Assignment 8 - Monte Carlo Integration

Aliaksandr Panko

May 19, 2018

Contents

1 Objectives

There are three light switchers for three lights. Initially, all of the lights are off. At times 1,2,3... a switch is chosen at random and is then flicked (changing it from Off to On, or from On to Off)

1. What is the state space?
2. Give a graphical representation of the Markov chain describing this process

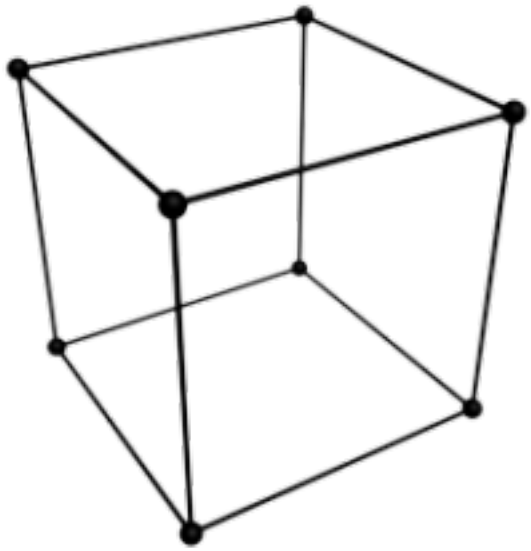
2 Solution

Since we have 3 switchers, we have $X = (x_1, x_2, x_3)$ where x_i represents the state(ON or OFF) of the switcher 'i'. This gives us the space:

S = (000,001,010,011,100,101,110,111)

Here 110 means that switchers 1 and 2 are ON and the switcher 3 is OFF. There are no other possible states of 3 switchers.

Since every time only 1 switcher is changed and overall 8 states, we can represent the process as a cube with 8 corners. In this case switching the light means moving from current corner to a new one along one of the edges.



3 Applications

3.1 Markov Chains

Markov chains are an important mathematical tool in stochastic processes. The underlying idea is the Markov Property, in order words, that some predictions about stochastic processes can be simplified by viewing the future as independent of the past, given the present state of the process. This is used to simplify predictions about the future state of a stochastic process.

Let us introduce some examples:

1. **Credit risk measurement**.
   Here the transition matrix represents the likelihood of the future evolution of the ratings. Basically it represents the probabilities that a certain company, country, etc. will either remain in their current state, or transition into a new state. However, determining of these probabilities is rather difficult: historical data and opinions from experts are usually used to do it.

2. **Market trends predictions**
   Here transition matrix contains probabilities of market states (Bull markets, Bear markets, Stagnant markets).

3. **Option pricing**
   Option pricing is based on trees that represent different paths for underlying asset. This process is also based on probabilities of going up and down in price. Random walk in general is based on Markov chain idea.

3.2 Monte Carlo Integration

- Monte Carlo methods are used in finance and mathematical finance to value and analyze (complex) instruments, portfolios and investments by simulating the various sources of uncertainty affecting their value, and then determining the distribution of their value over the range of resultant outcomes.

- Monte Carlo Methods are used construct "stochastic" or probabilistic financial models as opposed to the traditional static and deterministic models.

- In valuing an option on equity, the simulation generates several thousand possible (but random) price paths for the underlying share, with the associated exercise value (i.e. "payoff") of the option for each path. These payoffs are then averaged and discounted to today, and this result is the value of the option today.

- Monte Carlo Methods are used for portfolio evaluation.[17] Here, for each sample, the correlated behaviour of the factors impacting the component instruments is simulated over time, the resultant value of each instrument is calculated, and the portfolio value is then observed.

- Discrete event simulation and in particular in stress testing the design (heavily used by banks)