions with Stephen Mellas and Jim Riccobono at AQR Capital Management.

- **Assets, funds, and investors.** Institutional investors often consider an asset manager’s overall assets under management, the distribution of assets across fund types, client types, and location.
- **People.** Key personnel, overall headcount information, headcount by major departments, stability of senior people.
- **Client servicing.** Services and information disclosed to investors, ongoing performance attribution, market updates, etc.
- **History, culture, and ownership.** When was the asset management firm founded, how has it evolved, general investment culture, ownership of the asset management firm, and do the portfolio managers invest in their own funds.

- **Evaluating the specific fund.**

- **Terms.** Fund structure (e.g., master-feeder), investment minimum, fees, high water marks, hurdle rate, other fees (e.g., operating expenses, audit fees, administrative fees, fund organizational expenses, legal fees, sales fees, salaries), transparency of positions and exposures.
- **Redemption terms.** Any fees payable, lock-ups, gating provisions, can the investment manager suspend redemptions or pay redemption proceeds in-kind, and other restrictions.
- **Asset and investors.** Net asset value, number of investors, do any investors in the fund experience fee or redemption terms that differ materially from the standard ones?

- **Evaluating the investment process.**

- **Track record.** Past performance numbers and possible performance attribution.
- **Instruments.** The securities traded and geographical regions.
- **Team.** Investment personnel, experience, education, turnover.
- **Investment thesis and economic reasoning.** What is the underlying source of profit, i.e., why should the investment strategy be expected to be profitable? Who takes the other side of the trade and why? Has the strategy worked historically?
- **Investment process.** The analysis of the investment thesis and process is naturally one of the most important parts of finding an asset managers. Investors analyze what drives the asset manager’s decisions to buy and sell, the investment process, what data is used, how is information gathered and analyzed, what systems are used, etc.
- **Portfolio characteristics.** Leverage, turnover, liquidity, typical number of positions and position limits.
- **Examples of past trades.** What motivated these trades, how do they reflect the general investment process, how were positions adjusted as events evolved.

- **Portfolio construction methodology.** How is the portfolio constructed, how are positions adjusted over time, how is risk measured, position limits, etc.
- **Trading methodology.** Connections to broker/dealers, staffing of trading desk and is it operating 24/7, possibly co-location on major exchanges, use of internal or external broker algorithms, etc.
- **Financing of trades.** Prime brokers relations, leverage.
- **Evaluating the risk management.**
 - **Risk management team.** Team members, independence, and authority.
 - **Risk measures.** Risk measures calculated, risk reports to investors, stress testing.
 - **Risk management.** How is risk managed, what actions are taken when risk limits are breached and who makes the decision.
- **Due diligence of operational issues and back office.**
 - **Operations overview.** Teams, functions, and segregation of duties.
 - **Lifecycle of a trade.** The different steps a trade makes as it flows through the asset manager's systems.
 - **Cash management.** Who can move cash, how, and controls around this process.
 - **Valuation.** Independent pricing sources, what level of PM input is there, what controls and policies ensure accurate pricing, who monitors this internally and externally.
 - **Reconciliation.** How frequency and granularly are cash and positions reconciled.
 - **Client service.** Reporting frequency, transparency levels, and other client services and reporting.
 - **Service providers.** The main service providers used and any major changes (recent or planned).
 - **Systems.** What are the major homegrown or vendor systems with possible live system demos.
 - **Counterparties.** Who are the main ones, how are they selected, how is counterparty risk managed and by whom.
 - **Asset verification.** Some large investors (and/or their consultants) will ask to speak directly to the asset manager's administrator in order to independently verify that assets are valued correctly.
- **Due diligence of compliance, corporate governance, and regulatory issues.**
 - **Overview.** Teams, functions, independence.

- **Regulators and regulatory reporting.** Who are the regulators for the fund, summary of recent visits/interactions, frequency of reporting.
- **Corporate governance.** Summary of policies and oversight.
- **Employee training.** Code of ethics and training.
- **Personal trading.** Policy, frequency, recent violations and the associated penalties for breach.
- **Litigation.** What litigation the firm has been involved with.
- **Due diligence of business continuity plan (BCP) and disaster recovery plan.**
 - **Plan overview.** Policy, staffing, and backup facilities.
 - **Testing.** Frequency of tests and intensity.
 - **Cybersecurity.** How IT systems and networks are defended and tested.

The search process for finding an asset manager is very different for retail investors. Clearly, there is no standard structure for the search process for retail investors, but here are some considerations:

- **Retail investors searching for an asset manager.**
 - **Online search.** Some retail investors can search for useful information about investing online and they can make their investment online. However, finding the right websites may require a significant search effort and, once located, finding and understanding the right information within the website can be difficult as discussed further below.
 - **Walking into a local branch of a financial institution.** Retail investors may prefer to invest in person, e.g., by walking into the local branch of a financial institution such as a bank, insurance provider, or investment firm. Visiting multiple financial institutions can be time consuming and confusing for retail investors.
 - **Brokers and intermediaries.** Bergstresser et al. (2009) report that a large fraction of mutual funds are sold via brokers and study the characteristics of these fund flows.
 - **Choosing from pension system menu.** Lastly, retail investors get exposure to asset management through their pension systems. In defined contributions pension schemes, retail investors must search through a menu of options for their preferred fund.
- **Searching for the relevant information.**
 - **Fees.** Choi et al. (2010) find experimental evidence that “search costs for fees matter.” In particular, their study “asked 730 experimental subjects to allocate \$10,000 among four real S&P 500 index funds. All subjects received the funds

prospectuses. To make choices incentive-compatible, subjects expected payments depended on the actual returns of their portfolios over a specified time period after the experimental session. ... In one treatment condition, we gave subjects a one-page ‘cheat sheet’ that summarized the funds front-end loads and expense ratios. ... We find that eliminating search costs for fees improved portfolio allocations.”

- **Fund objective and skill.** Choi et al. (2010) also find evidence that investors face search costs associated with respect to the funds’ objectives such as the meaning of an index fund. “In a second treatment condition, we distributed one page of answers to frequently asked questions (FAQs) about S&P 500 index funds. ... When we explained what S&P 500 index funds are in the FAQ treatment, portfolio fees dropped modestly, but the statistical significance of this drop is marginal.”
- **Price and net asset value.** In some countries, retail investors buy and sell a mutual fund shares as listed shares on an exchange. In this case, a central piece of information is the relation between the share price and mutual fund’s net asset value, but investors must search for these pieces of information on different websites and are often they not synchronous.

- **Understanding the relevant information.**

- **Financial literacy.** In their study on the choice of index funds, Choi et al. (2010) find that “fees paid decrease with financial literacy.” Simply understanding the relevant information and, in particular, the (lack of) importance of past returns is an important part of the issue.
- **Opportunity costs.** Even for financially literate investors, the non-trivial amount of time it takes to search for a good asset manager may be viewed as a significant opportunity cost given that people have other productive uses of their time and value leisure time.